Model Comparison

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Douter de tout ou tout croire sont deux solutions également commodes, qui nous dispensent de réfléchir.

–Henri Poincaré

Introduction to Model Comparison

Why compare models?

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Why compare models?

- All models are imperfect
- How good is our model *given the modelling goals?*

Before beginning, evaluate the goals of the comparison

- Predictive performance
- Hypothesis testing
- Reduction of overfitting

If you are asking yourself, “should I use A/B/DIC?”

Remember Betteridge’s law. . .

Comparing models

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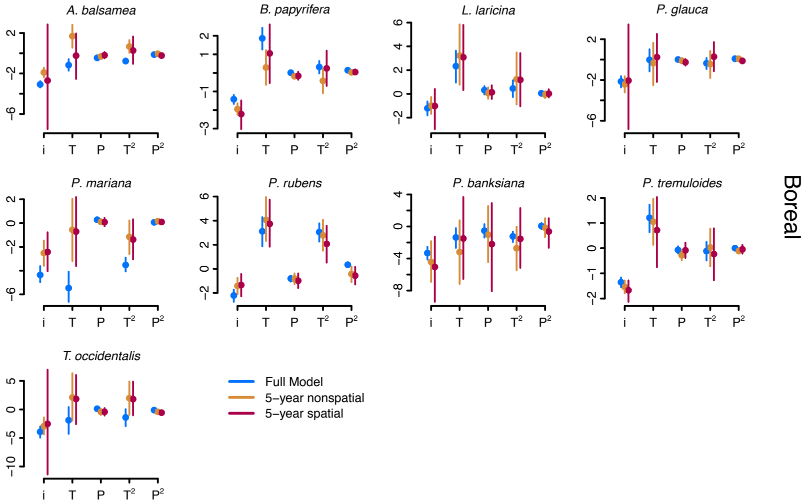
> *Any headline that ends in a question mark can be answered with the word "NO"*

Informal model comparison



Comparison through evaluation

If the goal is predictive performance, evaluate directly.

- Cross-validation
- k-fold cross validation

Cost: can be computationally intensive (especially for Bayesian). But you are already paying this cost (you ARE evaluating your models, right?)

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Requires selecting an evaluation score

- ROC/TSS (classification)
- RMSE (continuous)
- Goodness of fit
- ...

Bayesian predictive performance

Consider a regression model

$$
\begin{aligned}
\mathrm{pr}(\theta|y, x) &\propto \mathrm{pr}(y, x, |\theta)\mathrm{pr}(\theta) \\
y &\sim \mathcal{N}(\alpha + \beta x, \sigma)
\end{aligned}
$$

From a new value $\hat{x}$ we can compute a posterior prediction $\hat{y} = \alpha + \beta x$

Bayesian predictive performance

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Where is the prior?

Bayesian predictive performance

We want to summarize lppd taking into account:

- an entire set of prediction points $\hat{x} = \{x_1, x_2, \dots x_n\}$
- the entire posterior distribution of θ
    - (or, realistically, a set of S draws from the posterior distribution)

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To compare two competing models θ_1 and θ_2, simply compute lppd_{θ_1} and lppd_{θ_2}, the "better" model (for prediction) is the one with a larger lppd.

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Considering the lpd (using the calibration data), it can be proven, when θ_2 is *strictly nested* within θ_1, that $\mathrm{lpd}_{\theta_1} > \mathrm{lpd}_{\theta_2}$.

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Thus, we require a method for penalizing the larger (or more generally, more flexible) model to avoid simply overfitting, especially when validation data are unavailable.

AIC

$$\mathrm{AIC} = 2k - 2\log \mathrm{pr}(x|\hat{\theta})$$

- $\mathrm{pr}(x|\hat{\theta}) = \max(\mathrm{pr}(x|\theta))$ and k is the number of parameters.
- AIC increases as the model gets worse or the number of parameters gets larger
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What is the number of parameters in a hierarchical model?

DIC

$$D(\theta) = -2\log(\mathrm{pr}(x|\theta))$$

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We still penalize the model based on complexity, but we must estimate how many *effective* parameters there are:

$$p_D = \mathbb{E}[D(\theta)] - D(\mathbb{E}[\theta])$$

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$$p_D = \mathbb{E}[D(\theta)] - D(\mathbb{E}[\theta])$$

$$\mathrm{DIC} = D(\mathbb{E}[\theta]) + 2p_D$$

Pros:

- Easy to estimate
- Widely used and understood
- Effective for a variety of models regardless of nestedness or model size

Cons

- Not Bayesian
- Assume $\theta \sim \mathcal{MN}$
- Modest computational cost

Bayes factor

Consider two competing models θ_1 and θ_2

In classical likelihood statistics, we can compute the likelihood ratio:

$$\mathrm{LR} = \frac{\mathrm{MLE}(X|\theta_1)}{\mathrm{MLE}(X|\theta_2)}$$

Bayes factor

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In classical likelihood statistics, we can compute the likelihood ratio:

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A fully Bayesian approach is to take into account the entire posterior distribution of both models:

$$K = \frac{pr(\theta_1|X)}{pr(\theta_2|X)}$$

Bayes factor

For a single posterior estimate of each model:

$$
\begin{aligned}
K &= \frac{pr(\theta_1|X)}{pr(\theta_2|X)} \\
&= \frac{pr(X|\theta_1)pr(\theta_1)}{pr(X|\theta_2)pr(\theta_2)}
\end{aligned}
$$

To account for the entire distribution:

$$
\begin{aligned}
K &= \frac{\int pr(\theta_1|X)d\theta_1}{\int pr(\theta_2|X)d\theta_2} \\
&= \frac{\int pr(X|\theta_1)pr(\theta_1)d\theta_1}{\int pr(X|\theta_2)pr(\theta_2)d\theta_2}
\end{aligned}
$$

And others

- Bayesian model averaging
- Reversible jump MCMC