This print-out should have 30 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

Find an equation for the tangent to the graph of f at the point $P(2,\, f(2))$ when

$$f(x) \;=\; \frac{5}{1-3x}\,.$$

1. $y \;=\; \frac{3}{5}x - \frac{11}{5}$ **correct**

2. $y + \frac{3}{5}x + \frac{2}{5} \;=\; 0$

3. $y \;=\; 3x - 7$

4. $y \;=\; \frac{1}{5}x - \frac{9}{5}$

5. $y + \frac{1}{5}x + \frac{3}{5} \;=\; 0$

Explanation:

If $x = 2$, then $f(2) = -1$, so we have to find an equation for the tangent line to the graph of

$$f(x) \;=\; \frac{5}{1-3x}$$

at the point $(2, -1)$. Now the Newtonian quotient for f at a general point $(x,\, f(x))$ is given by

$$\frac{f(x+h) - f(x)}{h}\,.$$

First let's compute the numerator of the Newtonian Quotient:

$$f(x+h) - f(x) \;=\; \frac{5}{1-3(x+h)} - \frac{5}{1-3x}$$

$$=\; \frac{5(1-3x) - 5\{1-3(x+h)\}}{(1-3x)\,\{1-3(x+h)\}}$$

$$=\; \frac{15h}{(1-3h)\,(1-3(x+h))}\,.$$

Thus

$$\frac{f(x+h) - f(x)}{h} \;=\; \frac{15}{(1-3x)\,(1-3(x+h))}\,.$$

Hence

$$f'(x) \;=\; \lim_{h\to 0} \frac{15}{(1-3(x+h))(1-3x)}$$

$$=\; \frac{15}{(1-3x)^2}\,.$$

At $x = 2$, therefore,

$$f'(2) \;=\; \frac{15}{(1-6)^2} \;=\; \frac{3}{5}\,,$$

so by the point slope formula an equation for the tangent line at $(2, -1)$ is

$$y + 1 \;=\; \frac{3}{5}(x-2)$$

which after simplification becomes

$$\boxed{y \;=\; \frac{3}{5}x - \frac{11}{5}}\,.$$

002 10.0 points

Find the value of $f'(4)$ when

$$f(x) \;=\; \frac{5}{3}x^{3/2} + 8x^{1/2}\,.$$

1. $f'(4) \;=\; \frac{11}{2}$

2. $f'(4) \;=\; \frac{13}{2}$

3. $f'(4) \;=\; 7$ **correct**

4. $f'(4) \;=\; 6$

5. $f'(4) \;=\; \frac{15}{2}$

Explanation:

Since

$$\frac{d}{dx}x^r \;=\; rx^{r-1}\,,$$

we see that

$$f'(x) \;=\; \frac{5}{2}x^{1/2} + 4x^{-1/2}\,.$$

At $x = 4$, therefore,

$$\boxed{f'(4) \;=\; 7}\,.$$

003 10.0 points

Find the derivative of f when

$$f(x) \;=\; \sqrt{x}(x+5)\,.$$

1. $f'(x) \;=\; \dfrac{3x-5}{x\sqrt{x}}$

2. $f'(x) \;=\; \dfrac{2x+5}{x\sqrt{x}}$

3. $f'(x) \;=\; \dfrac{2x-5}{x\sqrt{x}}$

4. $f'(x) \;=\; \dfrac{3x+5}{2\sqrt{x}}$ **correct**

5. $f'(x) \;=\; \dfrac{3x-5}{2\sqrt{x}}$

6. $f'(x) \;=\; \dfrac{2x+5}{2\sqrt{x}}$

Explanation:

By the Product Rule

$$f'(x) \;=\; \frac{x+5}{2\sqrt{x}} + \sqrt{x}\,.$$

After simplification this becomes

$$\boxed{f'(x) \;=\; \frac{x+5+2x}{2\sqrt{x}} \;=\; \frac{3x+5}{2\sqrt{x}}}\,.$$

004 10.0 points

Find $f'(x)$ when

$$f(x) = \frac{2x-3}{4x+3}\,.$$

1. $f'(x) = \dfrac{20}{(2x-3)^2}$

2. $f'(x) = -\dfrac{18}{(4x+3)^2}$

3. $f'(x) = -\dfrac{20}{(4x+3)^2}$

4. $f'(x) = \dfrac{20}{4x+3}$

5. $f'(x) = \dfrac{18}{4x+3}$

6. $f'(x) = \dfrac{18}{(4x+3)^2}$ **correct**

Explanation:

By the quotient rule for differentiation,

$$f'(x) \;=\; \frac{2(4x+3) - 4(2x-3)}{(4x+3)^2}\,.$$

Consequently

$$\boxed{f'(x) \;=\; \frac{18}{(4x+3)^2}}\,.$$

005 10.0 points

Find the derivative of

$$g(x) \;=\; \left(\frac{x+3}{x+1}\right)(2x-7)\,.$$

1. $g'(x) \;=\; \dfrac{2x^2-4x-20}{(x+1)^2}$

2. $g'(x) \;=\; \dfrac{x^2+4x-20}{(x+1)^2}$

3. $g'(x) \;=\; \dfrac{2x^2+4x+20}{x+1}$

4. $g'(x) \;=\; \dfrac{2x^2+4x+20}{(x+1)^2}$ **correct**

5. $g'(x) \;=\; \dfrac{2x^2-4x-20}{x+1}$

6. $g'(x) \;=\; \dfrac{x^2-4x+20}{x+1}$

Explanation:

By the Quotient and Product Rules we see that

$$g'(x) \;=\; 2\left(\frac{x+3}{x+1}\right)$$

$$+\,(2x-7)\left(\frac{(x+1)-(x+3)}{(x+1)^2}\right)$$

$$=\; 2\left(\frac{x+3}{x+1}\right) + 2\left(\frac{2x-7}{(x+1)^2}\right)$$

$$=\; \frac{2(x+3)(x+1) + 2(2x-7)}{(x+1)^2}\,.$$

But

$$2(x+3)(x+1) - 2(2x-7)$$

$$=\; 2x^2 + 4x + 20\,.$$

Consequently

$$\boxed{g'(x) \;=\; \frac{2x^2+4x+20}{(x+1)^2}}\,.$$

006 10.0 points

Find $ax + b$ so that

$$\frac{d}{dx}\left(\frac{2x^2-x-4}{2x+3}\right) \;=\; \frac{4x^2+ax+b}{(2x+3)^2}\,.$$

1. $ax + b \;=\; 12x + 6$

2. $ax + b \;=\; 12x + 5$ **correct**

3. $ax + b \;=\; 12x + 7$

4. $ax + b \;=\; 18x + 6$

5. $ax + b \;=\; 18x + 7$

Explanation:

By the Quotient rule

$$\frac{d}{dx}\left(\frac{2x^2-x-4}{2x+3}\right)$$

$$=\; \frac{(4x-1)(2x+3) - 2(2x^2-x-4)}{(2x+3)^2}$$

$$=\; \frac{4x^2+12x+5}{(2x+3)^2}.$$

Consequently,

$$\boxed{ax + b \;=\; 12x + 5}\,.$$

007 10.0 points

Find the value of $F'(1)$ when

$$F(x) \;=\; \frac{f(x)}{f(x) - g(x)}$$

and

$$f(1) \;=\; 5, \qquad f'(1) \;=\; 2,$$

$$g(1) \;=\; 1, \qquad g'(1) \;=\; 1\,.$$

1. $F'(1) \;=\; \frac{3}{16}$ **correct**

2. $F'(1) \;=\; \frac{9}{16}$

3. $F'(1) \;=\; \frac{7}{16}$

4. $F'(1) \;=\; -\frac{7}{16}$

5. $F'(1) \;=\; -\frac{3}{16}$

Explanation:

By the Quotient Rule,

$$F'(x)$$

$$=\; \frac{f'(x)\,(f(x)-g(x)) - f(x)\,(f'(x)-g'(x))}{\left(f(x)-g(x)\right)^2}$$

$$=\; \frac{f(x)\,g'(x) - f'(x)\,g(x)}{\left(f(x)-g(x)\right)^2}\,,$$

Consequently,

$$\boxed{F'(1) \;=\; \frac{3}{16}}\,.$$

008 10.0 points

Find the value of $F'(2)$ when

$$F(x) \;=\; \frac{f(x)}{x}$$

and

$$f(2) \;=\; 2, \qquad f'(2) \;=\; 3\,.$$

1. $F'(2) \;=\; \frac{3}{2}$

2. $F'(2) \;=\; \frac{5}{4}$

3. $F'(2) \;=\; \frac{3}{4}$

4. $F'(2) \;=\; 1$ **correct**

5. $F'(2) \;=\; \frac{7}{4}$

Explanation:

By the Quotient Rule,

$$F'(x) \;=\; \frac{xf'(x) - f(x)}{x^2}\,.$$

At $x = 2$, therefore,

$$F'(2) \;=\; \frac{2f'(2) - f(2)}{2^2}\,.$$

Consequently,

$$\boxed{F'(2) \;=\; 1}\,.$$

009 10.0 points

Determine $g'(x)$ when

$$g(x) \;=\; \frac{6 + xf(x)}{\sqrt{x}}\,,$$

and f is a differentiable function.

1. $g'(x) \;=\; \dfrac{2xf(x) + x^2f'(x) - 6}{\sqrt{x}}$

2. $g'(x) \;=\; \dfrac{xf(x) + 2x^2f'(x) - 6}{2x\sqrt{x}}$ **correct**

3. $g'(x) \;=\; \dfrac{xf(x) - 2x^2f'(x) + 6}{2x\sqrt{x}}$

4. $g'(x) \;=\; \dfrac{xf(x) + 2x^2f'(x) + 6}{\sqrt{x}}$

5. $g'(x) \;=\; \dfrac{2xf(x) + x^2f'(x) - 6}{x\sqrt{x}}$

6. $g'(x) \;=\; \dfrac{xf(x) - x^2f'(x) + 6}{x\sqrt{x}}$

Explanation:

By the Quotient and Power Rules

$$g'(x) \;=\; \frac{\sqrt{x}(f(x) + xf'(x)) - \dfrac{6 + xf(x)}{2\sqrt{x}}}{(\sqrt{x})^2}\,.$$

But after bringing the numerator to a common denominator and simplifying, the right hand side becomes

$$\frac{2x(f(x) + xf'(x)) - (6 + xf(x))}{2x\sqrt{x}}\,.$$

Consequently,

$$\boxed{g'(x) \;=\; \frac{xf(x) + 2x^2f'(x) - 6}{2x\sqrt{x}}}\,.$$

010 10.0 points

Find the value of

$$\lim_{x\to 3} \frac{f(x) - f(3)}{\sqrt{x} - \sqrt{3}}$$

whenever f is a differentiable function.

1. limit $=\; -f'(3)$

2. limit $=\; -2\sqrt{3}\,f'(3)$

3. limit $=\; -2\,f'(3)$

4. limit $=\; f'(3)$

5. limit $=\; 2\sqrt{3}\,f'(3)$ **correct**

6. limit $=\; 2\,f'(3)$

Explanation:

Since

$$\frac{x-3}{\sqrt{x}-\sqrt{3}} = \sqrt{x}+\sqrt{3},$$

we see that

$$\frac{f(x)-f(3)}{\sqrt{x}-\sqrt{3}} = \frac{f(x)-f(3)}{x-3}\left(\frac{x-3}{\sqrt{x}-\sqrt{3}}\right)$$

$$= \frac{f(x)-f(3)}{x-3}(\sqrt{x}+\sqrt{3}).$$

But

$$\lim_{x\to 3}\frac{f(x)-f(3)}{x-3} = f'(3),$$

while

$$\lim_{x\to 3}\sqrt{x}+\sqrt{3} = 2\sqrt{3}.$$

Consequently, by Properties of limits,

$$\boxed{\lim_{x\to 3}\frac{f(x)-f(3)}{\sqrt{x}-\sqrt{3}} = 2\sqrt{3}f'(3)}.$$

011 10.0 points

Determine $f'(x)$ when

$$f(x) = x^2\sin x + 2x\cos x.$$

Do not type "$f'(x) =$" as part of your answer, only the algebraic expression.

Correct answer: $(x^2+2)*cos(x)$.

Explanation:
By the Product Rule,

$$f'(x) = 2x\sin x + x^2\cos x$$
$$+ 2\cos x - 2x\sin x$$

Consequently,

$$\boxed{f'(x) = (x^2+2)\cos x}.$$

keywords: differentiation, trig function, product rule

012 10.0 points

Determine the derivative of

$$f(x) = \frac{e^x-1}{x-3}.$$

1. $f'(x) = \dfrac{(x-4)e^x+1}{x-3}$

2. $f'(x) = \dfrac{(x-4)e^x-1}{(x-3)^2}$

3. $f'(x) = \dfrac{(x-2)e^x-1}{(x-3)^2}$

4. $f'(x) = \dfrac{(x-4)e^x+1}{(x-3)^2}$ **correct**

5. $f'(x) = \dfrac{(x-2)e^x-1}{x-3}$

6. $f'(x) = \dfrac{(x-2)e^x+1}{x-3}$

Explanation:
Using

$$\frac{d}{dx}e^x = e^x, \qquad \frac{d}{dx}x^n = nx^{n-1},$$

and the Quotient Rule, we see that

$$f'(x) = \frac{e^x(x-3)-(e^x-1)}{(x-3)^2}.$$

Consequently,

$$\boxed{f'(x) = \frac{(x-4)e^x+1}{(x-3)^2}}.$$

013 10.0 points

Determine $f'(x)$ when

$$f(x) = \frac{\sin(x)-2}{\sin(x)+4}.$$

1. $f'(x) = \dfrac{2\cos(x)}{\sin(x)+4}$

2. $f'(x) = -\dfrac{2\cos(x)}{(\sin(x)+4)^2}$

3. $f'(x) = -\dfrac{6\cos(x)}{(\sin(x)+4)^2}$

4. $f'(x) = \dfrac{6\sin(x)\cos(x)}{\sin(x)+4}$

5. $f'(x) = -\dfrac{2\sin(x)\cos(x)}{\sin(x)+4}$

6. $f'(x) = \dfrac{6\cos(x)}{(\sin(x)+4)^2}$ **correct**

Explanation:
By the Quotient Rule,

$$f'(x) = \frac{(\sin(x)+4)\cos(x)-(\sin(x)-2)\cos(x)}{(\sin(x)+4)^2}.$$

But

$$(\sin(x)+4)\cos(x)-(\sin(x)-2)\cos(x) = 6\cos(x).$$

Thus

$$\boxed{f'(x) = \frac{6\cos(x)}{(\sin(x)+4)^2}}.$$

keywords: derivative of trig functions, derivative, quotient rule

014 10.0 points

Determine the derivative of

$$f(x) = x^2\sin x + 2x\cos x.$$

1. $f'(x) = (x^2+2)\sin x$

2. $f'(x) = -(x^2+2)\cos x$

3. $f'(x) = (x^2+2)\cos x$ **correct**

4. $f'(x) = (x^2-2)\cos x$

5. $f'(x) = -(x^2+2)\sin x$

6. $f'(x) = -(x^2-2)\sin x$

Explanation:
By the Product Rule,

$$f'(x) = 2x\sin x + x^2\cos x$$
$$+ 2\cos x - 2x\sin x.$$

Consequently,

$$\boxed{f'(x) = (x^2+2)\cos x}.$$

015 10.0 points

Find the derivative of f when

$$f(x) = \frac{1+2\sin x}{\cos x}.$$

1. $f'(x) = \dfrac{2+\sin x}{\cos^2 x}$ **correct**

2. $f'(x) = \dfrac{2\sin x-1}{\cos^2 x}$

3. $f'(x) = \dfrac{\sin x-2}{\cos^2 x}$

4. $f'(x) = \dfrac{1-2\cos x}{\sin^2 x}$

5. $f'(x) = -\dfrac{1+2\cos x}{\sin^2 x}$

6. $f'(x) = \dfrac{2\sin x+1}{\cos^2 x}$

7. $f'(x) = -\dfrac{2+\cos x}{\sin^2 x}$

8. $f'(x) = \dfrac{2-\cos x}{\sin^2 x}$

Explanation:
By the quotient rule,

$$f'(x) = \frac{2\cos^2 x + \sin x(1+2\sin x)}{\cos^2 x}$$

$$= \frac{2(\sin^2 x+\cos^2 x)+\sin x}{\cos^2 x}.$$

But $\cos^2 x + \sin^2 x = 1$. Consequently,

$$\boxed{f'(x) = \frac{2+\sin x}{\cos^2 x}}.$$

016 10.0 points

Find the derivative of

$$f(x) = \frac{\tan x}{5+\tan x}.$$

1. $f'(x) = \dfrac{5}{(5\cos x+\sin x)^2}$ **correct**

2. $f'(x) = \dfrac{\tan x}{(5\cos x+\sin x)^2}$

3. $f'(x) = \dfrac{\sec^2 x}{5+\sec^2 x}$

4. $f'(x) = \dfrac{5\sec^2 x}{(1+5\tan x)^2}$

5. $f'(x) = \dfrac{5\tan x}{(5+\tan x)^2}$

6. $f'(x) = -\dfrac{\sec^2 x}{(5+\sec^2 x)^2}$

Explanation:
It is more convenient to simplify first: since

$$\tan x = \frac{\sin x}{\cos x},$$

we see that

$$\frac{\tan x}{5+\tan x} = \frac{\dfrac{\sin x}{\cos x}}{5+\dfrac{\sin x}{\cos x}}.$$

Thus

$$f(x) = \frac{\sin x}{5\cos x+\sin x}.$$

By the Product Rule, therefore,

$$f'(x) = \frac{\cos x}{5\cos x+\sin x}$$
$$- \frac{\sin x(\cos x - 5\sin x)}{(5\cos x+\sin x)^2}$$
$$= \frac{\cos x(5\cos x+\sin x)}{(5\cos x+\sin x)^2}$$
$$- \frac{\sin x(\cos x-5\sin x)}{(5\cos x+\sin x)^2}.$$

Consequently,

$$\boxed{f'(x) = \frac{5}{(5\cos x+\sin x)^2}}.$$

017 10.0 points

Find the derivative of

$$f(x) = \frac{\cos x+\sin x}{x^3}.$$

1. $f'(x) = \dfrac{(x-2)\sin x-(x+2)\cos x}{x^3}$

2. $f'(x) = \dfrac{(3-x)\sin x-(x+3)\cos x}{x^4}$

3. $f'(x) = \dfrac{(2-x)\sin x-(x+2)\cos x}{x^3}$

4. $f'(x) = \dfrac{(x-3)\cos x-(x+3)\sin x}{x^4}$ **correct**

5. $f'(x) = \dfrac{(x+2)\cos x+(x-2)\sin x}{x^3}$

6. $f'(x) = \dfrac{(x+3)\cos x+(x-3)\sin x}{x^4}$

Explanation:
By the Quotient Rule,

$$f'(x) = \frac{x^3(-\sin x+\cos x)-3x^2(\cos x+\sin x)}{x^6}.$$

Consequently,

$$\boxed{f'(x) = \frac{(x-3)\cos x-(x+3)\sin x}{x^4}}.$$

keywords:

018 10.0 points

Find the derivative of f when

$$f(x) = 4x\cos 5x.$$

1. $f'(x) = 20\cos 5x + 5x\sin 5x$

2. $f'(x) = 4\cos 5x + 20x\sin 4x$

3. $f'(x) = 20\cos 5x - 4x\sin 5x$

4. $f'(x) = 4\cos 5x - 20x\sin 5x$ **correct**

5. $f'(x) = 4\cos 4x - 4x\sin 5x$

Explanation:
Using the formulas for the derivatives of sine and cosine together with the Chain Rule we see that

$$f'(x) = (4x)'\cos 5x + 4x(\cos 5x)'$$
$$= \boxed{4\cos 5x - 20x\sin 5x.}$$

019 10.0 points

Determine $f'(x)$ when

$$f(x) = 2\sin 3x + 3\cos 2x.$$

1. $f'(x) = 2\cos 3x + 3\cos 2x$

2. $f'(x) = 6(\cos 3x + \sin 2x)$

3. $f'(x) = -6(\sin 2x + \cos 3x)$

4. $f'(x) = -(3\sin 2x + 2\cos 3x)$

5. $f'(x) = 6(\cos 3x - \sin 2x)$ **correct**

6. $f'(x) = 2\cos 3x - 3\sin 2x$

Explanation:
Since

$$\frac{d}{dx}\sin x = \cos x, \quad \frac{d}{dx}\cos x = -\sin x,$$

the Chain Rule ensures that

$$\boxed{f'(x) = 6(\cos 3x - \sin 2x)}.$$

020 10.0 points

Find $f'(x)$ when

$$f(x) = \left(\frac{x-2}{x+1}\right)^2.$$

1. $f'(x) = \dfrac{6(x+1)}{(x-1)^3}$

2. $f'(x) = \dfrac{6(x-2)}{(x+1)^3}$ **correct**

3. $f'(x) = -\dfrac{4(x-2)}{(x+1)^3}$

4. $f'(x) = -\dfrac{6(x+2)}{(x-1)^3}$

5. $f'(x) = \dfrac{4(x-1)}{(x+1)^3}$

6. $f'(x) = -\dfrac{4(x+1)}{(x-1)^3}$

Explanation:
By the Chain and Quotient Rules,

$$f'(x) = 2\left(\frac{x-2}{x+1}\right)\frac{(x+1)-(x-2)}{(x+1)^2}.$$

Consequently,

$$\boxed{f'(x) = \frac{6(x-2)}{(x+1)^3}}.$$

021 10.0 points

Find $f'(x)$ when

$$f(x) = \sqrt{x^2-2x}.$$

1. $f'(x) = \dfrac{1}{2}(x-1)\sqrt{x^2-2x}$

2. $f'(x) = \dfrac{x-1}{2\sqrt{x^2-2x}}$

3. $f'(x) = 2(x-1)\sqrt{x^2-2x}$

4. $f'(x) \;=\; \dfrac{2(x-1)}{\sqrt{x^2-2x}}$

5. $f'(x) \;=\; (x-1)\sqrt{x^2-2x}$

6. $f'(x) \;=\; \dfrac{x-1}{\sqrt{x^2-2x}}$ **correct**

Explanation:
By the Chain Rule,

$$f'(x) \;=\; \frac{1}{2\sqrt{x^2-2x}}(2x-2)\,.$$

Consequently,

$$\boxed{f'(x) \;=\; \frac{x-1}{\sqrt{x^2-2x}}}\,.$$

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022 10.0 points

Find the first derivative of f when

$$f(x) \;=\; 3\cos(2x) - \sin^2(x)\,.$$

1. $f'(x) \;=\; 14\sin(2x)$

2. $f'(x) \;=\; -14\cos(2x)$

3. $f'(x) \;=\; -14\sin(2x)$

4. $f'(x) \;=\; -7\sin(2x)$ **correct**

5. $f'(x) \;=\; 7\sin(2x)$

6. $f'(x) \;=\; -7\cos(2x)$

Explanation:
By the Chain Rule we see that

$$f'(x) \;=\; -6\sin(2x) - 2\sin(x)\cos(x)\,.$$

Now

$$2\sin(x)\cos(x) \;=\; \sin(2x)\,.$$

Consequently,

$$\boxed{f'(x) \;=\; -7\sin(2x)}\,.$$

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023 10.0 points

Find the value of $f'(-1)$ when

$$f(x) \;=\; \left(x + \frac{4}{x}\right)^5.$$

Correct answer: -9375.

Explanation:
Using the chain rule and the fact that

$$(x^\alpha)' = \alpha x^{\alpha-1}\,,$$

we see that

$$f'(x) \;=\; 5\left(x + \frac{4}{x}\right)^4\left(1 - \frac{4}{x^2}\right).$$

Consequently, at $x = -1$

$$\boxed{f'(-1) \;=\; -9375}\,.$$

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024 10.0 points

Find the derivative of f when

$$f(x) \;=\; \left(x^{7/2} + 3x^{-7/2}\right)^2.$$

1. $f'(x) \;=\; 7\left(\dfrac{x^7+3}{x^7}\right)$

2. $f'(x) \;=\; 8\left(\dfrac{1-3\,x^{-14}}{x^7}\right)$

3. $f'(x) \;=\; 7\left(\dfrac{x^{14}-9}{x^8}\right)$ **correct**

4. $f'(x) \;=\; 7\left(\dfrac{x^{14}-3}{x^7}\right)$

5. $f'(x) \;=\; 8\left(\dfrac{x^{14}+9}{x^8}\right)$

6. $f'(x) \;=\; 8\left(\dfrac{1+3\,x^{-14}}{x^7}\right)$

Explanation:

After expansion,

$$\left(x^{7/2} + 3x^{-7/2}\right)^2 \;=\; x^7 + 6 + 9x^{-7}$$

Thus

$$f'(x) \;=\; 7x^6 - 63x^{-8} \;=\; 7x^6 - \frac{63}{x^8}\,.$$

Consequently,

$$\boxed{f'(x) \;=\; 7\left(x^6 - \frac{9}{x^8}\right) \;=\; 7\left(\frac{x^{14}-9}{x^8}\right)}\,.$$

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025 10.0 points

Find the derivative of f when

$$f(x) \;=\; \left(x^1 + 2\,x^{-1}\right)^2.$$

1. $f'(x) \;=\; \dfrac{1}{x}\left(x^2 + 4\,x^{-2}\right)$

2. $f'(x) \;=\; \dfrac{2}{x}\left(x^2 - 2\,x^{-2}\right)$

3. $f'(x) \;=\; \dfrac{2}{x}\left(x^2 + 4\,x^{-2}\right)$

4. $f'(x) \;=\; \dfrac{2}{x}\left(x^2 + 2\,x^{-2}\right)$

5. $f'(x) \;=\; \dfrac{2}{x}\left(x^2 - 4\,x^{-2}\right)$ **correct**

6. $f'(x) \;=\; \dfrac{1}{x}\left(x^2 - 4\,x^{-2}\right)$

Explanation:
After expansion,

$$\left(x^1 + 2x^{-1}\right)^2 \;=\; x^2 + 4 + 4x^{-2}\,.$$

Thus

$$f'(x) \;=\; 2\left(x^{1/1} - 4x^{-3}\right),$$

which can also be written as

$$\boxed{f'(x) \;=\; \frac{2}{x}\left(x^2 - 4x^{-2}\right)}\,.$$

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026 10.0 points

Find the value of $f'(1)$ when

$$f(x) \;=\; 4(x^2+8)^{1/2} + \frac{1}{x}\,.$$

1. $f'(1) \;=\; -\dfrac{2}{3}$

2. $f'(1) \;=\; -\dfrac{1}{3}$

3. $f'(1) \;=\; \dfrac{1}{3}$ **correct**

4. $f'(1) \;=\; 0$

5. $f'(1) \;=\; \dfrac{2}{3}$

Explanation:
Using the Chain Rule and the fact that

$$\frac{d}{dx}\,x^r \;=\; r\,x^{r-1}$$

holds for all values of r, we see that

$$f'(x) \;=\; \frac{4x}{(x^2+8)^{1/2}} - \frac{1}{x^2}\,.$$

At $x = 1$, therefore,

$$\boxed{f'(1) \;=\; \frac{1}{3}}\,.$$

---

027 10.0 points

Find $f'(x)$ when

$$f(x) \;=\; \left(\frac{x}{3x^2+1}\right)^2.$$

1. $f'(x) \;=\; \dfrac{x(1-3x)}{(3x^2+1)^2}$

2. $f'(x) \;=\; \dfrac{2x(1-3x^2)}{(3x^2+1)^3}$ **correct**

3. $f'(x) \;=\; \dfrac{2x(1-3x)}{(3x^2+1)^2}$

4. $f'(x) \;=\; \dfrac{2(1-3x^2)}{(3x^2+1)^3}$

5. $f'(x) \;=\; \dfrac{x(1-3x^2)}{(3x^2+1)^3}$

6. $f'(x) \;=\; \dfrac{2(1-3x^2)}{(3x^2+1)^2}$

Explanation:
By the Power rule,

$$f'(x) \;=\; 2\Big(\frac{x}{3x^2+1}\Big) \times \frac{d}{dx}\Big(\frac{x}{3x^2+1}\Big).$$

But, by the Quotient rule,

$$\frac{d}{dx}\Big(\frac{x}{3x^2+1}\Big) \;=\; \frac{(3x^2+1)-6x^2}{(3x^2+1)^2}.$$

Consequently,

$$\boxed{f'(x) \;=\; \frac{2x(1-3x^2)}{(3x^2+1)^3}.}$$

---

028 10.0 points

Find the derivative of f when

$$f(x) \;=\; \frac{1}{(1-3x^2)^3}\,.$$

1. $f'(x) \;=\; -\dfrac{6x}{(1-3x^2)^3}$

2. $f'(x) \;=\; \dfrac{18x}{(1-3x^2)^4}$ **correct**

3. $f'(x) \;=\; \dfrac{6x}{(1-3x^2)^4}$

4. $f'(x) \;=\; -6x(1-3x^2)^3$

5. $f'(x) \;=\; 18x(1-3x^2)^3$

6. $f'(x) \;=\; -\dfrac{18x}{(1-3x^2)^4}$

Explanation:
By the Chain rule,

$$f'(x) \;=\; 3\Big(\frac{6x}{(1-3x^2)^4}\Big).$$

Consequently,

$$\boxed{f'(x) \;=\; \frac{18x}{(1-3x^2)^4}}\,.$$

---

029 10.0 points

Find $f'(x)$ when

$$f(x) \;=\; 3\sec^2 x + 2\tan^2 x\,.$$

1. $f'(x) \;=\; 10\sec^2 x\tan x$ **correct**

2. $f'(x) \;=\; -2\tan^2 x\sec x$

3. $f'(x) \;=\; -2\sec^2 x\tan x$

4. $f'(x) \;=\; 2\sec^2 x\tan x$

5. $f'(x) \;=\; 2\tan^2 x\sec x$

6. $f'(x) \;=\; 10\tan^2 x\sec x$

Explanation:
Since

$$\frac{d}{dx}\sec x = \sec x\tan x, \qquad \frac{d}{dx}\tan x = \sec^2 x,$$

the Chain Rule ensures that

$$f'(x) \;=\; 6\sec^2 x\tan x + 4\tan x\sec^2 x\,.$$

Consequently,

$$\boxed{f'(x) \;=\; 10\sec^2 x\tan x}\,.$$

---

030 10.0 points

Find the derivative of f when

$$f(x) \;=\; 20(x+4)^{1/5}(x-5)^{1/4}\,.$$

1. $f'(x) \;=\; \dfrac{20x}{(x+4)^{\frac{4}{5}}(x-5)^{\frac{3}{4}}}$

2. $f'(x) \;=\; \dfrac{9x}{(x+4)^{\frac{4}{5}}(x-5)^{\frac{3}{4}}}$ **correct**

3. $f'(x) \;=\; \dfrac{20x}{(x+4)^{\frac{6}{5}}(x-5)^{\frac{3}{4}}}$

4. $f'(x) \;=\; \dfrac{x}{(x+4)^{\frac{4}{5}}(x-5)^{\frac{3}{4}}}$

5. $f'(x) \;=\; \dfrac{9x}{(x+4)^{\frac{4}{5}}(x-5)^{\frac{5}{4}}}$

Explanation:
By the Product and power rules,

$$\begin{aligned} f'(x) \;&=\; \frac{4(x-5)^{1/4}}{(x+4)^{\frac{4}{5}}} + \frac{5(x+4)^{1/5}}{(x-5)^{\frac{3}{4}}} \\ &=\; \frac{4(x-5)+5(x+4)}{(x+4)^{\frac{4}{5}}(x-5)^{\frac{3}{4}}}. \end{aligned}$$

Thus

$$\boxed{f'(x) \;=\; \frac{9x}{(x+4)^{\frac{4}{5}}(x-5)^{\frac{3}{4}}}}\,.$$