

1 最小二乘拟合

1. $p(x) = a_0 + a_1x$

$$\begin{bmatrix} 5 & 16600 \\ 16600 & 59740000 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 28.3 \\ 83520 \end{bmatrix}$$

$$p(x) = 13.1465 - 2.255 \times 10^{-3}$$

2. $p(x) = a_0 + a_1x$

$$\begin{aligned} & \int_{-1}^1 (a_0 + a_1x - \sin x)^2 dx \\ &= \int_{-1}^1 (a_0^2 + a_1^2x^2 + 2a_0a_1x + \sin^2 x - 2a_0 \sin x - 2a_1x \sin x) dx \\ &= 2a_0^2 + \frac{2a_1^2}{3} + 1 - \frac{\sin 2}{2} + 4a_1 \cos 1 - 4a_1 \sin 1 \end{aligned}$$

$$\frac{\delta Q}{\delta a_0} = 4a_0 = 0$$

$$\frac{\delta Q}{\delta a_1} = \frac{4}{3}a_1 + 4 \cos 1 - 4 \sin 1 = 0$$

$$a_0 = 0 \quad a_1 = 2(\sin 1 - \cos 1)$$

3.

$$\begin{bmatrix} 5 & -0.5 \\ -0.5 & \frac{15}{8} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 9.32 \\ 2.855 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 2.0715 \\ 2.0751 \end{bmatrix}$$

$$\varphi_1(x) = 2.0715 + 2.0751x$$

$$\text{最小平方误差} = 0.1476356188$$

$$\begin{bmatrix} 5 & -0.5 & \frac{15}{8} \\ -0.5 & \frac{15}{8} & \frac{25}{8} \\ \frac{15}{8} & \frac{25}{8} & \frac{177}{128} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 9.32 \\ 2.855 \\ 2.71375 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -9.6945 \times 10^{-3} \\ -2.163 \\ 4.4197 \end{bmatrix}$$

$$\varphi_2(x) = 4.4197x^2 - 2.163x - 9.6945 \times 10^{-3}$$

$$\text{最小二乘误差} = 62.6693$$

4.

$$\begin{bmatrix} 5 & 0 & 34 \\ 0 & 34 & 36 \\ 34 & 36 & 370 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 58.3 \\ 43.2 \\ 563 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 3.5 \\ 0 \\ 1.2 \end{bmatrix}$$

$$\varphi(x) = 1.2x^2 + 3.5$$

$$5. \quad Q(x) = 3.5821a^2 + 0.4179b^2 + 2.2750ab - 7.5016a - 2.0438b + 4.4324$$

$$\begin{cases} 2 \times 3.5821a + 2.2750b - 7.5016 &= 0 \\ 2 \times 0.4179b + 2.2750a - 2.0438 &= 0 \end{cases} \quad (1)$$

$$a = 1.9948 \quad b = -2.9846$$

$$\varphi(x) = 1.9948 \cos x - 2.9846 \sin x$$

$$6. \quad y = ae^{bx} \Leftrightarrow \ln y = \ln a + bx, \text{ set } z = \ln y$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.7 & 0.5 & 0.25 & 0.75 \end{bmatrix} \begin{bmatrix} 1 & 0.7 \\ 1 & 0.5 \\ 1 & 0.25 \\ 1 & 0.75 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.7 & 0.5 & 0.25 & 0.75 \end{bmatrix} \begin{bmatrix} \ln(0.99) \\ \ln(1.21) \\ \ln(2.57) \\ \ln(4.23) \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2.2 \\ 2.2 & 1.365 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 2.56667 \\ 1.4059 \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0.66213 \\ -0.03722 \end{bmatrix}$$

$$\ln a = 0.66214 \quad a = 1.9389$$

$$y = 1.9389e^{-0.03722x}$$

7.

$$\begin{aligned}
 Q(x) &= \sum_{i=1}^m (x_i - ay_i - bx_iy_i)^2 \\
 &= \sum_{i=1}^m (ay_i + bx_iy_i - x_i)^2 \\
 A &= \begin{bmatrix} 0.6087 & 1.2783 \\ 0.6849 & 1.7122 \\ 0.7369 & 2.0630 \\ 0.8111 & 2.5955 \end{bmatrix} \quad Y = \begin{bmatrix} 2.1 \\ 2.5 \\ 2.8 \\ 3.2 \end{bmatrix} \\
 \Rightarrow \begin{bmatrix} 2.0403 & 5.5761 \\ 5.5761 & 15.5586 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} &= \begin{bmatrix} 7.6491 \\ 21.0472 \end{bmatrix}
 \end{aligned}$$

$$a = 2.52622 \quad b = 0.4474$$

$$f(x) = \frac{x}{2.5262 + 0.4474x}$$

8.

(1).

$$\begin{aligned}
 \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 4 \end{bmatrix} \\
 \begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 21 \\ 20 \end{bmatrix} \\
 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} \frac{26}{11} \\ \frac{15}{11} \end{bmatrix}
 \end{aligned}$$

(2).

$$\begin{aligned}
 \begin{bmatrix} 1 & 1 & 2 & 1 \\ -2 & 5 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & 5 \\ 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 1 & 1 & 2 & 1 \\ -2 & 5 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 13.1 \\ 7.9 \\ 5.1 \end{bmatrix} \\
 \begin{bmatrix} 7 & 6 \\ 6 & 31 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 35 \\ 76.5 \end{bmatrix} \\
 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 3.4586 \\ 1.7983 \end{bmatrix}
 \end{aligned}$$