This print-out should have 16 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 10.0 points

When f is defined by

$$f(x) = \sqrt{x},$$

find a so $f'(a)$ is five times the value of $f'(4)$.

1. $a = \dfrac{16}{25}$

2. $a = \dfrac{12}{25}$

3. $a = \dfrac{4}{25}$ **correct**

4. $a = \dfrac{8}{25}$

5. $a = \dfrac{2}{25}$

Explanation:

First we determine $f'(a)$ for a general value of a. Now

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

But when $f(x) = \sqrt{x}$,

$$\frac{f(x) - f(a)}{x - a} = \frac{\sqrt{x} - \sqrt{a}}{x - a}$$

$$= \frac{x - a}{(x - a)(\sqrt{x} + \sqrt{a})},$$

after rationalizing the numerator. Thus

$$f'(a) = \lim_{x \to a} \left(\frac{1}{\sqrt{x} + \sqrt{a}}\right) = \frac{1}{2\sqrt{a}}.$$

In this case

$$f'(a) = 5f'(4) \implies \frac{1}{2\sqrt{a}} = \frac{5}{2\sqrt{4}}.$$

Consequently,

$$\boxed{a = \frac{4}{(5)^2} = \frac{4}{25}}.$$

keywords: square root, definition of derivative, limit

002 10.0 points

The displacement (in feet) of a particle moving in a straight line is given by the equation of motion

$$s(t) = 3t^3 + 4t + 3,$$

where t is measured in seconds. Find the velocity, $v(1)$, of the particle after 1 second.

1. $v(1) = 14$ ft/sec

2. $v(1) = 11$ ft/sec

3. $v(1) = 15$ ft/sec

4. $v(1) = 13$ ft/sec **correct**

5. $v(1) = 12$ ft/sec

Explanation:

The velocity, $v(t)$, of the particle after t seconds is the derivative $s'(t)$ of the displacement

$$s(t) = 3t^3 + 4t + 3.$$

Thus

$$v(t) = 9t^2 + 4,$$

and so at $t = 1$,

$$\boxed{v(1) = 13 \text{ ft/sec}}.$$

003 10.0 points

Find the derivative of

$$f(x) = \frac{x^4}{12} + \frac{1}{4}x^{-4}.$$

1. $f'(x) = \dfrac{x^8 - 3}{3x^5}$ **correct**

2. $f'(x) = \dfrac{x^8 + 3}{3x^5}$

3. $f'(x) = \dfrac{x^7 - 3}{x^4}$

4. $f'(x) = \dfrac{x^8 - 1}{x^5}$

5. $f'(x) = \dfrac{x^7 + 1}{x^5}$

6. $f'(x) = \dfrac{x^7 + 1}{3x^4}$

Explanation:

Since

$$\frac{d}{dx} x^r = r\, x^{r-1},$$

it follows that

$$f'(x) = \frac{x^3}{3} - \frac{1}{x^5} = \frac{x^3 \cdot x^5 - 3}{3x^5}.$$

Consequently,

$$\boxed{f'(x) = \frac{x^8 - 3}{3x^5}}.$$

keywords: DerivFunc, DerivFuncExam,

004 10.0 points

Find the derivative of f when

$$f(x) = \frac{x^4}{16} + \frac{1}{2}x^{-4}.$$

1. $f'(x) = \dfrac{x^7 + 8}{4\,x^4}$

2. $f'(x) = \dfrac{x^7 - 8}{4\,x^4}$

3. $f'(x) = \dfrac{x^8 + 8}{4\,x^5}$

4. $f'(x) = \dfrac{x^8 - 2}{4\,x^5}$

5. $f'(x) = \dfrac{x^8 - 8}{4\,x^5}$ **correct**

Explanation:

Since

$$\frac{d}{dx} x^r = r\, x^{r-1},$$

it follows that

$$f'(x) = \frac{x^3}{4} - \frac{2}{x^5}.$$

Consequently,

$$\boxed{f'(x) = \frac{x^8 - 8}{4\,x^5}}.$$

keywords: derivatives, negative powers

005 10.0 points

Find the derivative of f when

$$f(x) = 4x^4 + \frac{4}{x^2} + 3\pi.$$

1. none of the other answers

2. $f'(x) = 2\Big(\dfrac{1 - 2x^6}{x^3}\Big)$

3. $f'(x) = 8\Big(\dfrac{2x^6 - 1}{x^3}\Big)$ **correct**

4. $f'(x) = 2\Big(\dfrac{2x^5 + 1}{x^2}\Big)$

5. $f'(x) = 8\Big(\dfrac{2x^6 + 1}{x^3}\Big)$

6. $f'(x) = 2\Big(\dfrac{2x^5 - 1}{x^2}\Big)$

7. $f'(x) = 8\Big(\dfrac{1 - 2x^6}{x^3}\Big)$

Explanation:

Since

$$\frac{d}{dx}(x^r) = rx^{r-1}$$

holds for all r, we see that

$$f'(x) = 16x^3 - \frac{8}{x^3}.$$

Consequently,

$$\boxed{f'(x) = 8\Big(\frac{2x^6 - 1}{x^3}\Big)}.$$

keywords: derivatives, negative powers

006 10.0 points

Find the value of the derivative of f at $x = -1$ when

$$f(x) = 2x + 3e^x.$$

1. $f'(-1) = 3e^{-1}$

2. $f'(-1) = 2 + 3e$

3. $f'(-1) = 2 - 3e$

4. $f'(-1) = 2 - 3e^{-1}$

5. $f'(-1) = 2 + 3e^{-1}$ **correct**

6. $f'(-1) = 2e$

Explanation:

After differentiation

$$f'(x) = 2 + 3e^x.$$

At $x = -1$, therefore,

$$\boxed{f'(-1) = 2 + 3e^{-1}}.$$

007 10.0 points

Determine the derivative of

$$f(x) = 3x^5 + 2e^x.$$

1. $f'(x) = 3x^5 + 2e^x$

2. $f'(x) = 5x^5 - 2e^x$

3. $f'(x) = 15x^4 - 2e^x$

4. $f'(x) = 5x^4 + 2e^x$

5. $f'(x) = 3x^5 - 2e^x$

6. $f'(x) = 15x^4 + 2e^x$ **correct**

Explanation:

Since

$$\frac{d}{dx}e^x = e^x, \qquad \frac{d}{dx}x^n = nx^{n-1},$$

we see that

$$\boxed{f'(x) = 15x^4 + 2e^x}.$$

008 10.0 points

Determine the derivative of

$$f(x) = 3e^x - x^5.$$

1. $f'(x) = 3e^x - 5x^4$ **correct**

2. $f'(x) = 3e^x + 5x^4$

3. $f'(x) = 3e^{x-1} + 5x^4$

4. $f'(x) = 3e^{x-1} + 5x^5$

5. $f'(x) = 3e^{x-1} - 5x^5$

6. $f'(x) = 3e^x - 5x^5$

Explanation:

Since

$$\frac{d}{dx}e^x = e^x, \qquad \frac{d}{dx}x^n = nx^{n-1},$$

we see that

$$\boxed{f'(x) = 3e^x - 5x^4}.$$

009 10.0 points

Determine the derivative of

$$f(x) = 2x + e^{x-1}.$$

1. $f'(x) = 2 - e^{x-1}$

2. $f'(x) = 2 - e^{x-2}$

3. $f'(x) = 2x + e^x$

4. $f'(x) = 2x - e^x$

5. $f'(x) = 2x + e^{x-2}$

6. $f'(x) = 2 + e^{x-1}$ **correct**

Explanation:

Since $e^{x-1} = e^x e^{-1} = e^{-1}e^x$ and

$$\frac{d}{dx}e^x = e^x, \qquad \frac{d}{dx}x^n = nx^{n-1},$$

we see that

$$\frac{d}{dx}e^{x-1} = \frac{d}{dx}e^{-1}e^x = e^{-1}e^x = e^{x-1}.$$

Consequently,

$$\boxed{f'(x) = 2 + e^{x-1}}.$$

010 10.0 points

Find the value of $f'(4)$ when

$$f(x) = \frac{2}{3}x^{3/2} + 8x^{1/2}.$$

1. $f'(4) = \dfrac{9}{2}$

2. $f'(4) = \dfrac{7}{2}$

3. $f'(4) = 3$

4. $f'(4) = 4$ **correct**

5. $f'(4) = \dfrac{5}{2}$

Explanation:

Since

$$\frac{d}{dx} x^r = rx^{r-1},$$

we see that

$$f'(x) = x^{1/2} + 4x^{-1/2}.$$

At $x = 4$, therefore,

$$\boxed{f'(4) = 4}.$$

011 10.0 points

Determine the derivative of f when

$$f(x) = \left(\frac{5}{6}\right)^{2/3}.$$

1. $f'(x) = 0$ **correct**

2. $f'(x) = \dfrac{2}{3}\left(\dfrac{5}{6}\right)^{-1/3}$

3. $f'(x) = \left(\dfrac{5}{6}\right)x^{-1/3}$

4. $f'(x)$ does not exist

5. $f'(x) = \dfrac{5}{9}x^{-1/3}$

Explanation:

The derivative of any constant function is zero. Consequently,

$$\boxed{f'(x) = 0}.$$

012 10.0 points

Determine $f'(x)$ when

$$f(x) = -2x^7 + 3x^3 + 8\pi.$$

1. $f'(x) = -14x^6 + 9x^2$ **correct**

2. $f'(x) = -14x^6 + 9x^2 + 8\pi x$

3. $f'(x) = -2x^7 + 9x^2 + 8$

4. $f'(x) = -14x^6 + 9x^2 + 8\pi$

5. $f'(x) = -14x^6 + 3x^3$

Explanation:
By linearity of differentiation and the rule

$$(x^n)' = n \cdot x^{n-1},$$

we see that

$$\boxed{f'(x) = -14x^6 + 9x^2}.$$

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013 10.0 points

Find the derivative of

$$f(x) = x^{\frac{1}{6}} - 5x^{-\frac{1}{6}} + 4.$$

1. $f'(x) = \dfrac{x^{\frac{1}{3}} - 5}{6x^{\frac{5}{6}}}$

2. $f'(x) = \dfrac{x^{\frac{1}{3}} + 5}{6x^{\frac{7}{6}}}$ **correct**

3. $f'(x) = \dfrac{x^{\frac{1}{6}} + 5}{6x^{\frac{5}{6}}}$

4. $f'(x) = \dfrac{x^{\frac{1}{3}} + 5}{5x^{\frac{7}{6}}}$

5. $f'(x) = \dfrac{x^{\frac{1}{3}} - 5}{6x^{\frac{7}{6}}}$

Explanation:
Since

$$\frac{d}{dx}(x^r) = rx^{r-1},$$

we see that

$$f'(x) = \frac{1}{6}\left(\frac{1}{x^{\frac{5}{6}}} + \frac{5}{x^{\frac{7}{6}}}\right).$$

Consequently,

$$\boxed{f'(x) = \frac{x^{\frac{1}{3}} + 5}{6x^{\frac{7}{6}}}}.$$

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014 10.0 points

Find the derivative of

$$f(x) = \sqrt{x} + \frac{2}{\sqrt{x}}.$$

1. $f'(x) = \dfrac{x+2}{x\sqrt{x}}$

2. $f'(x) = \dfrac{x+2}{2x\sqrt{x}}$

3. $f'(x) = \dfrac{x-2}{2x\sqrt{x}}$ **correct**

4. $f'(x) = \dfrac{x-2}{x\sqrt{x}}$

5. $f'(x) = \dfrac{x-2}{2\sqrt{x}}$

6. $f'(x) = \dfrac{x+2}{\sqrt{x}}$

Explanation:
Since

$$\frac{d}{dx}\sqrt{x} = \frac{d}{dx}x^{1/2} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}},$$

while

$$\frac{d}{dx}\frac{1}{\sqrt{x}} = \frac{d}{dx}x^{-1/2} = -\frac{1}{2}x^{-3/2} = -\frac{1}{2x\sqrt{x}}.$$

Thus

$$f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{x\sqrt{x}}.$$

Consequently,

$$\boxed{f'(x) = \frac{x-2}{2x\sqrt{x}}}.$$

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015 10.0 points

Find the derivative of g when

$$g(t) = \frac{4}{5t^5}.$$

1. $g'(t) = -\dfrac{4}{5}t^{-4}$

2. $g'(t) = \dfrac{4}{5}t^{-6}$

3. $g'(t) = 4t^{-6}$

4. $g'(t) = -\dfrac{4}{5}t^{-6}$

5. $g'(t) = -4t^{-4}$

6. $g'(t) = -4t^{-6}$ **correct**

Explanation:
Since

$$\frac{d}{dt}\, a\, t^r = r\, a\, t^{r-1}$$

for all $r \neq 0$ and all constants a, we see that

$$\boxed{g'(t) = -\frac{4}{5}\left(\frac{5}{t^6}\right) = -4t^{-6}}.$$

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016 10.0 points

Find the derivative of

$$f(x) = \frac{\sqrt{6}}{x^6}.$$

1. $f'(x) = -\dfrac{6\sqrt{6}}{x^7}$ **correct**

2. $f'(x) = \dfrac{6\sqrt{6}}{x^5}$

3. $f'(x) = \dfrac{6\sqrt{6}}{x^7}$

4. $f'(x) = \dfrac{\sqrt{6}}{6x^5}$

5. $f'(x) = \dfrac{7\sqrt{6}}{x^7}$

Explanation:
Since

$$f(x) = \frac{\sqrt{6}}{x^6} = \sqrt{6}\, x^{-6},$$

we see that

$$f'(x) = -6\sqrt{6}\, x^{-7}.$$

Consequently,

$$\boxed{f'(x) = -\frac{6\sqrt{6}}{x^7}}.$$