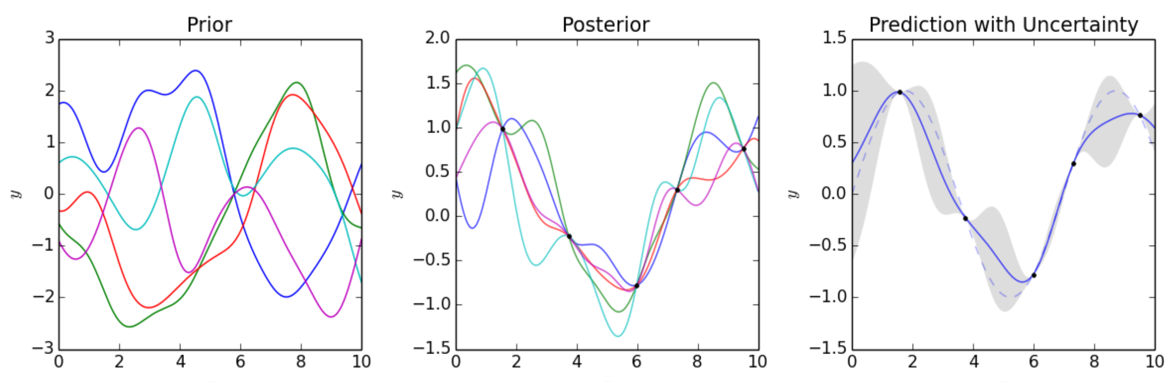
Gaussian Process Interpolation for Uncertainty Estimation in Image Registration

ECSE 626 – Project presentation



Guideline

- Image Registration
- Gaussian process Interpolation
- Method
- Preliminary Results

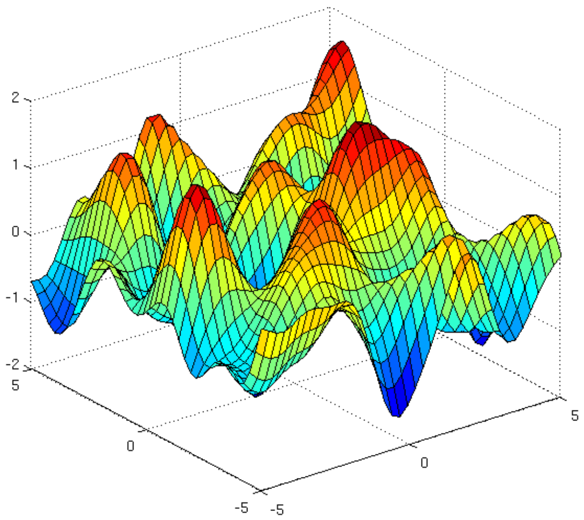
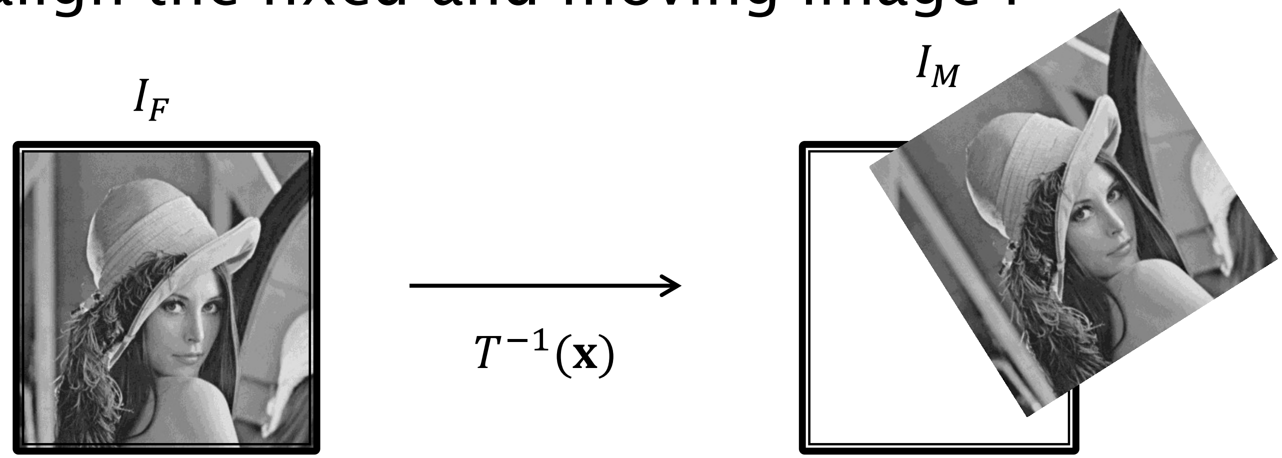


Image registration

- Registration
  - What T align the fixed and moving image ?

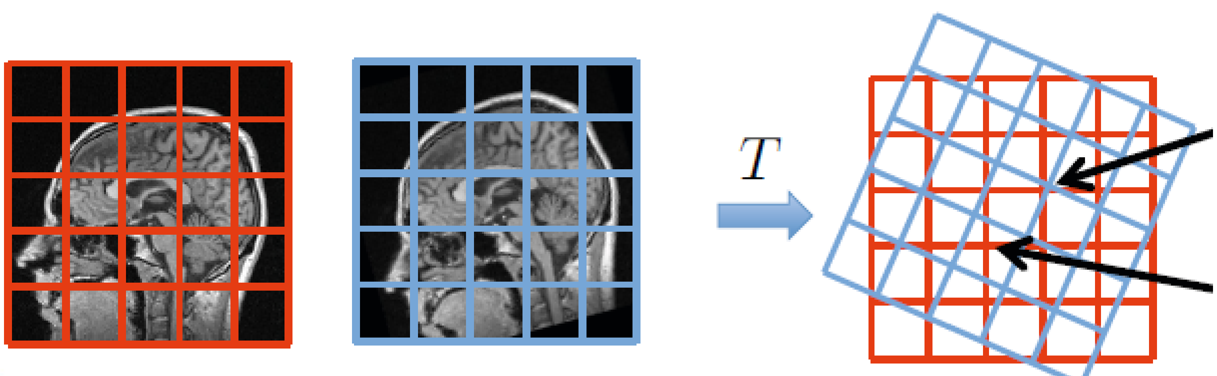


  - After random initialization of a tested $T_t(\mathbf{x})$
  - What is the best T_t that maximizes similarity S of I_F and $T_t(I_M)$

$$\hat{T} = \underset{T_t}{\operatorname{argmax}}\left(S(I_F, T_t(I_M))\right)$$

Image registration

- Major problem
  - Each image is divided into a grid
  - To compute $S(I_F(x), T_t(I_M(x)))$ comparison of pixel value at the same position x
  - Impossible to compare grid points of I_M and and I_F !

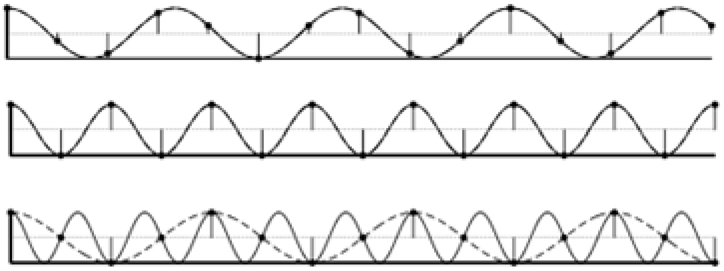


[1]

Image Registration

- Interpolation :
  - Construction of the continuous version of a discrete signal
  - Convolution with a kernel through all the observations
  - Interpolator with Nyquist frequency [2]

fe > fs/2



https://commons.wikimedia.org/wiki/File:Nyquist_Aliasing.svg

Image registration

- Example of aliased image (too low image resolution for the details)

https://commons.wikimedia.org/wiki/File:Nyquist_Aliasi
http://forum.hardware.fr/hfr/VideoSon/Traitement-Video/unik-artefacts-videos-sujet_142007_1.htm ng.svg

Image registration

- Quadratic interpolation

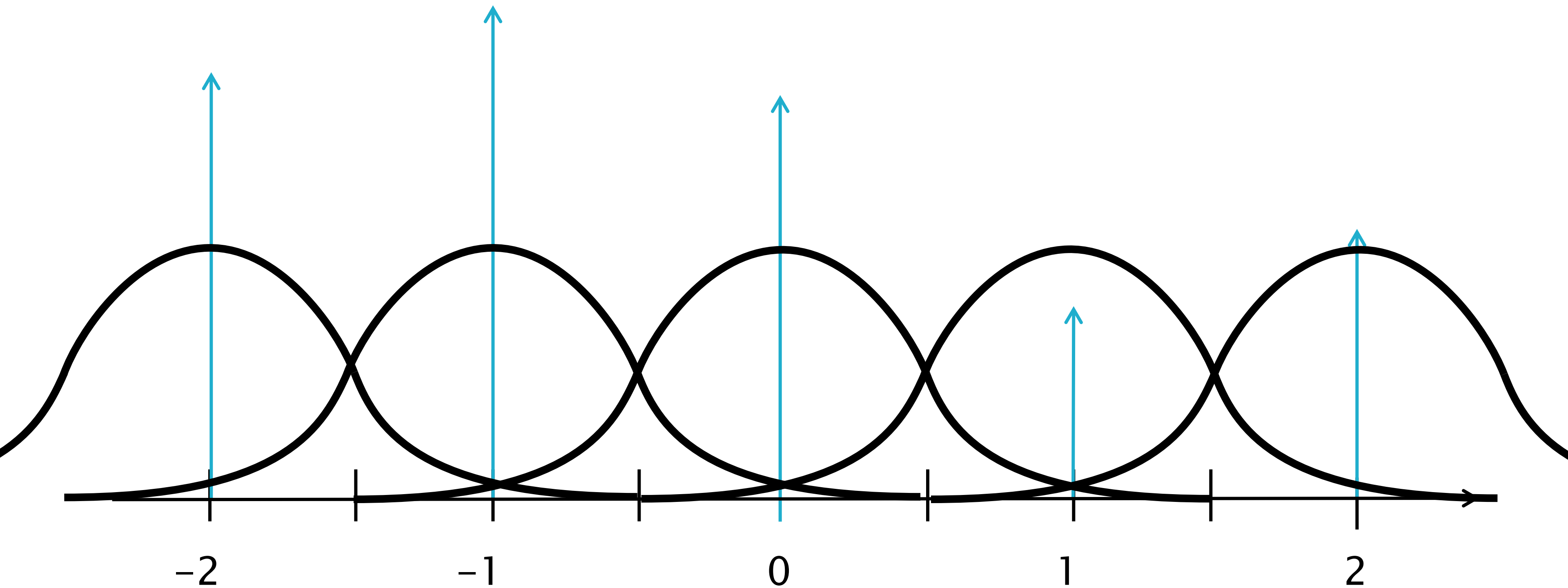


Image registration

- Resampling of $x = 0.3$

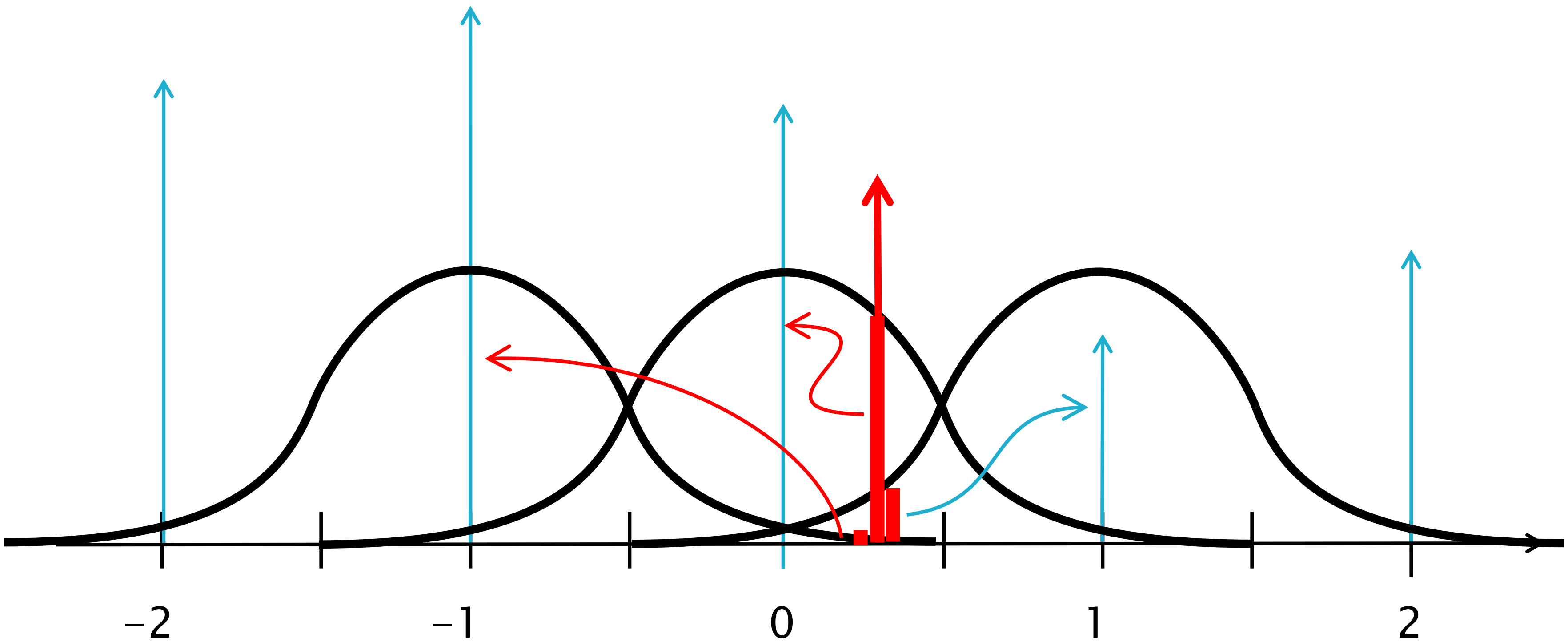


Image registration

- Interpolated values

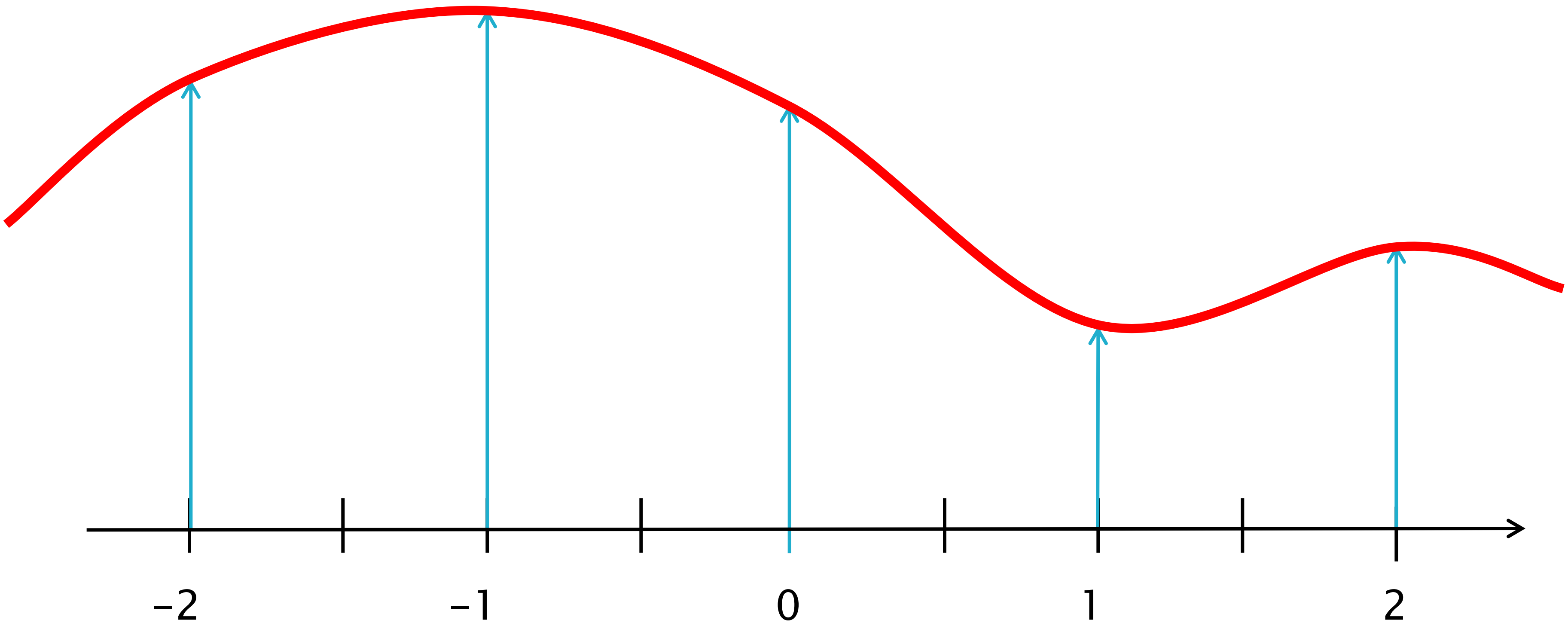
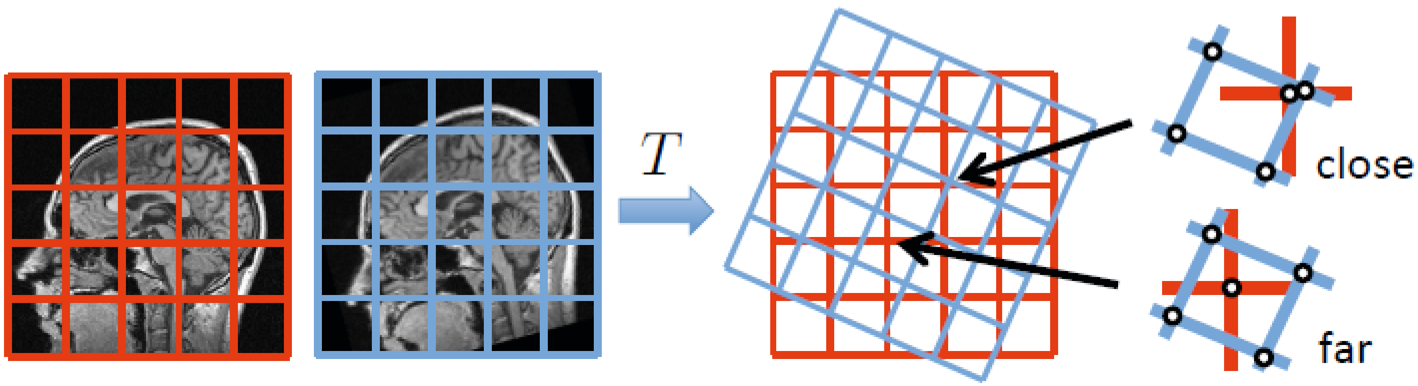


Image registration

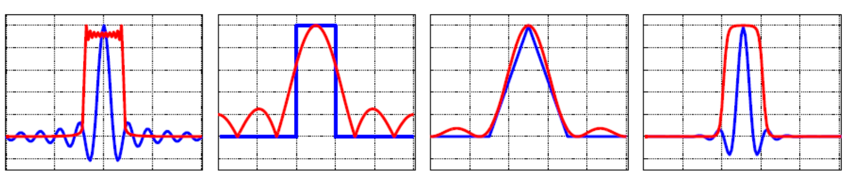
- Drawback of all the approaches
  - Difficult because interpolation error varies among grid points



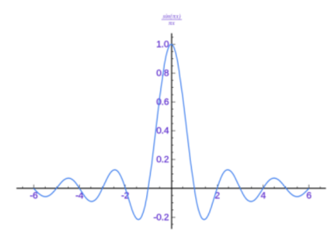
  - How to model the interpolation uncertainty ?

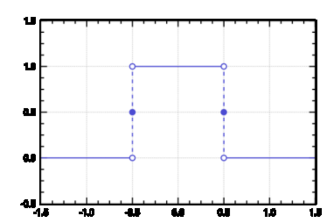
Gaussian Process Interpolation

- Gaussian process
  - $\mathcal{GP}(m,k)$: Stochastic process defined by mean function $m(x)$ and covariance function $k(x,x')$ [3]
  - Also called « Kriging » (used in geostatistic) [4]
- Why Gaussian Process interpolator
  - Because it estimate uncertainty and is closed to the optimal interpolator !

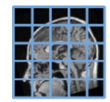
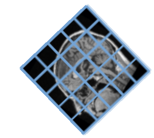


(a) Truncated sinc (b) Nearest (c) Linear (d) GP





Method : Bayesian Regression

- Fixed grid of image I_F : X^* 
- Grid of image I_M : Ω_{I_M} =>  $T(\Omega_{I_M}) = X$ 

Assumptions

- $\mathcal{GP}(0, k)$ prior over resampled image I_M^*
- Gaussian noise $\varepsilon = \mathcal{N}(0, \sigma_{I_M}^2)$ on measurements I_M

Posterior of resampled values I_M^*

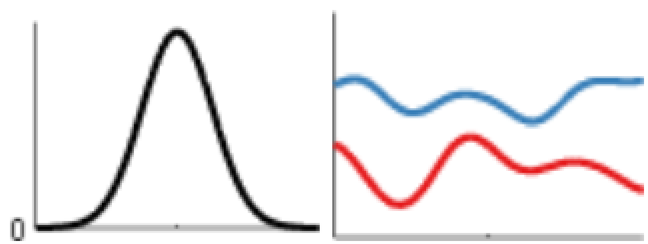
- $p(I_M^* | I_M, X, X^*) = \mathcal{N}(\mu_{I_M^*}, \Sigma_{I_M}^*)$

$$\mu_{I_M^*} = k(X, X^*).\left[k(X, X) + \sigma_{I_M}^2 \mathbf{I}\right]^{-1}.I_M$$

$$\Sigma_{I_M}^* = k(X^*, X^*) - k(X, X^*).\left[k(X, X) + \sigma_{I_M}^2 \mathbf{I}\right]^{-1}.k(X, X^*)$$

Method : Bayesian Regression

- Choice of covariance function $k(x, x')$
  - The most important choice of $\mathcal{GP}$!
  - Gaussian kernel : $k(x, x') = \exp(-\frac{\|x-x'\|^2}{2l^2})$
  - Because it captures the neighbours relation
  - Parametrized by the length l



http://people.seas.harvard.edu/~dduvenaud/cookbook/

Method : Similarity measure

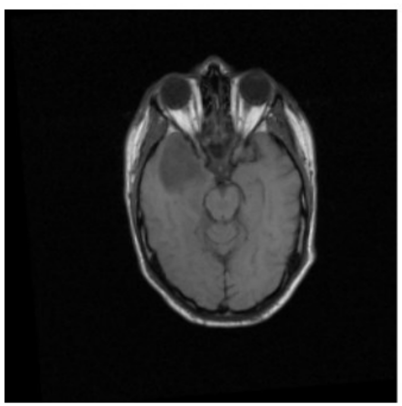
- Generative model instead of MAP
  - $p\left(I_F, I_M, I_M^*; T, \sigma_{I_M}, \sigma_{I_F}, l\right) = \underbrace{p\left(I_M^* \middle| I_M; T, \sigma_{I_M}, l\right)}_{\text{Previous posterior}} . \underbrace{p(I_F | I_M^*)}_{\text{Likelihood}}$
  - After marginalization over I_M^*
  - $\hat{T} = \underset{T}{\operatorname{argmax}} \left[\underbrace{\log\left(p\left(I_F, I_M; T, \sigma_{I_M}, \sigma_{I_F}, l\right)\right)} \right]$

$$\boxed{\log\left((2\pi)^{-\frac{k}{2}} |\Sigma|^{-\frac{1}{2}}\right) - \frac{1}{2}\left(I_F - \mu_{I_M^*}\right)^t \Sigma^{-1} \left(I_F - \mu_{I_M^*}\right)}$$

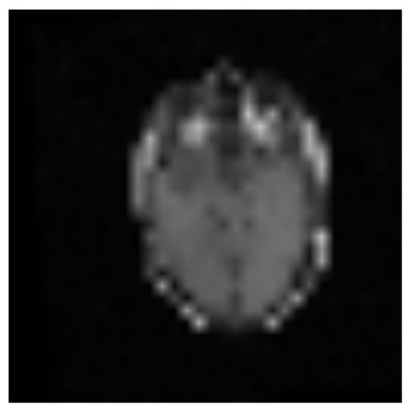
$\Sigma = \Sigma_{I_M}^* + \sigma_{I_M}^2$

Preliminary results

- Data
  - MRI–T1 3D volume from RIRE dataset [5]
  - Downsampling of one slice in one direction by 5
  - Random T for moving image



slice no3

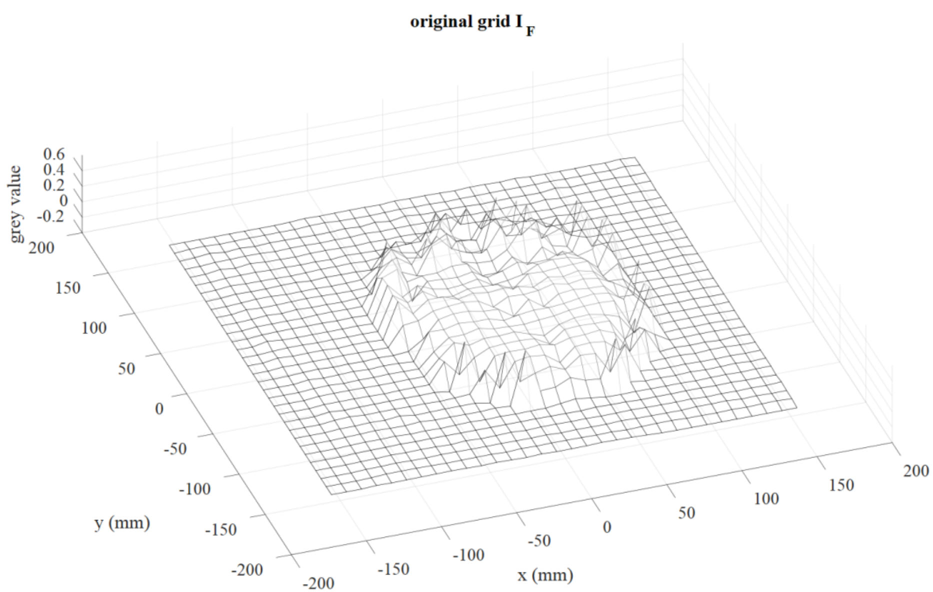


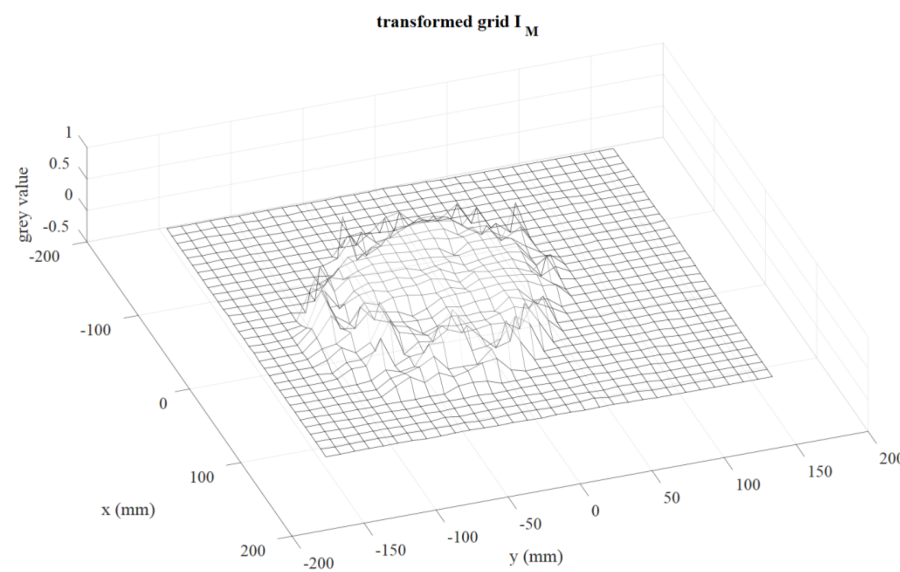
downsampled in x direction

im transformed by 0 0 -126.0484°

Preliminary results

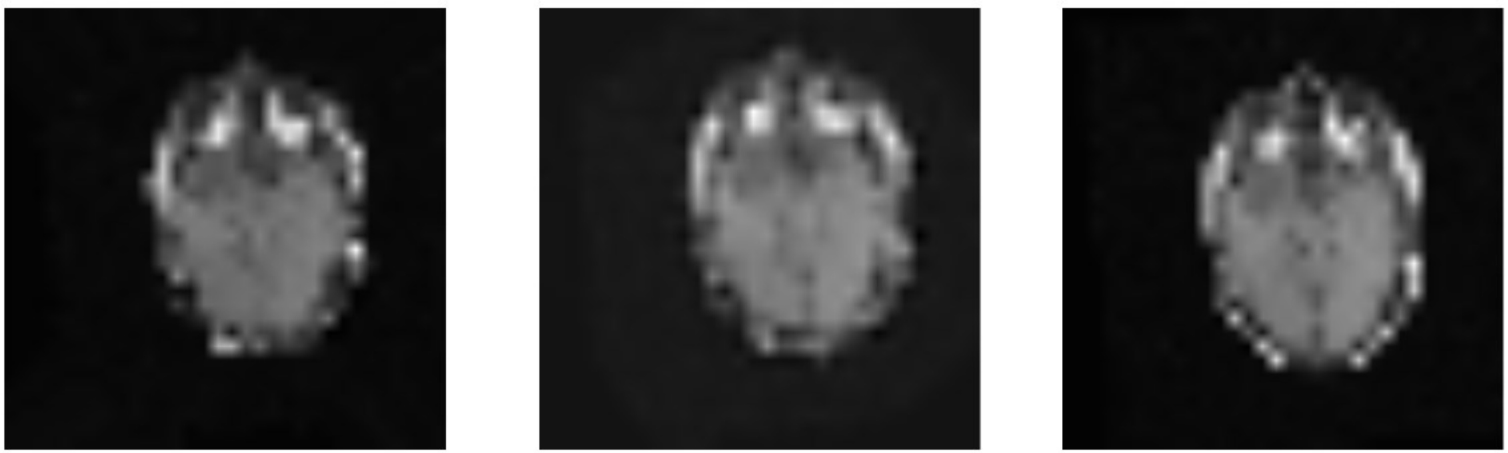
- Data
  - $[32 \times 32]$ grid for fixed image : X
  - $[32 \times 32]$ grid for moving image : Ω_{I_M}





Preliminary results

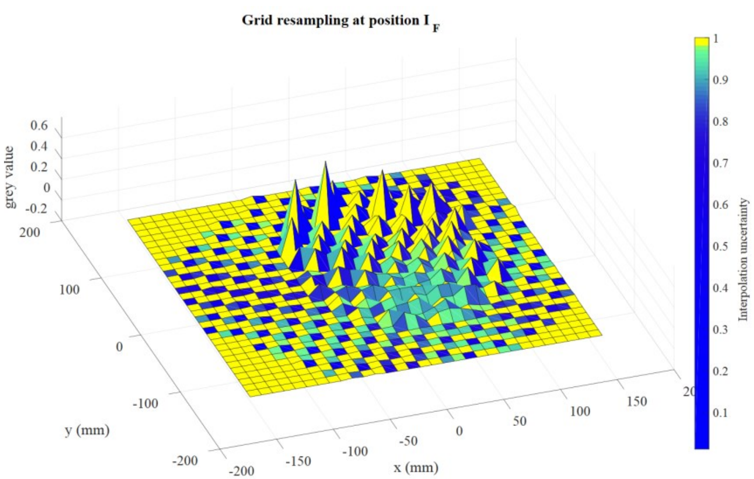
- Interpolation results
  - With lengthscale $l = 6{,}25$ and noise $\sigma_{I_M} = 0{,}1$



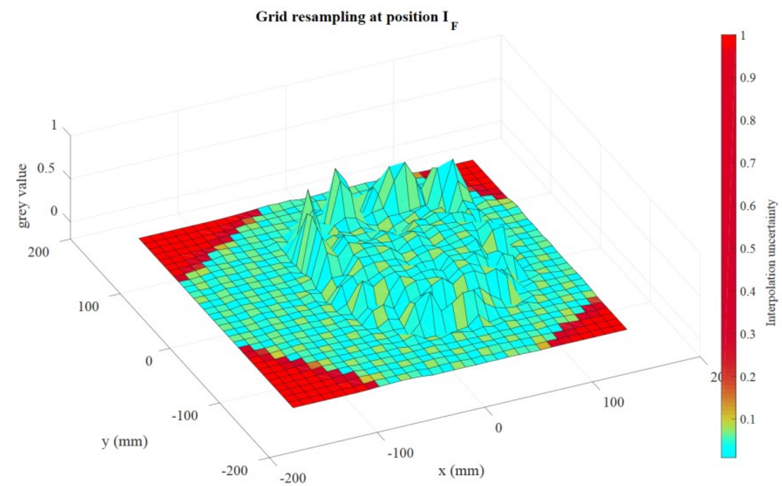
transformed Moving image   Interpolated values   Fixed image

Preliminary results

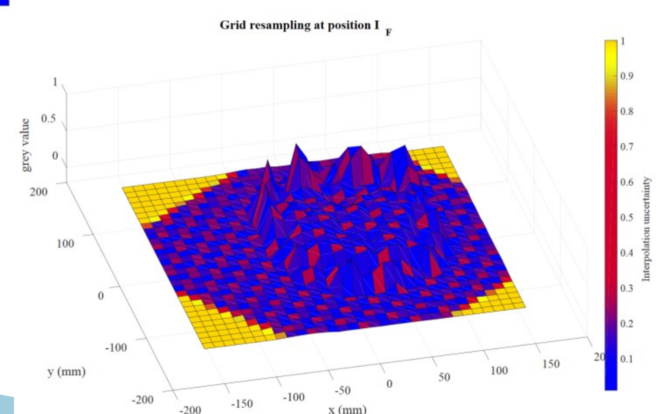
- Interpolation results
  - Influence of l : smoothness ↗ and uncertainty ↘



$l = 2$



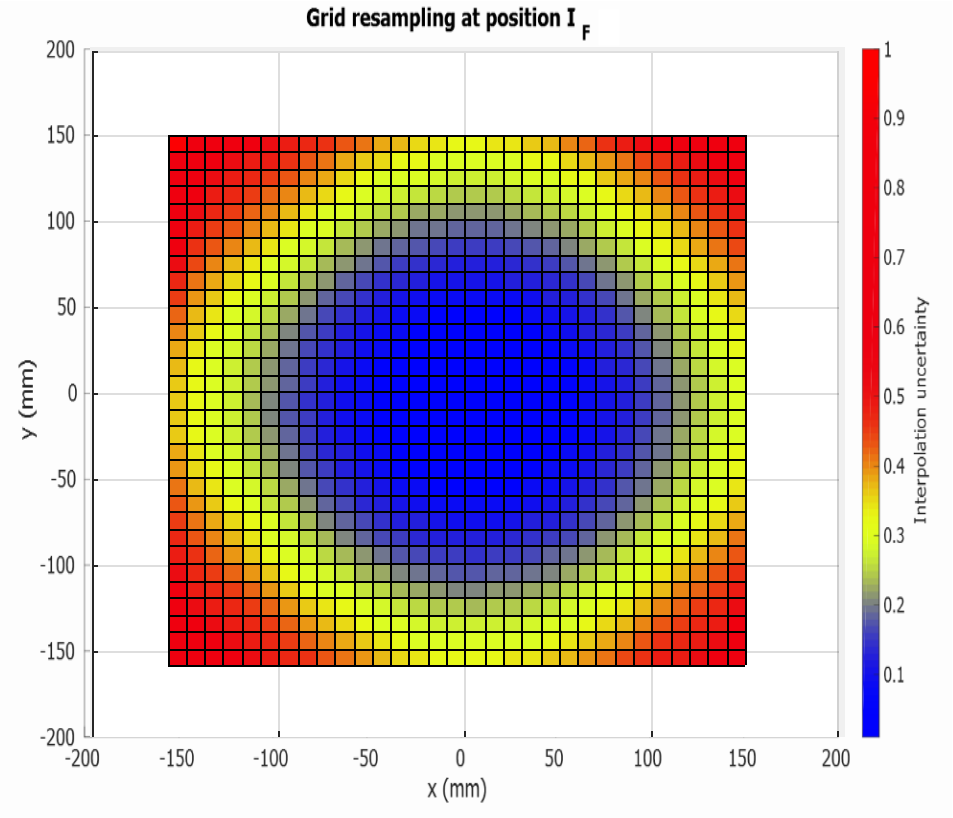
$l = 8$



$l = 6$

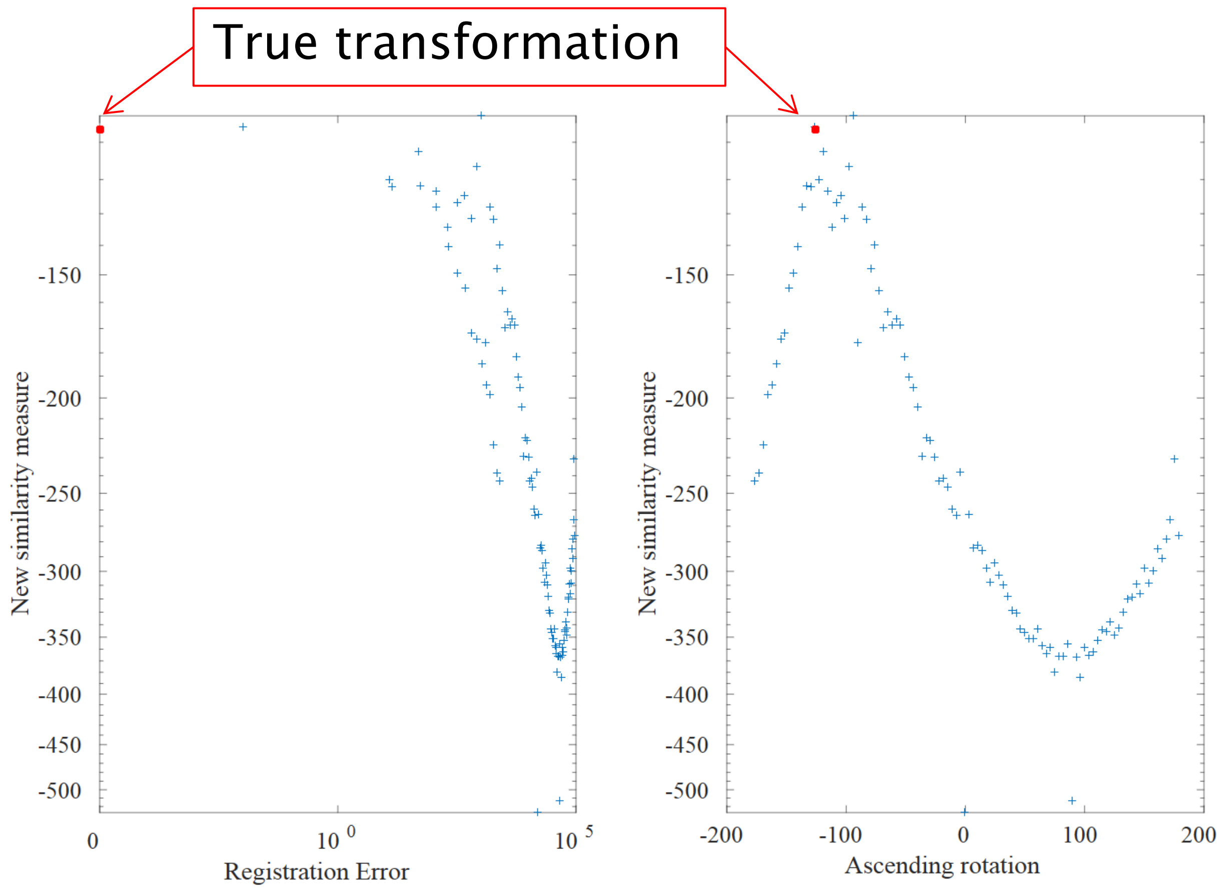
Preliminary results

- Influence of rotation



Preliminary results

- Cost function
  - Test with $T_x = 0$; $T_y = 0$; $T_{\theta_z} = -106°$



Future work

- Still work to do…
  - Best parameter l by maximization of BIC criteria
  - Qualitative comparison of interpolation methods
  - Quantitative comparison of interpolation (NN, cubic, spline) with X-corr and $\mathcal{GP}$ similarity measure

Conclusion

- Advantages
  - Compact and easy adjustable
  - Accurate with measure of uncertainty
  - $\mathcal{GP}$ is a growing field

- Drawbacks
  - Complexity
  - High memory requirement
  - Covariance $\Sigma_{I_M}^{*}$ sensible to noise
  - Instability because of high dimensionnality

References

- [1] Wachinger, Christian, et al. "Gaussian process interpolation for uncertainty estimation in image registration." *Medical Image Computing and Computer-Assisted Intervention–MICCAI 2014*. Springer International Publishing, 2014. 267-274.

- [2] Lehmann, Thomas M., Claudia Gönner, and Klaus Spitzer. "Survey: Interpolation methods in medical image processing." *Medical Imaging, IEEE Transactions on* 18.11 (1999): 1049-1075.

- [3] Rasmussen, Carl Edward. "Gaussian processes for machine learning." (2006).

- [4] Van Beers, Wim, and Jack PC Kleijnen. "Kriging interpolation in simulation: a survey." *Simulation Conference, 2004. Proceedings of the 2004 Winter*. Vol. 1. IEEE, 2004.

- [5] Fitzpatrick, J. Michael, Jay B. West, and Calvin R. Maurer Jr. "Predicting error in rigid-body point-based registration." *Medical Imaging, IEEE Transactions on* 17.5 (1998): 694-702.

Discussion

Annexe : Matrix computation on *Matlab*

```
% define the coordinates
% along x and y
x=[-3:1:3];
y=[-3:1:2];

% define the coordinates
% along the x-y plane
[xx,yy]=meshgrid(x,y)
```

x =

1	2	3	4	5	6	7
-3	-2	-1	0	1	2	3

y =

1	2	3	4	5	6
-3	-2	-1	0	1	2

xx =

-3	-2	-1	0	1	2	3
-3	-2	-1	0	1	2	3
-3	-2	-1	0	1	2	3
-3	-2	-1	0	1	2	3
-3	-2	-1	0	1	2	3
-3	-2	-1	0	1	2	3

yy =

-3	-3	-3	-3	-3	-3	-3
-2	-2	-2	-2	-2	-2	-2
-1	-1	-1	-1	-1	-1	-1
0	0	0	0	0	0	0
1	1	1	1	1	1	1
2	2	2	2	2	2	2