# LAPLACE APPROXIMATION

## BAYESIAN STATISTICS FOR ECOLOGISTS

IGB 18. TO 26. NOVEMBER 2019

# MCMC METHODS ARE SLOW

- Markov chains take time to learn the distribution
- With lots of parameters, memory requirements can be intense
- Sometimes, fast and approximate is good enough

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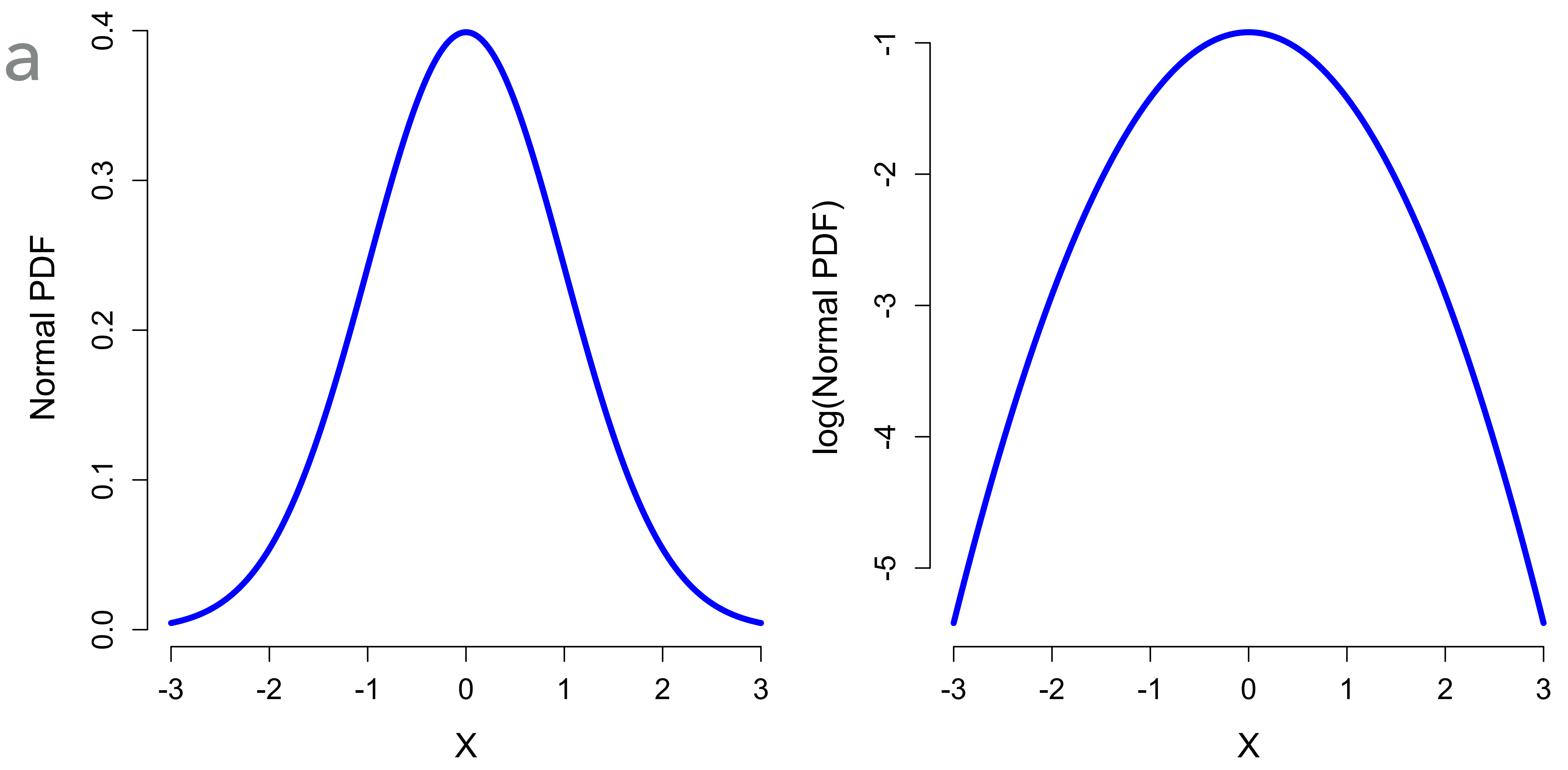
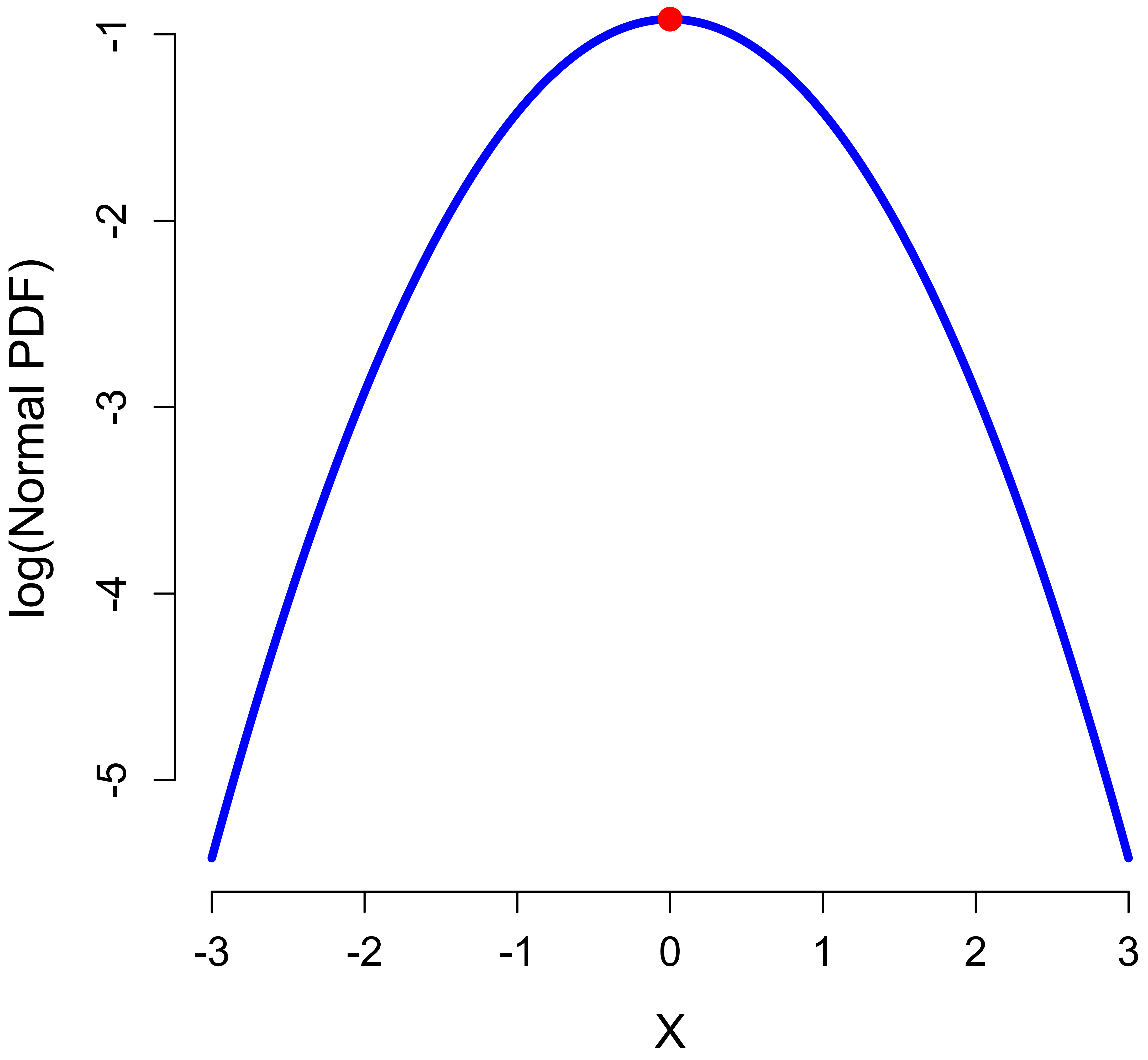
- The log of a gaussian is a parabola



# LAPLACE APPROXIMATION

- LA first finds the maximum of the log posterior (maximum a posteriori inference)



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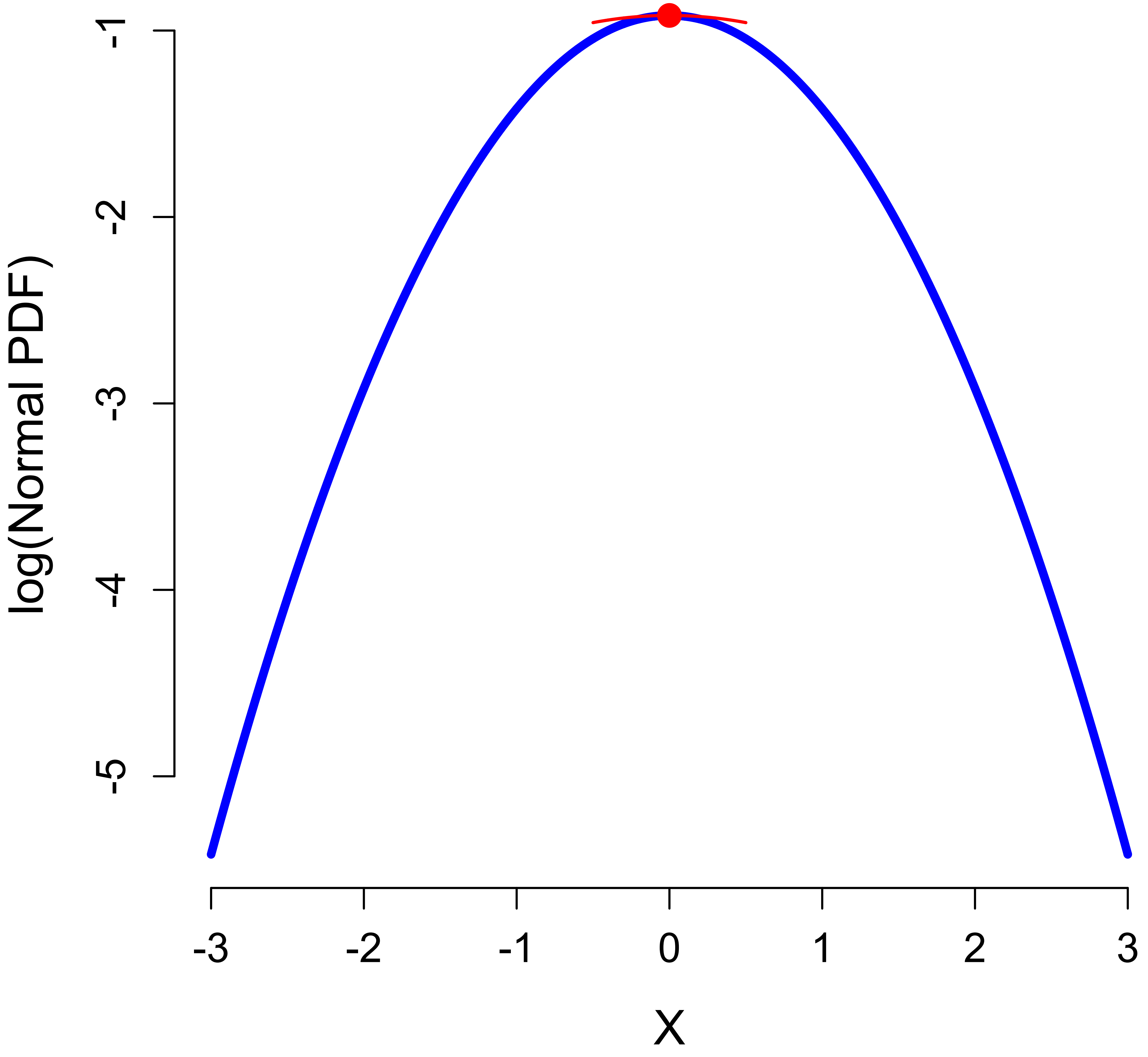

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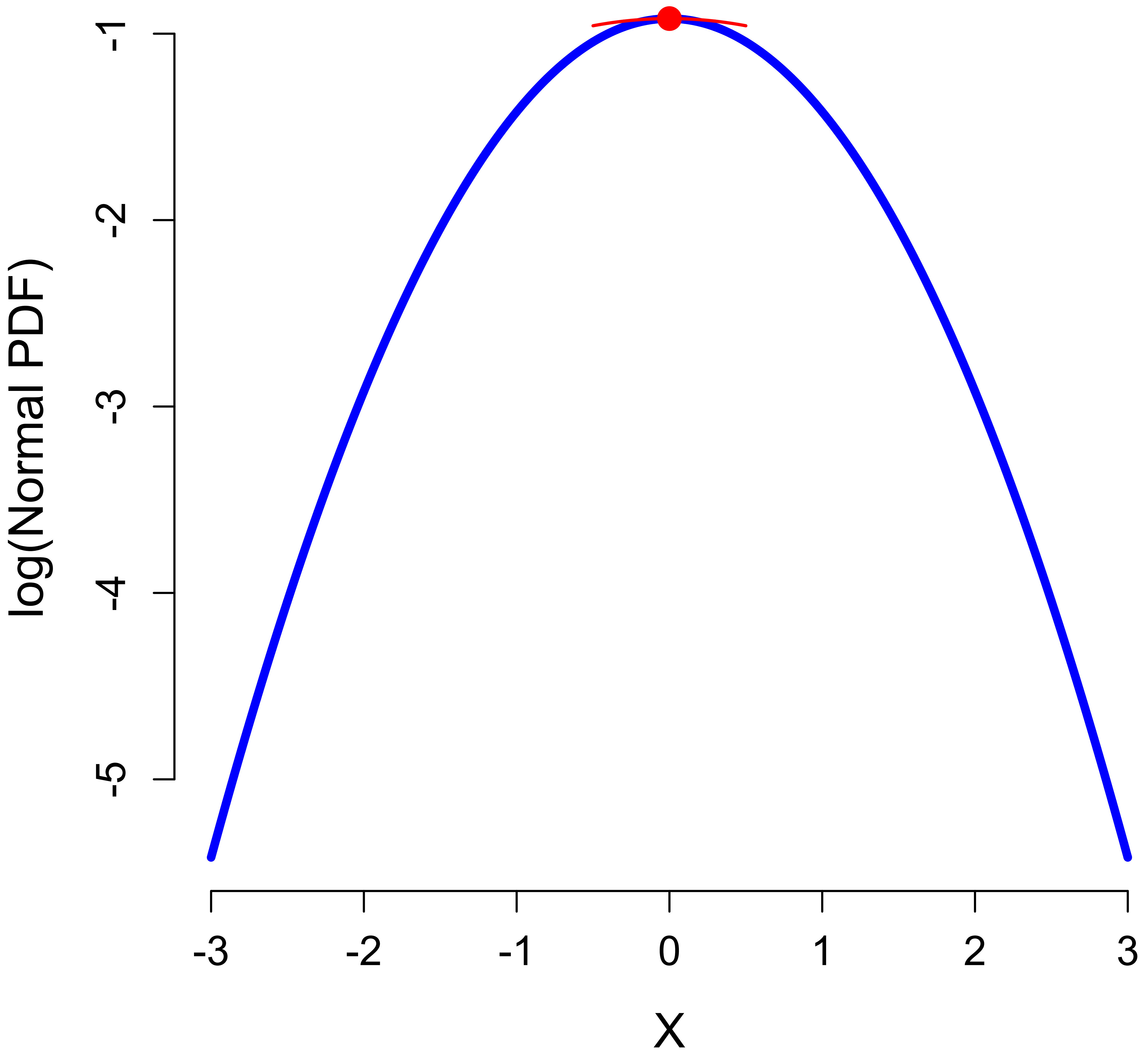

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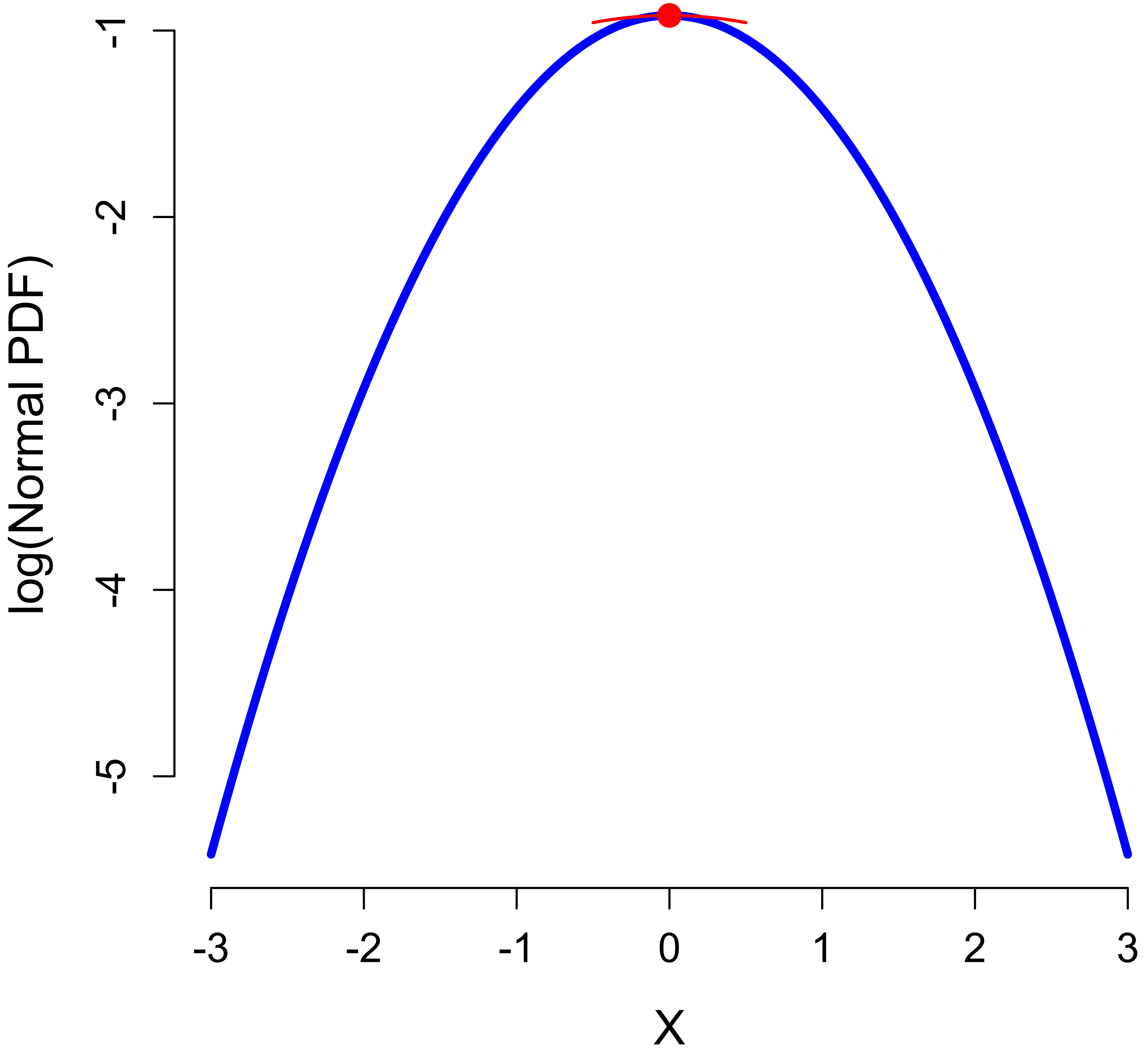
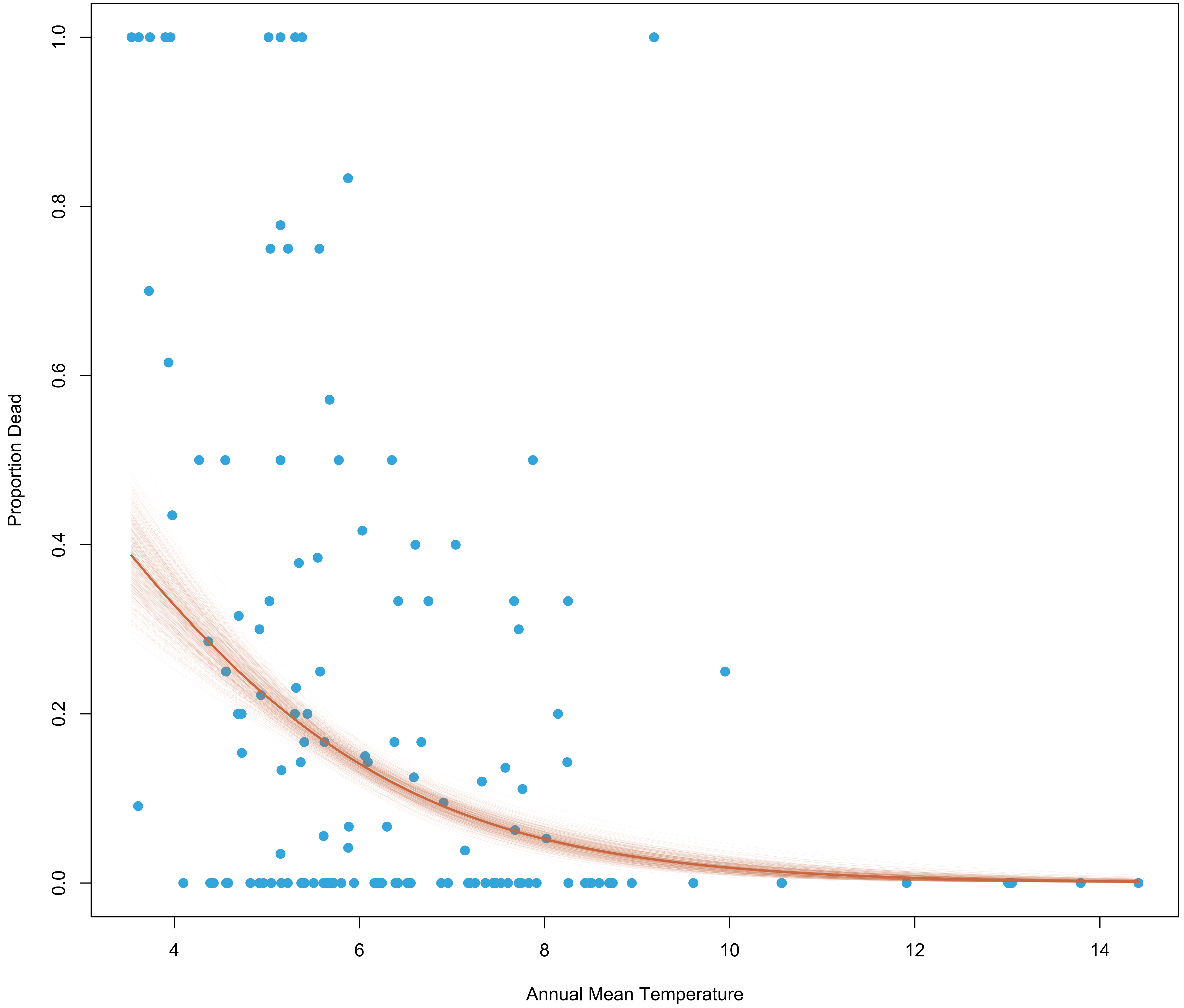
- We then estimate the **hessian matrix** to approximate the curvature at the maximum
- The hessian can be solved to estimate a **variance-covariance matrix** for all parameters
- Finally, we draw samples from a multivariate normal distribution to make posterior inference



# GLM COMPARISON

- Using Stan, we found a clear negative relationship between mortality and temperature.
- How do the results compare using Laplace approximation?



# BINOMIAL GLM

- This model has two parameters: a slope and an intercept

```
log_posterior <- function(params, data) {
    ntrees <- data$n  # number of trees in each plot
    died <- data$died # number of trees that died
    temperature <- data$annual_mean_temp

    intercept <- params[1]
    slope <- params[2]

    # using a logit link function
     mu <- plogis(intercept + slope * temperature)

    ll <- sum(dbinom(died, ntrees, mu, log=TRUE))
    lp <- dnorm(intercept, 0, 10, log = TRUE) +
          dnorm(slope, 0, 5, log=TRUE)

    return(ll + lp)
}
```

# BINOMIAL GLM

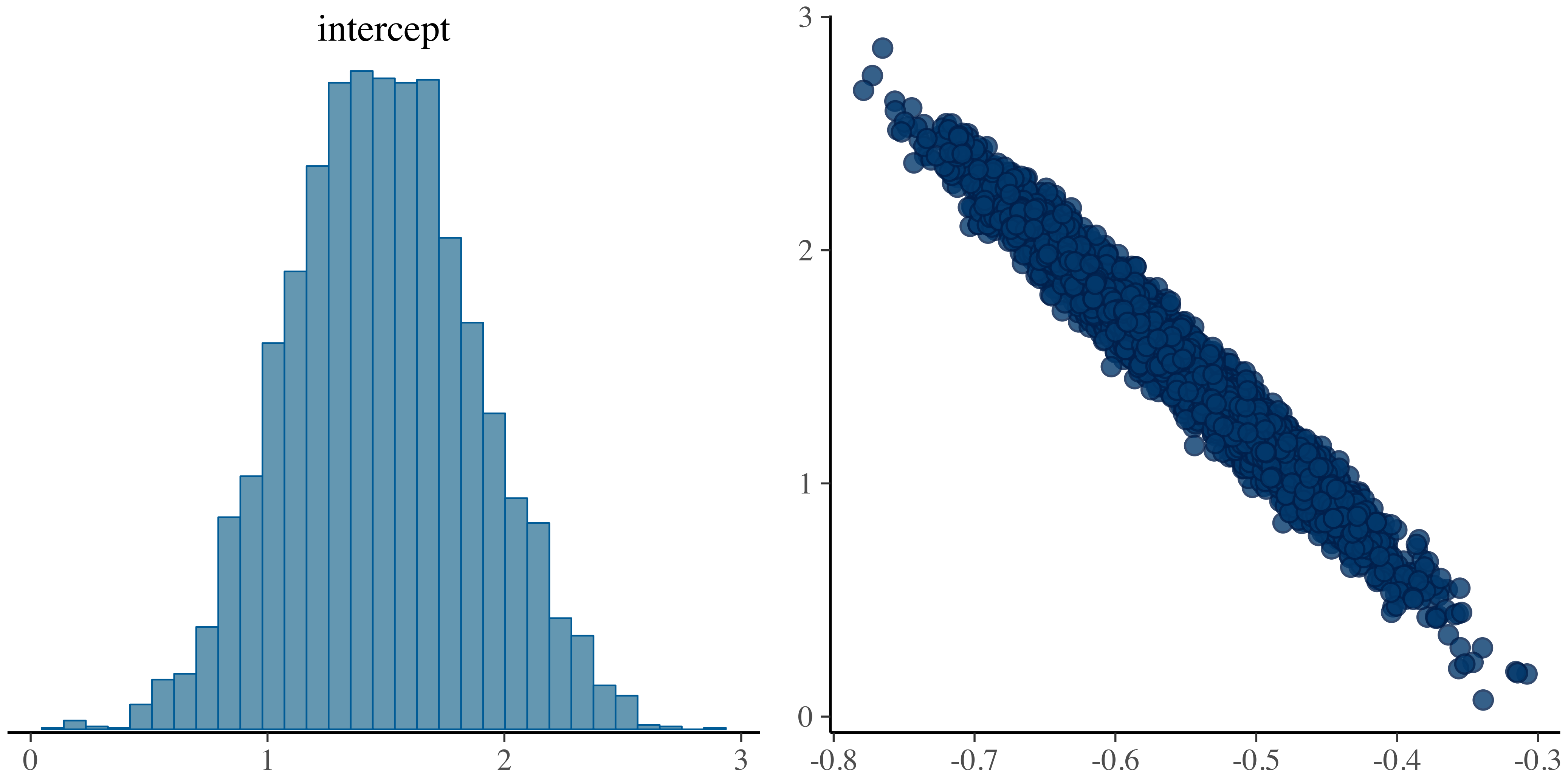
- Then it is simply a matter of using optim with hessian=TRUE
- MAP estimate & standard errors is nearly identical to the means from Stan
- Solve the hessian matrix to get the standard error
- Can use rmvnorm to get posterior samples
- Using Laplace approximation is 240-3300x faster

```
inits <- c(rnorm(1, 0, 10), rnorm(1, 0, 5))

fit <- optim(inits, log_posterior, control =
     list(fnscale = -1), hessian = TRUE, data = dat)
fit$par
vcv_mat <- solve(-fit$hessian)
## the standard error of each param
sqrt(diag(vcv_mat))
```

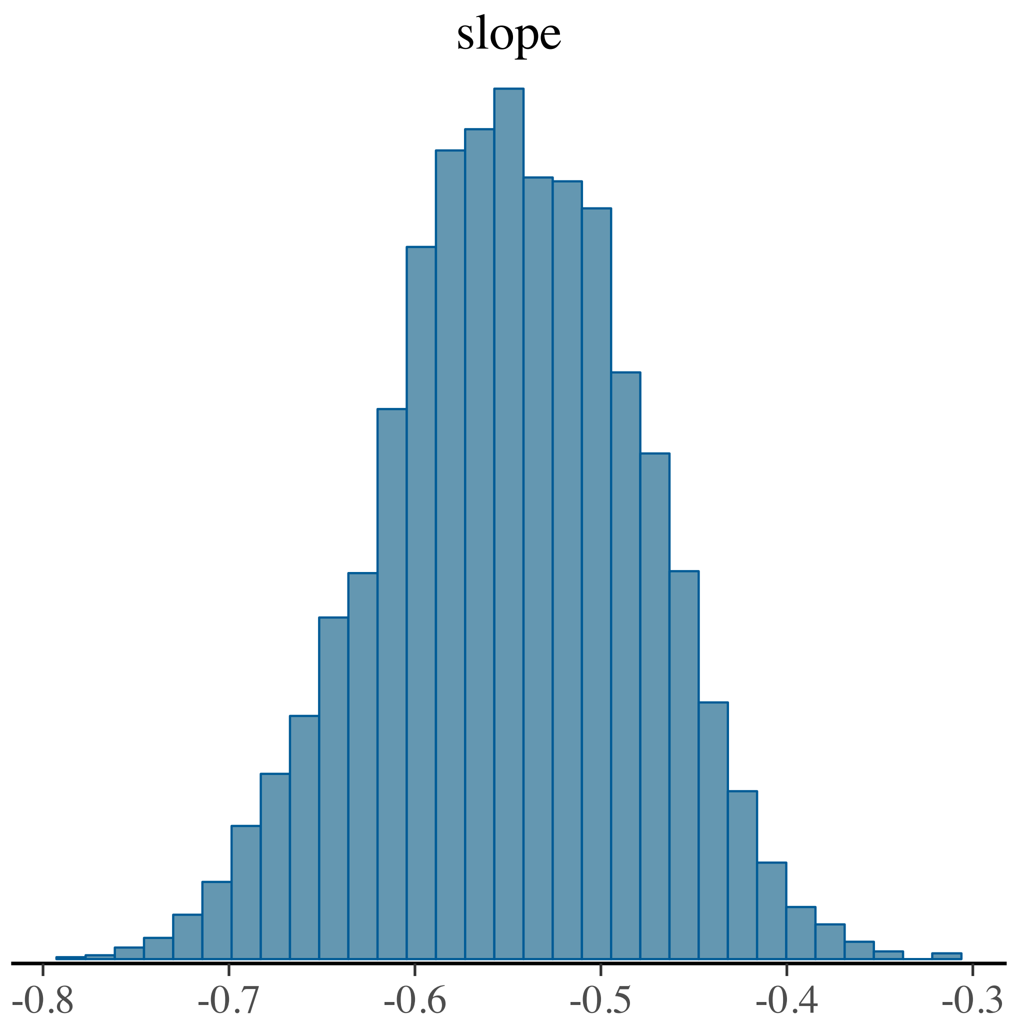
| | Stan | Laplace |
|---|---|---|
| intercept | 1.484<br>(0.39) | 1.478<br>(0.40) |
| slope | -0.549<br>(0.069) | -0.547<br>(0.071) |
| **Time to fit (s)** | 2.407<br>(33.192) | 0.010 |

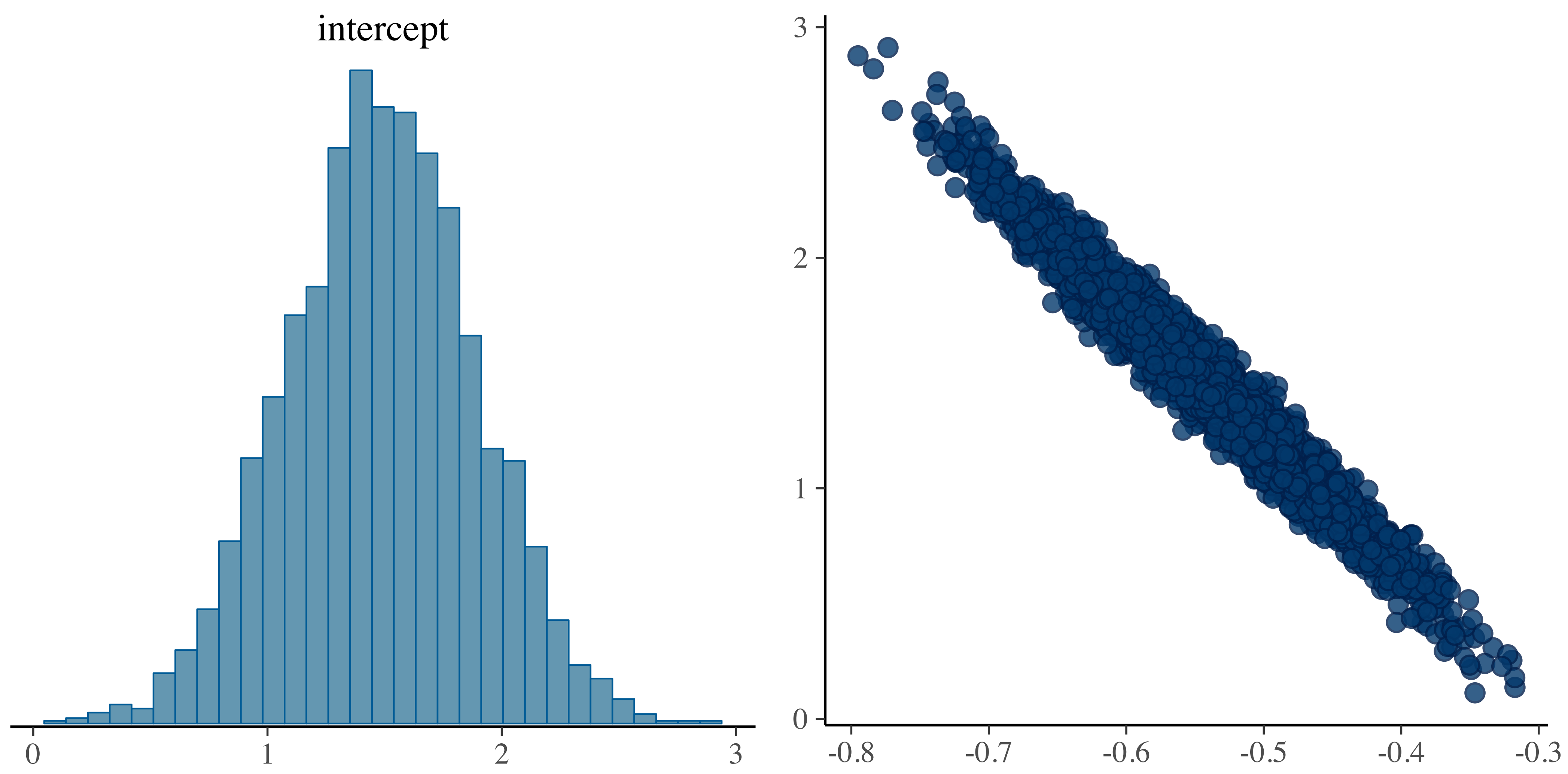
# GLM PAIRS PLOTS




Stan

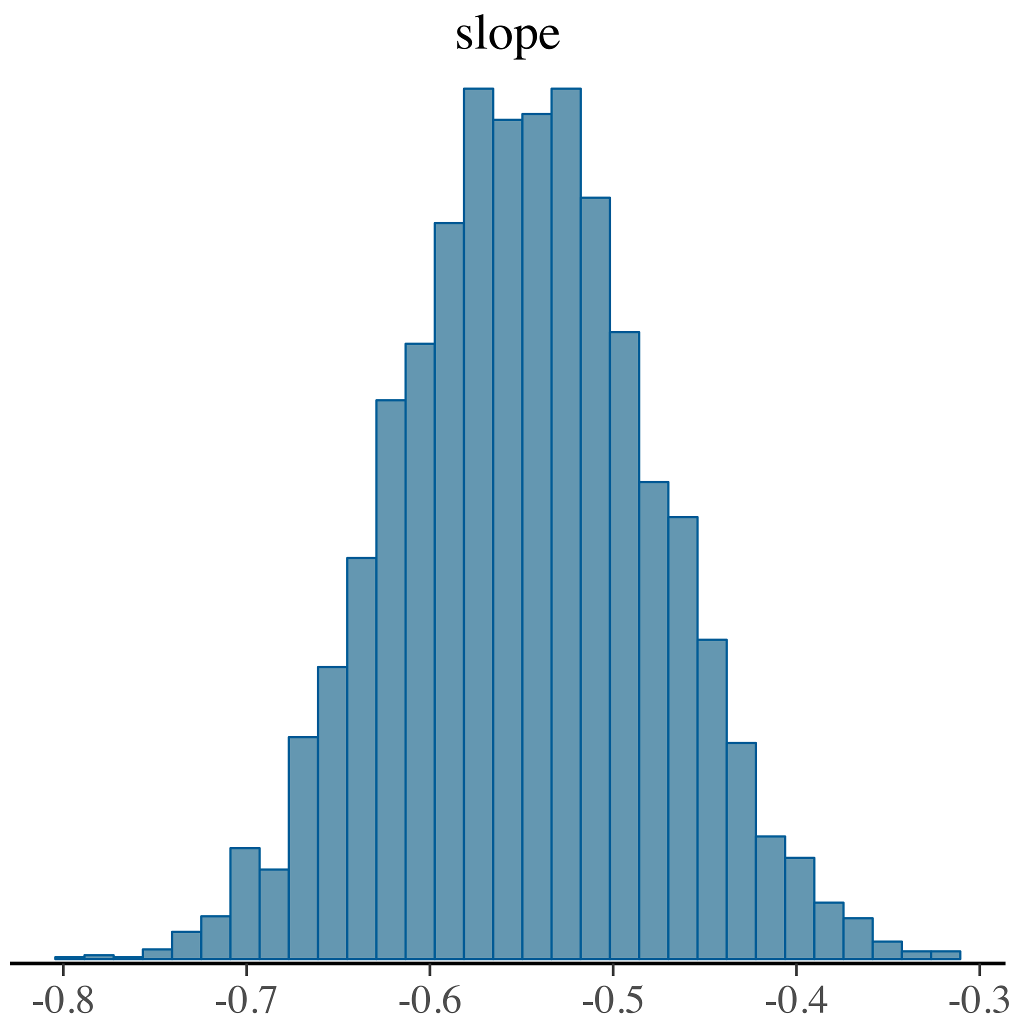
r = -0.978




Laplace

r = -0.979



# PREDICTIONS COMPARISON