第三章：非线性方程求解（保留三位小数）

1. 取 $x_0=\frac{\sqrt{3}}{3}$，由迭代格式得：

k	x	$\lvert x_{k+1}-x_k\rvert$	k	x	$\lvert x_{k+1}-x_k\rvert$
0	$\frac{\sqrt{3}}{3}$	1.3660	4	1.4972	$0.0001<10^{-3}$
1	1.3660	0.1131	5	1.4973	
2	1.4791	0.0167			
3.	1.4958	0.0014			

故 $f(x)$ 在 $[1,2]$ 的根为 1.4973.

2. 1) 将迭代格式写为 $x_{n+1}=\frac{2x_n^3-5x_n^2+42}{19}$. $\varphi_1(x)=\frac{2x^3-5x^2+42}{19}$.

$\varphi_1'(x)=\frac{6x^2-10x}{19}$ 在 $x=3$ 附近 $\lvert\varphi_1'(x)\rvert=\frac{24}{19}>1$ 故迭代格式在 $x=3.0$ 处发散.

2) 将迭代格式写为 $x_{n+1}=\sqrt{\frac{2x_n^3-19x_n+42}{5}}$ $\varphi_2(x)=\sqrt{\frac{2x^3-19x+42}{5}}$

$\varphi_2'(x)=\frac{1}{2\sqrt{\frac{2x^3-19x+42}{5}}}\cdot\frac{6x^2-19}{5}$ 在 $x=3$ 附近 $\lvert\varphi_2'(3)\rvert=1.25>1$

故迭代格式在 $x=3.0$ 处发散.

3) 将迭代格式写为 $x_{n+1}=\sqrt[3]{\frac{5x_n^2+19x_n-42}{2}}$ $\varphi_3(x)=\sqrt[3]{\frac{5x^2+19x-42}{2}}$

$\varphi_3'(x)=\frac{1}{3}\left(\frac{5x^2+19x-42}{2}\right)^{-\frac{2}{3}}\cdot\left(5x+\frac{19}{2}\right)$ $\varphi_3''(x)<0$

当 $x\in(2.8,4)$ 时 $\lvert\varphi_3'(x)\rvert\le\lvert\varphi_3'(2.8)\rvert=0.911<1$ 而且 $\varphi_3(x)\in(2.93,3.849)\in(2.8,4)$

由压缩映射定理 迭代格式在 $x=3$ 处收敛.

3. 由二分法可得下表

k	x	$f(x)$	求解区间	$\lvert x_k-x_{k-1}\rvert$
0	0	-1		
1	1	1	$[0,1]$	
2	0.5	-0.625	$[0.5,1]$	0.5
3.	0.75	-0.015625	$[0.75,1]$	0.25
4.	0.875	$\frac{223}{512}$	$[0.75,0.875]$	0.125
5	0.8125	$\frac{805}{4096}$	$[0.75,0.8125]$	0.0625
6.	0.78125	0.0377	$[0.75\ \ 0.765625]$	0.03125
7.	0.765625	0.02450	$[0.75\ \ 0.7578125]$	0.015625
8	$\frac{193}{256}$	-0.00312	$[\cancel{0.75}\,\frac{193}{256},\ \frac{97}{128}]$	$\frac{1}{128}$

9. $\frac{387}{512}$ 0.00316 $[\frac{193}{256}, \frac{387}{512}]$ $\frac{1}{256}$

10. $\frac{773}{1024}$ 0.0000166 $<10^{-3}$

故用二分法求得的$f(x)$在$[0,1]$上的根为 $\frac{773}{1024}$.

4.

k	x_k	$\lvert f(x_k)\rvert$
0	4.	5
1	3.375	$\frac{25}{64}$
2	$\frac{1433}{432}$	0.00335
3	3.31662	$0.0000318<10^{-4}$

故由牛顿迭代法计算得到 $\sqrt{11}=3.31662$

5. Newton 迭代法的迭代格式为：~~设~~

$$x_{k+1}=x_k-\frac{f(x_k)}{f(x_k)'}\cancel{=x_k-\ln a\frac{a^n-\sqrt[5]{9}}{\ln a\, a^n}}=x_k-\cancel{\frac{1}{\ln a}(1-\frac{\sqrt[5]{9}}{\sqrt[n]{a}})}$$

k x_k

~~5. 设 $f(x)=\sqrt[x]{a}-\sqrt[n]{a}$~~

设$f(x)=x^n-a$ 则$\sqrt[n]{a}$为$f(x)$的正零点.

牛顿迭代格式为 $x_{k+1}=x_k-\frac{f(x_k)}{f'(x_k)}=x_k-\frac{x_k^n-a}{nx_k^{n-1}}=\frac{n-1}{n}x_k+\frac{a}{n}\frac{1}{x_k^{n-1}}$

取$n=5$ $a=9$ 则 $x_{k+1}=\frac{4}{5}x_k+\frac{9}{5}\frac{1}{x_k^4}$

k	x_k	$f(x_k)$	$\lvert x_{k+1}-x_k\rvert$
0	2	23	0.2875
1	1.7125	5.728	0.1332
2	1.5793	0.825	0.0265
3.	1.5528	0.0277	$0.001<10^{-2}$
4.	1.5518		

故用Newton迭代法求得 $\sqrt[5]{9}\approx1.5518$

6. Newton 迭代格式为 $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{x_k^3 - 3x_k - 2}{3x_k^2 - 3} = \frac{2}{3}\cdot\frac{x_k^2 - x_k + 1}{x_k - 1}$

k	x_k	$\lvert f(x_k)\rvert$
0	1.5	3.125
1	2.3333	$\lvert 3.7033\rvert$
2	2.0555	0.5182
3	2.0019	0.01712
4	2.0000	$0 < 10^{-4}$

故用Newton迭代法求得 $f(x)$的根为 2.0000

7. 弦截法的迭代格式为

$x_{k+1} = x_k - f(x_k)\cdot\frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}$

k	x_k	$\lvert f(x_k)\rvert$
0	1	$\lvert -4\rvert = 4$
1	3	16
2	1.4	$\lvert -3.456\rvert = 3.456$
3	1.6842	$\lvert -2.2753\rvert$
4	2.2319	2.4222
5	1.9495	$\lvert -0.4393\rvert$
6	1.9929	$\lvert -0.0636\rvert$
7	2.0003	0.0027
8	2.0000	$0 < 10^{-4}$

故由弦截法计算$f(x)$的根为 2.0000

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8. $J(x,y)=\begin{pmatrix}\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y}\\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y}\end{pmatrix}=\begin{pmatrix}2x & 2y\\ 3x^2 & -1\end{pmatrix}$ $J(x_0,y_0)=\begin{pmatrix}1.6 & 1.2\\ 1.92 & -1\end{pmatrix}$ $f(x_0,y_0)=0$ $g(x_0,y_0)=-0.088$

解 $J(x_0,y_0)\begin{pmatrix}\Delta x\\ \Delta y\end{pmatrix}=\begin{pmatrix}-f(x_0,y_0)\\ -g(x_0,y_0)\end{pmatrix}=\begin{pmatrix}1.6 & 1.2\\ 1.92 & -1\end{pmatrix}\begin{pmatrix}\Delta x\\ \Delta y\end{pmatrix}=\begin{pmatrix}0\\ 0.088\end{pmatrix}$ 得 $\begin{pmatrix}\Delta x\\ \Delta y\end{pmatrix}=\begin{pmatrix}0.027\\ -0.036\end{pmatrix}$

$w_1=(x_0,y_0)^T+(\Delta x\ \ \Delta y)^T=(0.827\ \ 0.564)$

$J(x_1,y_1)=\begin{pmatrix}1.654 & 1.128\\ 2.0518 & -1\end{pmatrix}$ $f(x_1,y_1)=0.002$ $g(x_1,y_1)=0.0016$ 得 $\begin{pmatrix}\Delta x\\ \Delta y\end{pmatrix}=\begin{pmatrix}-9.6\times10^{-4}\\ 3.7\times10^{-4}\end{pmatrix}$ $|\Delta w|<10^{-3}$

故由牛顿迭代法求得非线性方程组的解为 $(0.827\ \ 0.564)$