插值（保留三位小数）

1. $l_0(x)=\frac{(x-2)(x-3)}{(-1-2)(-1-3)}=\frac{1}{12}(x-2)(x-3)$

$l_1(x)=\frac{(x+1)(x-3)}{(2+1)(2-3)}=-\frac{1}{3}(x+1)(x-3)$

$l_2(x)=\frac{(x+1)(x-2)}{(3+1)(3-2)}=\frac{1}{4}(x+1)(x-2)$

$L_2(x)=l_0(x)f(x_0)+l_1(x)f(x_1)+l_2(x)f(x_2)=\frac{1}{4}(x-2)(x-3)-\frac{5}{3}(x+1)(x-3)+\frac{7}{4}(x+1)(x-2)$

$L_2(0)=3$

2. $l_0(x)=\frac{(x-2)(x-5)}{(-2-2)(-2-5)}=\frac{1}{28}(x-2)(x-5)$ $l_1(x)=\frac{(x+2)(x-5)}{(2+2)(2-5)}=-\frac{1}{12}(x+2)(x-5)$

$l_2(x)=\frac{(x+2)(x-2)}{(5+2)(5-2)}=\frac{1}{21}(x+2)(x-2)$

$L_2(x)=-\frac{1}{4}(x+2)(x-5)+\frac{2}{7}(x+2)(x-2)$ $L_2(-1.2)=\frac{89}{175}=0.509$

$L_2(1.2)=\frac{404}{175}=2.309$

3. (1) $l_0(x)=\frac{x(x-\frac{1}{2})(x-1)}{-1\times(-1-\frac{1}{2})(-1-1)}=-\frac{1}{3}x(x-\frac{1}{2})(x-1)$

$l_1(x)=\frac{(x+1)(x-\frac{1}{2})(x-1)}{(0+1)(0-\frac{1}{2})(0-1)}=2(x+1)(x-\frac{1}{2})(x-1)$

$l_2(x)=\frac{(x+1)x(x-1)}{(\frac{1}{2}+1)(\frac{1}{2}+0)(\frac{1}{2}-1)}=-\frac{8}{3}(x+1)x(x-1)$

$l_3(x)=\frac{(x+1)(x-0)(x-\frac{1}{2})}{(1+1)(1-0)(1-\frac{1}{2})}=x(x+1)(x-\frac{1}{2})$

$L_3(x)=-x(x-\frac{1}{2})(x-1)-(x+1)(x-\frac{1}{2})(x-1)+x(x+1)(x-\frac{1}{2})$

(2) $l_0(x)=\frac{x(x-2)(x-3)}{-1(-1-2)(-1-3)}=-\frac{1}{12}x(x-2)(x-3)$ $l_1(x)=\frac{(x+1)(x-2)(x-3)}{(0+1)(0-2)(0-3)}=\frac{1}{6}(x+1)(x-2)(x-3)$

$l_2(x)=\frac{(x+1)x(x-3)}{(2+1)2(2-3)}=-\frac{1}{6}(x+1)x(x-3)$ $l_3(x)=\frac{(x+1)x(x-2)}{(3+1)3(3-2)}=\frac{1}{12}(x+1)x(x-2)$

$L_3(x)=-\frac{1}{6}x(x-2)(x-3)\cancel{+}-\frac{1}{6}x(x+1)(x-3)+\frac{1}{4}x(x+1)(x-2)$

4. 存疑？ M_2？

5. $l_0(x)=\dfrac{(x-100)(x-121)}{(81-100)(81-121)}=\dfrac{1}{760}(x-100)(x-121)$ $l_1(x)=\dfrac{(x-81)(x-121)}{(100-81)(100-121)}=\dfrac{-1}{399}(x-81)(x-121)$

$l_2(x)=\dfrac{(x-81)(x-100)}{(121-81)(121-100)}=\dfrac{1}{840}(x-81)(x-100)$ $L_2(x)=\dfrac{9}{760}(x-100)(x-121)+\dfrac{-10}{399}(x-81)(x-121)$

$+\dfrac{11}{840}(x-81)(x-100)$ $L_2(105)=10.248$

$R_2(x)=\dfrac{f^{(3)}(\xi)}{3!}(x-81)(x-100)(x-121)$ $\cancel{R_2(105)}=\cdots$

$f(x)=\sqrt{x}$ $f^{(3)}(x)=\dfrac{3}{8}x^{-\frac{5}{2}}$ $x\in(81,121)$ 则 $f^{(3)}(x)\in(6.35\times10^{-6},\ 2.33\times10^{-6})$ (区间导数3

则 $R_2(105)\in(-2.032\times10^{-3},\ -7.456\times10^{-4})$ 实际误差：-1.049×10^{-3} 在误差界中

6. 差商表如下：

	x	$f(x)$			
x_0	-1	3			
x_1	2	5	$f[x_0,x_1]=-\frac{2}{3}$		
x_2	3	7	$f[x_1,x_2]=2$	$f[x_0,x_1,x_2]=\frac{2}{3}$	
x_3	4	5	$f[x_2,x_3]=-2$	$f[x_1,x_2,x_3]=-2$	$f[x_0,x_1,x_2,x_3]=\frac{8}{15}$

$N_3(x)=3-\frac{2}{3}(x+1)+\frac{2}{3}(x+1)(x-2)+\frac{8}{15}(x+1)(x-2)(x-3)$

$N_3(1.2)=2.050$

7. $N_3(x)=1+2(x-4)+(x-1)(x-4)-(x-1)(x-3)(x-4)$

又 $N_3(2)=-7$

$f[1,2,3,4]=\dfrac{f[1,3,4]-f[2,3,4]}{1-2}=-1$

得 $f[2,3,4]=0$

8. 取插值点 $(a,f(a))$，$(b,f(b))$ 设步长为 $h=b-a$

则 $R_1(x)=\dfrac{f''(\xi)}{2!}(x-a)(x-b)=\dfrac{-\sin\xi}{2}(x-a)(x-b)$

$|R_1(x)|\le\dfrac{1}{2}\cdot\dfrac{(b-a)^2}{4}=\dfrac{h^2}{8}\le10^{-5}$ 得 $h\le0.00894$

9. $f[2^0,2^1] = \frac{f(2)-f(1)}{2-1} = -2089 \quad f[2^0,2^1\cdots2^7] = a_7 = 1$

$f[2^0,2^1,\cdots,2^8] = 0$.

10. $\begin{cases} S_0(x) = \frac{x-1.1}{1.05-1.1}\times2.12 + \frac{x-1.05}{1.10-1.05}\times2.20 & x\in(1.05,1.10) \quad f(1.075)=2.16 \\ S_1(x) = \frac{x-1.15}{1.10-1.15}\times2.20 + \frac{x-1.10}{1.15-1.10}\times2.17 & x\in(1.10,1.15) \\ S_2(x) = \frac{x-1.20}{1.15-1.20}\times2.17 + \frac{x-1.15}{1.20-1.15}\times2.32 & x\in(1.15,1.20) \quad f(1.175) = 2.245 \end{cases}$

11.

0 $f(0)$

1 $f(1)$ $f[0,1] = f(1)-f(0)$

1 $f(1)$ $f[1,1] = f'(1)$ $\quad f[0,1,1] = \frac{f'(1)-f(1)+f(0)}{1-0} = f'(1)-f(1)+f(0)$

$H_2(x) = f(0) + [f(1)-f(0)]x + [f'(1)-f(1)+f(0)]x(x-1)$.

$R_2(x) = \frac{f^{(3)}(\xi)}{3!}x(x-1)^2$

12. x $f(x)$

3 5

5 15 $f[5,3]=5$

5 15 $f[3,5]=7$ $f[3,5,5]=1$

$H_2(x) = 5+5(x-3)+(x-3)(x-5)$ $\quad H_2(3.7) = 5+5\times0.7+0.7\times-2.3 = 6.89$.

$R_2(x) = \frac{f^{(3)}(\xi)}{3!}(x-3)(x-5)^2$.

13. x $f(x)$

0 $f(0)$

1 $f(1)$ $f[0,1] = f(1)-f(0)$

3 $f(3)$ $f[1,3] = \frac{f(3)-f(1)}{2}$ $\quad f[0,1,3] = \frac{f(3)-3f(1)+2f(0)}{6}$.

3 $f(3)$ $f[3,3] = f'(3)$ $\quad f[1,3,3] = \frac{2f'(3)-f(3)+f(1)}{4}$ $\quad f[0,1,3,3]$

$= \frac{6f'(3)-5f(3)+9f(1)-4f(0)}{36}$

$H_3(x) = f(0)+f[0,1]x+f[0,1,3]x(x-1)+f[0,1,3,3]x(x-1)(x-3)$

$R_3(x) = \frac{f^{(4)}(\xi)}{4!}x(x-1)(x-3)^2$

14.

$H_3(x) = 1 + (-0.25)x + 0.125\, x(x-1)(x-3)$

$R_3(x) = \dfrac{f^{(4)}(\xi)}{4!}\, x(x-1)(x-3)^2$

15.

x	$f(x)$				
1	0.5				
1	0.5	$f[1,1]=0.5$			
2	1	$f[1,2]=0.5$	$f[1,1,2]=0$		
2	1	$f[2,2]=-1$	$f[1,2,2]=-1.5$	$f[1,1,2,2]=-1.5$	
2	1	$f[2,2]=-1$	$f[2,2,2]=1$	$f[1,2,2,2]=2.5$	$f[1,1,2,2,2]=4$

$H_4(x) = 0.5 + 0.5(x-1) - 1.5(x-1)^2(x-2) + 4(x-1)^2(x-2)^2$

$R_4(x) = \dfrac{f^{(5)}(\xi)}{5!}(x-1)^2(x-2)^3$

16. $h_0 = 1 \quad h_1 = 2 \quad h_2 = 1 \quad \begin{cases} \lambda_1 = \frac{2}{3} \\ \mu_1 = \frac{1}{3} \end{cases} \quad \begin{cases} \lambda_2 = \frac{1}{3} \\ \mu_2 = \frac{2}{3} \end{cases} \quad \begin{matrix} d_1 = 6f[-2,-1,1] = -12 \\ d_2 = 6f[-1,1,2] = 12 \end{matrix}$

由 $M_0 = M_3 = 0$ 的边界条件得

$$\begin{pmatrix} 2 & \lambda_1 \\ \mu_2 & 2 \end{pmatrix}\begin{pmatrix} M_1 \\ M_2 \end{pmatrix} = \begin{pmatrix} 2 & \frac{2}{3} \\ \frac{2}{3} & 2 \end{pmatrix}\begin{pmatrix} M_1 \\ M_2 \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} -12 \\ 12 \end{pmatrix} \Rightarrow \begin{pmatrix} M_1 \\ M_2 \end{pmatrix} = \begin{pmatrix} -9 \\ 9 \end{pmatrix}$$

则

$$S(x) = \begin{cases} -\frac{3}{2}(x+2)^3 + 4(x+1) + \frac{9}{2}(x+2) & x \in [-2,-1] \\ \frac{3}{2}x^3 - \frac{x}{2} + 4 & x \in [-1,1] \\ \frac{3}{2}(2-x)^3 + \frac{17}{2}x - 5 & x \in [1,2] \end{cases}$$

$S(0) = 4$.

17. $h_0=1 \quad h_1=1 \quad h_2=2.$ $\begin{cases}\lambda_1=\frac{1}{2}\\ \mu_1=\frac{1}{2}\end{cases}$ $\begin{cases}\lambda_2=\frac{2}{3}\\ \mu_2=\frac{1}{3}\end{cases}$

$d_0=6f[x_0,x_0,x_1]=-24.$ $d_1=6f[-1,0,1]=0$ $d_2=6f[0,1,3]=23$ $d_3=6f[1,3,3]=49.5$

则得

$$\begin{pmatrix}2 & 1 & & \\ 0.5 & 2 & 0.5 & \\ & \frac{1}{3} & 2 & \frac{2}{3} \\ & & 1 & 2\end{pmatrix}\begin{pmatrix}M_0\\M_1\\M_2\\M_3\end{pmatrix}=\begin{pmatrix}-24\\0\\23\\49.5\end{pmatrix} \Rightarrow \begin{pmatrix}M_0\\M_1\\M_2\\M_3\end{pmatrix}=\begin{pmatrix}-13.227\\2.4545\\3.409\\23.045\end{pmatrix}$$

$S(x)=\begin{cases}\text{代入公式（过于繁琐，省略）.}\end{cases}$