6.(1). 3阶代数精度的Gauss-Legendre积分公式共需2个积分节点..

令 $L_2(x) = \frac{1}{2^2\, 2!}\frac{d^2(x^2-1)^2}{dx^2} = 0$ 得 $x_0 = -\frac{\sqrt{3}}{3}$ $x_1 = \frac{\sqrt{3}}{3}$

积分系数: $\alpha_0 = \int_{-1}^{1} \frac{x-x_1}{x_0-x_1}dx = \int_{-1}^{1} \frac{x-\frac{\sqrt{3}}{3}}{-\frac{2\sqrt{3}}{3}}dx = 1$

$\alpha_1 = \int_{-1}^{1} \frac{x-x_0}{x_1-x_0}dx = \int_{-1}^{1} \frac{x+\frac{\sqrt{3}}{3}}{\frac{2\sqrt{3}}{3}}dx = 1$

故在[-1,1]上的Gauss积分公式为 $G_2(f) = f(-\frac{\sqrt{3}}{3}) + f(\frac{\sqrt{3}}{3}) = 2$

?(2) $\alpha_0 = \int_{-3}^{1} \frac{x-x_1}{x_0-x_1}dx = \int_{-3}^{1} \frac{x-\frac{\sqrt{3}}{3}}{-\frac{2\sqrt{3}}{3}}dx = 2+2\sqrt{3}$

$\alpha_1 = \int_{-3}^{1} \frac{x-x_0}{x_1-x_0}dx = \int_{-3}^{1} \frac{x+\frac{\sqrt{3}}{3}}{\frac{2\sqrt{3}}{3}}dx = 2-2\sqrt{3}$

则在[-3,1]上的Gauss积分公式为 $G_2(f) = 2[(2+2\sqrt{3})f(-1+2\cdot\frac{-\sqrt{3}}{3}) + (2-2\sqrt{3})f(-1+\frac{2\sqrt{3}}{3})]$

$= -531.556$

11. 向前差商: $f'(0.02) = \frac{f(0.04)-f(0.02)}{0.02} = \frac{7-9}{0.02} = 100.$

向后差商: $f'(0.06) = \frac{f(0.06)-f(0.04)}{0.02} = \frac{10-7}{0.02} = 150$

12. $f'(0.10) = \frac{f(0.10)-f(0.00)}{0.10} = \frac{1.50-1.70}{0.10} = -2.$

$f'(0.20) = \frac{f(0.20)-f(0.10)}{0.10} = \frac{1.60-1.50}{0.1} = 1$

$f''(0.20) = \frac{f'(0.20)-f'(0.10)}{0.1} = 30$

$f'(0.30) = \frac{f(0.30)-f(0.20)}{0.10} = \frac{2.00-1.60}{0.1} = 4$

$f'(0.40) = \frac{f(0.40)-f(0.30)}{0.10} = \frac{1.90-2.00}{0.1} = 1$

$f''(0.40) = \frac{f'(0.40)-f'(0.30)}{0.1} = -30$

14. 三点公式: 取 $x_0 = 0.51$ $x_1 = 0.53$ $x_2 = 0.55$.

则 $f'(x) = L_2'(x) = \frac{f(x_0)}{2h^2}(2x-x_1-x_2) - \frac{f(x_1)}{2h^2}(2x-x_0-x_2) + \frac{f(x_2)}{2h^2}(2x-x_0-x_1).$

则 f