

第二章:最小二乘拟合.(保留三位小数)

$$1. \sum_{i=1}^5 x_i = \dots, \sum_{i=1}^5 y_i =$$

$$1. \sum_{i=1}^5 (x_i - 1900) \times 0.45 = 3195 \quad \sum_{i=1}^5 [(x_i - 1900) \times 0.45]^2 = 2978775$$

设拟合直线为 $y = a + bx$. 则法方程为:

$$\begin{pmatrix} 5 & 3195 \\ 3195 & 2978775 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 28.3 \\ 13387.5 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 8.862 \\ -0.005 \end{pmatrix}$$

$$则拟合曲线为 $y = 8.862 - 0.005(x - 255) = 13.137 - 0.005x$.$$

2. 取如下五点, 设拟合直线为 $y = a + bx$

x_i	0	$\frac{2}{5}$	$-\frac{2}{5}$	$\frac{2}{5}$	$-\frac{2}{5}$	$\sum_{i=1}^5 x_i = 0$	$\sum_{i=1}^5 x_i^2 = 1.782$
y_i	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\sum_{i=1}^5 y_i = 0$	$\sum_{i=1}^5 y_i x_i = 1.634$

则法方程为:

$$\begin{pmatrix} 5 & 0 \\ 0 & 1.782 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 1.634 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0.9169 \end{pmatrix}$$

$$即 $y = 0.917x$.$$

3. 1° 一次:

$$\sum_{i=1}^5 x_i = -0.5 \quad \sum_{i=1}^5 x_i^2 = 1.375 \quad \sum_{i=1}^5 y_i = 9.32 \quad \sum_{i=1}^5 x_i y_i = 2.255$$

$$则法方程为: \begin{pmatrix} 5 & -0.5 \\ -0.5 & 1.375 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 9.32 \\ 2.255 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2.072 \\ 2.075 \end{pmatrix}$$

$$则 $\hat{y} = 2.072 + 2.075x$ 则最小平方误差为 $\sum_{i=1}^5 (\hat{y}_i - y_i)^2 = 0.148$.$$

2° 二次 法方程为:

$$\begin{pmatrix} 5 & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \sum y_i \\ \sum y_i x_i \\ \sum y_i x_i^2 \end{pmatrix} = \begin{pmatrix} 9.32 \\ 2.255 \\ 2.713875 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1.907 \\ 2.204 \\ 0.472 \end{pmatrix}$$

$$则 $\hat{y} = 1.907 + 2.204x + 0.472x^2$ 则最小平方误差为 $\sum_{i=1}^5 (\hat{y}_i - y_i)^2 = 0.027$$$

4. 法方程为

$$\begin{pmatrix} 5 & \sum_{i=1}^5 x_i^2 \\ \sum_{i=1}^5 x_i^2 & \sum_{i=1}^5 x_i^4 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 5 & 34 \\ 34 & 370 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^5 y_i \\ \sum_{i=1}^5 y_i x_i \end{pmatrix} = \begin{pmatrix} 58.3 \\ 563 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3.5 \\ 1.2 \end{pmatrix}$$

$$\text{则 } \hat{y} = 3.5 + 1.2x^2$$

5. 法方程为

$$\begin{pmatrix} \sum_{i=1}^4 \cos^2 x_i & \sum_{i=1}^4 \cos x_i \sin x_i \\ \sum_{i=1}^4 \cos x_i \sin x_i & \sum_{i=1}^4 \sin^2 x_i \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 & 0.0218 \\ 0.0218 & 1.348 \times 10^{-4} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^4 y_i \cos x_i \\ \sum_{i=1}^4 y_i \sin x_i \end{pmatrix} = \begin{pmatrix} 3.9 \\ 0.0181 \end{pmatrix}$$

$$\text{得 } a = 2.05 \quad b = -197.3 \quad \text{则 } \hat{y} = 2.05 \cos x - 197.3 \sin x$$

6. 令 $z = \ln y$ 则得下表 得 $z = \ln a + bx$

$$\text{则 } x_i \quad 0.70 \quad 0.50 \quad 0.25 \quad 0.75$$

$$y_i \quad 0.99 \quad 1.21 \quad 2.57 \quad 4.23$$

$$z_i \quad -0.01 \quad 0.191 \quad 0.944 \quad 1.442 \quad 2.567$$

$$\text{得法方程 } \begin{pmatrix} 4 & \sum_{i=1}^4 x_i \\ \sum_{i=1}^4 x_i & \sum_{i=1}^4 x_i^2 \end{pmatrix} \begin{pmatrix} \ln a \\ b \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^4 y_i \\ \sum_{i=1}^4 y_i x_i \end{pmatrix} = \begin{pmatrix} 14.825 \\ 14.06 \end{pmatrix} \Rightarrow \begin{pmatrix} \ln a \\ b \end{pmatrix} = \begin{pmatrix} 14.825 \\ -22.86 \end{pmatrix} \begin{pmatrix} 0.6625 \\ -0.037 \end{pmatrix}$$

$$\Rightarrow a = e^{\frac{14.825}{0.6625}} = e^{22.36} \Rightarrow a = 1.940 \quad \text{得 } y = 1.940 e^{-0.037x}$$

7. 令 $z = \frac{1}{y} = b + \frac{a}{x}$ 则可得下表

$$\begin{array}{c|cccc} x_i & 2.1 & 2.5 & 2.8 & 3.2 \\ y_i & 0.627 & 0.6249 & 0.7368 & 0.8111 \\ z_i & 1.64828 & 1.4601 & 1.3572 & 1.2329 \end{array} \quad \begin{pmatrix} 4 & \sum_{i=1}^4 \frac{1}{x_i} \\ \sum_{i=1}^4 \frac{1}{x_i} & \sum_{i=1}^4 \frac{1}{x_i^2} \end{pmatrix} \begin{pmatrix} b \\ a \end{pmatrix} = \begin{pmatrix} 4 & 1.5458 \\ 1.5458 & 0.6120 \end{pmatrix} \begin{pmatrix} b \\ a \end{pmatrix} = \begin{pmatrix} 5.693 \\ 2.2363 \end{pmatrix}$$

$$\text{得 } b = 0.4657 \quad a = 2.4779 \quad \text{则 } \hat{y} = \frac{x}{2.4779 + 0.4657x}$$

$$8. 1) A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \quad A^T = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \quad A^T A x = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ 6 \\ 4 \end{pmatrix} = \begin{pmatrix} 21 \\ 20 \end{pmatrix}$$

$$\begin{pmatrix} 6 & 5 \\ 5 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 21 \\ 20 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2.3636 \\ 1.3636 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} 1 & -2 \\ 1 & 5 \\ 2 & 1 \\ 1 & 1 \end{pmatrix} \quad A^T = \begin{pmatrix} 1 & 1 & 2 & 1 \\ -2 & 5 & 1 & 1 \end{pmatrix}$$

$$A^T A X = \begin{pmatrix} 1 & 1 & 2 & 1 \\ -2 & 5 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 1 & 5 \\ 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 6 & 21 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 2 & 1 \\ -2 & 5 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 13.1 \\ 7.9 \\ 5.1 \end{pmatrix} = \begin{pmatrix} 35 \\ 76.5 \end{pmatrix}$$

$$\text{得} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2.514 \\ 2.901 \end{pmatrix} \begin{pmatrix} 3.475 \\ 1.7879 \end{pmatrix} \begin{pmatrix} 3.459 \\ 1.798 \end{pmatrix}$$