This print-out should have 30 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Find y' when

$$xy + 5x + 4x^2 = 5\,.$$

1. $y' = \dfrac{5 + 4x - y}{x}$

2. $y' = \dfrac{y + 5 + 4x}{x}$

3. $y' = -(y + 5 + 8x)$

4. $y' = -\dfrac{y + 5 + 4x}{x}$

5. $y' = \dfrac{y + 5 + 8x}{x}$

6. $y' = -\dfrac{y + 5 + 8x}{x}$ **correct**

Explanation:

Differentiating implicitly with respect to x we see that

$$\frac{d}{dx}\left(xy + 5x + 4x^2\right) = \frac{d}{dx}(5)\,.$$

Thus

$$(xy' + y) + 5 + 8x = 0\,,$$

and so

$$xy' = -y - 5 - 8x\,.$$

Consequently,

$$\boxed{y' = -\frac{y + 5 + 8x}{x}}\,.$$

002 10.0 points

Find dy/dx when

$$2x^2 + 3y^2 = 4\,.$$

1. $\dfrac{dy}{dx} = 3xy$

2. $\dfrac{dy}{dx} = -\dfrac{2x}{y}$

3. $\dfrac{dy}{dx} = \dfrac{x}{3y}$

4. $\dfrac{dy}{dx} = -\dfrac{2x}{3y}$ **correct**

5. $\dfrac{dy}{dx} = \dfrac{2x}{3y}$

6. $\dfrac{dy}{dx} = -2xy$

Explanation:

Diferentiating

$$2x^2 + 3y^2 = 4$$

implicitly with respect to x we see that

$$4x + 6y\frac{dy}{dx} = 0\,.$$

Consequently,

$$\boxed{\frac{dy}{dx} = -\frac{4x}{6y} = -\frac{2x}{3y}}\,.$$

003 10.0 points

Find $\dfrac{dy}{dx}$ when

$$\frac{3}{\sqrt{x}} + \frac{4}{\sqrt{y}} = 5\,.$$

1. $\dfrac{dy}{dx} = -\dfrac{4}{3}\left(\dfrac{x}{y}\right)^{3/2}$

2. $\dfrac{dy}{dx} = \dfrac{3}{4}(xy)^{1/2}$

3. $\dfrac{dy}{dx} = \dfrac{4}{3}\left(\dfrac{x}{y}\right)^{3/2}$

4. $\dfrac{dy}{dx} = \dfrac{4}{3}(xy)^{1/2}$

5. $\dfrac{dy}{dx} = -\dfrac{3}{4}\left(\dfrac{y}{x}\right)^{3/2}$ **correct**

6. $\dfrac{dy}{dx} = \dfrac{3}{4}\Big(\dfrac{y}{x}\Big)^{3/2}$

Explanation:

Differentiating implicitly with respect to x, we see that

$$-\frac{1}{2}\Big(\frac{3}{x\sqrt{x}} + \frac{4}{y\sqrt{y}}\frac{dy}{dx}\Big) = 0\,.$$

Consequently,

$$\boxed{\frac{dy}{dx} = -\frac{3}{4}\Big(\frac{y}{x}\Big)^{3/2}}\,.$$

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004 10.0 points

Find $\dfrac{dy}{dx}$ when

$$3\sqrt{x} + 2\sqrt{y} = 4\,.$$

1. $\dfrac{dy}{dx} = 3 + \dfrac{4}{\sqrt{x}}$

2. $\dfrac{dy}{dx} = -\dfrac{3}{2}\Big(3 - \dfrac{4}{\sqrt{x}}\Big)$

3. $\dfrac{dy}{dx} = -\dfrac{3}{2}\Big(4 - \dfrac{3}{\sqrt{x}}\Big)$

4. $\dfrac{dy}{dx} = -\dfrac{3}{4}\Big(\dfrac{4-3\sqrt{x}}{\sqrt{x}}\Big)$ **correct**

5. $\dfrac{dy}{dx} = 4 - \dfrac{3}{\sqrt{x}}$

6. $\dfrac{dy}{dx} = \dfrac{3}{4}\Big(4 + \dfrac{3}{\sqrt{x}}\Big)$

Explanation:

Differentiating implicitly with respect to x we see that

$$\frac{1}{2}\Big(\frac{3}{\sqrt{x}} + \frac{2}{\sqrt{y}}\frac{dy}{dx}\Big) = 0\,.$$

Thus

$$\frac{dy}{dx} = -\frac{3}{2}\Big(\frac{\sqrt{y}}{\sqrt{x}}\Big)\,.$$

But

$$\sqrt{y} = \frac{4-3\sqrt{x}}{2}\,,$$

so

$$\boxed{\frac{dy}{dx} = -\frac{3}{4}\Big(\frac{4-3\sqrt{x}}{\sqrt{x}}\Big)}\,.$$

In this case, we can also begin by solving for y, using the Chain Rule, and simplifying:

$$y = \Big(\frac{4-3\sqrt{x}}{2}\Big)^2$$

$$\frac{dy}{dx} = 2\Big(\frac{4-3\sqrt{x}}{2}\Big)\Big(\frac{-3}{2\cdot 2\sqrt{x}}\Big)$$

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005 10.0 points

If y is defined implicitly by

$$y^2 - xy - 12 = 0\,,$$

find the value of dy/dx at (4, 6).

1. $\dfrac{dy}{dx}\Big|_{(4,\,6)} = \dfrac{3}{4}$ **correct**

2. $\dfrac{dy}{dx}\Big|_{(4,\,6)} = \dfrac{7}{8}$

3. $\dfrac{dy}{dx}\Big|_{(4,\,6)} = -\dfrac{3}{4}$

4. $\dfrac{dy}{dx}\Big|_{(4,\,6)} = \dfrac{2}{3}$

5. $\dfrac{dy}{dx}\Big|_{(4,\,6)} = -\dfrac{7}{8}$

Explanation:

Differentiating implicitly with respect to x we see that

$$2y\frac{dy}{dx} - y - x\frac{dy}{dx} = 0\,.$$

Thus

$$\frac{dy}{dx} = \frac{y}{2y-x}\,.$$

At (4, 6), therefore,

$$\boxed{\frac{dy}{dx}\Big|_{(4,\,6)} = \frac{3}{4}}\,.$$

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006 10.0 points

If $y = y(x)$ is defined implicitly by

$$3y^2 + xy - 2 = 0,$$

find the value of dy/dx at the point $(1, -1)$.

1. $\left.\frac{dy}{dx}\right|_{(1,-1)} = -\frac{1}{5}$ **correct**

2. $\left.\frac{dy}{dx}\right|_{(1,-1)} = \frac{2}{5}$

3. $\left.\frac{dy}{dx}\right|_{(1,-1)} = -\frac{3}{5}$

4. $\left.\frac{dy}{dx}\right|_{(1,-1)} = \frac{1}{5}$

5. $\left.\frac{dy}{dx}\right|_{(1,-1)} = -\frac{2}{5}$

6. $\left.\frac{dy}{dx}\right|_{(1,-1)} = \frac{3}{5}$

Explanation:

Differentiating implicitly with respect to x we see that

$$6y\frac{dy}{dx} + y + x\frac{dy}{dx} = 0,$$

so

$$\frac{dy}{dx} = -\frac{y}{6y+x}.$$

At $(1, -1)$, therefore,

$$\boxed{\left.\frac{dy}{dx}\right|_{(1,-1)} = -\frac{1}{5}}.$$

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007 10.0 points

Find $\frac{dy}{dx}$ when

$$2x^2 - xy + y^2 = 0.$$

1. $\frac{dy}{dx} = \frac{2y+x}{y-x}$

2. $\frac{dy}{dx} = \frac{2y-x}{y+x}$

3. $\frac{dy}{dx} = \frac{4x+y}{x-2y}$

4. $\frac{dy}{dx} = \frac{2y-x}{y-x}$

5. $\frac{dy}{dx} = \frac{4x-y}{x-2y}$ **correct**

6. $\frac{dy}{dx} = \frac{4x-y}{x+2y}$

Explanation:

Differentiating

$$2x^2 - xy + y^2 = 0$$

implicitly we see that

$$4x - (xy' + y) + 2yy' = 0.$$

Thus

$$(4x - y) - y'(x - 2y) = 0.$$

Consequently,

$$\boxed{\frac{dy}{dx} = \frac{4x-y}{x-2y}}.$$

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008 10.0 points

Find $\frac{dy}{dx}$ when

$$\tan(xy) = 2x + y.$$

1. $\frac{dy}{dx} = \frac{1 - x\sec^2(xy)}{y\sec^2(xy) + 2}$

2. $\frac{dy}{dx} = \frac{1 - x\sec^2(xy)}{y\sec^2(xy) - 2}$

3. $\frac{dy}{dx} = \frac{2 - y\sec^2(xy)}{x\sec^2(xy) + 1}$

4. $\frac{dy}{dx} = \frac{2 - y\sec^2(xy)}{x\sec^2(xy) - 1}$ **correct**

5. $\frac{dy}{dx} = \frac{2 + y\sec^2(xy)}{x\sec^2(xy) - 1}$

6. $\frac{dy}{dx} = \frac{1 + x\sec^2(xy)}{y\sec^2(xy) + 2}$

Explanation:

Differentiating implicitly with respect to x, we see that

$$\sec^2(xy)\Big(y + x\frac{dy}{dx}\Big) = 2 + \frac{dy}{dx}.$$

After rearranging, this becomes

$$\frac{dy}{dx}\Big(x\sec^2(xy) - 1\Big) = 2 - y\sec^2(xy).$$

Consequently,

$$\boxed{\frac{dy}{dx} = \frac{2 - y\sec^2(xy)}{x\sec^2(xy) - 1}}.$$

keywords:

009 10.0 points

Determine dy/dx when

$$5\cos x\sin y = 1.$$

1. $\dfrac{dy}{dx} = \tan x\tan y$ **correct**
2. $\dfrac{dy}{dx} = \tan x$
3. $\dfrac{dy}{dx} = \cot x\tan y$
4. $\dfrac{dy}{dx} = \tan xy$
5. $\dfrac{dy}{dx} = \cot x\cot y$

Explanation:

Differentiating implicitly with respect to x we see that

$$5\Big\{\cos x\cos y\frac{dy}{dx} - \sin y\sin x\Big\} = 0.$$

Thus

$$\frac{dy}{dx}\cos x\cos y = \sin x\sin y.$$

Consequently,

$$\boxed{\frac{dy}{dx} = \frac{\sin x\sin y}{\cos x\cos y} = \tan x\tan y}.$$

010 10.0 points

Determine dy/dx when

$$y\cos(x^2) = 5.$$

1. $\dfrac{dy}{dx} = -2xy\cot(x^2)$
2. $\dfrac{dy}{dx} = 2xy\cot(x^2)$
3. $\dfrac{dy}{dx} = 2xy\cos(x^2)$
4. $\dfrac{dy}{dx} = -2xy\tan(x^2)$
5. $\dfrac{dy}{dx} = 2xy\tan(x^2)$ **correct**
6. $\dfrac{dy}{dx} = -2xy\sin(x^2)$

Explanation:

After implicit differentiation with respect to x we see that

$$-2xy\sin(x^2) + y'\cos(x^2) = 0.$$

Consequently,

$$\boxed{\frac{dy}{dx} = \frac{2xy\sin(x^2)}{\cos(x^2)} = 2xy\tan(x^2)}.$$

011 10.0 points

Find the equation of the tangent line to the graph of

$$3y^2 - xy - 8 = 0,$$

at the point $P = (10, 4)$.

1. $5y = 2x$
2. $7y + 2x = 8$
3. $7y = 2x + 8$ **correct**

4. $13y = 4x + 12$

5. $13y + 4x = 12$

Explanation:
Differentiating implicitly with respect to x we see that

$$6y\frac{dy}{dx} - y - x\frac{dy}{dx} = 0,$$

so

$$\frac{dy}{dx} = \frac{y}{6y - x}.$$

At $P = (10, 4)$, therefore,

$$\left.\frac{dy}{dx}\right|_P = \frac{2}{7}.$$

Thus by the point slope formula, the equation of the tangent line at P is given by

$$y - 4 = \frac{2}{7}(x - 10).$$

Consequently,

$$\boxed{7y = 2x + 8}.$$

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012 10.0 points

Find an equation for the tangent line to the curve

$$26x^2 + 5xy + 7y^2 = 38$$

at the point (1, 1).

1. $y = 8x + 4$

2. $y = -8x + 4$

3. $y = -3x + 4$ **correct**

4. $y = 6x - 7$

5. $y = -6x + 6$

6. $y = 3x + 9$

Explanation:
Differentiating implicitly, we see that

$$\begin{aligned} 26x^2 + 5xy + 7y^2 &= 38 \\ 52x + 5xy' + 5y \cdot 1 + 14yy' &= 0 \\ 5xy' + 14yy' &= -52x - 5y \\ y'(5x + 14y) &= -52x - 5y \\ y' &= \frac{-52x - 5y}{5x + 14y} \end{aligned}$$

When $x = 1$ and $y = 1$, we have

$$y' = \frac{-52 - 5}{5 + 14} = \frac{-57}{19} = -3$$

so an equation of the tangent line is

$$\begin{aligned} y - 1 &= -3(x - 1) \\ y &= -3x + 4 \end{aligned}$$

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keywords:

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013 10.0 points

The curve with equation

$$y^2 = 10x^4 - x^2$$

is called a **kampyle of Eudoxus**.

Find an equation of the tangent line to this curve at the point $(1, 3)$.

1. $y = -\frac{25}{3}x + \frac{28}{3}$

2. $y = \frac{25}{3}x - \frac{19}{3}$

3. $y = -\frac{19}{3}x - \frac{10}{3}$

4. $y = \frac{25}{3}x + \frac{28}{3}$

5. $y = \frac{19}{3}x - \frac{10}{3}$ **correct**

Explanation:

$$\begin{aligned} y^2 &= 10x^4 - x^2 \\ 2yy' &= 10\left(4x^3\right) - 2x \\ y' &= \frac{20x^3 - x}{y} \end{aligned}$$

So at the point $(1, 3)$ we have

$$y' = \frac{20\,(1)^3 - 1}{3} = \frac{19}{3}$$

and an equation of the tangent line is

$$\begin{aligned} y - 3 &= \frac{19}{3}\,(x - 1) \\ y &= \frac{19}{3}x - \frac{10}{3} \end{aligned}$$

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014 10.0 points

Determine the derivative of

$$f(x) \;=\; 5\arcsin\left(\frac{x}{3}\right).$$

1. $f'(x) \;=\; \dfrac{15}{\sqrt{9 - x^2}}$

2. $f'(x) \;=\; \dfrac{3}{\sqrt{9 - x^2}}$

3. $f'(x) \;=\; \dfrac{5}{\sqrt{1 - x^2}}$

4. $f'(x) \;=\; \dfrac{3}{\sqrt{1 - x^2}}$

5. $f'(x) \;=\; \dfrac{5}{\sqrt{9 - x^2}}$ **correct**

6. $f'(x) \;=\; \dfrac{15}{\sqrt{1 - x^2}}$

Explanation:

Use of

$$\frac{d}{dx}\,\arcsin(x) \;=\; \frac{1}{\sqrt{1 - x^2}}\,,$$

together with the Chain Rule shows that

$$f'(x) \;=\; \frac{5}{\sqrt{1 - (x/3)^2}}\left(\frac{1}{3}\right).$$

Consequently,

$$\boxed{f'(x) \;=\; \frac{5}{\sqrt{9 - x^2}}}\,.$$

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015 10.0 points

Find the derivative of

$$f(x) \;=\; \left(\sin^{-1}(3x)\right)^2.$$

1. $f'(x) \;=\; 6\cos(3x)\,\sin(3x)$

2. $f'(x) \;=\; \dfrac{3}{\sqrt{1 - 9x^2}}\,\sin^{-1}(3x)$

3. $f'(x) \;=\; \cos(3x)\,\sin(3x)$

4. $f'(x) \;=\; \dfrac{6}{\sqrt{1 - 9x^2}}\,\sin^{-1}(3x)$ **correct**

5. $f'(x) \;=\; \dfrac{6}{\sqrt{9 - x^2}}\,\sin^{-1}(3x)$

6. $f'(x) \;=\; \dfrac{3}{\sqrt{9 - x^2}}\,\sin^{-1}(3x)$

Explanation:

The Chain Rule together with

$$\frac{d}{dx}\left(\sin^{-1}(ax)\right) \;=\; \frac{a}{\sqrt{1 - a^2x^2}}$$

shows that

$$\boxed{f'(x) \;=\; \frac{6}{\sqrt{1 - 9x^2}}\,\sin^{-1}(3x)}\,.$$

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016 10.0 points

Find the derivative of

$$f(x) \;=\; \sin^{-1}(e^{3x}).$$

1. $f'(x) \;=\; \dfrac{3}{\sqrt{1 - e^{6x}}}$

2. $f'(x) \;=\; \dfrac{3}{1 + e^{6x}}$

3. $f'(x) = \dfrac{1}{\sqrt{1-e^{6x}}}$

4. $f'(x) = \dfrac{1}{1+e^{6x}}$

5. $f'(x) = \dfrac{3e^{3x}}{1+e^{6x}}$

6. $f'(x) = \dfrac{e^{3x}}{1+e^{6x}}$

7. $f'(x) = \dfrac{e^{3x}}{\sqrt{1-e^{6x}}}$

8. $f'(x) = \dfrac{3e^{3x}}{\sqrt{1-e^{6x}}}$ **correct**

Explanation:
Since

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx}e^{ax} = ae^{ax},$$

the Chain Rule ensures that

$$\boxed{f'(x) = \frac{3e^{3x}}{\sqrt{1-e^{6x}}}}.$$

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017 10.0 points

Find the derivative of

$$f(x) = \tan^{-1}(e^{3x}).$$

1. $f'(x) = \dfrac{3e^{3x}}{\sqrt{1-e^{6x}}}$

2. $f'(x) = \dfrac{e^{3x}}{\sqrt{1-e^{6x}}}$

3. $f'(x) = \dfrac{1}{1+e^{6x}}$

4. $f'(x) = \dfrac{3}{1+e^{6x}}$

5. $f'(x) = \dfrac{3}{\sqrt{1-e^{6x}}}$

6. $f'(x) = \dfrac{1}{\sqrt{1-e^{6x}}}$

7. $f'(x) = \dfrac{e^{3x}}{1+e^{6x}}$

8. $f'(x) = \dfrac{3e^{3x}}{1+e^{6x}}$ **correct**

Explanation:
Since

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}, \quad \frac{d}{dx}e^{ax} = ae^{ax},$$

the Chain Rule ensures that

$$\boxed{f'(x) = \frac{3e^{3x}}{1+e^{6x}}}.$$

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018 10.0 points

Find the derivative of f when

$$f(x) = 2\sin^{-1}\frac{x}{2} - \sqrt{4-x^2}.$$

1. $f'(x) = \dfrac{x}{\sqrt{4-x^2}}$

2. $f'(x) = \dfrac{2}{\sqrt{4-x^2}}$

3. $f'(x) = \sqrt{\dfrac{2+x}{2-x}}$ **correct**

4. $f'(x) = \dfrac{1}{\sqrt{2-x}}$

5. $f'(x) = \dfrac{1}{\sqrt{2+x}}$

6. $f'(x) = \sqrt{\dfrac{2-x}{2+x}}$

Explanation:
By the Chain Rule,

$$\begin{aligned} f'(x) &= \frac{2}{\sqrt{4-x^2}} + \frac{x}{\sqrt{4-x^2}} \\ &= \frac{2+x}{\sqrt{4-x^2}}. \end{aligned}$$

On the other hand,

$$4-x^2 = (2-x)(2+x).$$

Consequently,

$$\boxed{f'(x) = \sqrt{\frac{2+x}{2-x}}}.$$

019 10.0 points

Find the derivative of f when

$$f(\theta) = \ln(\sin 3\theta).$$

1. $f'(\theta) = \dfrac{1}{\cos 3\theta}$

2. $f'(\theta) = 3\tan 3\theta$

3. $f'(\theta) = 3\cot 3\theta$ **correct**

4. $f'(\theta) = \dfrac{3}{\sin 3\theta}$

5. $f'(\theta) = -\tan 3\theta$

6. $f'(\theta) = \cot 3\theta$

Explanation:
By the Chain Rule,

$$f'(\theta) = \frac{1}{\sin(3\theta)}\frac{d}{d\theta}(\sin 3\theta) = \frac{3\cos 3\theta}{\sin 3\theta}.$$

Consequently,

$$\boxed{f'(\theta) = 3\cot 3\theta}.$$

020 10.0 points

Find the derivative of f when

$$f(\theta) = \ln(\cos 3\theta).$$

1. $f'(\theta) = -\dfrac{1}{\sin 3\theta}$

2. $f'(\theta) = \cot 3\theta$

3. $f'(\theta) = -3\tan 3\theta$ **correct**

4. $f'(\theta) = 3\tan 3\theta$

5. $f'(\theta) = -3\cot 3\theta$

6. $f'(\theta) = \dfrac{3}{\cos 3\theta}$

Explanation:
By the Chain Rule,

$$f'(\theta) = \frac{1}{\cos(3\theta)}\frac{d}{d\theta}(\cos 3\theta) = -\frac{3\sin 3\theta}{\cos 3\theta}.$$

Consequently,

$$\boxed{f'(\theta) = -3\tan 3\theta}.$$

021 10.0 points

Differentiate the function

$$f(x) = \cos(\ln 6x).$$

1. $f'(x) = -\dfrac{6\sin(\ln 6x)}{x}$

2. $f'(x) = \dfrac{1}{\cos(\ln 6\,x)}$

3. $f'(x) = -\sin(\ln 6\,x)$

4. $f'(x) = \dfrac{\sin(\ln 6\,x)}{x}$

5. $f'(x) = -\dfrac{\sin(\ln 6\,x)}{x}$ **correct**

6. $f'(x) = \dfrac{6\sin(\ln 6x)}{x}$

Explanation:
By the Chain Rule

$$\boxed{f'(x) = -\frac{\sin(\ln 6x)}{x}}.$$

022 10.0 points

Find the slope of the line tangent to the graph of

$$\ln(xy) + 2x = 0$$

at the point where $x = 1$.

1. slope $= -3e^2$

2. slope $= -\frac{3}{2}e^2$

3. slope $= 3e^{-2}$

4. slope $= \frac{3}{2}e^{-2}$

5. slope $= -3e^{-2}$ **correct**

6. slope $= \frac{3}{2}e^2$

Explanation:

Differentiating implicitly with respect to x we see that

$$\frac{1}{xy}\Big(y + x\frac{dy}{dx}\Big) + 2 = 0,$$

in which case

$$\frac{dy}{dx} = -\frac{y(1+2x)}{x} = -\frac{e^{-2x}(1+2x)}{x^2}$$

because, by exponentiation,

$$y = \frac{e^{-2x}}{x}.$$

Consequently, at $x = 1$,

$$\boxed{\text{slope} = \frac{dy}{dx}\Big|_{x=1} = -3e^{-2}}.$$

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023 10.0 points

Determine the value of $f'''(1)$ when

$$f(x) = 5\ln(x+3).$$

1. $f'''(1) = \frac{5}{16}$

2. $f'''(1) = -\frac{5}{64}$

3. $f'''(1) = -\frac{5}{32}$

4. $f'''(1) = -\frac{5}{16}$

5. $f'''(1) = \frac{5}{64}$

6. $f'''(1) = \frac{5}{32}$ **correct**

Explanation:

After successive applications of the Chain Rule to f we see that

$$f'(x) = \frac{5}{x+3}, \quad f''(x) = -\frac{5}{(x+3)^2},$$

and

$$f'''(x) = \frac{10}{(x+3)^3}.$$

At $x = 1$, therefore,

$$\boxed{f'''(1) = \frac{5}{32}}.$$

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024 10.0 points

Determine the value of $f''(1)$ when

$$f(x) = 4\ln(2x+1).$$

1. $f''(1) = -\frac{16}{3}$

2. $f''(1) = \frac{32}{9}$

3. $f''(1) = \frac{16}{9}$

4. $f''(1) = -\frac{16}{9}$ **correct**

5. $f''(1) = -\frac{32}{9}$

6. $f''(1) = \frac{16}{3}$

Explanation:

After successive applications of the Chain Rule to f we see that

$$f'(x) = \frac{8}{2x+1}, \quad f''(x) = -\frac{16}{(2x+1)^2}.$$

At $x = 1$, therefore,

$$\boxed{f''(1) = -\frac{16}{9}}.$$

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025 10.0 points

Find the derivative of

$$f(t) = \frac{1+\ln t}{4-\ln t}.$$

1. $f'(t) = -\dfrac{5}{t(4-\ln t)^2}$
2. $f'(t) = \dfrac{5}{t(4-\ln t)^2}$ **correct**
3. $f'(t) = \dfrac{4}{t(1+\ln t)^2}$
4. $f'(t) = -\dfrac{4\ln t}{t(1+\ln t)^2}$
5. $f'(t) = -\dfrac{5}{(4-\ln t)^2}$
6. $f'(t) = \dfrac{4\ln t}{(1+\ln t)^2}$

Explanation:

By the Quotient Rule,

$$f'(t) = \frac{(4-\ln t)(1/t)+(1+\ln t)(1/t)}{(4-\ln t)^2} = \frac{(4-\ln t)+(1+\ln t)}{t(4-\ln t)^2}.$$

Consequently,

$$\boxed{f'(t) = \frac{5}{t(4-\ln t)^2}}.$$

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026 10.0 points

Find the derivative of

$$f(x) = 4\ln(2x+\sqrt{6+4x^2}\,).$$

1. $f'(x) = \dfrac{8}{\sqrt{6+4x^2}}$ **correct**
2. $f'(x) = \dfrac{8}{6+4x^2}$
3. $f'(x) = -\dfrac{8}{\sqrt{6+4x^2}}$
4. $f'(x) = \dfrac{4}{2x+\sqrt{6+4x^2}}$
5. $f'(x) = 8\sqrt{6+4x^2}$

Explanation:

By the Chain rule,

$$\frac{d}{dx}\ln(2x+\sqrt{6+2x^2}\,) = \left(2+\frac{4x}{\sqrt{6+4x^2}}\right)\left(\frac{1}{2x+\sqrt{6+4x^2}}\right) = \frac{2}{\sqrt{6+4x^2}}.$$

Consequently,

$$\boxed{f'(x) = \frac{8}{\sqrt{6+4x^2}}}.$$

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027 10.0 points

Determine $f'(x)$ when

$$f(x) = e^{(3\ln(x^5))}.$$

1. $f'(x) = \dfrac{3}{x^2}e^{3\ln(x^5)}$
2. $f'(x) = e^{15/x}$
3. $f'(x) = 15x^{14}$ **correct**
4. $f'(x) = 15(\ln x)e^{3\ln(x^5)}$
5. $f'(x) = \dfrac{1}{x}e^{3\ln(x^5)}$

6. $f'(x) \;=\; 14x^{15}$

Explanation:
Since

$$r\ln x \;=\; \ln x^r\,, \qquad e^{\ln x} \;=\; x\,,$$

we see that

$$f(x) \;=\; e^{(\ln x^{15})} \;=\; x^{15}\,.$$

Consequently,

$$\boxed{f'(x) \;=\; 15x^{14}}\,.$$

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028 10.0 points

Calculate $f'(\ln 3)$ when

$$f(x) \;=\; \ln\left(\sqrt{8+e^x}\right).$$

1. $f'(\ln 3) \;=\; \dfrac{3}{44}$

2. $f'(\ln 3) \;=\; -\dfrac{3}{22}$

3. $f'(\ln 3) \;=\; \dfrac{3}{22}$ **correct**

4. $f'(\ln 3) \;=\; \dfrac{3}{11}$

5. $f'(\ln 3) \;=\; \dfrac{1}{22}$

Explanation:
By properties of logarithms,

$$f(x) \;=\; \frac{1}{2}\ln\left(8+e^x\right).$$

Using the Chain Rule, we now see that

$$\frac{df}{dx} \;=\; \frac{e^x}{2(8+e^x)}\,.$$

Thus, at $x = \ln 3$,

$$\boxed{f'(\ln 3) \;=\; \frac{3}{2(8+3)} = \frac{3}{22}}$$

since $e^{\ln x} = x$.

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029 10.0 points

Determine the value of the third derivative of f at $x = 1$ when

$$f(x) \;=\; 3\ln(3x+2),$$

1. $f'''(x) = -\dfrac{162}{125}$

2. $f'''(x) = \dfrac{486}{125}$

3. $f'''(x) = \dfrac{81}{125}$

4. $f'''(x) = -\dfrac{81}{125}$

5. $f'''(x) = \dfrac{162}{125}$ **correct**

Explanation:

After successive applications of the Chain Rule

$$f'(x) = \frac{9}{3x+2}, \quad f''(x) = -\frac{27}{(3x+2)^2},$$

and

$$f'''(x) = \frac{162}{(3x+2)^3}.$$

The value of f''' at $x = 1$ is thus given by

$$\boxed{f'''(1) = \frac{162}{125}.}$$

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030 10.0 points

Determine $f'(e)$ when

$$f(x) \;=\; x^2(2+(\ln x)^3).$$

1. $f'(e) \;=\; 8e$

2. $f'(e) = 5e$

3. $f'(e) = 9e$ **correct**

4. $f'(e) = 7e$

5. $f'(e) = 6e$

Explanation:

Using the Product and Power rules we see that

$$\begin{aligned} f'(x) &= 2x(2 + (\ln x)^3) + \frac{3x^2(\ln x)^2}{x} \\ &= x(4 + 2(\ln x)^3 + 3(\ln x)^2). \end{aligned}$$

At $x = e$, therefore,

$$\boxed{f'(e) = 9e.}$$