Multivariate Spatial-Dependence Modelling with Satellite Data

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Introduction

Spatial-statistical methods like `kriging` leverage spatial variation in the process of interest to produce de-noised and gap-filled predictions along with their statistical uncertainty. When dependence between multiple processes is identified, multivariate methods like `cokriging` offer improvement in accuracy and efficiency.

NASA's OCO-2 mission (Eldering et al. 2017) monitors column-averaged carbon dioxide (**XCO_2**) and solar-induced fluorescence (**SIF**). Modelling the observed dependence between spatial processes will aid the mission's goal of quantifying the global geographic distribution of XCO_2 and SIF.

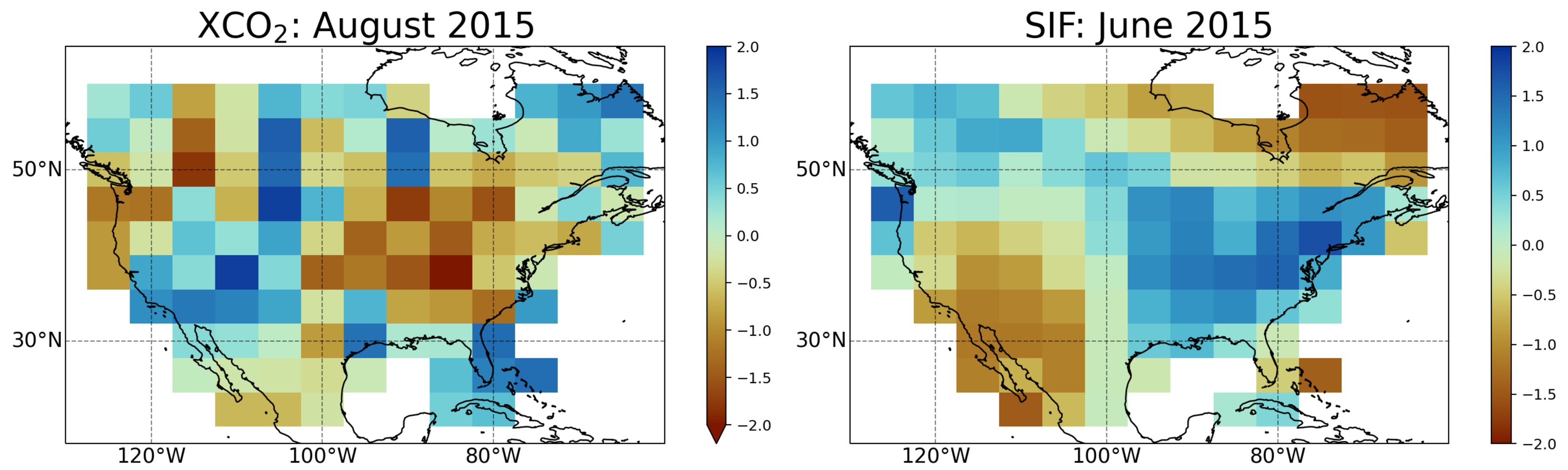


Figure 1: 4x5-degree monthly average residuals over North America for XCO2 in August 2015 (left) and SIF in June 2015 (right). Spatial residuals standardised by removal of temporal and spatial trends then scaled.

Methods

Let $Y_1(\cdot), \ldots, Y_p(\cdot)$ describe p processes of interest. Two perspectives on multivariate spatial dependence:

1. Joint approach (e.g., Genton and Kleiber 2015) with joint distribution $[Y_1(\cdot), \ldots, Y_p(\cdot)]$
2. Conditional approach (e.g., Cressie and Zammit-Mangion 2016) with conditional distributions $[Y_j(\cdot) \mid Y_i : i = 1, \ldots, p; i \neq j]$

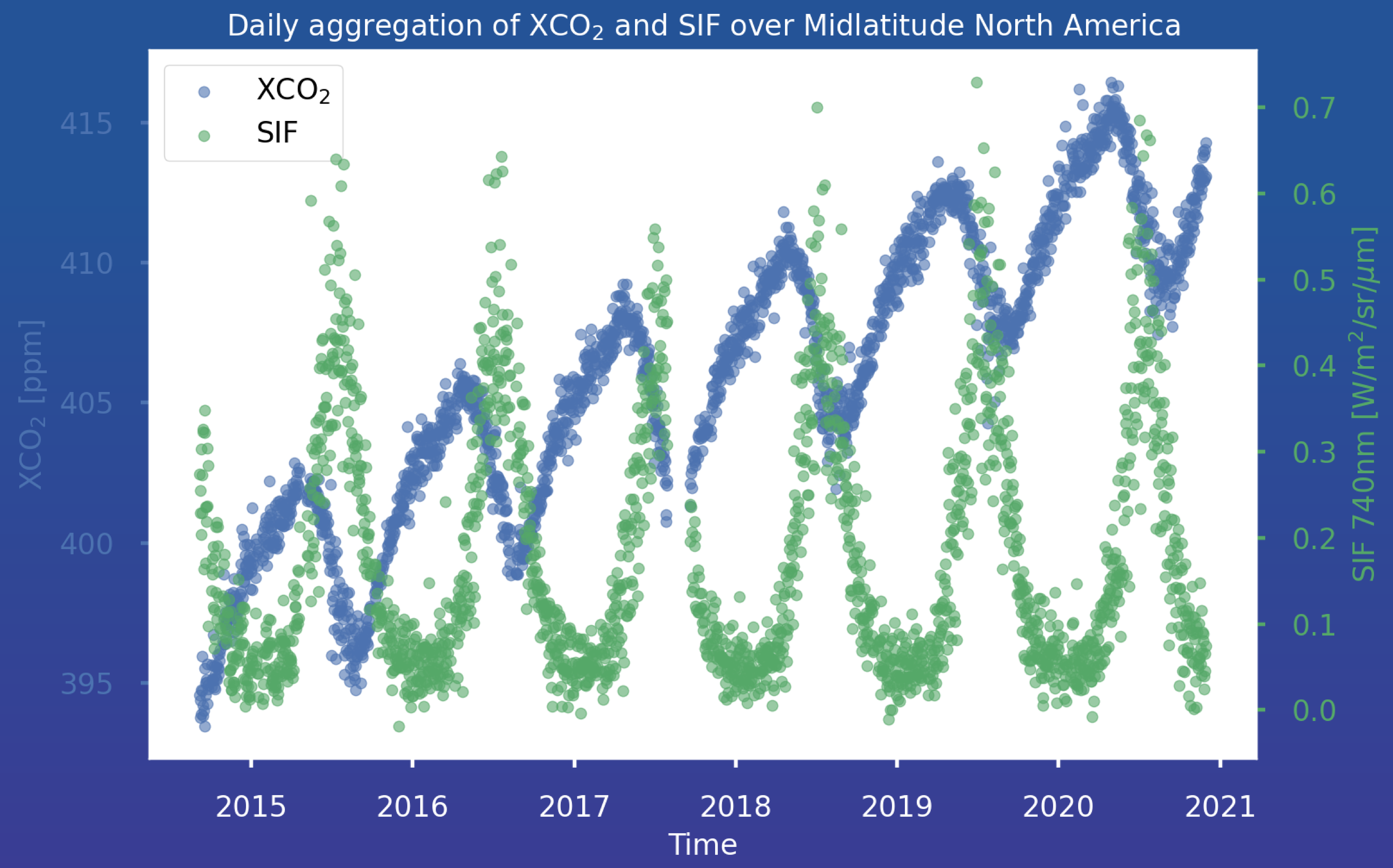
First, we consider approach (1) using the multivariate Matérn model. For spatial displacement $\mathbf{h} \in \mathbb{R}^d$, the covariance model is

$$C^{\circ}_{ij}(\mathbf{h}) = C^{\circ}_{ji}(\mathbf{h}) = \begin{cases} \sigma_i^2 M(\mathbf{h} \mid \nu_i, \ell_i); & i = j \\ \rho_{ij}\sigma_i\sigma_j M(\mathbf{h} \mid \nu_{ij}, \ell_{ij}); & i \neq j \end{cases}$$

$$M(\mathbf{h} \mid \nu, \ell) = \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\frac{\sqrt{2\nu}}{\ell} ||\mathbf{h}|| \right)^{\nu} K_{\nu} \left(\frac{\sqrt{2\nu}}{\ell} ||\mathbf{h}|| \right)$$

Model parameters are fit to empirical (cross-) semivariograms simultaneously by *composite* weighted least squares (Cressie 1985).

The inverse relationship between XCO_2 and SIF can be exploited to obtain better spatial predictions than modelling either process independently.



Results

- Excellent fit to (cross-) semivariograms near origin – the most important feature for cokriging. Mediocre fit at larger separation distances due to Matérn-model limitations.
- Cross-dependence captured in fit will lead to improvement with spatial predictions over univariate models. The next step is to compute cokriging predictions using the fitted model.
- Future work will improve the `bivariate spatial` model using the `conditional approach` (2) in Methods.

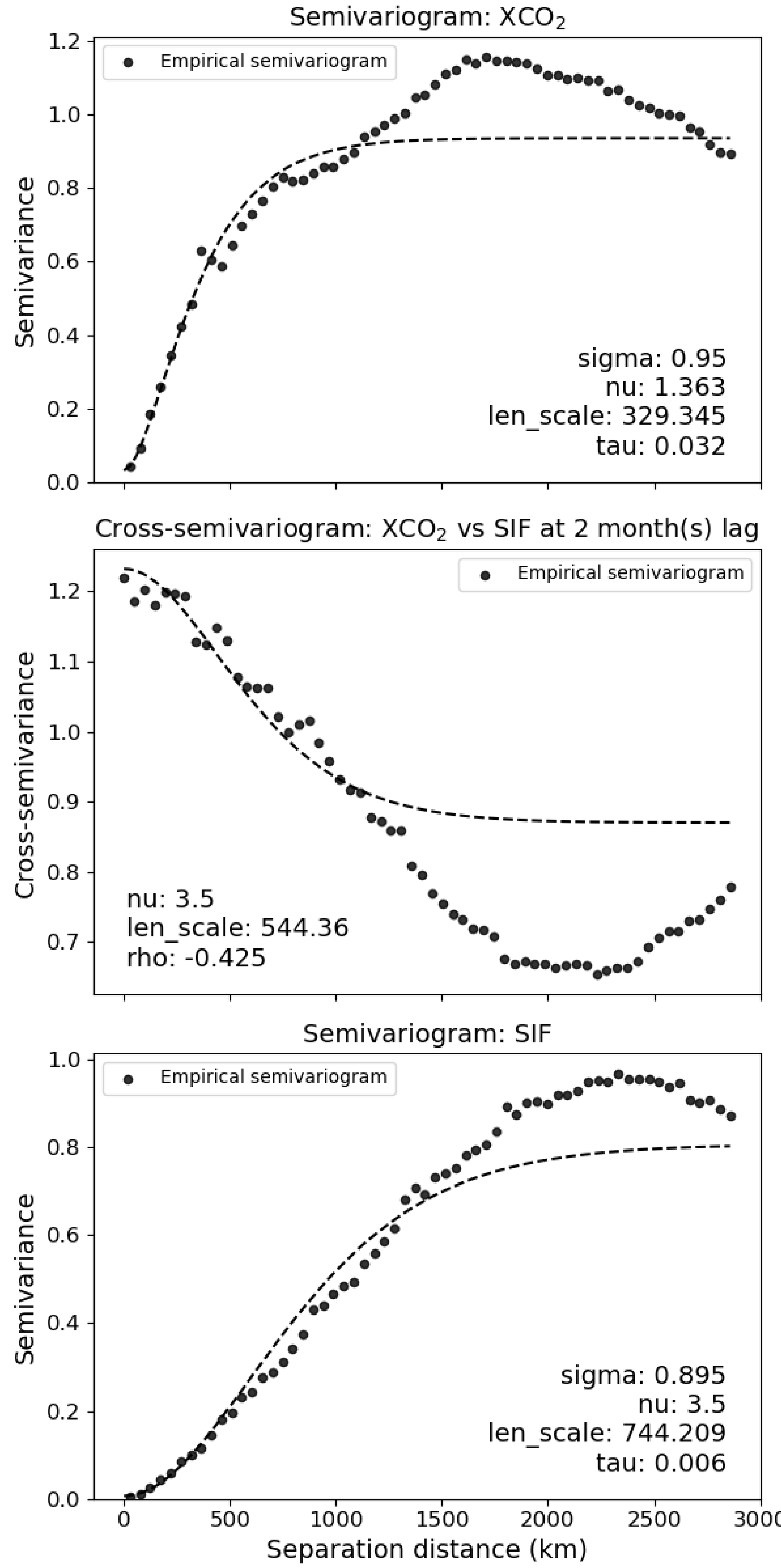


Figure 2: Fitted semivariograms and cross-semivariogram for the XCO2 and SIF spatial residuals in Figure 1 using a bin width of 47 km.

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