1 插值

1.

$$l_0(x) = \frac{(x-2)(x-3)}{(-1-2)(-1-3)} = \frac{1}{12}(x^2-5x+6)$$

$$l_1(x) = \frac{(x+1)(x-3)}{(2+1)(2-3)} = -\frac{1}{3}(x^2-2x-3)$$

$$l_2(x) = \frac{(x+1)(x-2)}{(3+1)(3-2)} = \frac{1}{4}(x^2-x-2)$$

$$L_2(x) = \frac{1}{4}(x^2-5x+6) - \frac{5}{3}(x^2-2x-3) + \frac{7}{4}(x^2-x-2) = \frac{1}{3}x^2 + \frac{1}{3}x + 3$$

$$L_2(0) = 3$$

2.

$$l_0(x) = \frac{(x-2)(x-5)}{(-2-2)(-2-5)} = \frac{1}{28}(x^2-7x+10)$$
$$l_1(x) = \frac{(x+2)(x-5)}{(2+2)(x-5)} = -\frac{1}{12}(x^2-3x-10)$$
$$l_2(x) = \frac{(x+2)(x-2)}{5+2)(5-2)} = \frac{1}{21}(x^2-4)$$

$$L_0(x) = \frac{1}{28}x^2 + \frac{3}{4}x + \frac{19}{14}$$
$$L_2(-1.2) = \frac{89}{175} = 0.5086$$
$$L_2(1.2) = \frac{404}{175} = 2.3086$$

3.

(1).

$$
\begin{aligned}
l_0(x) &= \frac{x(x-\frac{1}{2})(x-1)}{-1\times(-1-\frac{1}{2})(-1-1)} = -\frac{1}{3}(x^3-\frac{3}{2}x^2+\frac{1}{2}x) \\
l_1(x) &= \frac{(x+1)(x-\frac{1}{2})(x-1)}{\frac{1}{2}} = 2(x^3-\frac{1}{2}x^2-x+\frac{1}{2}) \\
l_2(x) &= \frac{(x+1)x(x-1)}{\frac{3}{2}\times\frac{1}{2}\times-\frac{1}{2}} = -\frac{8}{3}(x^3-x) \\
l_3(x) &= \frac{(x+1)x(x-\frac{1}{2})}{1} = x^3+\frac{1}{2}x^2-\frac{1}{2}x \\
L_3(x) &= -x^3+\frac{5}{2}x^2-\frac{1}{2}
\end{aligned}
$$

(2).

$$
\begin{aligned}
l_0(x) &= -\frac{1}{12}(x^3-5x^2+6x) \\
l_1(x) &= \frac{1}{6}(x^3-4x^2+x+6) \\
l_2(x) &= -\frac{1}{6}(x^3-2x^2-3x) \\
l_3(x) &= \frac{1}{12}(x^3-x^2-2x) \\
L_3(x) &= -\frac{1}{3}x^3+\frac{11}{12}x^2-x
\end{aligned}
$$

4. 不会

5.

$$
\begin{aligned}
l_0(x) &= \frac{1}{760}(x^2-221x+12100) \\
l_1(x) &= -\frac{1}{399}(x^2-202x+9801) \\
l_2(x) &= \frac{1}{840}(x^2-181x+8100)
\end{aligned}
$$

$$
\begin{aligned}
L_2(x) &= -\frac{1}{7980}x^2+\frac{601}{7980}x+\frac{495}{133} \\
L_2(105) &= \frac{1363}{133} = 10.2481
\end{aligned}
$$

$\sqrt{105} \approx 10.2469$

误差界=

$$R_n(x) \le \frac{(\sqrt{\xi})'''}{3!}(x-81)(x-100)(x-121) \qquad \xi \in [81,121]$$

所以

$$R_n(105) \in [\frac{-120}{161051}, \frac{-40}{19687}]$$

6. 如下表：

i	x_i	$f(x_i)$	一阶	二阶	三阶
0	-1	3			
1	2	5	$\frac{2}{3}$		
2	3	7	2	$\frac{1}{3}$	
3	4	5	-2	-2	$-\frac{7}{15}$

$$N_3(x) = -\frac{7}{15}x^3 + \frac{11}{5}x^2 - \frac{2}{15}x + \frac{1}{5}$$
$$f(1.2) \approx N(1.2) = \frac{1501}{625} = 2.4016$$

7.

$$\begin{aligned}
N_3(x) &= 1 + (x-4)\times 2 + (x-1)(x-4) + (x-1)(x-4)(x-3)\times -1 \\
&= 1 + 2x - 8 + x^2 - 5x + 4 - x^3 + 8x^2 - 19x + 12 \\
&= -x^3 + 9x^2 - 22x + 9 \\
& f(2) \approx N_3(2) = -7 \\
& [1,2,3,4] = \frac{f[1,3,4] - f[2,3,4]}{1-2} = -1 \\
& \Rightarrow f[2,3,4] = 0
\end{aligned}$$

8.

$$\epsilon \le 10^{-5}$$
$$M_2 = \max_{a \le x \le b} |f''(x)| = 1$$
$$|f(x) - p(x)| = \frac{M_2}{8}(x_{i+1} - x_i)^2 \le 10^{-5}$$
$$\Delta x \le \sqrt{8 \times 10^{-5}} \approx 9 \times 10^{-3}$$

9.

$$f[2^0,2^1]=\frac{f(1)-f(2)}{1-2}=-2975+886=-2089$$
$$f[2^0,2^1,...,2^7]=1$$
$$f[2^0,2^1,...,2^8]=0$$

10.

$$p(x)=\begin{cases}1.6x+0.44, & 1.05\le x\le 1.10\\ -0.6x+2.86, & 1.1\le x\le 1.15\\ 3x-1.28, & 1.15\le x\le 1.2\end{cases} \tag{1}$$

$f(1.075)=2.16$
$f(1.175)=2.245$

11. $P_2(x)=f(0)h_0(x)+f(1)h_1(x)+f'(1)g_1(x)$

$$\begin{array}{lll} h_0(0)=1 & h_0(1)=0 & h_0'(1)=0\\ h_1(0)=0 & h_1(1)=1 & h_1'(1)=0\\ g_1(0)=0 & g_1(1)=0 & g_1'(1)=1\end{array}$$

$$h_0(x)=(x-1)^2$$
$$h_1(x)=-x(x-2)$$
$$g_1(x)=x(x-1)$$

$P_2(x)=f(0)(x-1)^2-f(1)x(x-2)+f'(1)x(x-1)$
$R_2(x)=\frac{f^{(3)}(\xi)}{3!}(x-0)(x-1)^2$

12. $P_2(x)=f(3)h_0(x)+f(5)h_1(x)+f'(5)g_2(x)$

$$\begin{array}{lll} h_0(3)=1 & h_0(5)=0 & h_0'(5)=0\\ h_1(3)=0 & h_1(5)=1 & h_1'(5)=0\\ g_2(3)=0 & g_2(5)=0 & g_2'(5)=1\end{array}$$

$$h_0(x) = \frac{1}{4}x^2 - \frac{5}{2}x + \frac{25}{4}$$
$$h_1(x) = -\frac{1}{4}x^2 + \frac{5}{2}x - \frac{21}{4}$$
$$g_1(x) = \frac{1}{2}x^2 - 4x + \frac{15}{2}$$

$P_2(x) = x^2 - 3x + 5$
$R_2(x) = \frac{f^{(3)}(\xi)}{3!}(x-3)(x-5)^2$
$f(3.7) \approx P_2(3.7) = 7.59$

13. $P_3(x) = f(0)h_0(x) + f(1)h_1(x) + f(3)h_2(x) + f'(3)g_3(x)$

$$\begin{array}{llll} h_0(0)=1 & h_0(1)=0 & h_0(3)=0 & h_0'(3)=0 \\ h_1(0)=0 & h_1(1)=1 & h_1(3)=0 & h_1'(3)=0 \\ h_2(0)=0 & h_2(1)=0 & h_2(3)=1 & h_2'(3)=0 \\ g_3(0)=0 & g_3(1)=0 & g_3(3)=0 & g_3'(3)=1 \end{array}$$

解得：

$$h_0(x) = -\frac{1}{9}x^3 + \frac{7}{9}x^2 - \frac{5}{3}x + 1$$
$$h_1(x) = \frac{1}{4}x^3 - \frac{3}{2}x^2 + \frac{9}{4}x$$
$$h_2(x) = -\frac{5}{36}x^3 + \frac{13}{18}x^2 - \frac{7}{12}x$$
$$g_3(x) = \frac{1}{6}x^3 - \frac{2}{3}x^2 + \frac{1}{2}x$$

$P_3(x) = f(0)h_0(x) + ...$
$R_4(x) = \frac{f^{(4)}(\xi)}{4!}x(x-1)(x-3)^2$

14. 代入13题的结果： $P_3(x) = \frac{27}{200}x^3 - \frac{27}{50}x^2 + \frac{31}{200}x + 1$

15. $P_4(x) = f(1)h_0(x) + f(2)h_1(x) + f'(1)g_2(x) + f'(2)g_3(x) + f''(2)l_4(x)$

$$\begin{array}{lllll} h_0(1)=1 & h_0(2)=0 & h_0'(1)=0 & h_0'(2)=0 & h_0''(2)=0 \\ h_1(1)=0 & h_1(2)=1 & h_1'(1)=0 & h_1'(2)=0 & h_1''(2)=0 \\ g_2(1)=0 & g_2(2)=0 & g_2'(1)=1 & g_2'(2)=0 & g_2''(2)=0 \\ g_3(1)=0 & g_3(2)=0 & g_3'(1)=0 & g_3'(2)=1 & g_3''(2)=0 \\ l_4(1)=0 & l_4(2)=0 & l_4'(1)=0 & l_4'(2)=0 & l_4''(2)=1 \end{array}$$

解得：

$$h_0(x) = -3x^4 + 20x^3 - 48x^2 + 48x - 16$$
$$h_1(x) = 3x^4 - 20x^3 + 48x^2 - 48x + 17$$
$$g_2(x) = -x^4 + 7x^3 - 18x^2 + 20x - 8$$
$$g_3(x) = -2x^4 + 13x^3 - 30x^2 + 29x - 10$$
$$l_4(x) = \frac{1}{2}x^4 - 3x^3 + \frac{13}{2}x^2 - 6x + 2$$

$P_4(x) = \frac{7}{2}x^4 - \frac{45}{2}x^3 + \frac{103}{2}x^2 - 49x + 17$
$R_4(x) = \frac{f^(5)(\xi)}{5!}(x-1)^2(x-2)^3$

16. $h_0 = 1 \quad h_1 = 2 \quad h_2 = 1$

$$\begin{cases} \lambda_1 = \frac{2}{3} \\ \mu_1 = \frac{1}{3} \end{cases} \tag{2}$$

$$\begin{cases} \lambda_2 = \frac{1}{3} \\ \mu_2 = \frac{2}{3} \end{cases} \tag{3}$$

$d_1 = -12 \quad d_2 = 12$

$$\begin{bmatrix} 2 & \frac{2}{3} \\ \frac{2}{3} & 2 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} = \begin{bmatrix} -12 \\ 12 \end{bmatrix}$$

$\Rightarrow M_1 = -9, \qquad M_2 = 9$

$$S(x) = \begin{cases} -\frac{3}{2}(x+2)^3 + 4(x+1) + 3(x+2) + \frac{3}{2}(x+2) \\ \frac{3}{2}x^3 - \frac{1}{2}x + 4 \\ \frac{3}{2}(2-x)^3 + \frac{17}{2}x - 5 \end{cases} \tag{4}$$

17. $h_0 = 1, \quad h_1 = 1, \quad h_2 = 2$

$$\begin{cases} \lambda_1 = \frac{1}{2} \\ \mu_1 = \frac{1}{2} \end{cases} \tag{5}$$

$$\begin{cases} \lambda_2 = \frac{2}{3} \\ \mu_2 = \frac{1}{3} \end{cases} \tag{6}$$

$d_1 = 3(\frac{4-3}{1} - \frac{3-2}{1}) = 0$
$d_2 = 2(\frac{29-4}{2} - \frac{4-3}{1}) = 29 - 4 - 1 = 24$

$d_0 = 6(1-5) = -24$
$d_3 = 3(29 - \frac{25}{2}) = \frac{99}{2}$

$$\begin{bmatrix} 2 & 1 & & \\ \frac{1}{2} & 2 & \frac{1}{2} & \\ & \frac{1}{3} & 2 & \frac{2}{3} \\ & & 1 & 2 \end{bmatrix} \begin{bmatrix} M_0 \\ M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} -24 \\ 0 \\ 24 \\ \frac{99}{2} \end{bmatrix}$$

$$\begin{bmatrix} M_0 \\ M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} -13.14 \\ 2.273 \\ 4.045 \\ 22.722 \end{bmatrix}$$

后面不算了