1 非线性方程求解

1. $\varphi'(x) = \cos x \qquad |\varphi'(x)| < 1 \qquad x \in [0, 2]$

$$
\begin{aligned}
&x_0 = 1 \\
&x_1 = 1.341471 && \Delta = 0.341471 \\
&x_2 = 1.47382 && \Delta = 0.132349 \\
&x_3 = 1.495301 && \Delta = 0.021481 \\
&x_4 = 1.4971516 && \Delta = 0.0018506 \\
&x_5 = 1.4972895 && \Delta = 0.0001379 < 10^{-3}
\end{aligned}
$$

2.

(1).

$$
\begin{aligned}
x &= \frac{2}{19}x^3 - \frac{5}{19}x^2 + \frac{42}{19} \\
\varphi'(x) &= \frac{6}{19}x^2 - \frac{10}{19}x \\
\varphi'(3) &= 1.26 > 1
\end{aligned}
$$

不收敛

(2).

$$
\begin{aligned}
x &= \sqrt{\frac{2}{5}x^3 - \frac{19}{5}x + \frac{42}{5}} \\
\varphi'(x) &= \frac{\frac{6}{5}x^2 - \frac{19}{5}}{2\sqrt{\frac{2}{5}x^3 - \frac{19}{5}x + \frac{42}{5}}} \\
\varphi'(3) &= \frac{7\sqrt{195}}{78} = 1.2532 > 1
\end{aligned}
$$

(3).

$$
\begin{aligned}
x &= \sqrt[3]{\frac{5}{2}x^2 + \frac{19}{2}x - 21} \\
\varphi'(x) &= \frac{5x + \frac{19}{2}}{(\frac{5}{2}x^2 + \frac{19}{2}x - 21)^{\frac{2}{3}}} \\
\varphi'(3) &= 2.53757 > 1
\end{aligned}
$$

k	x	f(x)	求解区间	Δx
0	0	-1		
1	1	1	$[-1,1]$	
2	$\frac{1}{2}$	$-\frac{5}{8}$	$[\frac{1}{2},1]$	$\frac{1}{2}$
3	$\frac{3}{4}$	$-\frac{1}{64}$	$[\frac{3}{4},1]$	$\frac{1}{4}$
4	$\frac{7}{8}$	$\frac{223}{512}$	$[\frac{3}{4},\frac{7}{8}]$	$\frac{1}{8}$
...	...	...	...	...
11	$\frac{773}{1024}$	1.6567×10^{-5}	$[\frac{193}{256},\frac{773}{1024}]$	$\frac{1}{1024}$

3.

4.

$$
\begin{aligned}
&x_0 = 4 \\
&x_1 = \frac{27}{8} && \Delta = \frac{5}{8} \\
&x_2 = \frac{1433}{432} && \Delta = \frac{25}{432} \\
&x_3 = 3.3166248 && \Delta = 5.048296^{-4} \\
&x_4 = 3.3166248 && \Delta = 1.9 \times 10^{-7}
\end{aligned}
$$

5. 设$f(x) = x^n - x$，则即为求f(x)零点
迭代公式$x_{k+1} = x_k - \frac{x_k^n - \alpha}{n x_k^{n-1}}$

$$
\begin{aligned}
&x_0 = 2 \\
&x_1 = \frac{137}{80} \\
&x_2 = 1.57929 \\
&x_3 = 1.552783 \\
&x_4 = 1.55184467 \\
&x_5 = 1.55184557
\end{aligned}
$$

6.

$$\varphi'(x) = \frac{(x^3 - 3x - 2)(6x)}{(3x^2 - 3)^2}$$

$$\varphi'(1.5) = |-2| > 1$$

$$\varphi(x) = x - \frac{x^3 - 3x - 2}{3x^2 - 3}$$

$$x_0 = 1.5$$
$$x_1 = \frac{7}{3}$$
$$x_2 = \frac{37}{18}$$
$$x_3 = \frac{1027}{513}$$
$$x_4 = \frac{791047}{395523}$$
$$x_5 = 2$$

7.

$$x_0 = 1$$
$$x_1 = 3$$
$$x_2 = \frac{7}{5}$$
$$x_3 = \frac{32}{19}$$
$$x_4 = \frac{4649}{2083}$$
$$x_5 = 1.94949$$
$$x_6 = 1.992856$$
$$x_7 = 2.000247664$$
$$x_8 = 1.999998816$$

8. 这题我算了半天没算的出来...