# BAYESIAN BEST PRACTICES

## BAYESIAN STATISTICS FOR ECOLOGISTS

IGB 18. TO 26. NOVEMBER 2019

# FOUR STEPS TO AN ANALYSIS

1. Specify the joint posterior distribution of outcomes (i.e., response variables) and all unknowns/parameters
2. Draw from posterior distribution using MCMC
3. Evaluate model and revise if necessary (return to step 1)
4. Use posterior predictive distribution for inference

http://mc-stan.org/rstanarm/articles/rstanarm.html

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  - Prefer to “scale” outcomes via a **link function**

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- Forget conjugacy unless you know what you are doing and why

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- Can be useful to draw out your model as a **digraph** to make sure you don't miss anything

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Y

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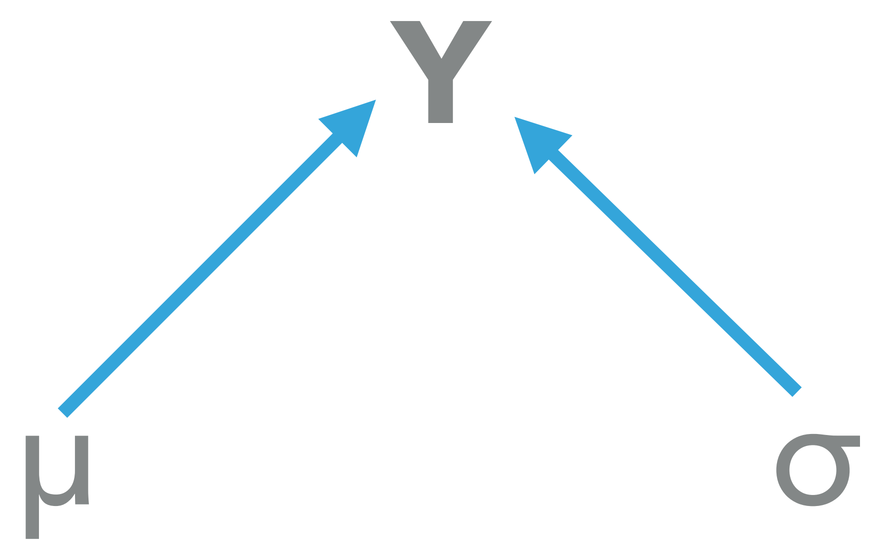

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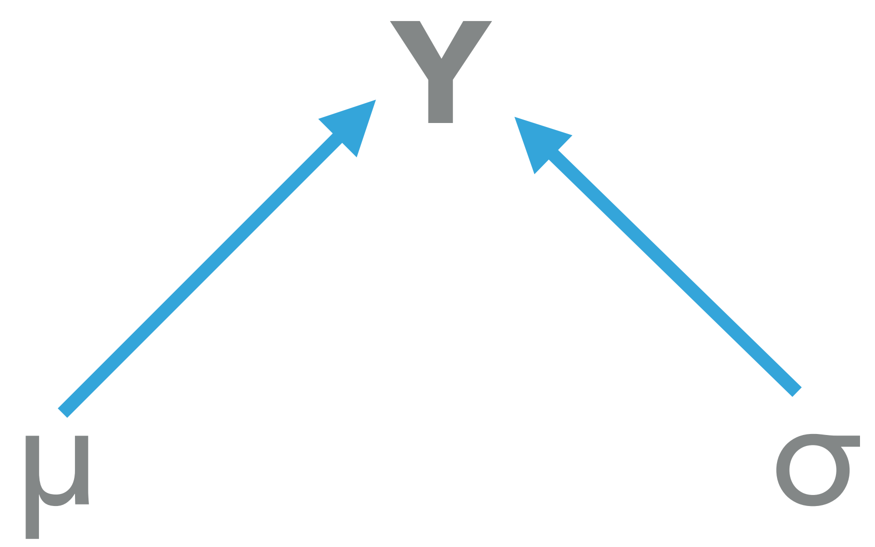

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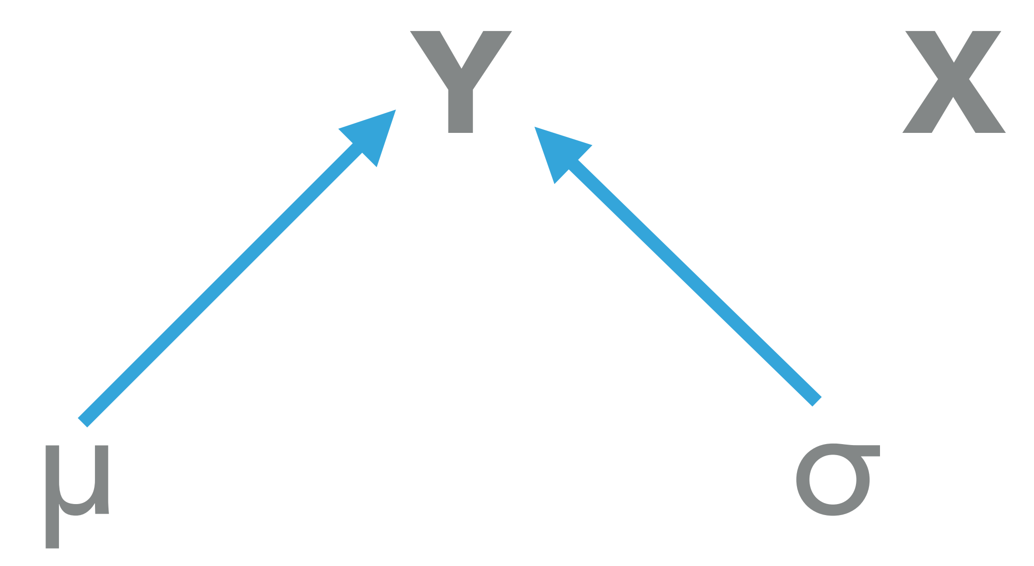

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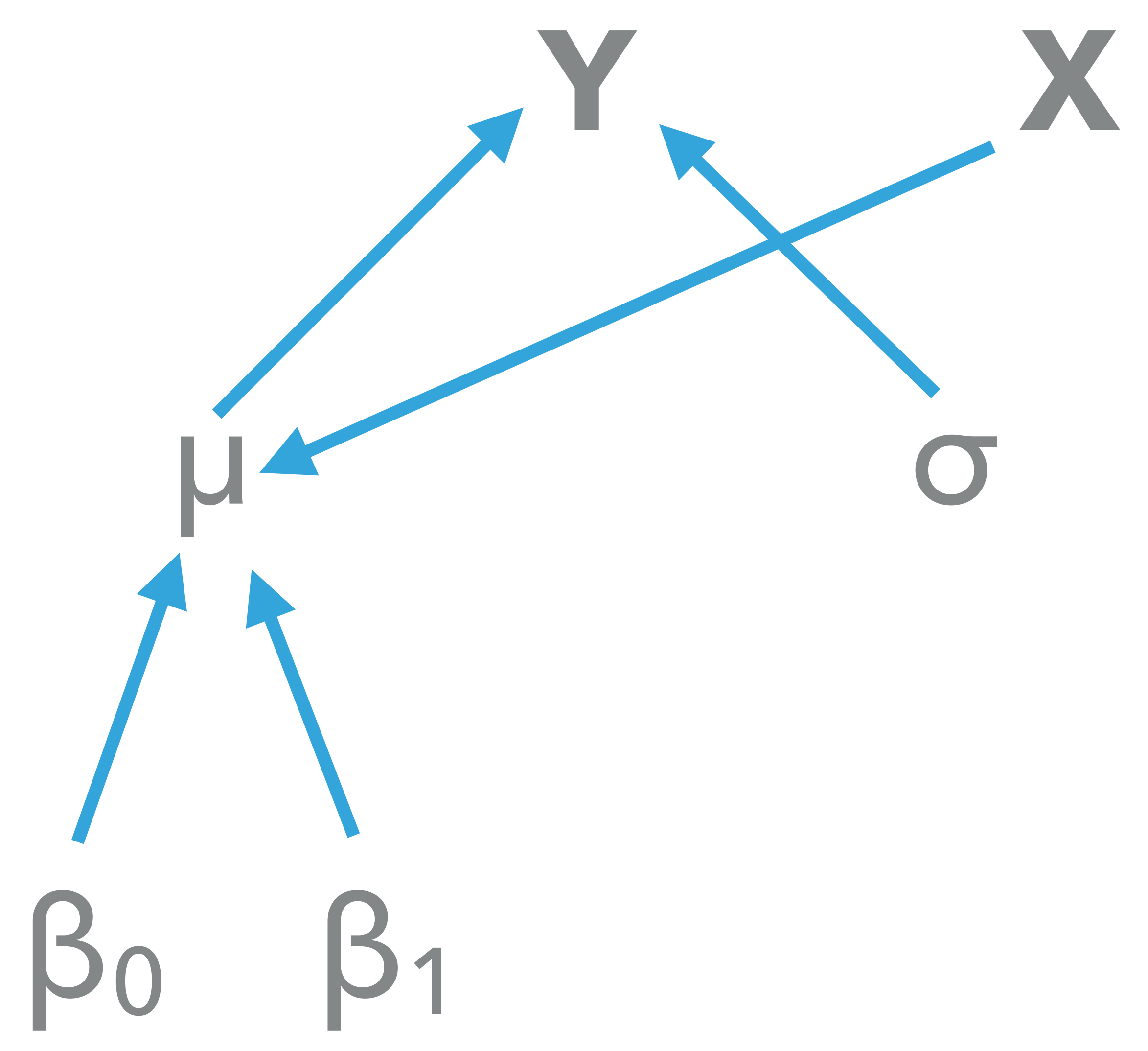

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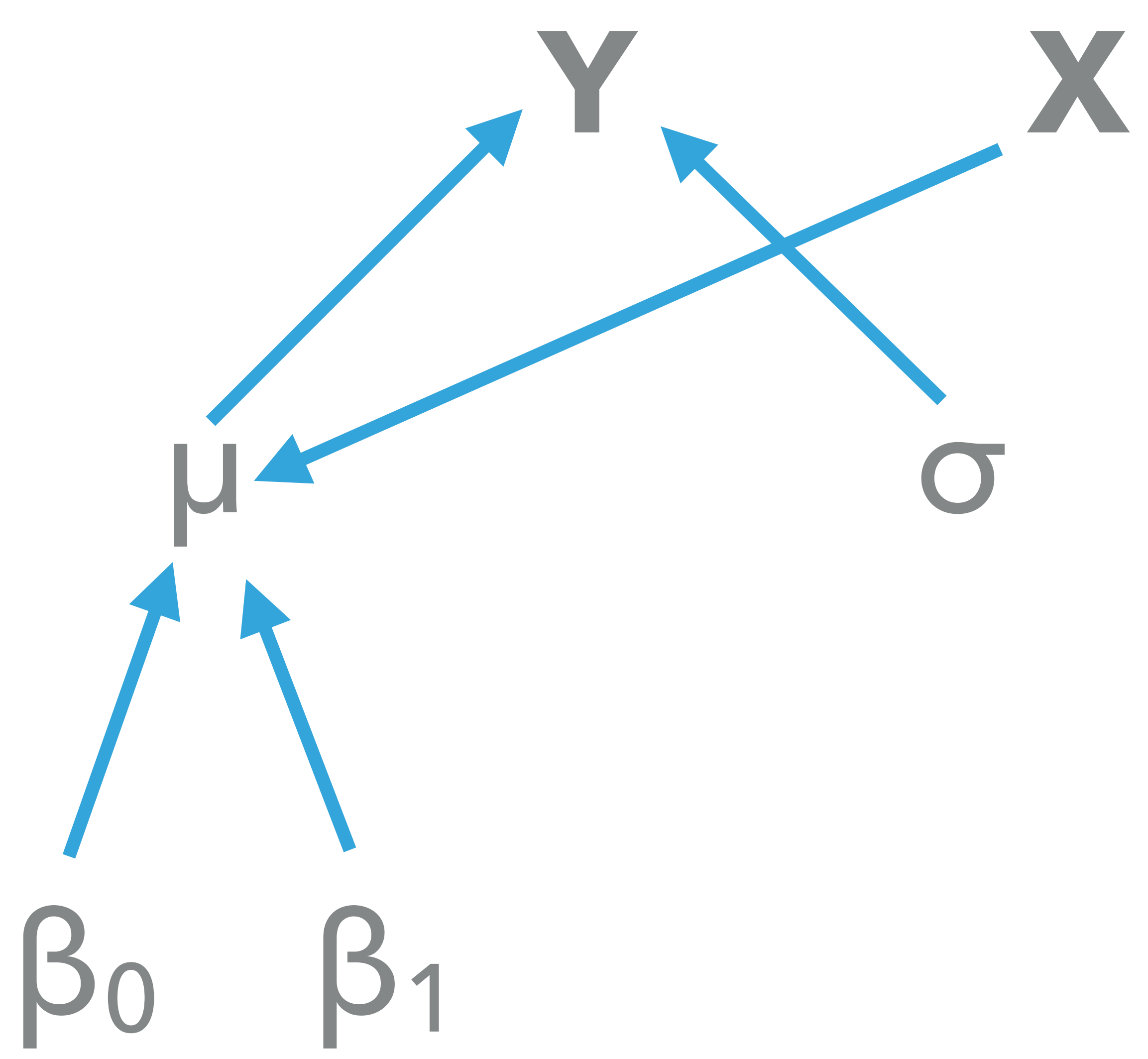

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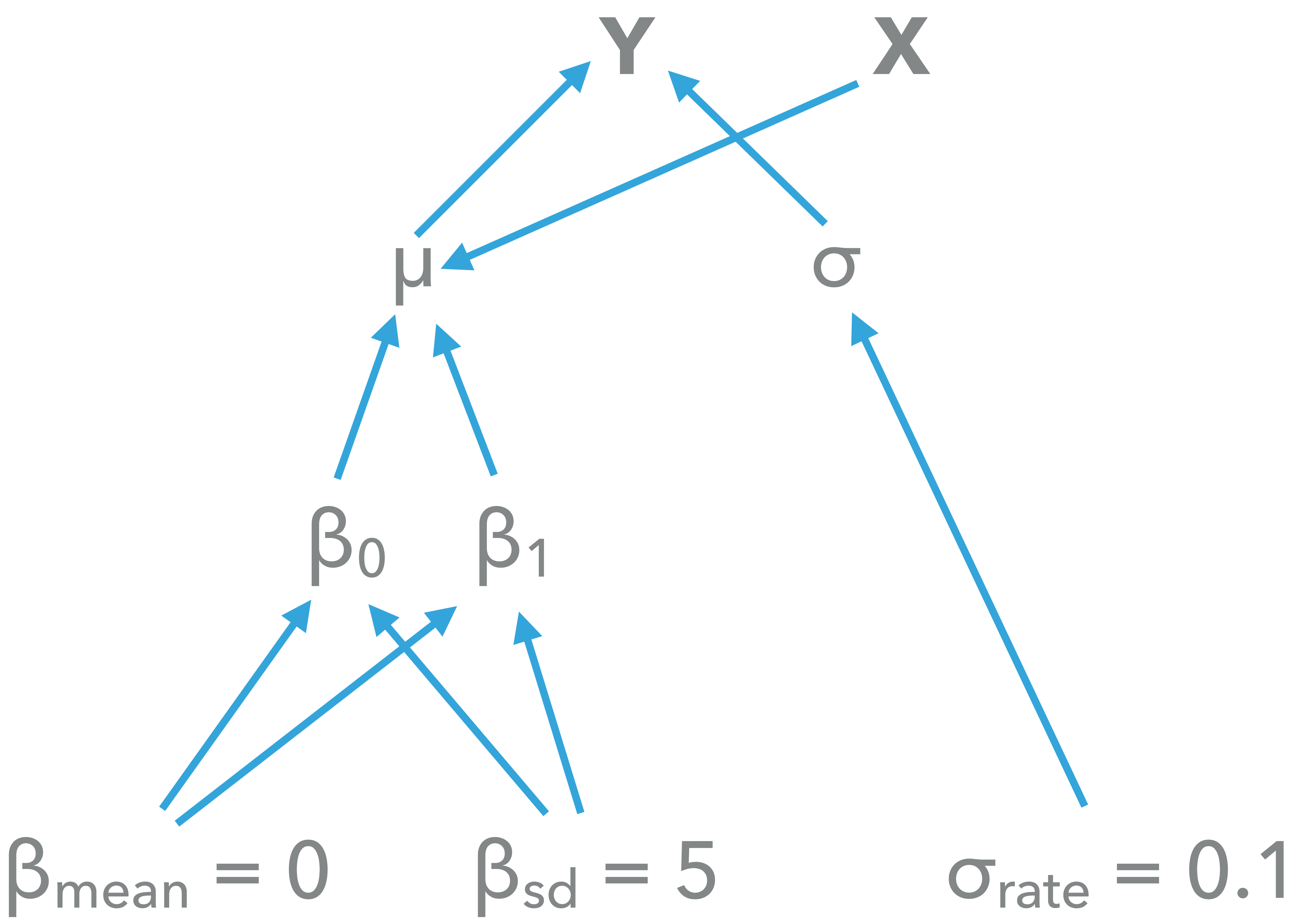

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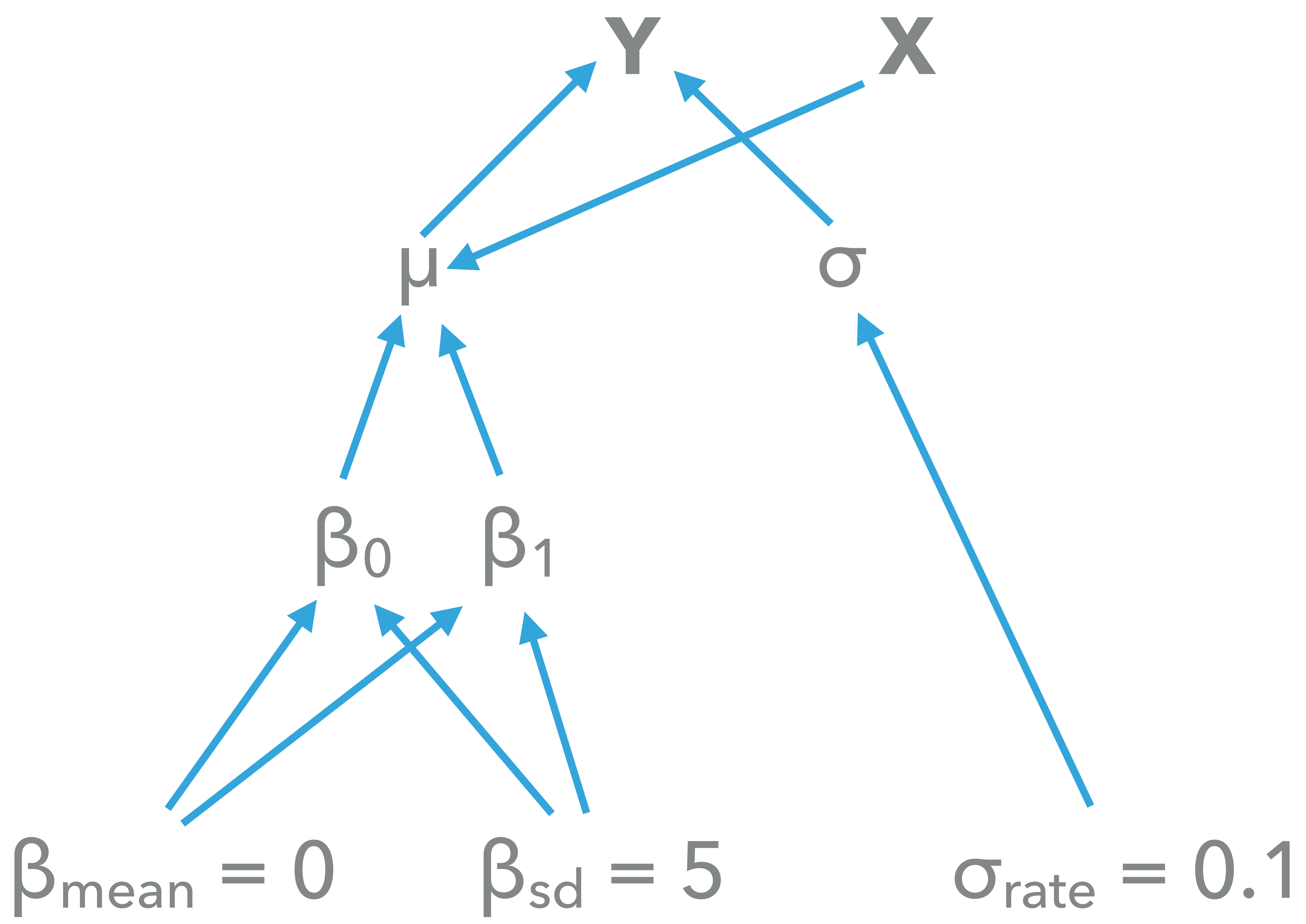

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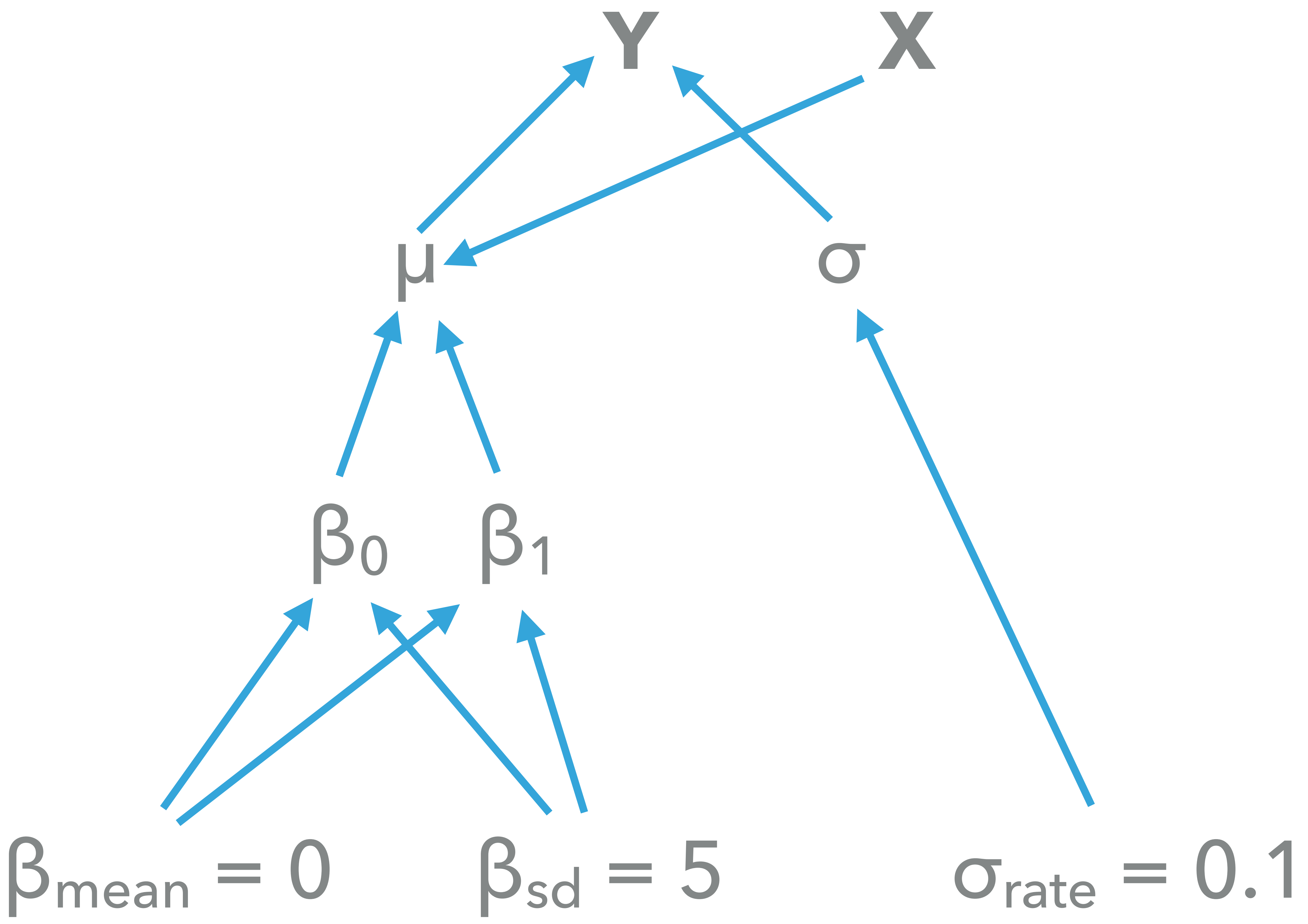

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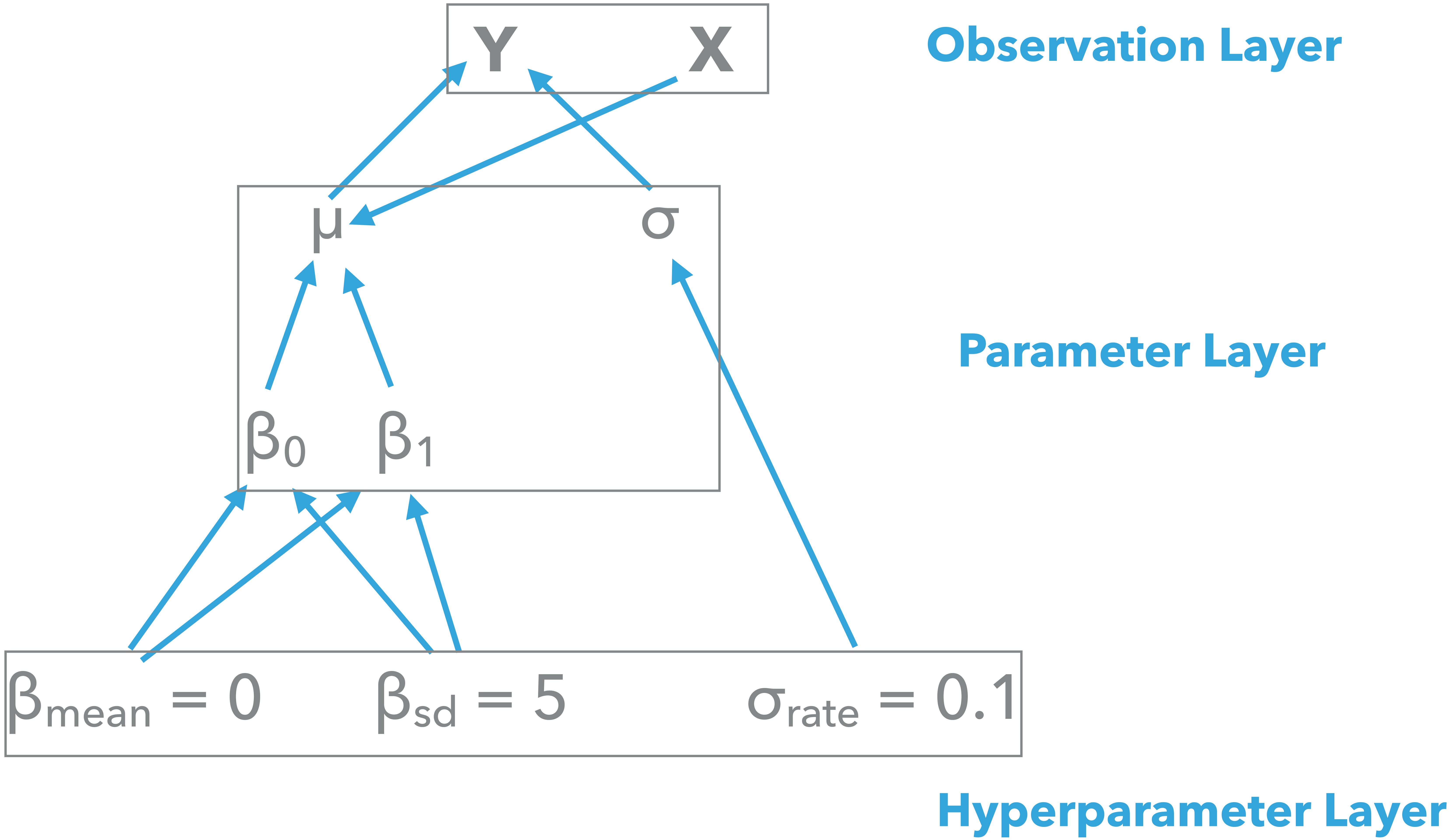

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- Select starting values and run until convergence (1.000s for Stan, 10.000s–100.000s or more for Metropolis)

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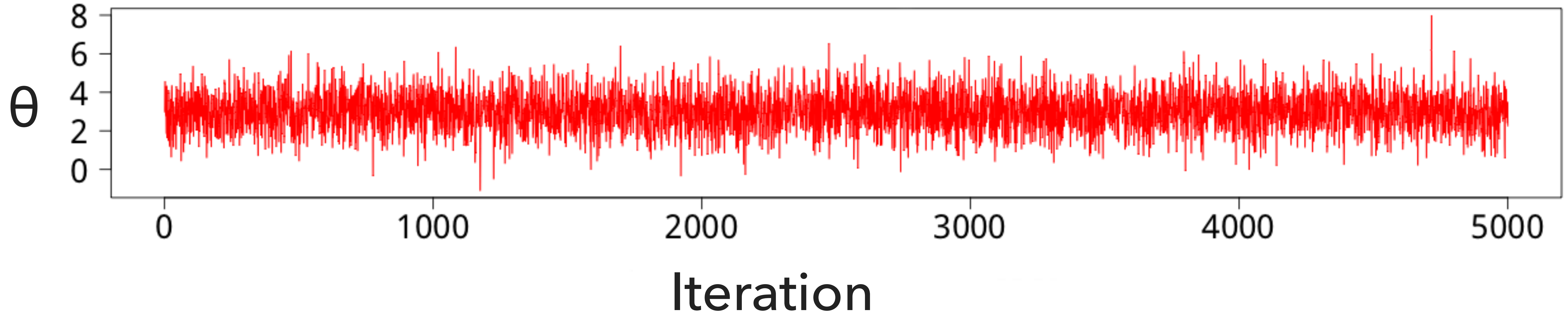
- **Trace plots** are one way to visually examine this property

# 3. TRACE PLOTS

- Low autocorrelation
- Thorough coverage of range of parameter values



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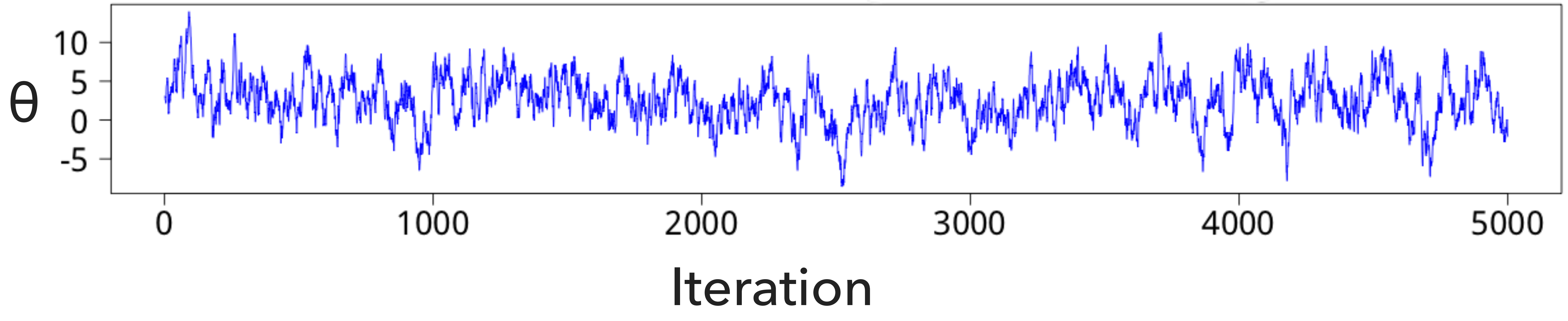
- High autocorrelation; can increase Metropolis step size
- Run longer
- Use thinning (not recommended)



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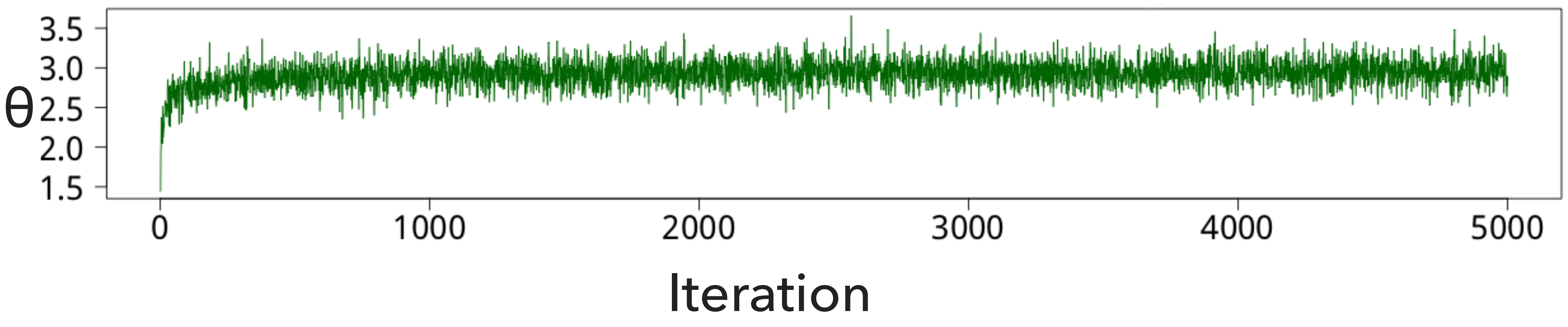
- Bad starting value
- Select new start
- Use burn-in (not the same as warm up!)



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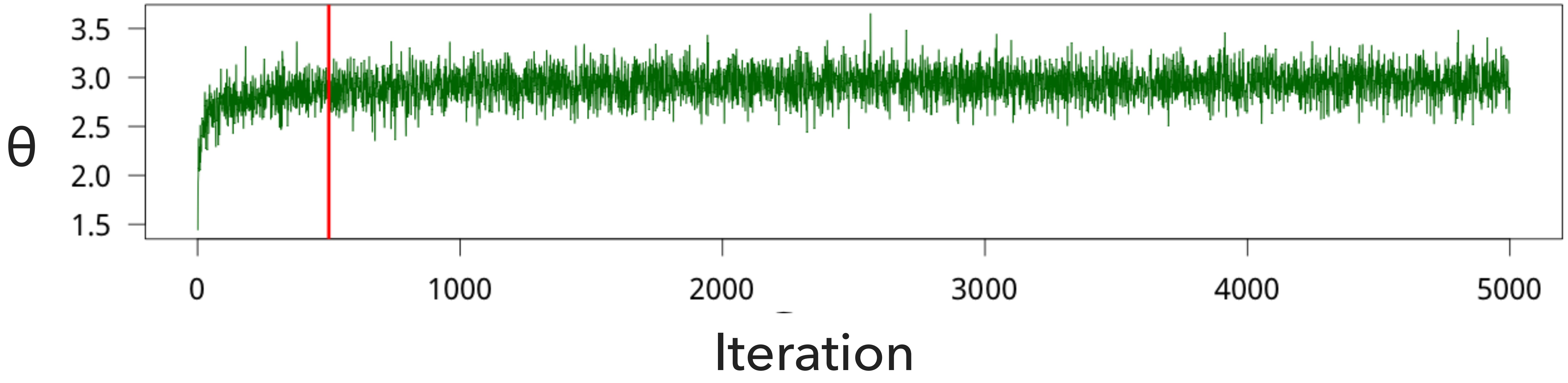
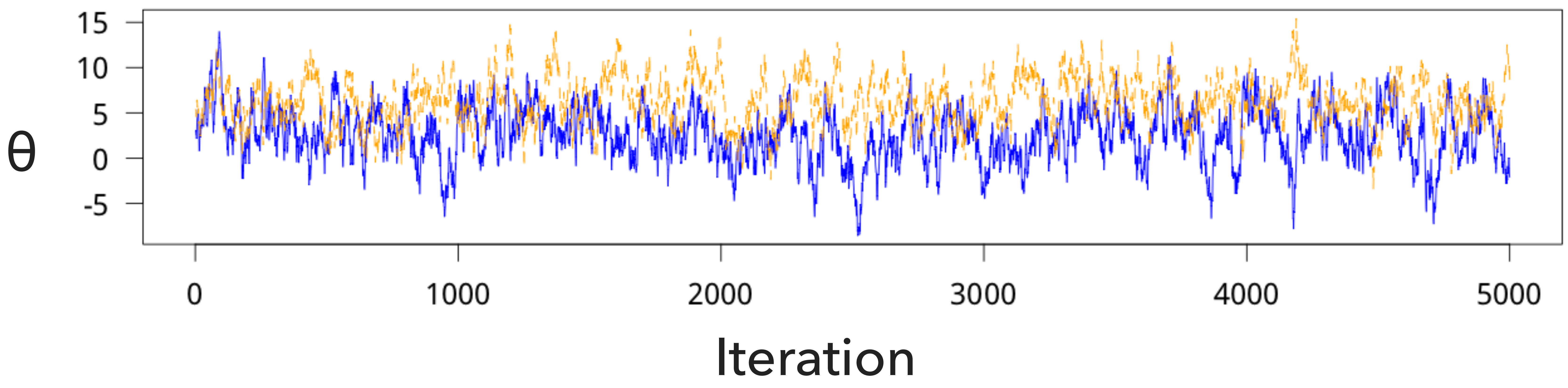
- A "burn-in" period selects a number of iterations to discard
- The idea is that the algorithm hasn't "forgotten" it's starting value
- Everything after burn-in approximates the stationary distribution



# 3. CONVERGENCE DIAGNOSTICS

- Using multiple chains allows us to compare within- and among-chain variance
- This is the **Gelman-Rubin statistic**; provided by Stan as r-hat
- Target value of 1.0. Less than 1.1 for all parameters is probably ok



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- Model evaluation is a more complex topic – briefly on next week, in more detail if time allows

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- **Posterior predictive distributions** – mean and standard error of outcomes
- **Posterior predictive simulations** – generate new outcomes from model (incorporates all uncertainty, including process error)