BAYESIAN BEST PRACTICES

BAYESIAN STATISTICS FOR ECOLOGISTS

IGB 12. TO 19. NOVEMBER 2018

FOUR STEPS TO AN ANALYSIS

1. Specify the joint posterior distribution of outcomes (i.e., response variables) and all unknowns/parameters

2. Draw from posterior distribution using MCMC

3. Evaluate model and revise if necessary (return to step 1)

4. Use posterior predictive distribution for inference

http://mc-stan.org/rstanarm/articles/rstanarm.html

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    - Prefer to "scale" outcomes via a **link function**

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- Forget conjugacy unless you know what you are doing and why

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- Can be useful to draw out your model as a **digraph** to make sure you don't miss anything

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- Select starting values and run until convergence (1.000s for Stan, 10.000s–100.000s or more for Metropolis)

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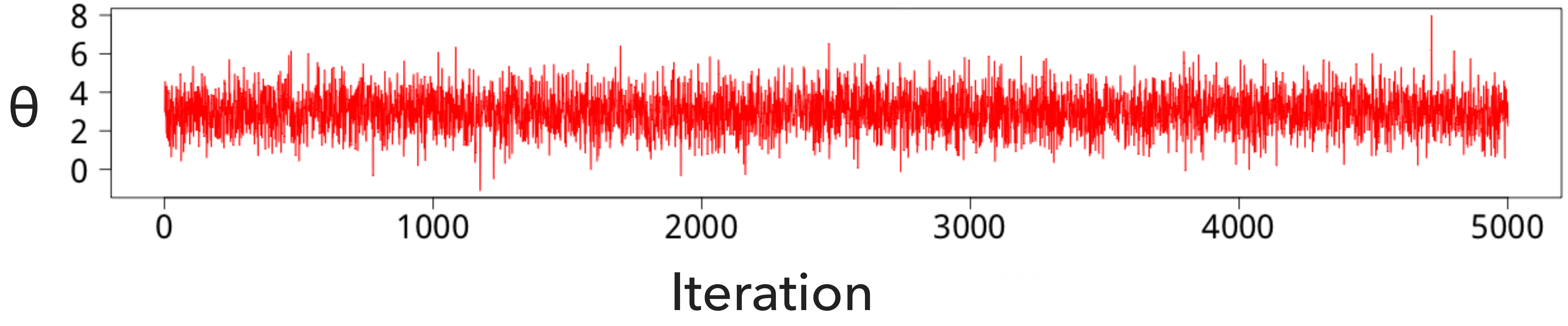
- **Trace plots** are one way to visually examine this property

3. TRACE PLOTS

- Low autocorrelation
- Thorough coverage of range of parameter values



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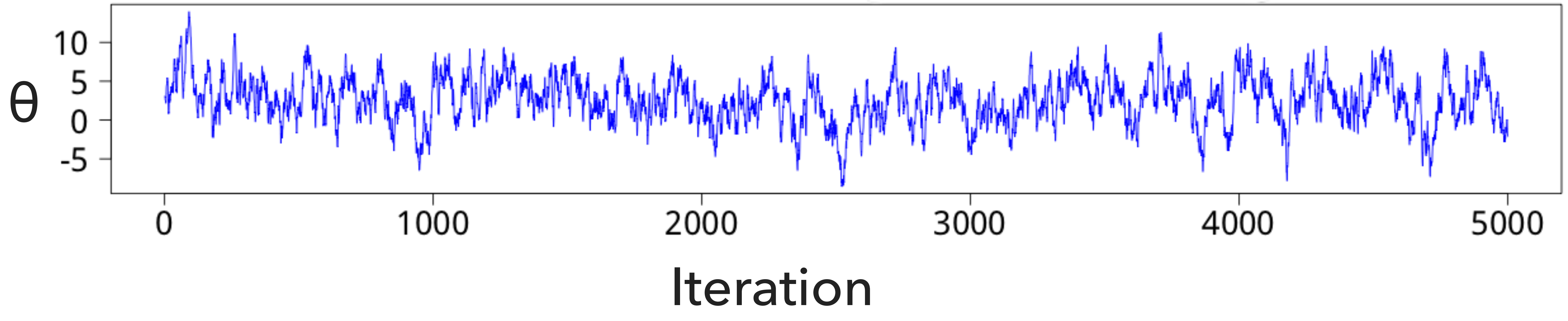
- High autocorrelation; can increase Metropolis step size
- Run longer
- Use thinning (not recommended)



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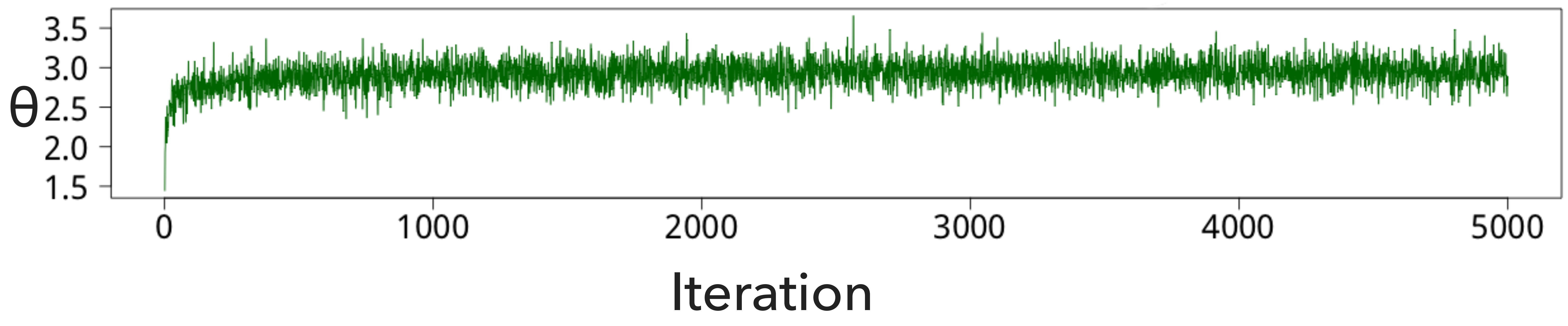
- Bad starting value
- Select new start
- Use burn-in (not the same as warm up!)



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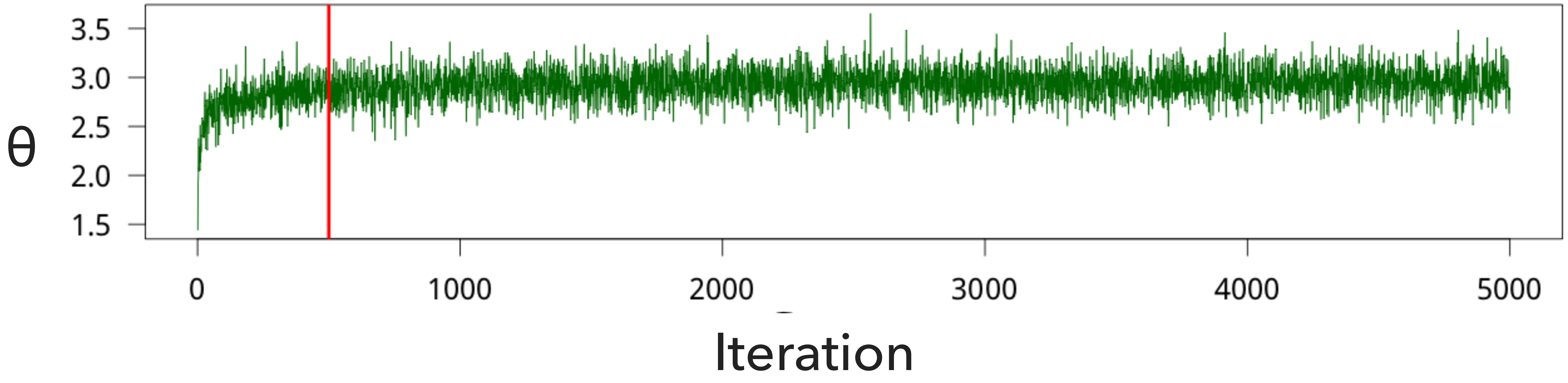
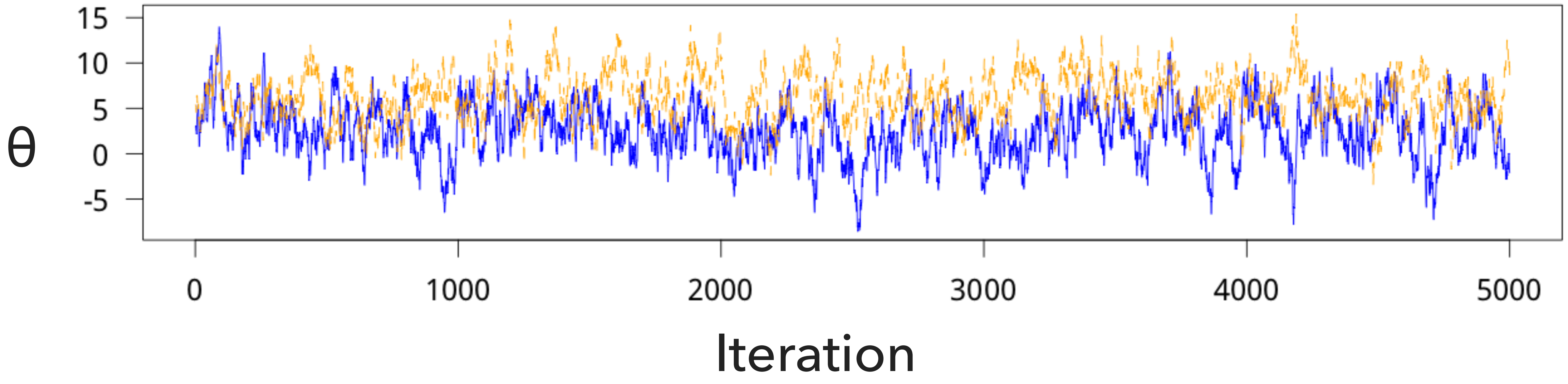
- A "burn-in" period selects a number of iterations to discard
- The idea is that the algorithm hasn't "forgotten" it's starting value
- Everything after burn-in approximates the stationary distribution



3. CONVERGENCE DIAGNOSTICS

- Using multiple chains allows us to compare within- and among-chain variance
- This is the **Gelman-Rubin statistic**; provided by Stan as r-hat
- Target value of 1.0. Less than 1.1 for all parameters is probably ok



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- Model evaluation is a more complex topic – briefly on Friday, in more detail if time allows

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- **Posterior predictive distributions** – mean and standard error of outcomes
- **Posterior predictive simulations** – generate new outcomes from model (incorporates all uncertainty, including process error)