**Measuring distances to the Palomar 5 and Palomar 13 globular clusters using RR Lyrae variable stars**

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**Abstract.** RR Lyrae stars resident in two globular clusters in proximity to the Milky Way Galaxy, Palomar 5 and Palomar 13 are analysed to calculate the distances to the globular clusters. PSF photometry was performed on a series of 12 epochs using data taken from the *Spitzer* IRAC in the mid-infrared bands, 3.6 µm (channel 1) and 4.5 µm (channel 2). Apparent magnitudes of detected stars were calculated and averaged using a Gaussian local estimation algorithm (GLOESS) which were corrected for extinction effects. PL relations were fit to the data in both clusters and distance moduli were obtained from the intercepts of the PL fit and subsequently their distances. Distances of *d* = 20.37 ± 0.36 (random) ± 1.13 (systematic) kpc for Pal 5 and *d* = 24.25 ± 0.44 (random) ± 1.35 (systematic) kpc for Pal 13 were found to be consistent with previous work.

**1. Introduction**

Einstein’s theory of general relativity (general relativity) sets out a different explanation for gravity than that described by Newton’s law of universal gravitation, which is given by

(1)

where is the gravitational constant, and are masses of an object and a test mass respectively and is the distance between the masses. General relativity introduces the equivalence principle whereby one cannot distinguish gravity from accelerated motion. In other words, a person in free-fall would feel no gravity. Therefore, a free-falling frame of reference is an inertial frame of reference. The theory describes gravity as the curvature of spacetime, spacetime is a four-dimensional description of space and time; this curvature is caused by mass[1].

A fundamental equation of general relativity is the Einstein field equation given by

(2)

where is the Einstein tensor, describing the curvature of spacetime, is the stress-energy tensor, describing sources of gravity, is the speed of light and is the gravitational constant. Equation (2) can be compared with (1) in that the former shows how spacetime is curved by sources of gravity, which is similar to the way Newton’s law measures the gravitational force between two objects. General relativity removes the idea that gravity is a force acting on a mass but instead the path followed by a mass in curved spacetime. However, (2) contains tensors and requires the knowledge of tensor analysis and will be avoided; a basic explanation is that and in (2) represent row and column indices respectively of matrices.

Gravitational waves were predicted by Albert Einstein in his theory of general relativity[2]. Gravitational waves arise due to changes in the curvature of spacetime caused by accelerating masses in certain scenarios, such as the coalescence of binary neutron stars, the merger of two black-holes and supernovae[1,3]. Gravitational waves can be visualised in a similar way to electromagnetic radiation and share some similarities in that gravitational waves are also transverse in nature. When masses are non-symmetrically accelerated, changes in spacetime in the form of gravitational waves propagate away from the event at the speed of light, this is observed as a strain on spacetime[4]. The propagation of gravitational waves at speeds other than the speed of light was discussed in a 1922 paper[5], stating a contradiction to general relativity. The speed of propagation of gravitational waves is therefore a clear test of general relativity as many alternate theories of gravity predict a difference in speed between electromagnetic radiation and gravitational waves from the same source[6], such as bimetric theories[7]. Therefore, a discovery that confirms the speed of gravitational waves to be the speed of light would result in the rejection of these other theories.

**2. RR Lyrae variables**

This section discusses reasons for the interest in the field of gravitational waves. The Laser Interferometer Gravitational-Wave Observatory (LIGO) detection of gravitational waves in 2015 has revived interest in the field with much ongoing research and many potential implications on current knowledge of gravity and cosmology.

Gravitational waves provide a test to the theory of general relativity. In the previous section, alternative theories to general relativity was briefly discussed, the discovery of gravitational waves and confirmation of their speed of propagation will help narrow down the range of possible theories of gravity. The detection of the first gravitational wave, GW150914, brought about the opportunity to test predictions of general relativity on binary black-hole (BBH) mergers. No evidence was found for deviations from the theory of general relativity[8], however this does not explicitly rule out other theories of gravity.

Extreme sensitivity is required to detect gravitational waves, most sources of gravitational waves produce strains of not more than one part in 1021[4] making direct detection of gravitational waves an impossibility until recently. LIGO is capable of strain sensitivities of 10-24 Hz-0.5[9] which corresponds to a change in the length of LIGO’s 4km interferometer arms by around 10-16 cm, a fraction of the width of proton[10]. This perspective gives insight into the challenges in detecting gravitational waves and the achievement that has been obtained by LIGO. In the future, LIGO will undergo an upgrade to increase sensitivity[11] and there are plans for a space-based gravitational wave observatory[12].

Further research on gravitational waves could result in the discovery of a gravitational wave background (GWB) which would provide more understanding of the early universe, particularly regarding inflation[13], serving as way to observe the universe before recombination, which is when the universe became transparent to electromagnetic radiation. Present LIGO sensitivities are unable to detect a GWB, however future developments may make this possible[14].

**3. Data selection and photometry**

*3.1. Data selection*

The data used in this project was collected from the *Spitzer* Space Telescope using the Infrared Array Camera (IRAC) using observations around the approximate centres of known globular clusters in the Milky Way Galaxy (MWG). In the case of this report, the Palomar 5 (Pal 5) and Palomar 13 (Pal 13) globular clusters were used. Data were collected in 12 time epochs in both the 3.6 µm and 4.5 µm bands, the only bands remaining operational on the telescope, in order to have additional data with which to calculate distances from. The data raw data was pre-processed before being converted into the FITS files used in the project. Basic Calibrated Data (BCD) images were created before the start of the project using the *Spitzer* Science Center (SSC) using an established pipeline in order the reduce the images into a usable state. This is done, for example, by removing biases from the camera, and flat-field corrections [Reach 05]. From here, mosaics of the data in each epoch and band were created, which is a stitching together of individual BCDs to create a larger frame of the areas of interest. This was, again, completed prior to receiving the data for use in the project and generously completed by our supervisor, Victoria Scowcroft. A master mosaic image was also created and provided in each band, which was used to generate a master source list of the stars within the frame. The master mosaic is particularly useful due to its lower signal-to-noise ratio (S/N) because the mosaic contains up to five well sampled BCDs. The data has been optimised for the 3.6 µm band and so this band has a lower signal-to-noise ratio than the 4.5 µm band [Garofalo 2018], which was noticed throughout the completion of this project.

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*3.2. Photometry*

Once the data is ready to be used, to measure distances to the globular clusters, photometry must first be performed on the data in order to obtain the apparent magnitudes of point sources detected in the frames. Photometry involves measuring the amount of light, the *Spitzer* data is measured in MJy and was converted into a counts measure using a conversion factor of exposure time divided by a flux conversion using details provided in the image header from the SSC pipeline. Conversion into counts was necessary to make the later calculation of magnitudes significantly easier.

There are two main methods for performing photometry that were considered in this project, aperture photometry and point-spread function (PSF) photometry. Aperture photometry involves drawing an aperture centred on the star of radius that is just large enough to capture light from the star, but not too small such that light from the star is omitted from this radius and then measuring the total amount of light, in counts, through that aperture. A second aperture, still centred on the star, is drawn but with a larger radius than the first such that a region of background light is captured. Combined these apertures form an annulus around the star of interest and the total count values through both are calculated. The average background value in the annulus is then subtracted from the inner aperture to give a background subtracted counts value for the star. This value will then be used to calculate the apparent magnitude of the source.

The other method considered was PSF photometry, which uses a point-spread function which is an amalgamation of the images of the point sources within the frame. The method for PSF photometry is similar to aperture photometry but instead follows an automated process of developing a point-spread function from the master mosaic frame, finding the sources of light, performing photometry on the detected sources, then subtracting the detected stars from the image and then run the process through another iteration [Stetson]. This procedure is particularly useful for crowded fields, of which globular clusters typically are, which provides a limitation for how far aperture photometry can be used. Therefore, PSF photometry has been used in the project.

A point-spread function was built from the master mosaic frame such that it can be used to identify point sources within the master frame, as well as the epoch frames, with profiles that are expected to be stars. A star detection algorithm, *DAOStarFinder*, [ref, also adnerson & king somewhere] was used to identify these initial point sources and requires several parameters to aid with its identification of stars. The first of which is the full width at half maximum (FWHM) value of the typical star in the sample. The radial profiles of several uncrowded stars were identified using a program called *Imexam* [ref] to gain an appreciation for the range of values for the FWHM, and a value of 5.0 was deemed acceptable. Another parameter is the threshold value, which determines which count values are deemed high enough to be a star. There is an efficiency problem here as there is a compromise between selecting a value too high such that actual stars are not detected but also if a value too low is chosen such that random noise is being detected as stars and the process becoming computationally time expensive. The threshold value was determined using the standard deviation of the image data and a sigma level of how many standard deviations should the data be clipped from, a sigma value of 6.0 was chosen for this data [EXPLAIN BETTER!!]. The next parameter is the *round* parameter which considers the profiles of the point sources and aims to discard elongated sources or sources where the light has a directional bias. This is important to ensure that, for example, galaxies are not counted as stars. The final main parameter is the *sharp* parameter which in some respects is the opposite of the *round* parameter and considers how fast the profile drops off, as opposed to any directional bias as in *round*, and aims to eliminate cosmic ray hits or bad pixels which may otherwise be counted as stars. These values were manually determined by inspection of the data and resultant PSF models and suitable values were found to avoid a trade-off between detecting stars erroneously or missing out sources that are valid stars. Once these values were determined, the PSF model was built, however using a high threshold value to select only uncrowded stars, typically around 100 stars per channel in the master frame. These stars were then individually checked and individual star frames with contamination from other stars, bad-pixels, or edge-of frame stars were removed and then the final PSF model built.

A master source list was created from the master mosaic frame by using the star detection algorithm and performing a PSF routine based on the PSF model built previously, however, in this case neglecting the photometry as it was not required for this frame. This list serves as a reference for the photometry performed on the stars in the 12 individual epochs and enable them to be matched to consistent star ID numbers between epochs because each run through the detection algorithm in different epochs frequently returns the stars in different orders. Coordinates in *x* and *y* for the stars detected were converted into right ascension and declination coordinates using the International Celestial Reference System (ICRS) based on information provided in the *Spitzer* image headers. The use of right ascension (RA) and declination (dec) is particularly useful as it allows direct comparisons between stars in the master frame and the epoch frames as well as objects identified in *Gaia* data release 2 (DR2) because the cartesian coordinates in each frame is unique to that frame and does not necessarily translate between the frames and the channels.

Once the master frame was created and saved, the PSF routine was performed on each epoch in turn. The process followed that described previously, following Stetson’s process [ref] whereby the stars were identified in the first epoch and photometry performed on the sources to obtain values for their flux at that moment in time and an error value provided [how? why?]. The coordinates of the stars were converted into RA and dec values and these were checked against the master source list and then joined as columns to the master table, forming the main table. This process was repeated in sequence up until the 12th and final epoch obtaining a final table with each source identified in the master table matched with up to count values for each star. If no match was found in a particular epoch, then no entry was made for that epoch when joined to the growing master table. This process was completed on both the 3.6 µm band (channel 1) and the 4.5 µm band (channel 2) giving two lists of sources identified, with a unique master ID, coordinates in RA and dec and values for apparent magnitude [insert ap. mag calculations before here!] and their corresponding uncertainties.

The next phase was identifying the RR Lyrae variables in each list. This was achieved by consulting a catalogue of variable stars created by Christine Clement for each globular cluster of interest [ref] as well as identifying stars classed as variable in a region of radius 10 arcmins around the centre of the cluster in *Gaia* DR2 [ref]. The coordinates of the stars in both the *Gaia* list and the catalogue of variable stars were matched to the main photometry tables created in both *Spitzer* channels to identify which table entries in the main tables corresponds to an RR Lyrae star. For the Pal 5 globular cluster, seven RRLs were identified in channel 2, five of which were also visible in channel 1 and for the Pal 13 globular cluster, **x** RRLs were identified in channel 2, **y** of which were also visible in channel 1.

The periods of the RRLs were obtained from the *Gaia* DR2 RR Lyrae list containing around 140000 RR Lyrae stars, if the star was not identified in this list by matching their *Gaia* source\_id value then the periods provided in the catalogue of variable stars was used. The period is important as in order to construct light curves for the stars and, to find the distance to a globular cluster, a period-luminosity (PL) relation must be fit.

*3.3. Light curves*

Light curves of the identified RRLs were constructed by plotting the apparent magnitude for each epoch against phase. The phase is calculated by taking the date and time the epoch frame was taken, converting this into a modified Julian date, using software within the AstroPy module and then dividing this by the period of the star and subtracting the floor value. The light curves contain several cycles by simply adding an integer value to the phase to create three periods as shown in figure **x**. By concatenating the phases in this way, it was easier to identify whether the end of the one period followed nicely into the start of the second period as the data collected only contained around one full period for each star. A fit was produced for each light curve using a Gaussian local estimation algorithm (GLOESS) [Persson 2004]. The GLOESS algorithm calculates the mean apparent magnitude of the RRL in its corresponding channels and this was then appended to arrays containing the period of the star.

**4. Period-Luminosity relations and distance measurements**

*4.1. Building PL relations for Pal 5 and Pal 13*

A PL relation shows the relationship between the magnitude of a variable star with its period [ref]. Only the apparent magnitude has been considered up until now and in order to calculate the distance to the globular clusters, the absolute magnitude must be known. This is achieved by comparing the data obtained for both Pal 5 and Pal 13 to a PL relation for RRL stars. The PL relations have been fit using the information provided for IRAC channels 3.6 µm and 4.5 µm in the Neeley et al. 2019 paper, hereby referred to as the Neeley paper [Neeley 2019].

Prior to applying the PL fit, the average apparent magnitudes, and uncertainties for each RRL in both channels were extracted from the GLOESS analysis described previously. The average magnitudes were corrected to account for extinction effects, which are caused by dust in the MWG resulting in a reddening of the apparent magnitudes which would influence the calculated distances. The corrections for dust extinction were obtained, for both clusters, from the IRSA Dust Extinction Service Queries built Chart

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The periods of each RRL were also extracted from either *Gaia* DR2 or the catalogue of variable stars and if necessary, they were fundamentalised. This applies to RRL type c stars which pulsate in the first overtone. These were fundamentalised using the following equation,

(2)

where *P* is the fundamentalised period and *PFO* is the first overtone period; the logarithms have been taken in preparation for the plot of the PL relation.

The PL relation from the Neeley paper takes the form,

(3)

where *M* is the absolute or apparent magnitude of an RRL in the sample, *a* is the intercept of the PL fit, *b* is the gradient of the PL fit and *P* is the period of the RRLs. The specific format of the PL fits can be found from the coefficients given in *Table 3* of the Neeley paper. For the two IRAC channels, the PL fits are,

(4)

(5)

and so, it is now possible to fit the PL relations to our data. This was done by holding the gradient *b* as constant and fixing it to the data but allowing the intercept *a* to vary. This means that the PL fits shown in figure x and figure y follow the form,

(6)

(7)

where *m* is the average apparent magnitude and is the intercept to be found by fixing the PL fit equations 6 and 7 to the corresponding IRAC channel. The intercept was found by using a least-squares fit and intercepts were found for Pal 5 in channel 1 as *a0* = 16.10 ± 0.03 mag and in channel 2 as *a0* = Chart, line chart

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**Figure 2**. Diagram of the LIGO interferometer. A simplified laser interferometer showing perpendicular arms, Lx and Ly, of 4km in length and a schematic diagram of distance between the observatories across the continental United States measured in light travel time[15].

*4.2. Calculating distances to Pal 5 and Pal 13*

The distance to a globular cluster can be calculated using a distance modulus [ref], which is the difference between the apparent magnitude and the absolute magnitude,

(8)

where *µ* is the distance modulus, *m* is apparent magnitude and *M* is absolute magnitude. Consider the PL relations in equations 4 to 7, for IRAC channel 1 the distance modulus can be calculated by subtracting equation 4 from equation 6, and likewise for IRAC channel 2 by subtracting equation 5 from equation 7, giving a general distance modulus, and substituting the relevant IRAC channel data of,

(9)

where *a0* is the calculated intercept from fitting the PL relation for each IRAC channel and *a* is the intercept given in the Neeley paper for the calibrated PL relation for each IRAC channel. Therefore, distance moduli values were calculated for Pal 5 to be *µ* = 16.51 ± 0.04 (± 0.12) mag in channel 1 and *µ* = 16.57 ± 0.04 (± 0.12) mag in channel 2, where the first uncertainty is the random uncertainty, defined as… , and the uncertainty in brackets is the systematic uncertainty, defined as… . Distance moduli were also calculated to Pal 13, giving *µ* = 16.88 ± 0.04 (± 0.12) mag in channel 1 and *µ* = 16.96 ± 0.04 (± 0.12) mag in channel 2.

The distance *d,* in parsecs (pc) can be calculated from the distance modulus by,

(10)

which can be rearranged to solve for *d* resulting in a final equation for the distance to be,

(11)

and so, distances to Pal 5 were calculated to be *d* = 20.04 ± 0.37 (± 1.11) kpc in channel 1 and *d* = 20.69 ± 0.35 (± 1.15) kpc in channel 2, as well as in Pal 13 to be *d* = 23.80 ± 0.38 (± 1.32) kpc in channel 1 and *d* = 24.69 ± 0.49 (± 1.37) kpc in channel 2. A summary of the results from this project is provided in table 1.

**Table 1.** Summary of the results and uncertainties for the intercept *m*, distance modulus *µ* and distance *d* obtained for Pal 5 (top) and Pal 13 (bottom).

|  |  |  |  |
| --- | --- | --- | --- |
| Pal 5 | | | |
| IRAC Channel | *m* [mag] | *µ* [mag] | *d* [kpc] |
| [3.6]  ± σrand (± σsys) | 16.10  ± 0.03 | 16.51  ± 0.04 (± 0.12) | 20.04  ± 0.37 (± 1.11) |
| [4.5]  ± σrand (± σsys) | 16.16  ± 0.03 | 16.57  ± 0.04 (± 0.12) | 20.69  ± 0.35 (± 1.15) |
| Pal 13 | | | |
| IRAC Channel | *m* [mag] | *µ* [mag] | *d* [kpc] |
| [3.6]  ± σrand (± σsys) | 16.47  ± 0.02 | 16.88  ± 0.04 (± 0.12) | 23.80  ± 0.38 (± 1.32) |
| [4.5]  ± σrand (± σsys) | 16.55  ± 0.03 | 16.96  ± 0.04 (± 0.12) | 24.69  ± 0.49 (± 1.37) |

**5. Discussion**

The distances to Pal 5 for both IRAC channels are within the bounds of the uncertainties when the random and systematic uncertainties are considered and so a mean distance to the Pal 5 globular cluster is calculated to be *dPal5* = 20.37 ± 0.36 (± 1.13) kpc. This value can be compared to previous efforts to calculate the distance to Pal 5, such as in the Price-Whelan *et al*. paper in 2019 which found a distance to Pal 5 to be 20.6 ± 0.2 kpc [ref]. The distance to Pal 5 calculated in this project is therefore consistent with the results obtained in [ref]. The differences in values for the distance to Pal 5 could be explained by the method used in [ref] considering the metallicity of the globular cluster. Other measurements of the distance to Pal 5 has been undertaken by K[ü](https://en.wiktionary.org/wiki/%C3%BC)pper *et al.* and found a distance of  kpc [ref] which does not agree with the value obtained in this report which could be because it appears extinction effects have not been considered.

The distances to Pal 13 in IRAC channel 1 and 2 are also in agreement and so an average distance to the cluster is *dPal13* = 24.25 ± 0.44 (± 1.35) kpc. A value of 24.3 ± 1.1 kpc was obtained by Cotê *et al*. [ref] which is consistent with the value obtained in this paper. Another distance of 23.6 ± 0.2 kpc was obtained by Shipp *et al*. [ref] which is again consistent with the distance to Pal 13 obtained. Therefore *dPal13 =* 24.25 ± 0.44 (± 1.35) kpc is consistent with recent efforts to calculate the distance to Pal 13.

There are a number of sources of uncertainty in the distance measurements obtained for both Pal 5 and Pal 13 which quote the random and systematic uncertainties separately. The random uncertainty in the distance modulus arises from the least-squares model used to calculate the intercept point of the PL fit and has been calculated by adding this in quadrature with the uncertainty in the calibrated absolute magnitude PL relation in the Neeley paper. This is influenced by photometric uncertainties in the apparent magnitudes of the RRLs which come from the background noise in the epoch images, as well blurring or smearing of stars, bad pixels and the potential for nearby bright sources to contaminate the field in the vicinity of the RRLs in the sample. Uncertainty in the dust extinction correction is also considered in the random uncertainty as it is primarily realised in the apparent magnitudes as opposed to the uncertainties of the PL fit itself.

Systematic uncertainties have been considered, which is the largest uncertainty as seen in the distance measurements. This arises from the calibration of the PL relations provided in the Neeley paper, where it is discussed that this is likely to do with the accuracy of the parallax values from *Gaia*.[REFINE EXPLAIN also NEELEY 2015].

The distance measurements obtained for both Pal 5 and Pal 13 have a total uncertainty of around 7% when propagated through to kpc. The large uncertainties obtained could be explained by the largest drawback of the analysis which is the small sample size. The channel 1 PL relations were fit four and three RRLs for Pal 5 and Pal 13 respectively and for channel 2 the PL relations were fit using seven and four RRLs respectively. It is remarkable that the resultant distance measurements are consistent with previous work completed which may owe to the relatively uncrowded clusters that Pal 5 and Pal13 are, but agreement is still observed for both globular clusters. Future work should focus on ways to perhaps mitigate some uncertainties, an example of this would be by considering the metallicities of the RRLs in the samples, in which case it is possible that the uncertainty could be reduced by up to 1% [Neeley 2019]. Further improvements could be made by refining the photometry.

Variability indices were also calculated for the entire sample of detected stars in both Pal 5 and Pal 13 [ref]. Analysis of standard deviation, IQR, and Welch-Stetson variability were considered in an attempt to identify and quantify statistically whether a star varies which would be useful to identify new variable stars in the cluster and confirm that the known RRLs do, indeed vary. Unfortunately, no progress was made here which may be due to the small amplitudes involved, particularly in the case of Pal 5 where most of the known RRLs are of type c.

**6. Future work**

The globular clusters analysed in this project contain a relatively small sample of RR Lyrae stars and attempts to avoid crowded fields has been made. The next step would be to apply this process to a more crowded globular cluster, such as IC4499, work has already started on this but is not yet complete. The IC4499 cluster contains at least 97 known RRLs from initial investigation of the catalogue of variable stars [ref] and so this may require adjusting the PSF photometry code to further optimise it for crowded fields.

A further development on this work would include considering the metallicities of the RRLs in the cluster which could improve the precision of the distance measurements, as was touched upon in the discussion and so this would be considered going forwards.

In terms of the wider field, looking to the future, improvements in uncertainties would be a main consideration, particularly concerning systematic uncertainties brought about from the calibrated PL relations. Improvements in parallax measurements could significantly reduce the uncertainties in distance measurements [ref]. This would be particularly exciting as this could help work towards refining the value and uncertainty in Hubble’s constant, as described in the introductory sections of this report, which would help improve the field’s understanding of the scale and evolution of the universe [ref].

**7. Conclusion**

Distances to the Pal 5 and Pal 13 globular clusters have been calculated using data collected from the *Spitzer* IRAC in the mid-infrared spectrum. An average distance of *dPal5* = 20.37 ± 0.36 (± 1.13) kpc to the Pal 5 globular cluster was obtained and found to be in agreement with the most recent measurement using a similar method when the random and systematic uncertainties are considered. An average distance of *dPal13* = 24.25 ± 0.44 (± 1.35) kpc to the Pal 13 globular cluster was calculated and in agreement with previous measurements undertaken.

These distance measurements were obtained by performing PSF photometry on mosaic images of individual epochs across a duration of at least one period for the largest RRL period in the frame using software coded in python. Apparent magnitudes of the detected sources were calculated and the RRLs within the frames identified and their data extracted and mean apparent magnitudes calculated by way of a gaussian local estimation algorithm. Extinction effects caused by dust in the MWG was corrected for and the apparent magnitudes adjusted as a result. PL relations were then fit to the data for each IRAC channel for both clusters using a calibrated PL relation from Neeley *et al.* [ref] to yield the intercept point of the fit using a least-squares model. All data points were within one standard deviation of the fit for both channels in Pal 5 and Pal 13 when considering uncertainty in the average apparent magnitude. The intercept was then used to calculate the distance moduli for each channel for both Pal 5 and Pal 13 and converted into distances in kpc.

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