

LTFAT Reference manual

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Chapter 1

LTFAT - Base routines

1.1 Basic routines

1.1.1 LTFATSTART - Start the LTFAT toolbox

Usage

```
ltfatstart;
```

Description

ltfatstart starts the LTFAT toolbox. This command must be run before using any of the functions in the toolbox.

To configure default options for functions, you can use the ltfatsetdefaults function in your startup script. A typical startup file could look like:

```
addpath('/path/to/my/work/ltfat');
ltfatstart;
ltfatsetdefaults('sgram','nocolorbar');
```

This will add the main LTFAT directory to your path, start the toolbox, and configure sgram to not display the colorbar.

The function walks the directory tree and adds a subdirectory to path if the directory contains a [subdirectory, init.m] script setting a status variable to some value greater than 0. status==1 identifies a toolbox module any other value just a directory to be added to path.

ltfatstart(0) suppresses any status messages.

!!WARNING for MATLAB users!!

The function indirectly calls clear all, which clears all your global and persistent variables. It comes with calling javaaddpath in blockproc/blockprocinit.m. You can avoid calling it by passing additional 'nojava' flag.

1.1.2 LTFATSTOP - Stops the LTFAT toolbox

Usage

```
ltfatstop;
```

Description

ltfatstop removes all LTFAT subdirectories from the path.

1.1.3 LTFATHHELP - Help on the LTFAT toolbox

Usage

```
ltfathelp;
v=ltfathelp('version');
mlist=ltfathelp('modules');
```

Description

`ltfathelp` displays some general help on the LTFAT toolbox.

`ltfathelp('version')` returns the version number.

`ltfathelp('modules')` returns a cell array of installed modules and corresponding version numbers.

1.1.4 LTFATMEX - Compile Mex/Oct interfaces

Usage

```
ltfatmex;
ltfatmex(...);
```

Description

`ltfatmex` compiles the C backend in order to speed up the execution of the toolbox. The C backend is linked to Matlab and Octave through Mex and Octave C++ interfaces. Please see INSTALL-Matlab or INSTALL-Octave for the requirements.

The action of `ltfatmex` is determined by one of the following flags:

'compile'	Compile stuff. This is the default.
'clean'	Removes the compiled functions.
'test'	Run some small tests that verify that the compiled functions work.

The target to work on is determined by one of the following flags.

General LTFAT:

'lib'	Perform action on the LTFAT C library.
'mex'	Perform action on the mex / oct interfaces.
'gpc'	Perform action on the GPC code for use with MULACLAB
'auto'	Choose automatically which targets to work on from the previous ones based on the operation system etc. This is the default.

Block-processing framework related:

'playrec'	Perform action on the playrec code for use with real-time block streaming framework.
'java'	Perform compilation of JAVA classes into the bytecode. The classes makes the GUI for the blockproc. framework.

Other:

'verbose' Print action details.

'debug' Build a debug version. This will disable compiler optimizations and include debug symbols.

1.1.5 LTFATBASEPATH - The base path of the LTFAT installation

Usage

```
bp = ltfatbasepath;
```

Description

`ltfatbasepath` returns the top level directory in which the LTFAT files are installed.

1.1.6 ISOCTAVE - True if the operating environment is octave

Usage

```
t=isoctave();
```

Description

`isoctave` returns 1 if the operating environment is Octave, otherwise it returns 0 (Matlab)

1.2 Parameter handling

1.2.1 LTFATARGHELPER - Parse arguments for LTFAT

Usage

```
[flags,varargout] = ltfatarghelper(posdepnames,definput,arglist,callfun);
```

Input parameters

posdepnames	Names of the position dependant parameters.
definput	Struct to define the allowed input
arglist	Commandline of the calling function (varargin)
callfun	Name of calling function (optional)

Output parameters

flags	Struct with information about flags.
keyvals	Struct with key / values.
varargout	The position dependant pars. properly initialized

Description

`[flags,keyvals]=ltfatarghelper(posdepnames,definput,arglist)` assists in parsing input parameters for a function in LTFAT. Parameters come in four categories:

- Position dependant parameters. These must not be strings. These are the first parameters passed to a function, and they are really just a short way of specifying key/value pairs. See below.
- Flags. These are single string appearing after the position dependant parameters.
- Key/value pairs. The key is always a string followed by the value, which can be anything.
- Expansions. These appear as flags, that expand into a pre-defined list of parameters. This is a short-hand way of specifying standard sets of flags and key/value pairs.

The parameters are parsed in order, so parameters appearing later in varargin will override previously set values.

The following example for calling `ltfatarghelper` is taken from dgt:

```
definput.keyvals.L=[];
definput.flags.phase={'freqinv','timeinv'};
[flags,kv]=ltfatarghelper({'L'},definput,varargin);
```

The first line defines a key/value pair with the key '`L`' having an initial value of [] (the empty matrix).

The second line defines a group of flags by the name of `phase`. The group `phase` contains the flags '`freqinv`' and '`timeinv`', which can both be specified on the command line by the user. The group-name `phase` is just for internal use, and does not appear to the user. The flag mentioned first in the list will be selected by default, and only one flag in a group can be selected at any time. A group can contain as many flags as desired.

The third line is the actual call to `ltfatarghelper` which defines the output `flags` and `kv`. The input `{'L'}` indicates that the value of the parameter '`L`' can also be given as the very first value in `varargin`.

The output struct `kv` contains the key/value pairs, so the value associated to '`L`' is stored in `kv.L`.

The output struct `flags` contains information about the flags chosen by the user. The value of `flags.phase` will be set to the selected flag in the group `phase` and additionally, the value of `flags.do_timeinv` will be 1 if '`timeinv`' was selected and 0 otherwise, and similarly for '`freqinv`'. This allows for easy checking of selected flags.

1.2.2 LTFATGETDEFAULTS - Get default parameters of function

`ltfatgetdefaults(fname)` returns the default parameters of the function `fname` as a cell array.

`ltfatgetdefaults('all')` returns all the set defaults.

1.2.3 LTFATSETDEFAULTS - Set default parameters of function

`ltfatsetdefaults(fname, ...)` sets the default parameters to be the parameters specified at the end of the list of input arguments.

`ltfatsetdefaults(fname)` clears any default parameters for the function `fname`.

`ltfatsetdefaults('clearall')` clears all defaults from all functions.

1.2.4 SCALARDISTRIBUTE - Copy scalar to array shape for parameter handling

Usage

```
[...] = scalardistribute(...);
```

Description

`[...] = scalardistribute(...)` copies the input parameters to the output parameters.

- If one of the input parameters is an array, all the output parameters will be column vectors containing the same number of elements. If one of the other input parameters is a scalar, it will be replicated to the correct length. This allows a scalar value to be repeated for all conditions.
- If two or more input parameters are arrays, they must have the exact same size. They will be converted to vectors and returned in the output parameters. This allows two arrays to co-vary at the same time.

This operator is useful for sanitizing input parameters: The user is allowed to enter scalars or arrays as input parameters. These inputs are in turn passed to `scaldardistribute`, which makes sure that the arrays have the same shape, and that scalars are replicated. The user of `scaldardistribute` can now generate conditions based on all the parameters, and be sure they have the right sizes.

As an example, consider:

```
[a,b,c] = scalardistribute(1,[2,3],[4,5])
```

This code produces the following output:

```
a =
```

```
1
1
```

```
b =
```

```
2
3
```

```
c =
```

```
4
5
```

1.3 Graphical user interfaces

1.3.1 MULACLAB - Graphical interface for audio processing using frame multipliers

Usage

```
mulaclab;
```

Description

When starting the interface, the user is asked to choose the processed signal, named original signal in the interface. Possible signals are .wav files and .mat files containing decompositions preliminarily saved using the mulaclab interface. The interface only handles monochannel signals. So for a multichannel .wav files, the first channel is used as the original signal.

After choosing the original signal, the user is presented with the main interface. This interface is divided in two areas:

- The right part of the figure contains the visualizations, which represent the spectrograms of the original and modified signals.

The 'Original signal' visualization is used to display the original signal spectrogram and to graphically define the symbol of the Gabor multiplier that will be applied on this signal.

The 'Overview of original signal' visualization also represents the original signal spectrogram and can be used for fast zooming and moving of the other visualizations. Zooming and moving is controlled by mouse interaction with the white rectangle displayed on this visualization.

The 'Modified signal' visualization is used to display the spectrogram of the modified signal after application of the multiplier.

It is possible to hide the 'Overview of original signal' and 'Modified signal' visualizations using the 'Visualization' menu.

- The left part of the figure contains panels with tools for user interaction.

The 'Audoplayer' panel contains the controls for audio playback of the original and modified signal.

The 'Visualization' panel contains tools used to adapt the display of the visualizations.

The 'Selection' panel contains tools and information concerning the multilayered selection used to graphically specify the symbol of the multiplier.

Known Matlab limitations:

- When using Matlab on Linux with multiple screens, there might be a Matlab bug preventing the display of the multiplier symbol. This can be solved by docking the figure.
- When using a Matlab version prior to 7.3 (R2006b), the rectangle displayed on the 'Overview of original signal' visualization is not automatically updated when using the zoom and pan tools of the 'Zoom' panel. It can be manually updated by re-clicking on the currently selected tool or by changing the current tool.

The Matlab Image Processing Toolbox is required by the mulaclab function.

MULACLAB uses the GPC library available from <http://www.cs.man.ac.uk/~toby/gpc/>. This library is distributed alongside LTFAT, but under different licensing conditions. Please see the `ltfat/thirdparty/gpc` file for the exact conditions.

Chapter 2

LTFAT - Gabor analysis

2.1 Basic Time/Frequency analysis

2.1.1 TCONV - Twisted convolution

Usage

```
h=tconv(f,g);
```

Description

`tconv(f,g)` computes the twisted convolution of the square matrices f and g .

Let $h = tconv(f, g)$ for f, g being $L \times L$ matrices. Then h is given by

$$h(m+1, n+1) = \sum_{l=0}^{L-1} \sum_{k=0}^{L-1} f(k+1, l+1) g(m-k+1, n-l+1) e^{-2\pi i(m-k)l/L}$$

where $m - k$ and $n - l$ are computed modulo L .

If both f and g are of class `sparse` then h will also be a sparse matrix. The number of non-zero elements of h is usually much larger than the numbers for f and g . Unless f and g are very sparse, it can be faster to convert them to full matrices before calling `tconv`.

The routine ?? can be used to calculate an inverse convolution. Define h and r by:

```
h=tconv(f,g);
r=tconv(spreadinv(f),h);
```

then r is equal to g .

2.1.2 DSFT - Discrete Symplectic Fourier Transform

Usage

```
C=dsft(F);
```

Description

`dsft(F)` computes the discrete symplectic Fourier transform of F . F must be a matrix or a 3D array. If F is a 3D array, the transformation is applied along the first two dimensions.

Let F be a $K \times L$ matrix. Then the DSFT of F is given by

$$C(m+1, n+1) = \frac{1}{\sqrt{KL}} \sum_{l=0}^{L-1} \sum_{k=0}^{K-1} F(k+1, l+1) e^{2\pi i(kn/K - lm/L)}$$

for $m = 0, \dots, L-1$ and $n = 0, \dots, K-1$.

The `dsft` is its own inverse.

References: [31]

2.1.3 ZAK - Zak transform

Usage

```
c=zak(f,a);
```

Description

`zak(f,a)` computes the Zak transform of f with parameter a . The coefficients are arranged in an $a \times L/a$ matrix, where L is the length of f .

If f is a matrix then the transformation is applied to each column. This is then indexed by the third dimension of the output.

Assume that $c = \text{zak}(f,a)$, where f is a column vector of length L and $N = L/a$. Then the following holds for $m = 0, \dots, a-1$ and $n = 0, \dots, N-1$

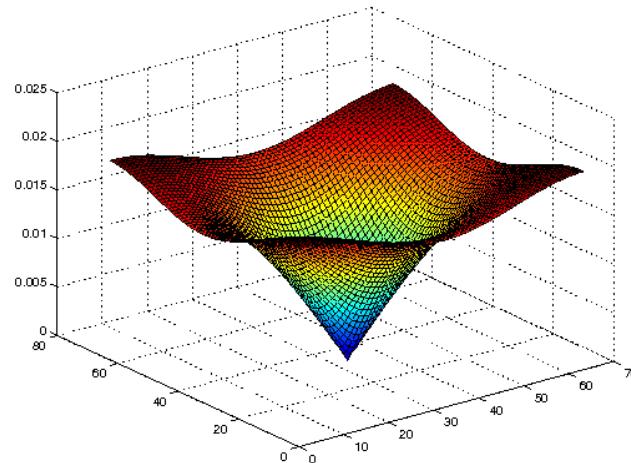
$$c(m+1, n+1) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} f(m-ka+1) e^{2\pi i nk/M}$$

Examples:

This figure shows the absolute value of the Zak-transform of a Gaussian. Notice that the Zak-transform is 0 in only a single point right in the middle of the plot

```
a=64;
L=a^2;
g=pgauss(L);
zg=zak(g,a);

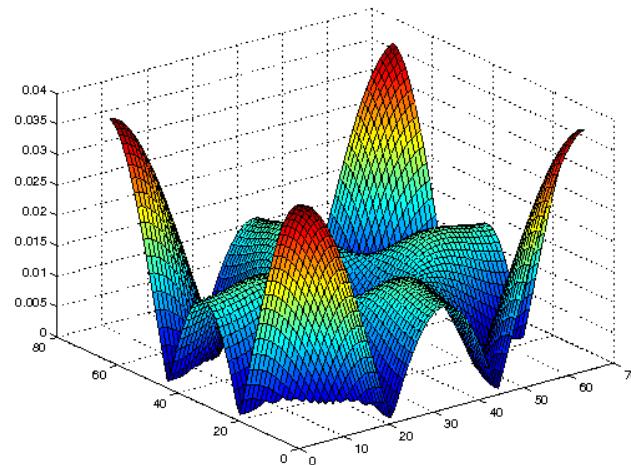
surf(abs(zg));
```



This figure shows the absolute value of the Zak-transform of a 4th order Hermite function. Notice how the Zak transform of the Hermite functions is zero on a circle centered on the corner

```
a=64;
L=a^2;
g=pherm(L,4);
zg=zak(g,a);

surf(abs(zg));
```



References: [47], [16]

2.1.4 IZAK - Inverse Zak transform

Usage

```
f=izak(c);
```

Description

`izak(c)` computes the inverse Zak transform of c . The parameter of the Zak transform is deduced from the size of c .

References: [47], [16]

2.1.5 COL2DIAG - Move columns of a matrix to diagonals

Usage

```
cout=col2diag(cin);
```

Description

`col2diag(cin)` will rearrange the elements in the square matrix `cin` so that columns of `cin` appears as diagonals. Column number n will appear as diagonal number $-n$ and $L - n$, where L is the size of the matrix.

The function is its own inverse.

`col2diag` performs the underlying coordinate transform for spreading function and Kohn-Nirenberg calculus in the finite, discrete setting.

2.1.6 S0NORM - S₀-norm of signal

Usage

```
y = s0norm(f);
y = s0norm(f, ...);
```

Description

`s0norm(f)` computes the S_0 -norm of a vector.

If the input is a matrix or ND-array, the S_0 -norm is computed along the first (non-singleton) dimension, and a vector of values is returned.

WARNING: The S_0 -norm is computed by computing a full Short-time Fourier transform of a signal, which can be quite time-consuming. Use this function with care for long signals.

`s0norm` takes the following flags at the end of the line of input parameters:

<code>'dim',d</code>	Work along specified dimension. The default value of [] means to work along the first non-singleton one.
<code>'rel'</code>	Return the result relative to the l^2 norm (the energy) of the signal.

2.2 Gabor systems

2.2.1 DGT - Discrete Gabor transform

Usage

```
c=dgt(f,g,a,M);
c=dgt(f,g,a,M,L);
c=dgt(f,g,a,M,'lt',lt);
[c,Ls]=dgt(...);
```

Input parameters

<code>f</code>	Input data.
<code>g</code>	Window function.
<code>a</code>	Length of time shift.
<code>M</code>	Number of channels.
<code>L</code>	Length of transform to do.
<code>lt</code>	Lattice type (for non-separable lattices).

Output parameters

<code>c</code>	$M \times N$ array of coefficients.
<code>Ls</code>	Length of input signal.

Description

`dgt(f,g,a,M)` computes the Gabor coefficients (also known as a windowed Fourier transform) of the input signal f with respect to the window g and parameters a and M . The output is a vector/matrix in a rectangular layout.

The length of the transform will be the smallest multiple of a and M that is larger than the signal. f will be zero-extended to the length of the transform. If f is a matrix, the transformation is applied to each column. The length of the transform done can be obtained by `L=size(c,2)*a;`

The window g may be a vector of numerical values, a text string or a cell array. See the help of `gabwin` for more details.

`dgt(f,g,a,M,L)` computes the Gabor coefficients as above, but does a transform of length L . f will be cut or zero-extended to length L before the transform is done.

`[c,Ls]=dgt(f,g,a,M)` or `[c,Ls]=dgt(f,g,a,M,L)` additionally returns the length of the input signal f . This is handy for reconstruction:

```
[c,Ls]=dgt(f,g,a,M);
fr=idgt(c,gd,a,Ls);
```

will reconstruct the signal f no matter what the length of f is, provided that gd is a dual window of g .

`[c, Ls, g]=dgt(...)` additionally outputs the window used in the transform. This is useful if the window was generated from a description in a string or cell array.

The Discrete Gabor Transform is defined as follows: Consider a window g and a one-dimensional signal f of length L and define $N = L/a$. The output from `c=dgt(f, g, a, M)` is then given by:

$$c(m+1, n+1) = \sum_{l=0}^{L-1} f(l+1) \overline{g(l-an+1)} e^{-2\pi i l m / M}$$

where $m = 0, \dots, M-1$ and $n = 0, \dots, N-1$ and $l-an$ is computed modulo L .

Non-separable lattices:

`dgt(f, g, a, M, 'lt', lt)` computes the DGT for a non-separable lattice given by the time-shift a , number of channels M and lattice type lt . Please see the help of `matrix2lattice` for a precise description of the parameter lt .

The non-separable discrete Gabor transform is defined as follows: Consider a window g and a one-dimensional signal f of length L and define $N = L/a$. The output from `c=dgt(f, g, a, M, L, lt)` is then given by:

$$c(m+1, n+1) = \sum_{l=0}^{L-1} f(l+1) \overline{g(l-an+1)} e^{-2\pi i l (m+w(n)) / M}$$

where $m = 0, \dots, M-1$ and $n = 0, \dots, N-1$ and $l-an$ are computed modulo L . The additional offset w is given by $w(n) = \text{mod}(n \cdot lt_1, lt_2)/lt_2$ in the formula above.

Additional parameters:

`dgt` takes the following flags at the end of the line of input arguments:

'freqinv'	Compute a DGT using a frequency-invariant phase. This is the default convention described above.
'timeinv'	Compute a DGT using a time-invariant phase. This convention is typically used in FIR-filter algorithms.

Examples:

In the following example we create a Hermite function, which is a complex-valued function with a circular spectrogram, and visualize the coefficients using both `imagesc` and `plotdgt`:

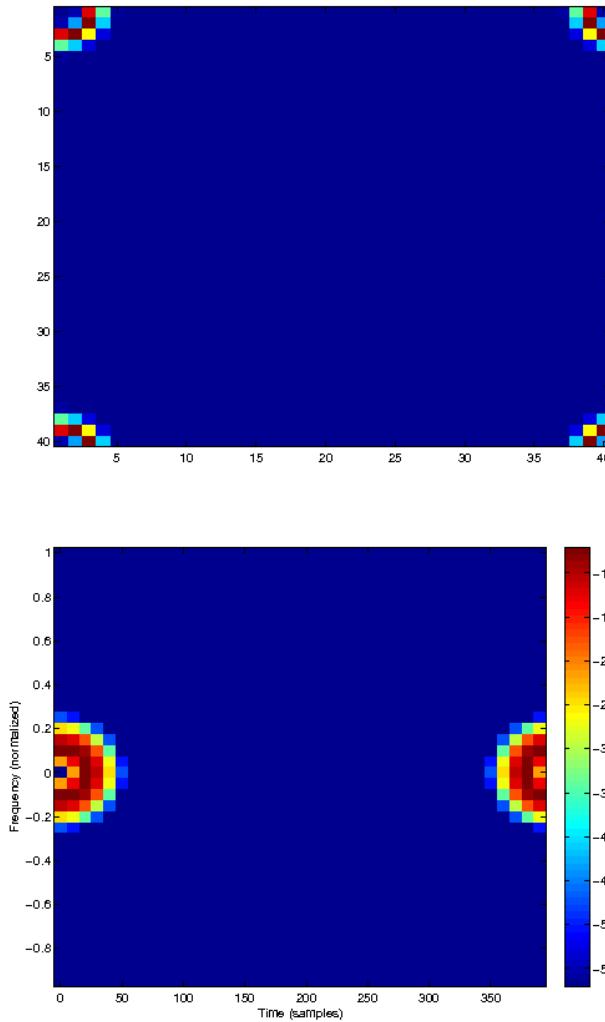
```

a=10;
M=40;
L=a*M;
h=pherm(L, 4); % 4th order hermite function.
c=dgt(h, 'gauss', a, M);

% Simple plot: The squared modulus of the coefficients on
% a linear scale
figure(1);
imagesc(abs(c).^2);

% Better plot: zero-frequency is displayed in the middle,
% and the coefficients are show on a logarithmic scale.
figure(2);
plotdgt(c, a, 'dynrange', 50);

```



References: [32], [40]

2.2.2 IDGT - Inverse discrete Gabor transform

Usage

```
f=idgt(c,g,a);
f=idgt(c,g,a,Ls);
f=idgt(c,g,a,Ls,lt);
```

Input parameters

c	Array of coefficients.
g	Window function.
a	Length of time shift.
Ls	Length of signal.
lt	Lattice type (for non-separable lattices)

Output parameters

f	Signal.
----------	---------

Description

`idgt(c, g, a)` computes the Gabor expansion of the input coefficients c with respect to the window g and time shift a . The number of channels is deduced from the size of the coefficients c .

`idgt(c, g, a, Ls)` does as above but cuts or extends f to length Ls .

`[f, g]=idgt(...)` additionally outputs the window used in the transform. This is useful if the window was generated from a description in a string or cell array.

For perfect reconstruction, the window used must be a dual window of the one used to generate the coefficients.

The window g may be a vector of numerical values, a text string or a cell array. See the help of `gabwin` for more details.

If g is a row vector, then the output will also be a row vector. If c is 3-dimensional, then `idgt` will return a matrix consisting of one column vector for each of the TF-planes in c .

Assume that `f=idgt(c, g, a, L)` for an array c of size $M \times N$. Then the following holds for $k = 0, \dots, L-1$:

$$f(l+1) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} c(m+1, n+1) e^{2\pi i m l / M} g(l - an + 1)$$

Non-separable lattices:

`idgt(c, g, a, 'lt', lt)` computes the Gabor expansion of the input coefficients c with respect to the window g , time shift a and lattice type lt . Please see the help of `matrix2lattice` for a precise description of the parameter lt .

Assume that `f=dgt(c, g, a, L, lt)` for an array c of size $M \times N$. Then the following holds for $k = 0, \dots, L-1$:

$$f(l+1) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} c(m+1, n+1) e^{2\pi i m l / M} g(l - an + 1)$$

Additional parameters:

`idgt` takes the following flags at the end of the line of input arguments:

'freqinv'	Compute an IDGT using a frequency-invariant phase. This is the default convention described above.
'timeinv'	Compute an IDGT using a time-invariant phase. This convention is typically used in FIR-filter algorithms.

Examples:

The following example demonstrates the basic principles for getting perfect reconstruction (short version):

```

f=greasy;           % test signal
a=32;              % time shift
M=64;              % frequency shift
gs={'blackman',128}; % synthesis window
ga={'dual',gs};    % analysis window

[c,Ls]=dgt(f,ga,a,M);    % analysis

% ... do interesting stuff to c at this point ...

r=idgt(c,gs,a,Ls); % synthesis

norm(f-r)            % test

```

This code produces the following output:

```
ans =
5.971590714093986e-15
```

The following example does the same as the previous one, with an explicit construction of the analysis and synthesis windows:

```
f=greasy; % test signal
a=32; % time shift
M=64; % frequency shift
Ls=length(f); % signal length

% Length of transform to do
L=dgtlength(Ls,a,M);

% Analysis and synthesis window
gs=firwin('blackman',128);
ga=gabdual(gs,a,M,L);

c=dgt(f,ga,a,M); % analysis

% ... do interesting stuff to c at this point ...

r=idgt(c,gs,a,Ls); % synthesis

norm(f-r) % test
```

This code produces the following output:

```
ans =
6.028280895685831e-15
```

2.2.3 ISGRAM - Spectrogram inversion

Usage

```
f=isgram(c,g,a);
f=isgram(c,g,a,Ls);
[f,relres,iter]=isgram(...);
```

Input parameters

c	Array of coefficients.
g	Window function.
a	Length of time shift.
Ls	length of signal.

Output parameters

f	Signal.
relres	Vector of residuals.
iter	Number of iterations done.

Description

`isgram(s,g,a)` attempts to invert a spectrogram computed by

```
s = abs(dgt(f,g,a,M)).^2;
```

using an iterative method.

`isgram(c,g,a,Ls)` does as above but cuts or extends f to length L_s .

If the phase of the spectrogram is known, it is much better to use `idgt`.

`[f,relres,iter]=isgram(...)` additionally return the residuals in a vector `relres` and the number of iteration steps `iter`.

Generally, if the spectrogram has not been modified, the iterative algorithm will converge slowly to the correct result. If the spectrogram has been modified, the algorithm is not guaranteed to converge at all.

`isgram` takes the following parameters at the end of the line of input arguments:

<code>'lt',lt</code>	Specify the lattice type. See the help on <code>matrix2latticetype</code> .
<code>'zero'</code>	Choose a starting phase of zero. This is the default
<code>'rand'</code>	Choose a random starting phase.
<code>'int'</code>	Construct a starting phase by integration. Only works for Gaussian windows.
<code>'griflim'</code>	Use the Griffin-Lim iterative method, this is the default.
<code>'bfgs'</code>	Use the limited-memory Broyden Fletcher Goldfarb Shanno (BFGS) method.
<code>'tol',t</code>	Stop if relative residual error is less than the specified tolerance.
<code>'maxit',n</code>	Do at most n iterations.
<code>'print'</code>	Display the progress.
<code>'quiet'</code>	Don't print anything, this is the default.
<code>'printstep',p</code>	If <code>'print'</code> is specified, then print every p'th iteration. Default value is p=10;

The BFGS method makes use of the `minFunc` software. To use the BFGS method, please install the `minFunc` software from: <http://www.cs.ubc.ca/~schmidtm/Software/minFunc.html>.

Examples:

To reconstruct the phase of 'greasy', use the following:

```
% Setup the problem and the coefficients
f=greasy;
g='gauss';
a=20; M=200;
c=dgt(f,g,a,M);
s=abs(c).^2;
theta=angle(c);

% Reconstruct and get spectrogram and angle
r=isgram(s,g,a);
c_r=dgt(r,g,a,M);
s_r=abs(c_r).^2;
theta_r=angle(c_r);

% Compute the angular difference
```

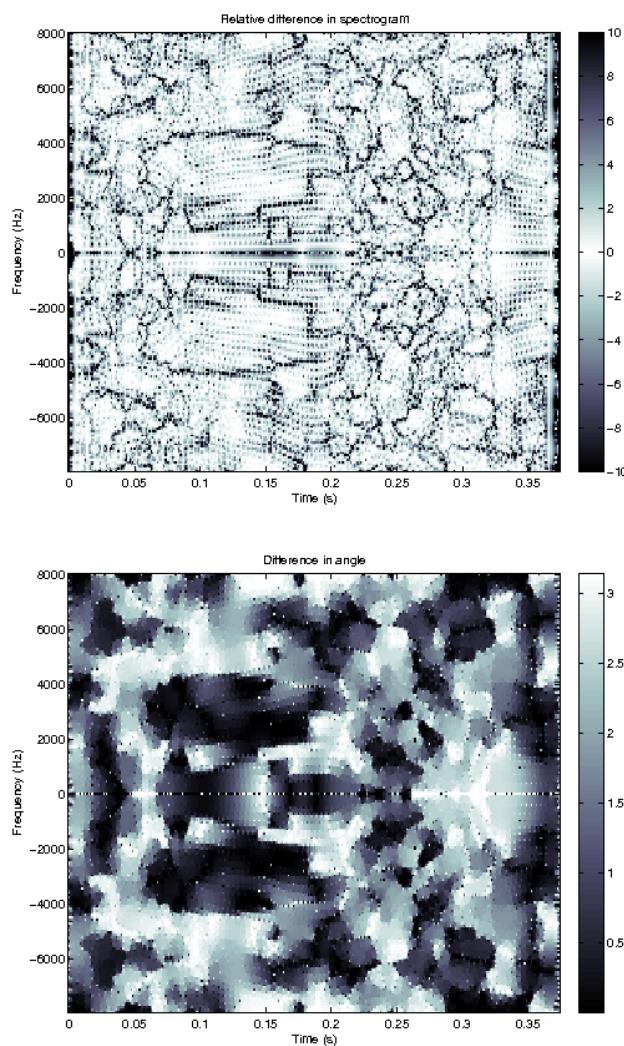
```

d1=abs(theta-theta_r);
d2=2*pi-d1;
anglediff=min(d1,d2);

% Plot the difference in spectrogram and phase
figure(1);
plotdgt(s./s_r,a,16000,'clim',[ -10,10]);
colormap([bone;flipud(bone)])
title('Relative difference in spectrogram');

figure(2);
plotdgt(anglediff,a,16000,'lin');
colormap(bone);
title('Difference in angle');

```



References: [39], [27], [59]

2.2.4 ISGRAMREAL - Spectrogram inversion (real signal)

Usage

```

f=isgramreal(s,g,a,M);
f=isgramreal(s,g,a,M,Ls);

```

```
[f, relres, iter]=isgramreal(...);
```

Input parameters

c	Array of coefficients.
g	Window function.
a	Length of time shift.
M	Number of channels.
Ls	length of signal.

Output parameters

f	Signal.
relres	Vector of residuals.
iter	Number of iterations done.

Description

`isgramreal(s, g, a, M)` attempts to invert a spectrogram computed by

```
s = abs(dgtreal(f, g, a, M)).^2;
```

by an iterative method.

`isgramreal(s, g, a, M, Ls)` does as above but cuts or extends f to length Ls .

If the phase of the spectrogram is known, it is much better to use `dgtreal`

`f, relres, iter]=isgramreal(...)` additionally returns the residuals in a vector `relres` and the number of iteration steps done.

Generally, if the spectrogram has not been modified, the iterative algorithm will converge slowly to the correct result. If the spectrogram has been modified, the algorithm is not guaranteed to converge at all.

`isgramreal` takes the following parameters at the end of the line of input arguments:

'lt',lt	Specify the lattice type. See the help on <code>matrix2latticetype</code> . Only the rectangular or quinque lattices can be specified.
'zero'	Choose a starting phase of zero. This is the default
'rand'	Choose a random starting phase.
'int'	Construct a starting phase by integration. Only works for Gaussian windows.
'griflim'	Use the Griffin-Lim iterative method, this is the default.
'bfgs'	Use the limited-memory Broyden Fletcher Goldfarb Shanno (BFGS) method.
'tol',t	Stop if relative residual error is less than the specified tolerance.
'maxit',n	Do at most n iterations.
'print'	Display the progress.
'quiet'	Don't print anything, this is the default.
'printstep',p	If 'print' is specified, then print every p'th iteration. Default value is p=10.

The BFGS method makes use of the `minFunc` software. To use the BFGS method, please install the `minFunc` software from: <http://www.cs.ubc.ca/~schmidtm/Software/minFunc.html>.

References: [39], [27], [59]

2.2.5 DGT2 - 2-D Discrete Gabor transform

Usage

```
c=dgt2(f,g,a,M);
c=dgt2(f,g1,g2,[a1,a2],[M1,M2]);
c=dgt2(f,g1,g2,[a1,a2],[M1,M2],[L1,L2]);
[c,Ls]=dgt2(f,g1,g2,[a1,a2],[M1,M2]);
[c,Ls]=dgt2(f,g1,g2,[a1,a2],[M1,M2],[L1,L2]);
```

Input parameters

f	Input data, matrix.
g, g1, g2	Window functions.
a, a1, a2	Length of time shifts.
M, M1, M2	Number of modulations.
L1, L2	Length of transform to do

Output parameters

c	array of coefficients.
Ls	Original size of input matrix.

Description

`dgt2(f,g,a,M)` will calculate a separable two-dimensional discrete Gabor transformation of the input signal f with respect to the window g and parameters a and M .

For each dimension, the length of the transform will be the smallest possible that is larger than the length of the signal along that dimension. f will be appropriately zero-extended.

`dgt2(f,g,a,M,L)` computes a Gabor transform as above, but does a transform of length L along each dimension. f will be cut or zero-extended to length L before the transform is done.

`[c,Ls]=dgt2(f,g,a,M)` or `[c,Ls]=dgt2(f,g,a,M,L)` additionally returns the length of the input signal f . This is handy for reconstruction:

```
[c,Ls]=dgt2(f,g,a,M);
fr=idgt2(c,gd,a,Ls);
```

will reconstruct the signal f no matter what the size of f is, provided that gd is a dual window of g .

`dgt2(f,g1,g2,a,M)` makes it possible to use a different window along the two dimensions.

The parameters a , M , L and Ls can also be vectors of length 2. In this case the first element will be used for the first dimension and the second element will be used for the second dimension.

The output c has 4 or 5 dimensions. The dimensions index the following properties:

1. Number of translation along 1st dimension of input.
2. Number of channel along 1st dimension of input
3. Number of translation along 2nd dimension of input.
4. Number of channel along 2nd dimension of input
5. Plane number, corresponds to 3rd dimension of input.

2.2.6 IDGT2 - 2D Inverse discrete Gabor transform

Usage

```
f=idgt2(c,g,a);
f=idgt2(c,g1,g2,a);
f=idgt2(c,g1,g2,[a1 a2]);
f=idgt2(c,g,a,Ls);
f=idgt2(c,g1,g2,a,Ls);
f=idgt2(c,g1,g2,[a1 a2],Ls);
```

Input parameters

c	Array of coefficients.
g, g1, g2	Window function(s).
a, a1, a2	Length(s) of time shift.
Ls	Length(s) of reconstructed signal (optional).

Output parameters

f	Output data, matrix.
----------	----------------------

Description

`idgt2(c,g,a,M)` will calculate a separable two dimensional inverse discrete Gabor transformation of the input coefficients c using the window g and parameters a , along each dimension. The number of channels is deduced from the size of the coefficients c .

`idgt2(c,g1,g2,a)` will do the same using the window $g1$ along the first dimension, and window $g2$ along the second dimension.

`idgt2(c,g,a,Ls)` or `idgt2(c,g1,g2,a,Ls)` will cut the signal to size Ls after the transformation is done.

The parameters a and Ls can also be vectors of length 2. In this case the first element will be used for the first dimension and the second element will be used for the second dimension.

2.2.7 DGTREAL - Discrete Gabor transform for real-valued signals

Usage

```
c=dgtreal(f,g,a,M);
c=dgtreal(f,g,a,M,L);
[c,Ls]=dgtreal(f,g,a,M);
[c,Ls]=dgtreal(f,g,a,M,L);
```

Input parameters

f	Input data
g	Window function.
a	Length of time shift.
M	Number of modulations.
L	Length of transform to do.

Output parameters

c $M \times N$ array of coefficients.

Ls Length of input signal.

Description

`dgtreal(f, g, a, M)` computes the Gabor coefficients (also known as a windowed Fourier transform) of the real-valued input signal f with respect to the real-valued window g and parameters a and M . The output is a vector/matrix in a rectangular layout.

As opposed to `dgt` only the coefficients of the positive frequencies of the output are returned. `dgtreal` will refuse to work for complex valued input signals.

The length of the transform will be the smallest multiple of a and M that is larger than the signal. f will be zero-extended to the length of the transform. If f is a matrix, the transformation is applied to each column. The length of the transform done can be obtained by `L=size(c, 2)*a`.

The window g may be a vector of numerical values, a text string or a cell array. See the help of `gabwin` for more details.

`dgtreal(f, g, a, M, L)` computes the Gabor coefficients as above, but does a transform of length L . f will be cut or zero-extended to length L before the transform is done.

`[c, Ls]=dgtreal(f, g, a, M)` or `[c, Ls]=dgtreal(f, g, a, M, L)` additionally returns the length of the input signal f . This is handy for reconstruction:

```
[c, Ls]=dgtreal(f, g, a, M);
fr=idgtreal(c, gd, a, M, Ls);
```

will reconstruct the signal f no matter what the length of f is, provided that gd is a dual window of g .

`[c, Ls, g]=dgtreal(...)` additionally outputs the window used in the transform. This is useful if the window was generated from a description in a string or cell array.

See the help on `dgt` for the definition of the discrete Gabor transform. This routine will return the coefficients for channel frequencies from 0 to `floor(M/2)`.

`dgtreal` takes the following flags at the end of the line of input arguments:

'freqinv' Compute a `dgtreal` using a frequency-invariant phase. This is the default convention described in the help for `dgt`.

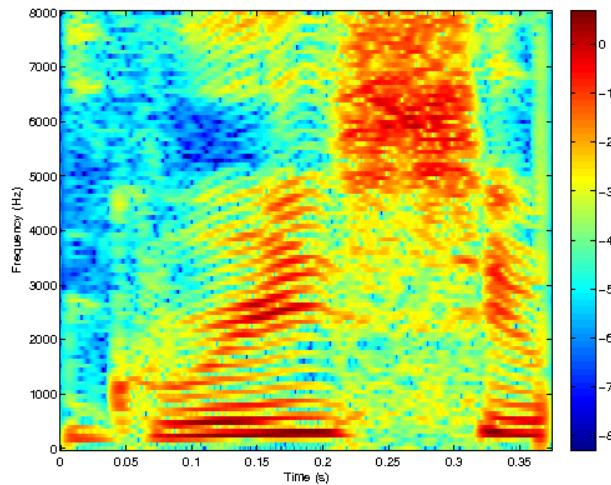
'timeinv' Compute a `dgtreal` using a time-invariant phase. This convention is typically used in filter bank algorithms.

`dgtreal` can be used to manually compute a spectrogram, if you want full control over the parameters and want to capture the output

```
f=greasy; % Input test signal
fs=16000; % The sampling rate of this particular test signal
a=10; % Downsampling factor in time
M=200; % Total number of channels, only 101 will be computed

% Compute the coefficients using a 20 ms long Hann window
c=dgtreal(f, {'hann', 0.02*fs}, a, M);

% Visualize the coefficients as a spectrogram
dynrange=90; % 90 dB dynamical range for the plotting
plotdgtreal(c, a, M, fs, dynrange);
```



References: [32], [40]

2.2.8 IDGTRREAL - Inverse discrete Gabor transform for real-valued signals

Usage

```
f=idgtreal(c,g,a,M);
f=idgtreal(c,g,a,M,Ls);
```

Input parameters

c	Array of coefficients.
g	Window function.
a	Length of time shift.
M	Number of channels.
Ls	length of signal.

Output parameters

f	Signal.
----------	---------

Description

`idgtreal(c, g, a, M)` computes the Gabor expansion of the input coefficients c with respect to the real-valued window g , time shift a and number of channels M . c is assumed to be the positive frequencies of the Gabor expansion of a real-valued signal.

It must hold that `size(c, 1) == floor(M/2) + 1`. Note that since the correct number of channels cannot be deduced from the input, `idgtreal` takes an additional parameter as opposed to `idgt`.

The window g may be a vector of numerical values, a text string or a cell array. See the help of `gabwin` for more details.

`idgtreal(c, g, a, M, Ls)` does as above but cuts or extends f to length Ls .

`[f, g] = idgtreal(...)` additionally outputs the window used in the transform. This is useful if the window was generated from a description in a string or cell array.

For perfect reconstruction, the window used must be a dual window of the one used to generate the coefficients.

If g is a row vector, then the output will also be a row vector. If c is 3-dimensional, then `idgtreal` will return a matrix consisting of one column vector for each of the TF-planes in c .

See the help on `idgt` for the precise definition of the inverse Gabor transform.

`idgtreal` takes the following flags at the end of the line of input arguments:

- | | |
|------------------|--|
| 'freqinv' | Use a frequency-invariant phase. This is the default convention described in the help for <code>dgt</code> . |
| 'timeinv' | Use a time-invariant phase. This convention is typically used in filter bank algorithms. |

Examples:

The following example demonstrates the basic principles for getting perfect reconstruction (short version):

```
f=greasy;           % test signal
a=32;              % time shift
M=64;              % frequency shift
gs={'blackman',128}; % synthesis window
ga={'dual',gs};    % analysis window

[c,Ls]=dgtreal(f,ga,a,M); % analysis

% ... do interesting stuff to c at this point ...

r=idgtreal(c,gs,a,M,Ls); % synthesis

norm(f-r)           % test
```

This code produces the following output:

```
ans =
5.926290334081506e-15
```

The following example does the same as the previous one, with an explicit construction of the analysis and synthesis windows:

```
f=greasy;           % test signal
a=32;              % time shift
M=64;              % frequency shift
Ls=length(f);      % signal length

% Length of transform to do
L=dgtlength(Ls,a,M);

% Analysis and synthesis window
gs=firwin('blackman',128);
ga=gabdual(gs,a,M,L);

c=dgtreal(f,ga,a,M); % analysis

% ... do interesting stuff to c at this point ...

r=idgtreal(c,gs,a,M,Ls); % synthesis

norm(f-r)           % test
```

This code produces the following output:

```
ans =
6.026604080005277e-15
```

2.2.9 GABWIN - Compute a Gabor window from text or cell array

Usage

```
[g,info] = gabwin(g,a,M,L);
```

Description

`[g,info]=gabwin(g,a,M,L)` computes a window that fits well with the specified number of channels M , time shift a and transform length L . The window itself is specified by a text description or a cell array containing additional parameters.

The window can be specified directly as a vector of numerical values. In this case, `gabwin` only checks assumptions about transform sizes etc.

`[g,info]=gabwin(g,a,M)` does the same, but the window must be a FIR window, as the transform length is unspecified.

`gabwin(g,a,M,L,lt)` or `gabwin(g,a,M,[],lt)` does as above but for a non-separable lattice specified by `lt`. Please see the help of `matrix2latticetype` for a precise description of the parameter `lt`.

The window can be specified as one of the following text strings:

'gauss'	Gaussian window fitted to the lattice, i.e. $tfr = a \cdot M / L$.
'dualgauss'	Canonical dual of Gaussian window.
'tight'	Tight window generated from a Gaussian.

In these cases, a long window is generated with a length of L .

It is also possible to specify one of the window names from `firwin`. In such a case, `gabwin` will generate the specified FIR window with a length of M .

The window can also be specified as cell array. The possibilities are:

- {'gauss', ...} Additional parameters are passed to `pgauss`.
- {'dual', ...} Canonical dual window of whatever follows. See the examples below.
- {'tight', ...} Canonical tight window of whatever follows.

It is also possible to specify one of the window names from `firwin` as the first field in the cell array. In this case, the remaining entries of the cell array are passed directly to `firwin`.

Some examples: To compute a Gaussian window of length L fitted for a system with time-shift a and M channels use:

```
g=gabwin('gauss',a,M,L);
```

To compute Gaussian window with equal time and frequency support irrespective of a and M :

```
g=gabwin({'gauss',1},a,M,L);
```

To compute the canonical dual of a Gaussian window fitted for a system with time-shift a and M channels:

```
gd=gabwin('gaussdual',a,M,L);
```

To compute the canonical tight window of the Gaussian window fitted for the system:

```
gd=gabwin({'tight','gauss'},a,M,L);
```

To compute the dual of a Hann window of length 20:

```
g=gabwin({'dual',{'hann',20}},a,M,L);
```

The structure `info` provides some information about the computed window:

info.gauss True if the window is a Gaussian.

info.tfr Time/frequency support ratio of the window. Set whenever it makes sense.

```
info.wasrow Input was a row window
info.isfir Input is an FIR window
info.isdual Output is the dual window of the auxiliary window.
info.istight Output is known to be a tight window.
info.auxinfo Info about auxiliary window.
info.g1 Length of window.
```

2.2.10 PROJKERN - Projection onto generating kernel space

Usage

```
cout=projkern(cin,a);
cout=projkern(cin,g,a);
cout=projkern(cin,ga,gs,a);
```

Input parameters

cin	Input coefficients
g	analysis/synthesis window
ga	analysis window
gs	synthesis window
a	Length of time shift.

Output parameters

cout	Output coefficients
-------------	---------------------

Description

`cout=projkern(cin,a)` projects a set of Gabor coefficients c onto the space of possible Gabor coefficients. This means that `cin` and `cout` synthesize to the same signal. A tight window generated from a Gaussian will be used for both analysis and synthesis.

The rationale for this function is as follows: Because the coefficient space of a Gabor frame is larger than the signal space (since the frame is redundant) then there are many coefficients that correspond to the same signal.

Therefore, you might desire to work with the coefficients `cin`, but you are in reality working with `cout`.

`cout=projkern(cin,g,a)` does the same, using the window `g` for analysis and synthesis.

`cout=projkern(cin,ga,gs,a)` does the same, but for different analysis `ga` and synthesis `gs` windows.

2.2.11 DGTLENGTH - DGT length from signal

Usage

```
L=dgtlength(Ls,a,M);
L=dgtlength(Ls,a,M,lt);
```

Description

`dgtlength(Ls, a, M)` returns the length of a Gabor system that is long enough to expand a signal of length L_s . Please see the help on `dgt` for an explanation of the parameters a and M .

If the returned length is longer than the signal length, the signal will be zero-padded by `dgt`.

A valid transform length must be divisible by both a and M . This means that the minimal admissible transform length is

```
Lsmallest = lcm(a, M);
```

and all valid transform lengths are multiples of $Lsmallest$

Non-separable lattices:

`dgtlength(Ls, a, M, lt)` does as above for a non-separable lattice with lattice-type lt . For non-separable lattices, there is the additional requirement on the transform length, that the structure of the lattice must be periodic. This gives a minimal transform length of

```
Lsmallest = lcm(a, M) * lt(2);
```

2.3 Wilson bases and WMDCT

2.3.1 DWILT - Discrete Wilson transform

Usage

```
c=dwilt(f, g, M);
c=dwilt(f, g, M, L);
[c, Ls]=dwilt(...);
```

Input parameters

f	Input data
g	Window function.
M	Number of bands.
L	Length of transform to do.

Output parameters

c	$2M \times N$ array of coefficients.
Ls	Length of input signal.

Description

`dwilt(f, g, M)` computes a discrete Wilson transform with M bands and window g .

The length of the transform will be the smallest possible that is larger than the signal. f will be zero-extended to the length of the transform. If f is a matrix, the transformation is applied to each column.

The window g may be a vector of numerical values, a text string or a cell array. See the help of `wilwin` for more details.

`dwilt(f, g, M, L)` computes the Wilson transform as above, but does a transform of length L . f will be cut or zero-extended to length L before the transform is done.

`[c, Ls]=dwilt(f, g, M)` or `[c, Ls]=dwilt(f, g, M, L)` additionally return the length of the input signal f . This is handy for reconstruction:

```
[c, Ls]=dwilt(f, g, M);
fr=idwilt(c, gd, M, Ls);
```

will reconstruct the signal f no matter what the length of f is, provided that gd is a dual Wilson window of g .

`[c, Ls, g]=dwilt(...)` additionally outputs the window used in the transform. This is useful if the window was generated from a description in a string or cell array.

A Wilson transform is also known as a maximally decimated, even-stacked cosine modulated filter bank.

Use the function `wil2rect` to visualize the coefficients or to work with the coefficients in the TF-plane.

Assume that the following code has been executed for a column vector f :

```
c=dwilt(f,g,M); % Compute a Wilson transform of f.
N=size(c,2)*2; % Number of translation coefficients.
```

The following holds for $m = 0, \dots, M-1$ and $n = 0, \dots, N/2-1$:

If $m = 0$:

$$c(1, n+1) = \sum_{l=0}^{L-1} f(l+1)g(l-2nM+1)$$

If m is odd and less than M

$$c(m+1, n+1) = \sqrt{2} \sum_{l=0}^{L-1} f(l+1) \sin\left(\pi \frac{m}{M} l\right) g(l-2nM+1)$$

$$c(m+M+1, n+1) = \sqrt{2} \sum_{l=0}^{L-1} f(l+1) \cos\left(\pi \frac{m}{M} l\right) g(l-(2n+1)M+1)$$

If m is even and less than M

$$c(m+1, n+1) = \sqrt{2} \sum_{l=0}^{L-1} f(l+1) \cos\left(\pi \frac{m}{M} l\right) g(l-2nM+1)$$

$$c(m+M+1, n+1) = \sqrt{2} \sum_{l=0}^{L-1} f(l+1) \sin\left(\pi \frac{m}{M} l\right) g(l-(2n+1)M+1)$$

if $m = M$ and M is even:

$$c(M+1, n+1) = \sum_{l=0}^{L-1} f(l+1)(-1)^l g(l-2nM+1)$$

else if $m = M$ and M is odd

$$c(M+1, n+1) = \sum_{k=0}^{L-1} f(l+1)(-1)^l g(l-(2n+1)M+1)$$

References: [14], [58], [26]

2.3.2 IDWILT - Inverse discrete Wilson transform

Usage

```
f=idwilt(c,g);
f=idwilt(c,g,Ls);
```

Input parameters

c $2M \times N$ array of coefficients.

g Window function.

Ls Final length of function (optional)

Output parameters

f	Input data
----------	------------

Description

`idwilt(c, g)` computes an inverse discrete Wilson transform with window g . The number of channels is deduced from the size of the coefficient array c .

The window g may be a vector of numerical values, a text string or a cell array. See the help of `wilwin` for more details.

`idwilt(f, g, Ls)` does the same, but cuts of zero-extend the final result to length L_s .

`[f, g]=idwilt(...)` additionally outputs the window used in the transform. This is usefull if the window was generated from a description in a string or cell array.

References: [14], [58]

2.3.3 DWILT2 - 2D Discrete Wilson transform**Usage**

```
c=dwilt2(f, g, M);
c=dwilt2(f, g1, g2, [M1, M2]);
c=dwilt2(f, g1, g2, [M1, M2], [L1, L2]);
[c, Ls]=dwilt2(f, g1, g2, [M1, M2], [L1, L2]);
```

Input parameters

f	Input data, matrix.
g, g1, g2	Window functions.
M, M1, M2	Number of bands.
L1, L2	Length of transform to do.

Output parameters

c	array of coefficients.
Ls	Original size of input matrix.

Description

`dwilt2(f, g, M)` calculates a two dimensional discrete Wilson transform of the input signal f using the window g and parameter M along each dimension.

For each dimension, the length of the transform will be the smallest possible that is larger than the length of the signal along that dimension. f will be appropriately zero-extended.

All windows must be whole-point even.

`dwilt2(f, g, M, L)` computes a Wilson transform as above, but does a transform of length L along each dimension. f will be cut or zero-extended to length L before the transform is done.

`[c, Ls]=dwilt(f, g, M)` or `[c, Ls]=dwilt(f, g, M, L)` additionally returns the length of the input signal f . This is handy for reconstruction.

`c=dwilt2(f, g1, g2, M)` makes it possible to use a different window along the two dimensions.

The parameters L , M and L_s can also be vectors of length 2. In this case the first element will be used for the first dimension and the second element will be used for the second dimension.

The output c has 4 or 5 dimensions. The dimensions index the following properties:

1. Number of translations along 1st dimension of input.
2. Number of channels along 1st dimension of input
3. Number of translations along 2nd dimension of input.

4. Number of channels along 2nd dimension of input
5. Plane number, corresponds to 3rd dimension of input.

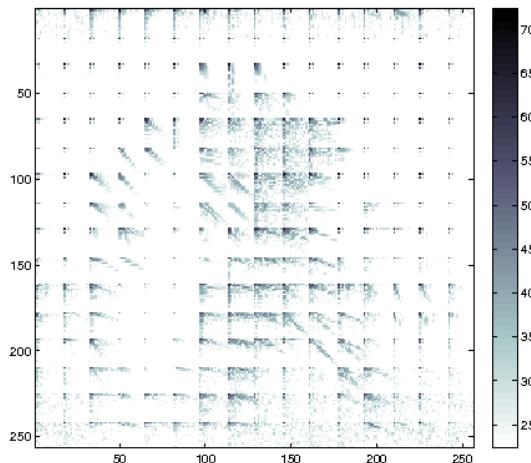
Examples:

The following example visualize the `dwilt2` coefficients of a test image. For clarity, only the 50 dB largest coefficients are show::

```
c=dwilt2(cameraman,'itersine',16);
c=reshape(c,256,256);

figure(1);
imagesc(cameraman), colormap(gray), axis('image');

figure(2);
cc=dynlimit(20*log10(abs(c)),50);
imagesc(cc), colormap(flipud(bone)), axis('image'), colorbar;
```



2.3.4 IDWILT2 - 2D Inverse Discrete Wilson transform

Usage

```
f=idwilt2(c,g);
```

```
f=idwilt2(c,g1,g2);
f=idwilt2(c,g1,g2,Ls);
```

Input parameters

c	Array of coefficients.
g, g1, g2	Window functions.
Ls	Size of reconstructed signal.

Output parameters

f	Output data, matrix.
----------	----------------------

Description

`idwilt2(c, g)` calculates a separable two dimensional inverse discrete Wilson transformation of the input coefficients c using the window g . The number of channels is deduced from the size of the coefficients c .

`idwilt2(c, g1, g2)` does the same using the window $g1$ along the first dimension, and window $g2$ along the second dimension.

`idwilt2(c, g1, g2, Ls)` cuts the signal to size Ls after the transformation is done.

2.3.5 WMDCT - Windowed MDCT transform**Usage**

```
c=wmdct(f,g,M);
c=wmdct(f,g,M,L);
[c,Ls]=wmdct(...);
```

Input parameters

f	Input data
g	Window function.
M	Number of bands.
L	Length of transform to do.

Output parameters

c	$M \times N$ array of coefficients.
Ls	Length of input signal.

Description

`wmdct(f, g, M)` computes a Windowed Modified Discrete Cosine Transform with M bands and window g .

The length of the transform will be the smallest possible that is larger than the signal. f will be zero-extended to the length of the transform. If f is a matrix, the transformation is applied to each column. g must be whole-point even.

The window g may be a vector of numerical values, a text string or a cell array. See the help of `wilwin` for more details.

`wmdct(f, g, M, L)` computes the MDCT transform as above, but does a transform of length L . f will be cut or zero-extended to length L before the transform is done.

`[c, Ls] = wmdct(f, g, M)` or `[c, Ls] = wmdct(f, g, M, L)'` additionally returns the length of the input signal f . This is handy for reconstruction:

```
[c, Ls] = wmdct(f, g, M);
fr = iwmdct(c, gd, M, Ls);
```

will reconstruct the signal f no matter what the length of f is, provided that gd is a dual Wilson window of g .

$[c, Ls, g] = wmdct(\dots)$ additionally outputs the window used in the transform. This is useful if the window was generated from a description in a string or cell array.

The WMDCT is sometimes known as an odd-stacked cosine modulated filter bank. The WMDCT defined by this routine is slightly different from the most common definition of the WMDCT, in order to be able to use the same window functions as the Wilson transform.

Assume that the following code has been executed for a column vector f of length L :

```
c = wmdct(f, g, M); % Compute the WMDCT of f.
N = size(c, 2); % Number of translation coefficients.
```

The following holds for $m = 0, \dots, M - 1$ and $n = 0, \dots, N - 1$:

If $m + n$ is even:

$$c(m+1, n+1) = \sqrt{2} \sum_{l=0}^{L-1} f(l+1) \cos\left(\frac{\pi}{M}\left(m+\frac{1}{2}\right)l + \frac{\pi}{4}\right) g(l-nM+1)$$

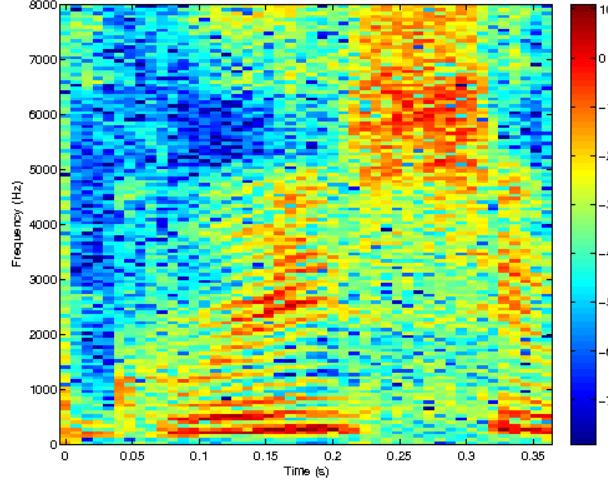
If $m + n$ is odd:

$$c(m+1, n+1) = \sqrt{2} \sum_{l=0}^{L-1} f(l+1) \sin\left(\frac{\pi}{M}\left(m+\frac{1}{2}\right)l + \frac{\pi}{4}\right) g(l-nM+1)$$

Examples:

The following example shows the WMDCT coefficients (128 channels) of the greasy test signal:

```
fs = 16000; % Sampling rate
c = wmdct(greasy, {'hann', 0.02*fs}, 128);
plotwmdct(c, fs, 90);
```



Compare the visual difference with the redundant expansion of the same signal given in the example of the dgtreal function.

References: [73], [74], [64], [15]

2.3.6 IWMDCT - Inverse MDCT

Usage

```
f = iwmdct(c, g);
f = iwmdct(c, g, Ls);
```

Input parameters

c	M*N array of coefficients.
g	Window function.
Ls	Final length of function (optional)

Output parameters

f	Input data
----------	------------

Description

`iwmdct (c, g)` computes an inverse windowed MDCT with window g . The number of channels is deduced from the size of the coefficient array c .

The window g may be a vector of numerical values, a text string or a cell array. See the help of `wilwin` for more details.

`iwmdct (f, g, Ls)` does the same, but cuts or zero-extends the final result to length Ls .

`[f, g]=iwmdct (...)` additionally outputs the window used in the transform. This is usefull if the window was generated from a description in a string or cell array.

References: [73], [74], [64], [15]

2.3.7 WMDCT2 - 2D Discrete windowed MDCT transform**Usage**

```
c=wmdct2 (f, g, M);
c=wmdct2 (f, g1, g2, [M1, M2]);
c=wmdct2 (f, g1, g2, [M1, M2], [L1, L2]);
[c, L]=wmdct2 (f, g1, g2, [M1, M2], [L1, L2]);
```

Input parameters

f	Input data, matrix.
g, g1, g2	Window functions.
M, M1, M2	Number of bands.
L1, L2	Length of transform to do.

Output parameters

c	array of coefficients.
Ls	Original size of input matrix.

Description

`wmdct2 (f, g, M)` calculates a two dimensional Modified Discrete Cosine transform of the input signal f using the window g and parameter M along each dimension.

For each dimension, the length of the transform will be the smallest possible that is larger than the length of the signal along that dimension. f will be appropriately zero-extended.

All windows must be whole-point even.

`wmdct2 (f, g, M, L)` computes a 2D windowed MDCT as above, but does a transform of length L along each dimension. f will be cut or zero-extended to length L before the transform is done.

`[c, Ls]=wmdct (f, g, M)` or `[c, Ls]=wmdct (f, g, M, L)` additionally return the length of the input signal f . This is handy for reconstruction.

`c=wmdct2 (f, g1, g2, M)` makes it possible to use different windows along the two dimensions.

The parameters L , M and L_s can also be vectors of length 2. In this case the first element will be used for the first dimension and the second element will be used for the second dimension.

The output c has 4 or 5 dimensions. The dimensions index the following properties:

1. Number of translation along 1st dimension of input.
2. Number of channel along 1st dimension of input
3. Number of translation along 2nd dimension of input.
4. Number of channel along 2nd dimension of input
5. Plane number, corresponds to 3rd dimension of input.

2.3.8 IWMDCT2 - 2D Inverse windowed MDCT transform

Usage

```
f=iwmdct2(c,g);
f=iwmdct2(c,g1,g2);
f=iwmdct2(c,g1,g2,Ls);
```

Input parameters

c	Array of coefficients.
g, g1, g2	Window functions.
Ls	Size of reconstructed signal.

Output parameters

f	Output data, matrix.
----------	----------------------

Description

`iwmdct2(c, g)` calculates a separable two dimensional inverse wmdct transform of the input coefficients c using the window g . The number of channels is deduced from the size of the coefficients c .

`iwmdct2(c, g1, g2)` does the same using the window $g1$ along the first dimension, and window $g2$ along the second dimension.

`iwmdct2(c, g1, g2, Ls)` cuts the signal to size Ls after the transform is done.

2.3.9 WIL2RECT - Arrange Wilson coefficients in a rectangular layout

Usage

```
c=wil2rect(c);
```

Description

`wil2rect(c)` rearranges the coefficients c in a rectangular shape. The coefficients must have been obtained from `dwilt`. After rearrangement the coefficients are placed correctly on the time/frequency-plane.

The rearranged array is larger than the input array: it contains zeros on the spots where the Wilson transform is missing a DC or Nyquest component.

2.3.10 RECT2WIL - Inverse of WIL2RECT

Usage

```
c=rect2wil(c);
```

Description

`rect2wil(c)` takes Wilson coefficients processed by `wil2rect` and puts them back in the compact form used by `dwilt` and `idwilt`. The coefficients can then be processed by `idwilt`.

2.3.11 WILWIN - Compute a Wilson/WMDCT window from text or cell array**Usage**

```
[g,info] = wilwin(g,M,L);
```

Description

`[g,info]=wilwin(g,M,L)` computes a window that fits well with the specified number of channels M and transform length L . The window itself is specified by a text description or a cell array containing additional parameters.

The window can be specified directly as a vector of numerical values. In this case, `wilwin` only checks assumptions about transform sizes etc.

`[g,info]=wilwin(g,M)` does the same, but the window must be a FIR window, as the transform length is unspecified.

The window can be specified as one of the following text strings:

'gauss'	Gaussian window with optimal concentration
'dualgauss'	Riesz dual of Gaussian window with optimal concentration.
'tight'	Window generating an orthonormal basis

In these cases, a long window is generated with a length of L .

It is also possible to specify one of the window names from `firwin`. In such a case, `wilwin` generates the specified FIR window with a length of M .

The window can also be specified as cell array. The possibilities are:

```
{'gauss',...} Additional parameters are passed to PGAUSS
{'dual',...} Dual window of whatever follows. See the examples below.
{'tight',...} Orthonormal window of whatever follows.
```

It is also possible to specify one of the window names from `firwin` as the first field in the cell array. In this case, the remaining entries of the cell array are passed directly to `firwin`.

Some examples:

```
g=wilwin('gauss',M,L);
```

This computes a Gaussian window of length L fitted for a system with M channels.

```
g=wilwin({'gauss',1},M,L);
```

This computes a Gaussian window with equal time and frequency support irrespective of M .

```
gd=wilwin('gaussdual',M,L);
```

This computes the dual of a Gaussian window fitted for a system with M channels.

```
gd=wilwin({'tight','gauss'},M,L);
```

This computes the orthonormal window of the Gaussian window fitted for the system.

```
g=wilwin({'dual',{'hann',20}},M,L);
```

This computes the dual of a Hann window of length 20.

The structure `info` provides some information about the computed window:

info.gauss True if the window is a Gaussian.

info.tfr Time/frequency support ratio of the window. Set whenever it makes sense.

info.wasrow Input was a row window

info.isfir Input is an FIR window

info.isdual Output is the dual window of the auxiliary window.

info.istight Output is known to be a tight window.

info.auxinfo Info about auxiliary window.

info.g1 Length of window.

2.3.12 DWILTLENGTH - DWILT/WMDCT length from signal

Usage

```
L=dwiltlength(Ls,M);
```

Description

`dwiltlength(Ls,M)` returns the length of a Wilson / WMDCT system with M channels system is long enough to expand a signal of length L_s . Please see the help on dwilt or wmdct for an explanation of the parameter M .

If the returned length is longer than the signal length, the signal will be zero-padded by dwilt or wmdct.

A valid transform length must be divisible by $2M$. This means that the minimal admissible transform length is

```
Lsmallest = 2*M;
```

and all valid transform lengths are multipla of $Lsmallest$

2.4 Reconstructing windows

2.4.1 GABDUAL - Canonical dual window of Gabor frame

Usage

```
gd=gabdual(g,a,M);
gd=gabdual(g,a,M,L);
gd=gabdual(g,a,M,'lt',lt);
```

Input parameters

g Gabor window.

a Length of time shift.

M Number of channels.

L Length of window. (optional)

lt Lattice type (for non-separable lattices).

Output parameters

gd Canonical dual window.

Description

`gabdual(g, a, M)` computes the canonical dual window of the discrete Gabor frame with window g and parameters a, M .

The window g may be a vector of numerical values, a text string or a cell array. See the help of `gabwin` for more details.

If the length of g is equal to M , then the input window is assumed to be an FIR window. In this case, the canonical dual window also has length of M . Otherwise the smallest possible transform length is chosen as the window length.

`gabdual(g, a, M, L)` returns a window that is the dual window for a system of length L . Unless the dual window is a FIR window, the dual window will have length L .

`gabdual(g, a, M, 'lt', lt)` does the same for a non-separable lattice specified by lt . Please see the help of `matrix2latticetype` for a precise description of the parameter lt .

If $a > M$ then the dual window of the Gabor Riesz sequence with window g and parameters a and M will be calculated.

Examples:

The following example shows the canonical dual window of the Gaussian window:

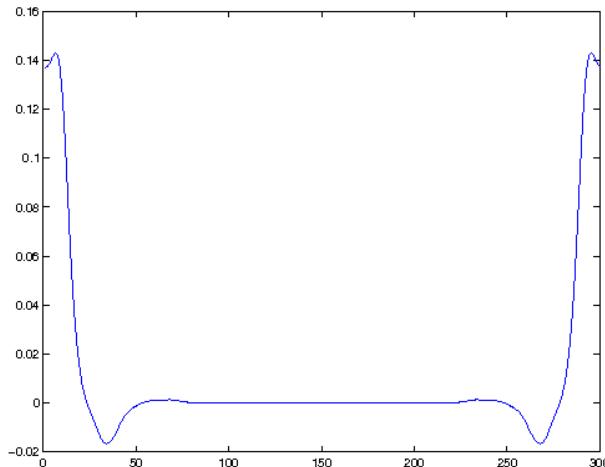
```

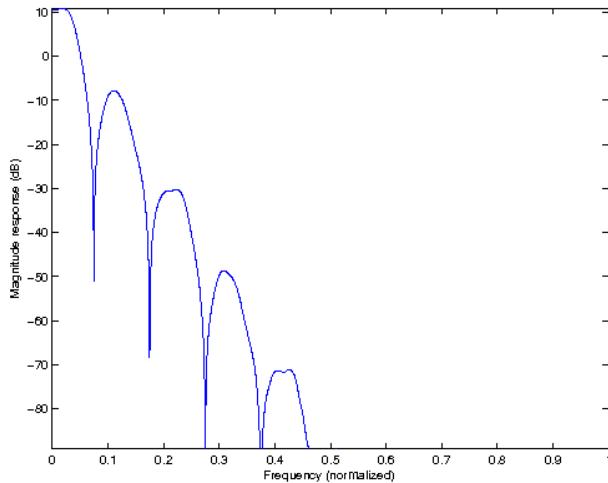
a=20;
M=30;
L=300;
g=pgauss(L,a*M/L);
gd=gabdual(g,a,M);

% Simple plot in the time-domain
figure(1);
plot(gd);

% Frequency domain
figure(2);
magresp(gd,'dynrange',100);

```





2.4.2 GABTIGHT - Canonical tight window of Gabor frame

Usage

```
gt=gabtight (a,M,L);
gt=gabtight (g,a,M);
gt=gabtight (g,a,M,L);
gd=gabtight (g,a,M,'lt',lt);
```

Input parameters

g	Gabor window.
a	Length of time shift.
M	Number of modulations.
L	Length of window. (optional)
lt	Lattice type (for non-separable lattices).

Output parameters

gt	Canonical tight window, column vector.
-----------	--

Description

`gabtight (a,M,L)` computes a nice tight window of length L for a lattice with parameters a, M . The window is not an FIR window, meaning that it will only generate a tight system if the system length is equal to L .

`gabtight (g,a,M)` computes the canonical tight window of the Gabor frame with window g and parameters a, M .

The window g may be a vector of numerical values, a text string or a cell array. See the help of `gabwin` for more details.

If the length of g is equal to M , then the input window is assumed to be a FIR window. In this case, the canonical dual window also has length of M . Otherwise the smallest possible transform length is chosen as the window length.

`gabtight (g,a,M,L)` returns a window that is tight for a system of length L . Unless the input window g is a FIR window, the returned tight window will have length L .

`gabtight (g,a,M,'lt',lt)` does the same for a non-separable lattice specified by lt . Please see the help of `matrix2latticetype` for a precise description of the parameter lt .

If $a > M$ then an orthonormal window of the Gabor Riesz sequence with window g and parameters a and M will be calculated.

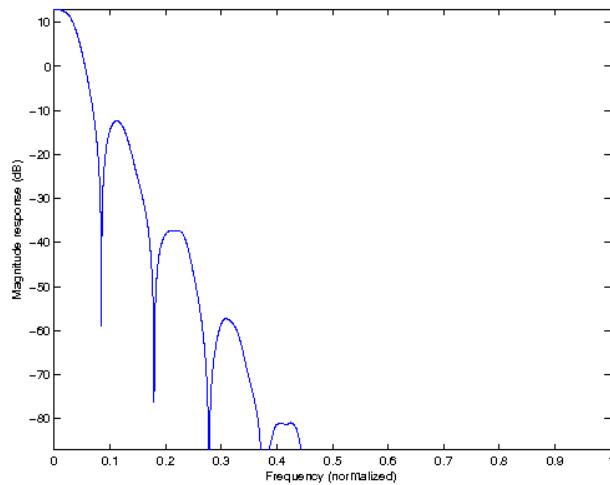
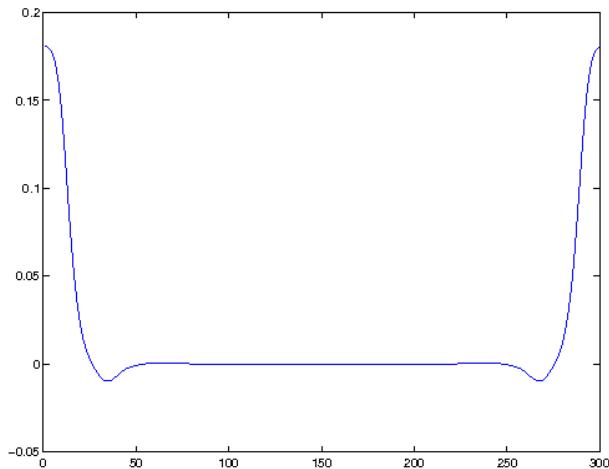
Examples:

The following example shows the canonical tight window of the Gaussian window. This is calculated by default by gabtight if no window is specified:

```
a=20;
M=30;
L=300;
gt=gabtight(a,M,L);

% Simple plot in the time-domain
figure(1);
plot(gt);

% Frequency domain
figure(2);
magresp(gt,'dynrange',100);
```



2.4.3 GABFIRDUAL - Compute FIR dual window

Usage

```
gd=gabfirdual(Ldual,g,a,M);
gd=gabfirdual(Ldual,g,a,M,varargin);
```

Input parameters

Ldual	Length of dual window
g	Window function
a	Time shift
M	Number of Channels
alpha1	Weight of l^1 -norm in the time domain
alpha2	Weight of l^1 -norm in the freq. domain

Output parameters

gd	Dual window
-----------	-------------

Description

`gabfirdual(Ldual,g,a,M)` computes an FIR window gd which is an approximate dual window of the Gabor system defined by g , a and M . The FIR dual window will be supported on $Ldual$ samples.

This function solve a convex optimization problem that can be written as:

$$\begin{aligned} gd = & \arg \min_x \|\alpha x\|_1 + \|\beta \mathcal{F}x\|_1 \\ & + \|\omega(x - g)\|_2^2 \\ & \delta \|x\|_{S0} + \mu \|\nabla x\|_2^2 + \gamma \|\nabla \mathcal{F}x\|_2^2 \end{aligned}$$

such that x is a dual window of g

Note: This function require the unlocbox. You can download it at <http://unlocbox.sourceforge.net>

The function uses an iterative algorithm to compute the approximate FIR dual. The algorithm can be controlled by the following flags:

'alpha',alpha	Weight in time. If it is a scalar, it represent the weights of the entire L1 function in time. If it is a vector, it is the associated weight assotiated to each component of the L1 norm (length: Ldual). Default value is $\alpha = 0$. Warning: this value should not be too big in order to avoid the the L1 norm proximal operator kill the signal. No L1-time constraint: $\alpha = 0$
'beta',beta	Weight in frequency. If it is a scalar, it represent the weights of the entire L1 function in frequency. If it is a vector, it is the associated weight assotiated to each component of the L1 norm in frequency. (length: Ldual). Default value is $\beta = 0$. Warning: this value should not be too big in order to avoid the the L1 norm proximal operator kill the signal. No L1-frequency constraint: $\beta = 0$

'omega',omega	Weight in time of the L2-norm. If it is a scalar, it represent the weights of the entire L2 function in time. If it is a vector, it is the associated weight assotiated to each component of the L2 norm (length: Ldual). Default value is $\omega = 0$. No L2-time constraint: $\omega = 0$
'glike',g_l	g_l is a windows in time. The algorithm try to shape the dual window like g_l . Normalization of g_l is done automatically. To use option omega should be different from 0. By default $g_d = 0$.
'mu',mu	Weight of the smooth constraint Default value is 1. No smooth constraint: $\mu = 0$
'gamma',gamma	Weight of the smooth constraint in frequency. Default value is 1. No smooth constraint: $\gamma = 0$
'delta',delta	Weight of the S0-norm. Default value is 0. No S0-norm: $\delta = 0$
'dual'	Look for a dual windows (default)
'painless'	Construct a starting guess using a painless-case approximation. This is the default
'zero'	Choose a starting guess of zero.
'rand'	Choose a random starting phase.
'tol',t	Stop if relative residual error is less than the specified tolerance.
'maxit',n	Do at most n iterations. default 200
'print'	Display the progress.
'debug'	Display all the progresses.
'quiet'	Don't print anything, this is the default.
'fast'	Fast algorithm, this is the default.
'slow'	Safer algorithm, you can try this if the fast algorithm is not working. Before using this, try to iterate more.
'printstep',p	If 'print' is specified, then print every p'th iteration. Default value is p=10;
'hardconstraint'	Force the projection at the end (default)
'softconstraint'	Do not force the projection at the end

2.4.4 GABOPTDUAL - Compute dual window

Usage

```
gd=gaboptdual(g,a,M);
gd=gaboptdual(g,a,M, varargin);
```

Input parameters

g	Window function
a	Time shift
M	Number of Channels

Output parameters

gd	Dual window
-----------	-------------

Description

`gaboptdual(g, a, M)` computes a window gd which is an approximate dual window of the Gabor system defined by g , a and M .

This function solve a convex optimization problem that can be written as:

$$\begin{aligned} gd = & \arg \min_x \|\alpha x\|_1 + \|\beta \mathcal{F}x\|_1 \\ & + \|\omega(x - g_l)\|_2^2 \\ & \delta \|x\|_{S0} + \mu \|\nabla x\|_2^2 + \gamma \|\nabla \mathcal{F}x\|_2^2 \end{aligned}$$

such that x is a dual window of g

Note: This function require the unlocbox. You can download it at <http://unlocbox.sourceforge.net>

The function uses an iterative algorithm to compute the approximate optimized dual. The algorithm can be controlled by the following flags:

'alpha',alpha	Weight in time. If it is a scalar, it represent the weights of the entire L1 function in time. If it is a vector, it is the associated weight assotiated to each component of the L1 norm (length: Ldual). Default value is $\alpha = 0$. Warning: this value should not be too big in order to avoid the the L1 norm proximal operator kill the signal. No L1-time constraint: $\alpha = 0$
'beta',beta	Weight in frequency. If it is a scalar, it represent the weights of the entire L1 function in frequency. If it is a vector, it is the associated weight assotiated to each component of the L1 norm in frequency. (length: Ldual). Default value is $\beta = 0$. Warning: this value should not be too big in order to avoid the the L1 norm proximal operator kill the signal. No L1-frequency constraint: $\beta = 0$
'omega',omega	Weight in time of the L2-norm. If it is a scalar, it represent the weights of the entire L2 function in time. If it is a vector, it is the associated weight assotiated to each component of the L2 norm (length: Ldual). Default value is $\omega = 0$. No L2-time constraint: $\omega = 0$
'glike',g_l	g_l is a windows in time. The algorithm try to shape the dual window like g_l . Normalization of g_l is done automatically. To use option omega should be different from 0. By default $g_d = 0$.
'mu',mu	Weight of the smooth constraint Default value is 1. No smooth constraint: $\mu = 0$
'gamma',gamma	Weight of the smooth constraint in frequency. Default value is 1. No smooth constraint: $\gamma = 0$
'delta',delta	Weight of the S0-norm. Default value is 0. No S0-norm: $\delta = 0$
'dual'	Look for a dual windows (default)
'painless'	Construct a starting guess using a painless-case approximation. This is the default

'zero'	Choose a starting guess of zero.
'rand'	Choose a random starting phase.
'tol',t	Stop if relative residual error is less than the specified tolerance.
'maxit',n	Do at most n iterations. default 200
'print'	Display the progress.
'debug'	Display all the progresses.
'quiet'	Don't print anything, this is the default.
'fast'	Fast algorithm, this is the default.
'slow'	Safer algorithm, you can try this if the fast algorithm is not working. Before using this, try to iterate more.
'printstep',p	If 'print' is specified, then print every p'th iteration. Default value is p=10;
'hardconstraint'	Force the projection at the end (default)
'softconstraint'	Do not force the projection at the end

2.4.5 GABFIRTIGHT - Compute FIR tight window

Usage

```
gt=gabfirtight (Lsupport, g, a, M) ;
gt=gabfirtight (Lsupport, g, a, M, varargin) ;
```

Input parameters

Lsupport	Length of the tight window
g	Initial window function
a	Time shift
M	Number of Channels

Output parameters

gt	Tight window
-----------	--------------

Description

`gabfirtight (Lsupport, g, a, M)` computes an FIR window gd which is tight. The FIR dual window will be supported on $Lsupport$ samples.

This function solve a convex optimization problem that can be written as:

$$\begin{aligned} gd = & \arg \min_x \|\alpha x\|_1 + \|\beta \mathcal{F}x\|_1 \\ & + \|\omega(x - g_l)\|_2^2 \\ & \delta \|x\|_{S0} + \mu \|\nabla x\|_2^2 + \gamma \|\nabla \mathcal{F}x\|_2^2 \end{aligned}$$

such that x is a tight FIR window

Note: This function require the unlocbox. You can download it at <http://unlocbox.sourceforge.net>

The function uses an iterative algorithm to compute the approximate FIR tight windows. Warning The algorithm solve a non convex problem and might be stuck in bad local minima. The algorithm can be controlled by the following flags:

'alpha',alpha	Weight in time. If it is a scalar, it represent the weights of the entire L1 function in time. If it is a vector, it is the associated weight assotiated to each component of the L1 norm (length: Ldual). Default value is $\alpha = 0$. Warning: this value should not be too big in order to avoid the the L1 norm proximal operator kill the signal. No L1-time constraint: $\alpha = 0$
'beta',beta	Weight in frequency. If it is a scalar, it represent the weights of the entire L1 function in frequency. If it is a vector, it is the associated weight assotiated to each component of the L1 norm in frequency. (length: Ldual). Default value is $\beta = 0$. Warning: this value should not be too big in order to avoid the the L1 norm proximal operator kill the signal. No L1-frequency constraint: $\beta = 0$
'omega',omega	Weight in time of the L2-norm. If it is a scalar, it represent the weights of the entire L2 function in time. If it is a vector, it is the associated weight assotiated to each component of the L2 norm (length: Ldual). Default value is $\omega = 0$. No L2-time constraint: $\omega = 0$
'glike',g_l	g_l is a windows in time. The algorithm try to shape the dual window like g_l . Normalization of g_l is done automatically. To use option omega should be different from 0. By default $g_d = 0$.
'mu',mu	Weight of the smooth constraint Default value is 1. No smooth constraint: $\mu = 0$
'gamma',gamma	Weight of the smooth constraint in frequency. Default value is 1. No smooth constraint: $\gamma = 0$
'delta',delta	Weight of the S0-norm. Default value is 0. No S0-norm: $\delta = 0$
'dual'	Look for a dual windows (default)
'painless'	Construct a starting guess using a painless-case approximation. This is the default
'zero'	Choose a starting guess of zero.
'rand'	Choose a random starting phase.
'tol',t	Stop if relative residual error is less than the specified tolerance.
'maxit',n	Do at most n iterations. default 200
'print'	Display the progress.
'debug'	Display all the progresses.
'quiet'	Don't print anything, this is the default.
'fast'	Fast algorithm, this is the default.
'slow'	Safer algorithm, you can try this if the fast algorithm is not working. Before using this, try to iterate more.
'printstep',p	If 'print' is specified, then print every p'th iteration. Default value is p=10;
'hardconstraint'	Force the projection at the end (default)
'softconstraint'	Do not force the projection at the end

2.4.6 GABOPTTIGHT - Compute a optimized tight window

Usage

```
gt=gabopttight(Ltight,g,a,M);
gt=gabopttight(Ltight,g,a,M,varargin);
```

Input parameters

g	Initial window function
a	Time shift
M	Number of Channels

Output parameters

gt	Tight window
-----------	--------------

Description

`gabopttight(g,a,M)` computes a tight window *gt* for a frame of parameter *a* and *M*

This function solves a convex optimization problem that can be written as:

$$\begin{aligned} \text{gd} = & \arg \min_x \|\alpha x\|_1 + \|\beta \mathcal{F}x\|_1 \\ & + \|\omega(x - g_l)\|_2^2 \\ & \delta \|x\|_{S_0} + \mu \|\nabla x\|_2^2 + \gamma \|\nabla \mathcal{F}x\|_2^2 \end{aligned}$$

such that *x* is tight window

Note: This function require the unlocbox. You can download it at <http://unlocbox.sourceforge.net>

The function uses an iterative algorithm to compute the approximate optimized tight window. Warning
The algorithm solve a non convex problem and might be stack in bad local minima. The algorithm can be controlled by the following flags:

'alpha',alpha	Weight in time. If it is a scalar, it represent the weights of the entire L1 function in time. If it is a vector, it is the associated weight assotiated to each component of the L1 norm (length: Ldual). Default value is $\alpha = 0$. Warning: this value should not be too big in order to avoid the the L1 norm proximal operator kill the signal. No L1-time constraint: $\alpha = 0$
'beta',beta	Weight in frequency. If it is a scalar, it represent the weights of the entire L1 function in frequency. If it is a vector, it is the associated weight assotiated to each component of the L1 norm in frequency. (length: Ldual). Default value is $\beta = 0$. Warning: this value should not be too big in order to avoid the the L1 norm proximal operator kill the signal. No L1-frequency constraint: $\beta = 0$
'omega',omega	Weight in time of the L2-norm. If it is a scalar, it represent the weights of the entire L2 function in time. If it is a vector, it is the associated weight assotiated to each component of the L2 norm (length: Ldual). Default value is $\omega = 0$. No L2-time constraint: $\omega = 0$

'glike',g_l	g_l is a windows in time. The algorithm try to shape the dual window like g_l . Normalization of g_l is done automatically. To use option omega should be different from 0. By default $g_d = 0$.
'mu', mu Weight of the smooth constraint Default value is 1.	No smooth constraint: $\mu = 0$
'gamma', gamma Weight of the smooth constraint in frequency. Default value is 1.	No smooth constraint: $\gamma = 0$
'delta', delta Weight of the S0-norm. Default value is 0.	No S0-norm: $\delta = 0$
'dual'	Look for a dual windows (default)
'painless'	Construct a starting guess using a painless-case approximation. This is the default
'zero'	Choose a starting guess of zero.
'rand'	Choose a random starting phase.
'tol',t	Stop if relative residual error is less than the specified tolerance.
'maxit',n	Do at most n iterations. default 200
'print'	Display the progress.
'debug'	Display all the progresses.
'quiet'	Don't print anything, this is the default.
'fast'	Fast algorithm, this is the default.
'slow'	Safer algorithm, you can try this if the fast algorithm is not working. Before using this, try to iterate more.
'printstep',p	If 'print' is specified, then print every p'th iteration. Default value is p=10;
'hardconstraint'	Force the projection at the end (default)
'softconstraint'	Do not force the projection at the end

2.4.7 GABCONVEXOPT - Compute a window using convex optimization

Usage

```
gout=gabconvexopt (g, a, M);
gout=gabconvexopt (g, a, M, varargin);
```

Input parameters

g	Window function /initial point (tight case)
a	Time shift
M	Number of Channels

Output parameters

gout	Output window
iter	Number of iterations
relres	Reconstruction error

Description

`gabconvexopt (g, a, M)` computes a window g_{out} which is the optimal solution of the convex optimization problem below

$$gd = \arg \min_x \|\alpha x\|_1 + \|\beta \mathcal{F}x\|_1$$

$$+ \|\omega(x - g_l)\|_2^2$$

$$\delta \|x\|_{S0} + \mu \|\nabla x\|_2^2 + \gamma \|\nabla \mathcal{F}x\|_2^2$$

such that x satisfies the constraints

Three constraints are possible:

- x is dual with respect of g
- x is tight
- x is compactly supported on L_{dual}

Note: This function require the unlocbox. You can download it at <http://unlocbox.sourceforge.net>

The function uses an iterative algorithm to compute the approximate. The algorithm can be controlled by the following flags:

'alpha',alpha Weight in time. If it is a scalar, it represent the weights of the entire L1 function in time. If it is a vector, it is the associated weight assotiated to each component of the L1 norm (length: L_{dual}). Default value is $\alpha = 0$. **Warning:** this value should not be too big in order to avoid the the L1 norm proximal operator kill the signal. No L1-time constraint: $\alpha = 0$

'beta',beta Weight in frequency. If it is a scalar, it represent the weights of the entire L1 function in frequency. If it is a vector, it is the associated weight assotiated to each component of the L1 norm in frequency. (length: L_{dual}). Default value is $\beta = 0$. **Warning:** this value should not be too big in order to avoid the the L1 norm proximal operator kill the signal. No L1-frequency constraint: $\beta = 0$

'omega',omega Weight in time of the L2-norm. If it is a scalar, it represent the weights of the entire L2 function in time. If it is a vector, it is the associated weight assotiated to each component of the L2 norm (length: L_{dual}). Default value is $\omega = 0$. No L2-time constraint: $\omega = 0$

'glike',g_l g_l is a windows in time. The algorithm try to shape the dual window like g_l . Normalization of g_l is done automatically. To use option omega should be different from 0. By default $g_d = 0$.

'mu', mu **Weight of the smooth constraint Default value is 1.** No smooth constraint: $\mu = 0$

'gamma', gamma **Weight of the smooth constraint in frequency. Default value is 1.** No smooth constraint: $\gamma = 0$

'delta', delta **Weight of the S0-norm. Default value is 0.** No S0-norm: $\delta = 0$

'support' L_{dual} Add a constraint on the support. The windows should be compactly supported on L_{dual} .

'tight'	Look for a tight windows
'dual'	Look for a dual windows (default)
'painless'	Construct a starting guess using a painless-case approximation. This is the default
'zero'	Choose a starting guess of zero.
'rand'	Choose a random starting phase.
'tol',t	Stop if relative residual error is less than the specified tolerance.
'maxit',n	Do at most n iterations. default 200
'print'	Display the progress.
'debug'	Display all the progresses.
'quiet'	Don't print anything, this is the default.
'fast'	Fast algorithm, this is the default.
'slow'	Safer algorithm, you can try this if the fast algorithm is not working. Before using this, try to iterate more.
'printstep',p	If 'print' is specified, then print every p'th iteration. Default value is p=10;
'hardconstraint'	Force the projection at the end (default)
'softconstraint'	Do not force the projection at the end

2.4.8 GABPROJDUAL - Gabor Dual window by projection

Usage

```
gd=gabprojdual(gm,g,a,M)
gd=gabprojdual(gm,g,a,M,L)
```

Input parameters

gm	Window to project.
g	Window function.
a	Length of time shift.
M	Number of modulations.
L	Length of transform to consider

Output parameters

gd	Dual window.
-----------	--------------

Description

`gabprojdual(gm,g,a,M)` calculates the dual window of the Gabor frame given by g , a and M closest to gm measured in the l^2 norm. The function projects the suggested window gm onto the subspace of admissible dual windows, hence the name of the function.

`gabprojdual(gm,g,a,M,L)` first extends the windows g and gm to length L .

`gabprojdual(...,'lt',lt)` does the same for a non-separable lattice specified by lt . Please see the help of `matrix2latticetype` for a precise description of the parameter lt .

2.4.9 GABMIXDUAL - Computes the mixdual of g1

Usage

```
gamma=mixdual(g1,g2,a,M)
```

Input parameters

g1	Window 1
g2	Window 2
a	Length of time shift.
M	Number of modulations.

Output parameters

gammaf	Mixdual of window 1.
---------------	----------------------

Description

`gabmixdual(g1,g2,a,M)` computes a dual window of $g1$ from a mix of the canonical dual windows of $g1$ and $g2$.

References: [94]

2.4.10 WILORTH - Wilson orthonormal window

Usage

```
gt=wilorth(M,L);
gt=wilorth(g,M);
gt=wilorth(g,M,L);
```

Input parameters

g	Auxiliary window window function (optional).
M	Number of modulations.
L	Length of window (optional).

Output parameters

gt	Window generating an orthonormal Wilson basis.
-----------	--

Description

`wilorth(M,L)` computes a nice window of length L generating an orthonormal Wilson or WMDCT basis with M frequency bands for signals of length L .

`wilorth(g,M)` computes a window generating an orthonormal basis from the window g and number of channels M .

The window g may be a vector of numerical values, a text string or a cell array. See the help of `wilwin` for more details.

If the length of g is equal to $2 \times M$, then the input window is assumed to be a FIR window. In this case, the orthonormal window also has length of $2 \times M$. Otherwise the smallest possible transform length is chosen as the window length.

`wilorth(g,M,L)` pads or truncates g to length L before calculating the orthonormal window. The output will also be of length L .

The input window g must be real whole-point even. If g is not whole-point even, the computed window will not generate an orthonormal system (i.e. reconstruction will not be perfect). For a random window g , the window closest to g that satisfies these restrictions can be found by

```
g_wpe = real(peven(g));
```

All Gabor windows in the toolbox satisfies these restrictions unless clearly stated otherwise.

2.4.11 WILDUAL - Wilson dual window

Usage

```
gamma=wildual(g,M);
gamma=wildual(g,M,L);
```

Input parameters

g	Gabor window.
M	Number of modulations.
L	Length of window. (optional)

Output parameters

gamma	Canonical dual window.
--------------	------------------------

Description

`wildual(g,M)` returns the dual window of the Wilson or WMDCT basis with window g , parameter M and length equal to the length of the window g .

The window g may be a vector of numerical values, a text string or a cell array. See the help of `wilwin` for more details.

If the length of g is equal to $2 \cdot M$ then the input window is assumed to be an FIR window. In this case, the dual window also has length of $2 \cdot M$. Otherwise the smallest possible transform length is chosen as the window length.

`wildual(g,M,L)` does the same, but now L is used as the length of the Wilson basis.

The input window g must be real and whole-point even. If g is not whole-point even, then reconstruction using the dual window will not be perfect. For a random window g , the window closest to g that satisfies these restrictions can be found by

```
g_wpe = real(peven(g));
```

All windows in the toolbox satisfies these restrictions unless clearly stated otherwise.

2.5 Conditions numbers

2.5.1 GABFRAMEBOUNDS - Calculate frame bounds of Gabor frame

Usage

```
fcond=gabframebounds(g,a,M);
[A,B]=gabframebounds(g,a,M);
[A,B]=gabframebounds(g,a,M,L);
[A,B]=gabframebounds(g,a,M,'lt',lt);
```

Input parameters

g	The window function.
a	Length of time shift.
M	Number of channels.
L	Length of transform to consider.
lt	Lattice type (for non-separable lattices).

Output parameters

fcond	Frame condition number (B/A)
A, B	Frame bounds.

Description

`gabframebounds(g, a, M)` calculates the ratio B/A of the frame bounds of the Gabor system with window g , and parameters a, M .

`[A, B]=gabframebounds(. . .)` returns the frame bounds A and B instead of just the ratio.

The window g may be a vector of numerical values, a text string or a cell array. See the help of `gabwin` for more details.

`gabframebounds(g, a, M, L)` will cut or zero-extend the window to length L .

`gabframebounds(g, a, M, 'lt', lt)` does the same for a non-separable lattice specified by lt . Please see the help of `matrix2latticetype` for a precise description of the parameter lt .

2.5.2 GABRIESZBOUNDS - Calculate Riesz sequence/basis bounds of Gabor frame**Usage**

```
fcond=gabrieszbounds(g, a, M) ;
[A, B]=gabrieszbounds(g, a, M) ;
[A, B]=gabrieszbounds(g, a, M, L) ;
```

Input parameters

g	The window function.
a	Length of time shift.
M	Number of channels.
L	Length of transform to consider.

Output parameters

fcond	Frame condition number (B/A)
A, B	Frame bounds.

Description

`gabrieszbounds(g, a, M)` calculates the ratio B/A of the Riesz bounds of the Gabor system with window g , and parameters a, M .

`[A, B]=gabrieszbounds(g, a, M)` calculates the Riesz bounds A and B instead of just the ratio.

The window g may be a vector of numerical values, a text string or a cell array. See the help of `gabwin` for more details.

If the optional parameter L is specified, the window is cut or zero-extended to length L .

2.5.3 WILBOUNDS - Calculate frame bounds of Wilson basis

Usage

```
[AF,BF]=wilbounds(g,M)
[AF,BF]=wilbounds(g,M,L)
```

Input parameters

g	Window function.
M	Number of channels.
L	Length of transform to do (optional)

Output parameters

AF, BF	Frame bounds.
---------------	---------------

Description

wilbounds(*g, M*) calculates the frame bounds of the Wilson frame operator of the Wilson basis with window *g* and *M* channels.

[*A, B*]=wilbounds(*g, a, M*) returns the frame bounds *A* and *B* instead of just the ratio.

The window *g* may be a vector of numerical values, a text string or a cell array. See the help of wilwin for more details.

If the length of *g* is equal to $2 \cdot M$ then the input window is assumed to be a FIR window. Otherwise the smallest possible transform length is chosen as the window length.

If the optional parameter *L* is specified, the window is cut or zero-extended to length *L*.

2.5.4 GABDUALNORM - Measure of how close a window is to being a dual window

Usage

```
dn=gabdualnorm(g,gamma,a,M);
dn=gabdualnorm(g,gamma,a,M,L);
dn=gabdualnorm(g,gamma,a,M,'lt',lt);
[scal,res]=gabdualnorm(...);
```

Input parameters

gamma	input window..
g	window function.
a	Length of time shift.
M	Number of modulations.
L	Length of transform to consider

Output parameters

dn	dual norm.
scal	Scaling factor
res	Residual

Description

`gabdualnorm(g, gamma, a, M)` calculates how close *gamma* is to being a dual window of the Gabor frame with window *g* and parameters *a* and *M*.

The windows *g* and *gamma* may be vectors of numerical values, text strings or cell arrays. See the help of `gabwin` for more details.

`[scal, res]=gabdualnorm(...)` computes two entities: *scal* determines if the windows are scaled correctly, it must be 1 for the windows to be dual. *res* is close to zero if the windows (scaled correctly) are dual windows.

`gabdualnorm(g, gamma, a, M, L)` does the same, but considers a transform length of *L*.

`gabdualnorm(g, gamma, a, M, 'lt', lt)` does the same for a non-separable lattice specified by *lt*. Please see the help of `matrix2latticetype` for a precise description of the parameter *lt*.

`gabdualnorm` can be used to get the maximum relative reconstruction error when using the two specified windows. Consider the following code for some signal *f*, windows *g*, *gamma*, parameters *a* and *M* and transform-length *L* (See help on `dgt` on how to obtain *L*):

```
fr=idgt(dgt(f,g,a,M),gamma,a);
er=norm(f-fr)/norm(f);
eest=gabdualnorm(g,gamma,a,M,L);
```

Then *er* < *eest* for all possible input signals *f*.

To get a similar estimate for an almost tight window *gt*, simply use

```
eest=gabdualnorm(gt,gt,a,M,L);
```

2.5.5 GABFRAMEDIAG - Diagonal of Gabor frame operator

Usage

```
d=gabframediag(g,a,M,L);
d=gabframediag(g,a,M,L,'lt',lt);
```

Input parameters

g	Window function.
a	Length of time shift.
M	Number of channels.
L	Length of transform to do.
lt	Lattice type (for non-separable lattices).

Output parameters

d	Diagonal stored as a column vector
----------	------------------------------------

Description

`gabframediag(g, a, M, L)` computes the diagonal of the Gabor frame operator with respect to the window *g* and parameters *a* and *M*. The diagonal is stored as column vector of length *L*.

The diagonal of the frame operator can for instance be used as a preconditioner.

2.5.6 WILFRAMEDIAG - Diagonal of Wilson and WMDCT frame operator

Usage

```
d=wilframediag(g,M,L);
```

Input parameters

g	Window function.
M	Number of channels.
L	Length of transform to do.

Output parameters

d	Diagonal stored as a column vector
----------	------------------------------------

Description

`wilframediag(g, M, L)` computes the diagonal of the Wilson or WMDCT frame operator with respect to the window g and number of channels M . The diagonal is stored as a column vector of length L .

The diagonal of the frame operator can for instance be used as a preconditioner.

2.6 Phase gradient methods and reassignment

2.6.1 GABPHASEGRAD - Phase gradient of the DGT

Usage

```
[tgrad, fgrad, c] = gabphasegrad('dgt', f, g, a, M);
[tgrad, fgrad] = gabphasegrad('phase', cphase, a);
[tgrad, fgrad] = gabphasegrad('abs', s, g, a);
```

Description

`[tgrad, fgrad]=gabphasegrad(method, ...)` computes the time-frequency gradient of the phase of the dgt of a signal. The derivative in time $tgrad$ is the relative instantaneous frequency while the frequency derivative $fgrad$ is the local group delay.

$tgrad$ and $fgrad$ measure the deviation from the current time and frequency, so a value of zero means that the instantaneous frequency is equal to the center frequency of the considered channel.

$tgrad$ is scaled such that distances are measured in samples. Similarly, $fgrad$ is scaled such that the Nyquist frequency (the highest possible frequency) corresponds to a value of $L/2$. The absolute time and frequency positions can be obtained as

```
tgradabs = bsxfun(@plus,tgrad,fftindex(M)*L/M); fgradabs = bsxfun(@plus,fgrad,(0:L/a-1)*a);
```

The computation of $tgrad$ and $fgrad$ is inaccurate when the absolute value of the Gabor coefficients is low. This is due to the fact the the phase of complex numbers close to the machine precision is almost random. Therefore, $tgrad$ and $fgrad$ may attain very large random values when `abs(c)` is close to zero.

The computation can be done using four different methods.

'dgt'	Directly from the signal using algorithm by Auger and Flandrin.
'cross'	Directly from the signal using algorithm by Nelson.
'phase'	From the phase of a DGT of the signal. This is the classic method used in the phase vocoder.
'abs'	From the absolute value of the DGT. Currently this method works only for Gaussian windows.

`[tgrad, fgrad]=gabphasegrad('dgt', f, g, a, M)` computes the time-frequency gradient using a DGT of the signal f . The DGT is computed using the window g on the lattice specified by the time shift a and the number of channels M . The algorithm used to perform this calculation computes several DGTs, and therefore this routine takes the exact same input parameters as `dgt`.

The window g may be specified as in `dgt`. If the window used is '`gauss`', the computation will be done by a faster algorithm.

`[tgrad, fgrad, c]=gabphasegrad('dgt', f, g, a, M)` additionally returns the Gabor coefficients c , as they are always computed as a byproduct of the algorithm.

`[tgrad, fgrad]=gabphasegrad('cross', f, g, a, M)` does the same as above but this time using algorithm by Nelson which is based on computing several DGTs.

`[tgrad, fgrad]=gabphasegrad('phase', cphase, a)` computes the phase gradient from the phase `cphase` of a DGT of the signal. The original DGT from which the phase is obtained must have been computed using a time-shift of a using the default phase convention ('`freqinv`') e.g.:

```
[tgrad, fgrad]=gabphasegrad('phase', angle(dgt(f, g, a, M)), a)
```

`[tgrad, fgrad]=gabphasegrad('abs', s, g, a)` computes the phase gradient from the spectrogram s . The spectrogram must have been computed using the window g and time-shift a e.g.:

```
[tgrad, fgrad]=gabphasegrad('abs', abs(dgt(f, g, a, M)), g, a)
```

`[tgrad, fgrad]=gabphasegrad('abs', s, g, a, difforder)` uses a centered finite difference scheme of order `difforder` to perform the needed numerical differentiation. Default is to use a 4th order scheme.

Currently the '`abs`' method only works if the window g is a Gaussian window specified as a string or cell array.

References: [10], [22], [34]

2.6.2 GABPHASEDERIV - DGT phase derivatives

Usage

```
[phased, c] = gabphasederiv(dflag, 'dgt', f, g, a, M);
phased      = gabphasederiv(dflag, 'cross', f, g, a, M)
phased      = gabphasederiv(dflag, 'phase', cphase, a, difforder);
phased      = gabphasederiv(dflag, 'abs', s, g, a);
[{:phased1, phased2, ...}] = gabphasederiv({:dflag1, dflag2, ...}, ...);
[{:phased1, phased2, ...}, c] = gabphasederiv({:dflag1, dflag2, ...}, 'dgt', ...);
```

Description

`phased=gabphasederiv(dflag, method, ...)` computes the time-frequency derivative `dflag` of the phase of the `dgt` of a signal using algorithm `method`.

The following strings can be used in place of `dflag`:

' t '	First phase derivative in time.
' f '	First phase derivative in frequency.
' tt '	Second phase derivative in time.
' ff '	Second phase derivative in frequency.
' tf '	or 'ft' Second order mixed phase derivative.

`phased` is scaled such that (possibly non-integer) distances are measured in samples. Similarly, the frequencies are scaled such that the Nyquist frequency (the highest possible frequency) corresponds to a value of $L/2$.

The computation of `phased` is inaccurate when the absolute value of the Gabor coefficients is low. This is due to the fact the the phase of complex numbers close to the machine precision is almost random. Therefore, `phased` attain very large random values when `abs(c)` is close to zero.

The phase derivative computation can be done using four different methods depending on the string `method`:

' dgt '	Directly from the signal using algorithm by Auger and Flandrin.
----------------	---

'cross'	Directly from the signal using algorithm by Nelson.
'phase'	From the unwrapped phase of a DGT of the signal using a finite differences scheme. This is the classic method used in the phase vocoder.
'abs'	From the absolute value of the DGT exploiting explicit dependency between partial derivatives of log-magnitudes and phase. Currently this method works only for Gaussian windows.

`phased=gabphasederiv(dflag,'dgt',f,g,a,M)` computes the time-frequency derivative using a DGT of the signal f . The DGT is computed using the window g on the lattice specified by the time shift a and the number of channels M . The algorithm used to perform this calculation computes several DGTs, and therefore this routine takes the exact same input parameters as `dgt`.

`[phased,c]=gabphasederiv(dflag,'dgt',f,g,a,M)` additionally returns the Gabor coefficients c , as they are always computed as a byproduct of the algorithm.

`phased=gabphasederiv(dflag,'cross',f,g,a,M)` does the same as above but this time using algorithm by Nelson which is based on computing several DGTs.

`phased=gabphasederiv(dflag,'phase',cphase,a)` computes the phase derivative from the phase $cphase$ of a DGT of the signal. The original DGT from which the phase is obtained must have been computed using a time-shift of a using the default phase convention ('`freqinv`') e.g.:

```
phased=gabphasederiv(dflag,'phase',angle(dgt(f,g,a,M)),a)
```

`phased=gabphasederiv(dflag,'abs',s,g,a)` computes the phase derivative from the absolute values of DGT coefficients s . The spectrogram must have been computed using the window g and time-shift a e.g.:

```
phased=gabphasederiv(dflag,'abs',abs(dgt(f,g,a,M)),g,a)
```

Currently the '`abs`' method only works if the window g is a Gaussian window specified as a string or a cell array.

`phased=gabphasederiv(dflag,'abs',s,g,a,difforder)` uses a centered finite difference scheme of order $difforder$ to perform the needed numerical differentiation. Default is to use a 4th order scheme.

Phase conventions

First derivatives in either direction are subject to phase convention. The following additional flags define the phase convention the original phase would have had:

'freqinv'	Derivatives reflect the frequency-invariant phase of <code>dgt</code> . This is the default.
'timeinv'	Derivatives reflect the time-invariant phase of <code>dgt</code> .
'symphase'	Derivatives reflect the symmetric phase of <code>dgt</code> .
'relative'	This is a combination of ' <code>freqinv</code> ' and ' <code>timeinv</code> '. It uses ' <code>timeinv</code> ' for derivatives along frequency and ' <code>freqinv</code> ' for derivatives along time and for the mixed derivative. This is useful for the reassignment functions.

Please see `ltfatnote042` for the description of relations between the phase derivatives with different phase conventions. Note that for the '`relative`' convention, the following holds:

```
gabphasederiv('t',...,'relative') == gabphasederiv('t',...,'freqinv')
gabphasederiv('f',...,'relative') == -gabphasederiv('f',...,'timeinv')
gabphasederiv('tt',...,'relative') == gabphasederiv('tt',...)
gabphasederiv('ff',...,'relative') == -gabphasederiv('ff',...)
gabphasederiv('tf',...,'relative') == gabphasederiv('tf',...,'freqinv')
```

Several derivatives at once

`phasedcell=gabphasederiv({dflag1,dflag2,...},...)` computes several phase derivatives at once while reusing some temporary computations thus saving computation time. `{dflag1,dflag2,...}` is a cell array of the derivative flags and cell elements of the returned `phasedcell` contain the corresponding derivatives i.e.:

```
[pderiv1,pderiv2,...] = deal(phasedcell{:});
```

`[phasedcell,c]=gabphasederiv({dflag1,dflag2,...},'dgt',...)` works the same as above but in addition returns coefficients `c` which are the byproduct of the '`dgt`' method.

Other flags and parameters work as before.

References: [10], [22], [34], [9], [33], [67], [75]

2.6.3 GABREASSIGN - Reassign time-frequency distribution

Usage

```
sr = gabreassign(s,tgrad,fgrad,a);
```

Description

`gabreassign(s,tgrad,fgrad,a)` reassigns the values of the positive time-frequency distribution `s` using the phase gradient given by `fgrad` and `tgrad`. The lattice is determined by the time shift `a` and the number of channels deduced from the size of `s`.

`fgrad` and `tgrad` can be obtained by the routine `gabphasegrad`.

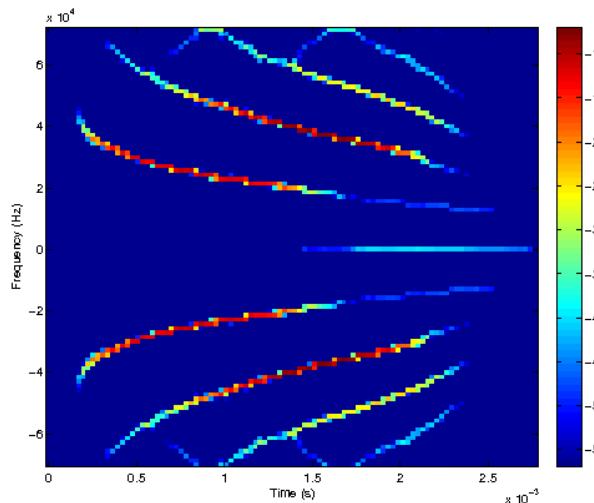
Examples:

The following example demonstrates how to manually create a reassigned spectrogram. An easier way is to just call `resgram`:

```
% Create reassigned vector field of the bat signal.
a=4; M=100;
[tgrad, fgrad, c] = gabphasegrad('dgt',bat,'gauss',a,M);

% Perform the actual reassignment
sr = gabreassign(abs(c).^2,tgrad,fgrad,a);

% Display it using plotdgt
plotdgt(sr,a,143000,50);
```



References: [10]

2.6.4 GABREASSIGNADJUST - Adjustable reassignment of a time-frequency distribution

Usage

```
sr = gabreassignadjust(s,pderivs,a,mu);
```

Description

`gabreassignadjust(s,pderivs,a,mu)` reassigns the values of the positive time-frequency distribution s using first and second order phase derivatives given by `pderivs` and parameter $\mu > 0$. The lattice is determined by the time shift a and the number of channels deduced from the size of s .

`pderivs` is a cell array of phase derivatives which can be obtained as follows:

```
pderivs = gabphasederiv({'t','f','tt','ff','tf'},...,'relative');
```

Please see help of `gabphasederiv` for description of the missing parameters.

`gabreassign(s,pderivs,a,mu,despeckle)` works as above, but some coefficients are removed prior to the reassignment process. More precisely a mixed phase derivative `pderivs{5}` is used to determine which coefficients m, n belong to sinusoidal components (such that $\text{abs}(1+pderivs{5})(m, n)$ is close to zero) and to impulsive components (such that $\text{abs}(pderivs{5})(m, n)$ is close to zero). Parameter `despeckle` determines a threshold on the previous quantities such that coefficients with higher associated values are set to zeros.

Algorithm

The routine uses the adjustable reassignment presented in the references.

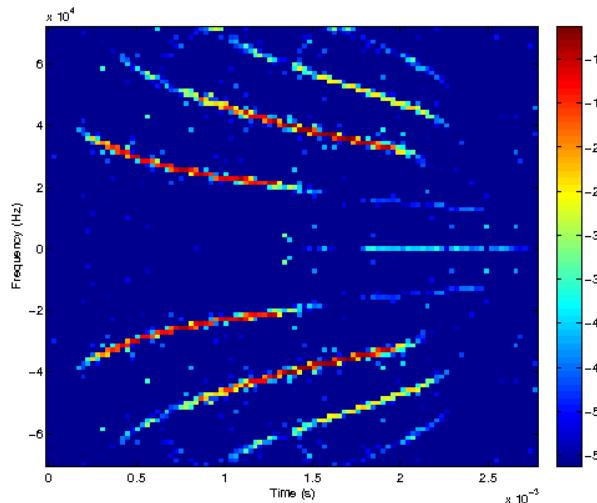
Examples:

The following example demonstrates how to manually create a reassigned spectrogram.:

```
% Compute the phase derivatives
a=4; M=100;
[pderivs, c] = gabphasederiv({'t','f','tt','ff','tf'},'dgt',bat,'gauss',a,M,'rela

% Reassignment parameter
mu = 0.1;
% Perform the actual reassignment
sr = gabreassignadjust(abs(c).^2,pderivs,a,mu);

% Display it using plotdgt
plotdgt(sr,a,143000,50);
```



References: [9], [8], [75]

2.7 Phase reconstruction

2.7.1 CONSTRUCTPHASE - Construct phase for DGT

Usage

```
c=constructphase(s,g,a);
c=constructphase(s,g,a,tol);
c=constructphase(c,g,a,tol,mask);
c=constructphase(c,g,a,tol,mask,usephase);
[c,newphase,usedmask,tgrad,fgrad] = constructphase(...);
```

Input parameters

s	Initial coefficients.
g	Analysis Gabor window.
a	Hop factor.
tol	Relative tolerance.
mask	Mask for selecting known phase.
usephase	Explicit known phase.

Output parameters

c	Coefficients with the constructed phase.
newphase	Just the (unwrapped) phase.
usedmask	Mask for selecting coefficients with the new phase.
tgrad	Relative time phase derivative.
fgrad	Relative frequency phase derivative.

Description

`constructphase(s, g, a)` will construct a suitable phase for the positive valued coefficients s .

If s is the absolute values of the Gabor coefficients of a signal obtained using the window g and time-shift a , i.e.:

```
c=dgt(f,g,a,M); s=abs(c);
```

then `constructphase(s, g, a)` will attempt to reconstruct c .

The window g must be Gaussian, i.e. g must have the value 'gauss' or be a cell array {'gauss', tfr}.

`constructphase(s, g, a, tol)` does as above, but sets the phase of coefficients less than tol to random values. By default, tol has the value 1e-10.

`constructphase(c, g, a, M, tol, mask)` accepts real or complex valued c and real valued $mask$ of the same size. Values in $mask$ which can be converted to logical true (anything other than 0) determine coefficients with known phase which is used in the output. Only the phase of remaining coefficients (for which $mask==0$) is computed.

`constructphasereal(c, g, a, M, tol, mask, usephase)` does the same as before but uses the known phase values from `usephase` rather than from c .

In addition, tol can be a vector containing decreasing values. In that case, the algorithm is run `numel(tol)` times, initialized with the result from the previous step in the 2nd and the further steps.

Further, the function accepts the following flags:

'freqinv' The constructed phase complies with the frequency invariant phase convention such that it can be directly used in `idgtreal`. This is the default.

'timeinv' The constructed phase complies with the time-invariant phase convention. The same flag must be used in the other functions e.g. `idgtreal`

This function requires a computational subroutine that is only available in C. Use `ltfatmex` to compile it.

References: [76]

2.7.2 CONSTRUCTPHASEREAL - Construct phase for DGTREAL

Usage

```
c=constructphasereal(s,g,a,M);
c=constructphasereal(s,g,a,M,tol);
c=constructphasereal(c,g,a,M,tol,mask);
c=constructphasereal(c,g,a,M,tol,mask,usephase);
[c,newphase,usedmask,tgrad,fgrad] = constructphasereal(...);
```

Input parameters

s	Initial coefficients.
g	Analysis Gabor window.
a	Hop factor.
M	Number of channels.
tol	Relative tolerance.
mask	Mask for selecting known phase.
usephase	Explicit known phase.

Output parameters

c	Coefficients with the constructed phase.
newphase	Just the (unwrapped) phase.
usedmask	Mask for selecting coefficients with the new phase.
tgrad	Relative time phase derivative.
fgrad	Relative frequency phase derivative.

Description

`constructphasereal(s, g, a, M)` will construct a suitable phase for the positive valued coefficients s .

If s contains the absolute values of the Gabor coefficients of a signal obtained using the window g , time-shift a and number of channels M , i.e.:

```
c=dgtreal(f,g,a,M); s=abs(c);
```

then `constructphasereal(s, g, a, M)` will attempt to reconstruct c .

The window g must be Gaussian, i.e. g must have the value 'gauss' or be a cell array {'gauss', ...}.

`constructphasereal(s, g, a, M, tol)` does as above, but sets the phase of coefficients less than tol to random values. By default, tol has the value 1e-10.

`constructphasereal(c, g, a, M, tol, mask)` accepts real or complex valued c and real valued $mask$ of the same size. Values in $mask$ which can be converted to logical true (anything other than 0) determine coefficients with known phase which is used in the output. Only the phase of remaining coefficients (for which $mask==0$) is computed.

`constructphasereal(c, g, a, M, tol, mask, usephase)` does the same as before but uses the known phase values from $usephase$ rather than from c .

In addition, tol can be a vector containing decreasing values. In that case, the algorithm is run `numel(tol)` times, initialized with the result from the previous step in the 2nd and the further steps.

Further, the function accepts the following flags:

'freqinv' The constructed phase complies with the frequency invariant phase convention such that it can be directly used in `idgtreal`. This is the default.

'timeinv' The constructed phase complies with the time-invariant phase convention. The same flag must be used in the other functions e.g. `idgtreal`

This function requires a computational subroutine that is only available in C. Use `lftfmex` to compile it.

References: [76]

2.8 Phase conversions

2.8.1 PHASELOCK - Phaselock Gabor coefficients

Usage

```
c=phaselock(c, a);
```

Description

`phaselock(c, a)` phaselocks the Gabor coefficients c . The coefficients must have been obtained from a `dgt` with parameter a .

Phaselocking the coefficients modifies them so as if they were obtained from a time-invariant Gabor system. A filter bank produces phase locked coefficients.

Phaselocking of Gabor coefficients correspond to the following transform: Consider a signal f of length L and define $N = L/a$. The output from `c=phaselock(dgt(f, g, a, M), a)` is given by

$$c(m+1, n+1) = \sum_{l=0}^{L-1} f(l+1) e^{-2\pi i m(l-na)/M} \overline{g(l-an+1)}$$

where $m = 0, \dots, M-1$ and $n = 0, \dots, N-1$ and $l-an$ is computed modulo L .

`phaselock(c, a, 'lt', lt)` does the same for a non-separable lattice specified by lt . Please see the help of `matrix2latticetype` for a precise description of the parameter lt .

References: [77]

2.8.2 PHASEUNLOCK - Undo phase lock of Gabor coefficients

Usage

```
c=phaseunlock(c, a);
```

Description

`phaseunlock(c, a)` removes phase locking from the Gabor coefficients c . The coefficient must have been obtained from a `dgt` with parameter a .

Phase locking the coefficients modifies them so as if they were obtained from a time-invariant Gabor system. A filter bank produces phase locked coefficients.

References: [77]

2.8.3 PHASELOCKREAL - Phaselock Gabor coefficients

Usage

```
c=phaselockreal(c, a, M);
```

Description

`phaselockreal(c, a, M)` phaselocks the Gabor coefficients c . The coefficients must have been obtained from a `dgtreal` with parameter a .

Phaselocking the coefficients modifies them so as if they were obtained from a time-invariant Gabor system. A filter bank produces phase locked coefficients.

Please see help of `phaselock` for more details.

References: [77]

2.8.4 PHASEUNLOCKREAL - Undo phase lock of Gabor coefficients

Usage

```
c=phaseunlockreal(c, a, M);
```

Description

`phaseunlockreal(c, a, M)` removes phase locking from the Gabor coefficients c . The coefficient must have been obtained from a `dgtreal` with parameter a .

Phase locking the coefficients modifies them so as if they were obtained from a time-invariant Gabor system. A filter bank produces phase locked coefficients.

References: [77]

2.8.5 SYMPHASE - Change Gabor coefficients to symmetric phase

Usage

```
c=sympphase(c, a);
```

Description

`sympphase(c, a)` alters the phase of the Gabor coefficients c so as if they were obtained from a Gabor transform based on symmetric time/frequency shifts. The coefficient must have been obtained from a `dgt` with parameter a .

Gabor coefficients with symmetric phase correspond to the following transform: Consider a signal f of length L and define $N = L/a$. The output from `c=sympphase(dgt(f, g, a, M), a)` is given by

$$c(m+1, n+1) = \sum_{l=0}^{L-1} f(l+1) e^{-2\pi i m(l-na/2)/M} \overline{g(l-an+1)}$$

where $m = 0, \dots, M-1$ and $n = 0, \dots, N-1$ and $l-an$ is computed modulo L .

`sympphase(c, a, 'lt', lt)` does the same for a non-separable lattice specified by lt . Please see the help of `matrix2latticetype` for a precise description of the parameter lt .

References: [22]

2.9 Support for non-separable lattices

2.9.1 MATRIX2LATTICETYPE - Convert matrix form to standard lattice description

Usage

```
[a, M, lt] = matrix2latticetype(L, V);
```

Description

`[a, M, lt]=matrix2latticetype(L, V)` converts a 2×2 integer matrix description into the standard description of a lattice using the a , M and lt . The conversion is *only* valid for the specified transform length L .

The lattice type lt is a 1×2 vector $[lt_1, lt_2]$ denoting an irreducible fraction lt_1/lt_2 . This fraction describes the distance in frequency (counted in frequency channels) that each coefficient is offset when moving in time by the time-shift of a . Some examples: $lt=[0 1]$ defines a square lattice, $lt=[1 2]$ defines the quinque (almost hexagonal) lattice, $lt=[1 3]$ describes a lattice with a $1/3$ frequency offset for each time shift and so forth.

An example:

```
[a, M, lt] = matrix2latticetype(120, [10 0; 5 10])
```

This code produces the following output:

```
a =
```

```
10
```

```
M =
```

```
12
```

```
lt =
```

```
1      2
```

Coefficient layout:

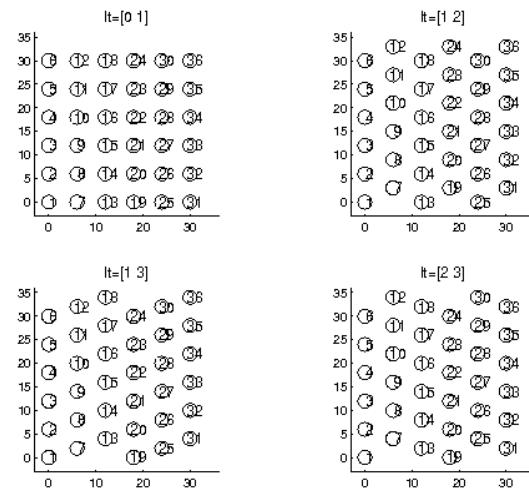
The following code generates plots which show the coefficient layout and enumeration of the first 4 lattices in the time-frequency plane:

```
a=6;
M=6;
L=36;
b=L/M;
N=L/a;
cw=3;
ftz=12;

[x,y]=meshgrid(a*(0:N-1),b*(0:M-1));

lt1=[0 1 1 2];
lt2=[1 2 3 3];

for fignum=1:4
    subplot(2,2,fignum);
    z=y;
    if lt2(fignum)>0
        z=z+mod(lt1(fignum)*x/lt2(fignum),b);
    end;
    for ii=1:M*N
        text(x(ii)-cw/4,z(ii),sprintf('%2.0i',ii),'FontSize',ftz);
        rectangle('Curvature',[1 1], 'Position',[x(ii)-cw/2,z(ii)-cw/2,cw,cw]);
    end;
    axis([-cw L -cw L]);
    axis('square');
    title(sprintf('lt=[%i %i]',lt1(fignum),lt2(fignum)),'FontSize',ftz);
end;
```

**2.9.2 LATTICETYPE2MATRIX - Convert lattice description to matrix form****Usage**

```
V=latticetype2matrix(L,a,M,lt);
```

Description

`V=latticetype2matrix(L,a,M,lt)` converts a standard description of a lattice using the a , M and lt parameters into a 2×2 integer matrix description. The conversion is *only* valid for the specified transform length L .

The output will be in lower triangular Hermite normal form.

For more information, see http://en.wikipedia.org/wiki/Hermite_normal_form.

An example:

```
V = latticetype2matrix(120,10,12,[1 2])
```

This code produces the following output:

```
V =
```

```
10      0
 5      10
```

2.9.3 SHEARFIND - Shears for transformation of a general lattice to separable**Usage**

```
[s0,s1,br] = shearfind(L,a,M,lt);
```

Description

`[s0,s1,br]=shearfind(L,a,M,lt)` computes three numbers, the first two represent a frequency and time shear respectively. With the returned choices of s_0 and s_1 one can transform an initial lattice given by a , M and lt into a separable (rectangular) lattice given by

$$a_r = \frac{aL}{b_r M}, \quad M_r = \frac{L}{b_r}.$$

If s_0 is non-zero, the transformation from general to separable lattice requires a frequency-side shear. Similarly, if s_1 is non-zero, a time-side shear is required.

2.9.4 NOSHEARLENGTH - Transform length that does not require a frequency shear**Usage**

```
L=noshearlength(Ls,a,M,lt)
```

Description

`noshearlength(Ls,a,M,lt)` computes the next larger transform length bigger or equal to L_s for which the shear algorithm does not require a frequency side shear for a non-separable Gabor system specified by a , M and lt .

This property makes computation of the canonical dual and tight Gabor windows `gabdual` and `gabtight` and the `dgt` for a full length window faster, if this transform length is chosen.

2.10 Plots**2.10.1 TFPLT - Plot coefficient matrix on the TF plane****Usage**

```
tfplot(coef,step,yr);
tfplot(coef,step,yr,...);
```

Description

`tfplot(coef, step, yr)` will plot a rectangular coefficient array on the TF-plane. The shift in samples between each column of coefficients is given by the variable `step`. The vector `yr` is a 1×2 vector containing the lowest and highest normalized frequency.

`C=tfplot(...)` returns the processed image data used in the plotting. Inputting this data directly to `imagesc` or similar functions will create the plot. This is usefull for custom post-processing of the image data.

`tfplot` is not meant to be called directly. Instead, it is called by other plotting routines to give a uniform display format.

`tfplot` (and all functions that call it) takes the following arguments.

'dynrange',r	Limit the dynamical range to <code>r</code> by using a colormap in the interval $[chigh-r, chigh]$, where <code>chigh</code> is the highest value in the plot. The default value of [] means to not limit the dynamical range.
'db'	Apply $20 \cdot \log_{10}$ to the coefficients. This makes it possible to see very weak phenomena, but it might show too much noise. A logarithmic scale is more adapted to perception of sound. This is the default.
'dbsq'	Apply $10 \cdot \log_{10}$ to the coefficients. Same as the ' <code>db</code> ' option, but assume that the input is already squared.
'lin'	Show the coefficients on a linear scale. This will display the raw input without any modifications. Only works for real-valued input.
'linsq'	Show the square of the coefficients on a linear scale.
'linabs'	Show the absolute value of the coefficients on a linear scale.
'tc'	Time centering. Move the beginning of the signal to the middle of the plot.
'clim',clim	Use a colormap ranging from <code>clim(1)</code> to <code>clim(2)</code> . These values are passed to <code>imagesc</code> . See the help on <code>imagesc</code> .
'image'	Use <code>imagesc</code> to display the plot. This is the default.
'contour'	Do a contour plot.
'surf'	Do a surf plot.
'colorbar'	Display the colorbar. This is the default.
'nocolorbar'	Do not display the colorbar.
'display'	Display the figure. This is the default.
'nodisplay'	Do not display figure. This is useful if you only want to obtain the output for further processing.

If both '`clim`' and '`dynrange`' are specified, then '`clim`' takes precedence.

It is possible to customize the text by setting the following values:

'time', t The word denoting time. Default is 'Time'

'frequency',f	The word denoting frequency. Default is 'Frequency'
'samples',s	The word denoting samples. Default is 'samples'
'normalized',n	Default value is 'normalized'.

2.10.2 PLOTDGT - Plot DGT coefficients

Usage

```
plotdgt(coef,a);
plotdgt(coef,a,fs);
plotdgt(coef,a,fs,dynrange);
```

Description

`plotdgt(coef,a)` plots the Gabor coefficients *coef*. The coefficients must have been produced with a time shift of *a*.

`plotdgt(coef,a,fs)` does the same assuming a sampling rate of *fs* Hz of the original signal.

`plotdgt(coef,a,fs,dynrange)` additionally limits the dynamic range.

The figure generated by this function places the zero-frequency in the center of the y-axis, with positive frequencies above and negative frequencies below.

`C=plotdgt(...)` returns the processed image data used in the plotting. Inputting this data directly to `imagesc` or similar functions will create the plot. This is useful for custom post-processing of the image data.

`plotdgt` supports all the optional parameters of `tfplot`. Please see the help of `tfplot` for an exhaustive list.

2.10.3 PLOTDGTRREAL - Plot DGTRREAL coefficients

Usage

```
plotdgtrreal(coef,a,M);
plotdgtrreal(coef,a,M,fs);
plotdgtrreal(coef,a,M,fs,dynrange);
```

Description

`plotdgtrreal(coef,a,M)` plots Gabor coefficient from `dgtreal`. The parameters *a* and *M* must match those from the call to `dgtreal`.

`plotdgtrreal(coef,a,M,fs)` does the same assuming a sampling rate of *fs* Hz of the original signal.

`plotdgtrreal(coef,a,M,fs,dynrange)` additionally limits the dynamic range.

`C=plotdgtrreal(...)` returns the processed image data used in the plotting. Inputting this data directly to `imagesc` or similar functions will create the plot. This is useful for custom post-processing of the image data.

`plotdgtrreal` supports all the optional parameters of `tfplot`. Please see the help of `tfplot` for an exhaustive list.

2.10.4 PLOTDWILT - Plot DWILT coefficients

Usage

```
plotdwilt(coef);
plotdwilt(coef,fs);
plotdwilt(coef,fs,dynrange);
```

Description

`plotdwilt(coef)` will plot coefficients from `dwilt`.

`plotdwilt(coef,fs)` will do the same assuming a sampling rate of *fs* Hz of the original signal. Since a Wilson representation does not contain coefficients for all positions on a rectangular TF-grid, there will be visible 'holes' among the lowest (DC) and highest (Nyquist rate) coefficients. See the help on `wil2rect`.

`plotdwilt(coef, fs, dynrange)` will additionally limit the dynamic range.

`C=plotdwilt(...)` returns the processed image data used in the plotting. Inputting this data directly to `imagesc` or similar functions will create the plot. This is useful for custom post-processing of the image data.

`plotdwilt` supports all the optional parameters of `tfplot`. Please see the help of `tfplot` for an exhaustive list.

2.10.5 PLOTWMDCT - Plot WMDCT coefficients

Usage

```
plotwmdct(coef);
plotwmdct(coef, fs);
plotwmdct(coef, fs, dynrange);
```

Description

`plotwmdct(coef)` plots coefficients from `wmdct`.

`plotwmdct(coef, fs)` does the same assuming a sampling rate of `fs` Hz of the original signal.

`plotwmdct(coef, fs, dynrange)` additionally limits the dynamic range.

`C=plotwmdct(...)` returns the processed image data used in the plotting. Inputting this data directly to `imagesc` or similar functions will create the plot. This is useful for custom post-processing of the image data.

`plotwmdct` supports all the optional parameters of `tfplot`. Please see the help of `tfplot` for an exhaustive list.

2.10.6 SGGRAM - Spectrogram

Usage

```
sgram(f, op1, op2, ... );
sgram(f, fs, op1, op2, ... );
C=sgram(f, ... );
```

Description

`sgram(f)` plots a spectrogram of `f` using a Discrete Gabor Transform (DGT).

`sgram(f, fs)` does the same for a signal with sampling rate `fs` (sampled with `fs` samples per second);

`sgram(f, fs, dynrange)` additionally limits the dynamic range of the plot. See the description of the '`dynrange`' parameter below.

`C=sgram(f, ...)` returns the image to be displayed as a matrix. Use this in conjunction with `imwrite` etc. These coefficients are **only** intended to be used by post-processing image tools. Numerical Gabor signal analysis and synthesis should **always** be done using the `dgt`, `idgt`, `dgtreal` and `idgtreal` functions.

Additional arguments can be supplied like this:

```
sgram(f, fs, 'dynrange', 50)
```

The arguments must be character strings possibly followed by an argument:

'dynrange',r Limit the dynamical range to `r` by using a colormap in the interval $[chigh - r, chigh]$, where `chigh` is the highest value in the plot. The default value of `[]` means to not limit the dynamical range.

'db' Apply $20 \cdot \log_{10}$ to the coefficients. This makes it possible to see very weak phenomena, but it might show too much noise. A logarithmic scale is more adapted to perception of sound. This is the default.

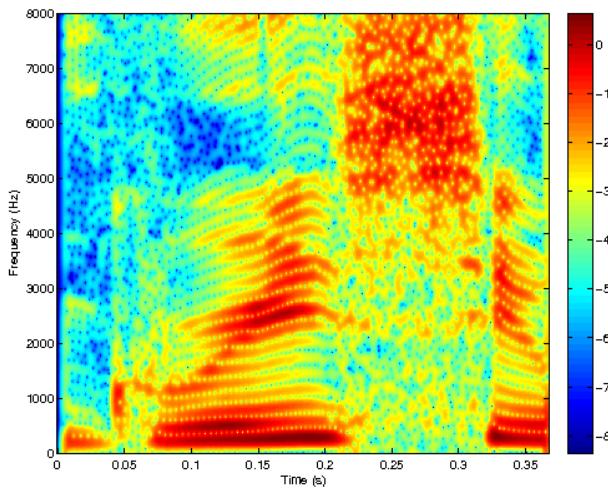
'lin'	Show the energy of the coefficients on a linear scale.
'tfr',v	Set the ratio of frequency resolution to time resolution. A value $v = 1$ is the default. Setting $v > 1$ will give better frequency resolution at the expense of a worse time resolution. A value of $0 < v < 1$ will do the opposite.
'wlen',s	Window length. Specifies the length of the window measured in samples. See help of pgauss on the exact details of the window length.
'posfreq'	Display only the positive frequencies. This is the default for real-valued signals.
'nf'	Display negative frequencies, with the zero-frequency centered in the middle. For real signals, this will just mirror the upper half plane. This is standard for complex signals.
'tc'	Time centering. Move the beginning of the signal to the middle of the plot. This is useful for visualizing the window functions of the toolbox.
'image'	Use imagesc to display the spectrogram. This is the default.
'clim',clim	Use a colormap ranging from $clim(1)$ to $clim(2)$. These values are passed to imagesc. See the help on imagesc.
'thr',r	Keep only the largest fraction r of the coefficients, and set the rest to zero.
'fmax',y	Display y as the highest frequency. Default value of [] means to use the Nyquist frequency.
'xres',xres	Approximate number of pixels along x-axis / time. The default value is 800
'yres',yres	Approximate number of pixels along y-axis / frequency. The Default value is 600
'contour'	Do a contour plot to display the spectrogram.
'surf'	Do a surf plot to display the spectrogram.
'mesh'	Do a mesh plot to display the spectrogram.
'colorbar'	Display the colorbar. This is the default.
'nocolorbar'	Do not display the colorbar.

In addition to these parameters, sgram accepts any of the flags from normalize. The window used to calculate the spectrogram will be normalized as specified.

Examples:

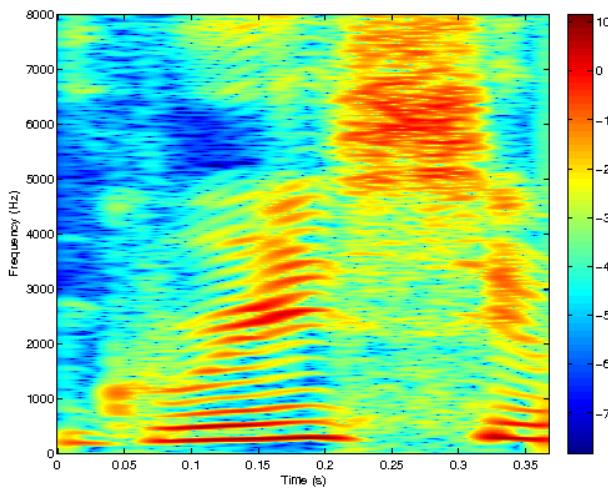
The greasy signal is sampled using a sampling rate of 16 kHz. To display a spectrogram of greasy with a dynamic range of 90 dB, use:

```
sgram(greasy,16000,90);
```



To create a spectrogram with a window length of 20ms (which is typically used in speech analysis) use

```
fs=16000;
sgram(greasy,fs,90,'wlen',round(20/1000*fs));
```



2.10.7 GABIMAGEPARS - Find Gabor parameters to generate image

Usage

```
[a,M,L,N,Ngood]=gabimagepars(Ls,x,y);
```

Description

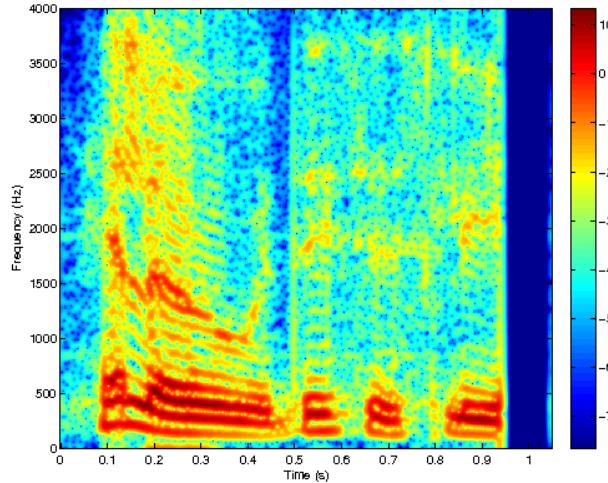
`[a,M,L,N,Ngood]=gabimagepars(Ls,x,y)` will compute a reasonable set of parameters a , M and L to produce a nice Gabor 'image' of a signal of length L_s . The approximate number of pixels in the time direction is given as x and the number of pixels in the frequency direction is given as y .

The output parameter N_{good} contains the number of time steps (columns in the coefficients matrix) that contains relevant information. The columns from N_{good} until N only contains information from a zero-extension of the signal.

If you use this function to calculate a grid size for analysis of a real-valued signal (using `dgtreal`), please input twice of the desired size y . This is because `dgtreal` only returns half as many coefficients in the frequency direction as `dgt`.

An example: We wish to compute a Gabor image of a real valued signal f of length 7500. The image should have an approximate resolution of 600×800 pixels:

```
[f,fs]=linus; f=f(4001:4000+7500);
[a,M,L,N,Ngood] = gabimagepars(7500,800,2*600);
c = dgtreal(f,'gauss',a,M);
plotdgtreal(c,a,M,fs,90);
```



The size of c is $(M/2) + 1 \times N$ equal to 601×700 pixels.

For this function to work properly, the specified numbers for x and y must not be large prime numbers.

2.10.8 RESGRAM - Reassigned spectrogram plot

Usage

```
resgram(f,op1,op2, ... );
resgram(f,fs,op1,op2, ... );
```

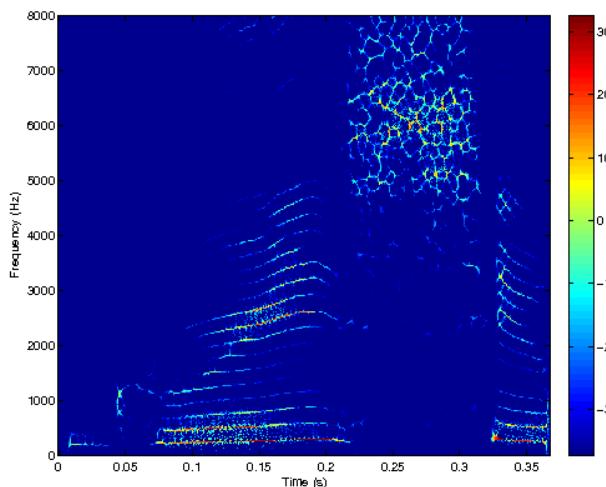
Description

`resgram(f)` plots a reassigned spectrogram of f .

`resgram(f,fs)` does the same for a signal with sampling rate fs (sampled with fs samples per second).

Because reassigned spectrograms can have an extreme dynamical range, consider always using the '`dynrange`' or '`clim`' options (see below) in conjunction with the '`db`' option (on by default). An example:

```
resgram(greasy,16000,'dynrange',70);
```



This will produce a reassigned spectrogram of the greasy signal without drowning the interesting features in noise.

`C=``resgram(f, ...)` returns the image to be displayed as a matrix. Use this in conjunction with `imwrite` etc. These coefficients are **only** intended to be used by post-processing image tools. Reassignment should be done using the `gabreassign` function instead.

`resgram` accepts the following additional arguments:

'dynrange',r	Limit the dynamical range to r by using a colormap in the interval $[chigh - r, chigh]$, where $chigh$ is the highest value in the plot. The default value of [] means to not limit the dynamical range.
'db'	Apply $20 \times \log_{10}$ to the coefficients. This makes it possible to see very weak phenomena, but it might show too much noise. A logarithmic scale is more adapted to perception of sound. This is the default.
'sharp',alpha	Set the sharpness of the plot. If $alpha = 0$ the regular spectrogram is obtained. $alpha = 1$ means full reassignment. Anything in between will produce a partially sharpened picture. Default is $alpha = 1$.
'lin'	Show the energy of the coefficients on a linear scale.
'tfr',v	Set the ratio of frequency resolution to time resolution. A value $v = 1$ is the default. Setting $v > 1$ will give better frequency resolution at the expense of a worse time resolution. A value of $0 < v < 1$ will do the opposite.
'wlen',s	Window length. Specifies the length of the window measured in samples. See help of <code>pgauss</code> on the exact details of the window length.
'posfreq'	Display only the positive frequencies. This is the default for real-valued signals.
'nf'	Display negative frequencies, with the zero-frequency centered in the middle. For real signals, this will just mirror the upper half plane. This is standard for complex signals.
'tc'	Time centering. Move the beginning of the signal to the middle of the plot. This is useful for visualizing the window functions of the toolbox.
'image'	Use <code>imagesc</code> to display the spectrogram. This is the default.

'clim',clim	Use a colormap ranging from <i>clim</i> (1) to <i>clim</i> (2). These values are passed to <code>imagesc</code> . See the help on <code>imagesc</code> .
'thr',r	Keep only the largest fraction <i>r</i> of the coefficients, and set the rest to zero.
'fmax',y	Display <i>y</i> as the highest frequency. Default value of [] means to use the Nyquist frequency.
'xres',xres	Approximate number of pixels along x-axis / time. The default value is 800
'yres',yres	Approximate number of pixels along y-axis / frequency. The default value is 600
'contour'	Do a contour plot to display the spectrogram.
'surf'	Do a surf plot to display the spectrogram.
'mesh'	Do a mesh plot to display the spectrogram.
'colorbar'	Display the colorbar. This is the default.
'nocolorbar'	Do not display the colorbar.

In addition to these parameters, `sgram` accepts any of the flags from `normalize`. The window used to calculate the spectrogram will be normalized as specified.

2.10.9 INSTFREQPLOT - Plot of instantaneous frequency

Usage

```
instfreqplot(f,op1,op2, ... );
instfreqplot(f,fs,op1,op2, ... );
```

Description

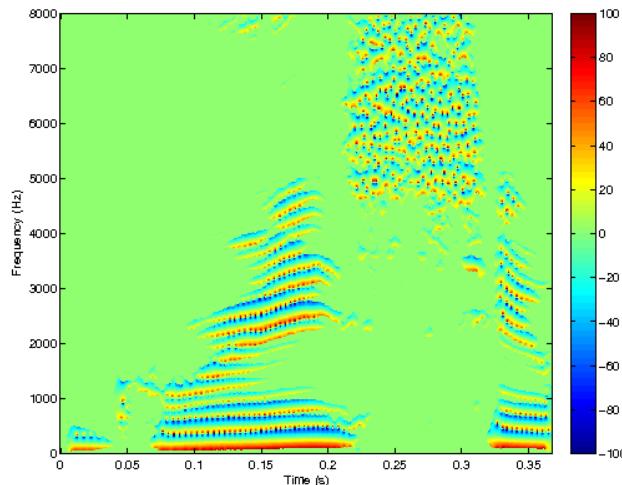
`instfreqplot(f)` plots the instantaneous frequency of *f* using a dgt.

`instfreqplot(f,fs)` does the same for a signal with sampling rate *fs* Hz.

The instantaneous frequency contains extreme spikes in regions where the spectrogram is close to zero. These points are usually uninteresting and destroy the visibility of the plot. Use the '`'thr'`' or '`'clim'`' or '`'climsym'`' options (see below) to remove these points.

An example:

```
instfreqplot(greasy,16000,'thr',.03,'climsym',100);
```



will produce a nice instantaneous frequency plot of the greasy signal.

`instfreqplot` accepts the following optional arguments:

'tfr',v	Set the ratio of frequency resolution to time resolution. A value $v = 1$ is the default. Setting $v > 1$ will give better frequency resolution at the expense of a worse time resolution. A value of $0 < v < 1$ will do the opposite.
'wlen',s	Window length. Specifies the length of the window measured in samples. See help of <code>pgauss</code> on the exact details of the window length.
'thr',r	Keep the coefficients with a magnitude larger than r times the largest magnitude. Set the instantaneous frequency of the rest of the coefficients to zero
'nf'	Display negative frequencies, with the zero-frequency centered in the middle. For real signals, this will just mirror the upper half plane. This is standard for complex signals.
'tc'	Time centering. Move the beginning of the signal to the middle of the plot. This is useful for visualizing the window functions of the toolbox.
'image'	Use <code>imagesc</code> to display the spectrogram. This is the default.
'dgt'	Use the ' <code>dgt</code> ' method to compute the instantaneous frequency. This is the default.
'clim',clim	Use a colormap ranging from <code>clim(1)</code> to <code>clim(2)</code> . These values are passed to <code>imagesc</code> . See the help on <code>imagesc</code> .
'climsym',cval	Use a colormap ranging from $-cval$ to <code>cval</code>
'fmax',y	Display <code>y</code> as the highest frequency.

2.10.10 PHASEPLOT - Phase plot

Usage

```
phaseplot(f,op1,op2, ... );
phaseplot(f,fs,op1,op2, ... );
```

Description

`phaseplot(f)` plots the phase of `f` using a `dgt`.

`phaseplot(f, fs)` does the same for a signal with sampling rate `fs` Hz.

`phaseplot` should only be used for short signals (shorter than the resolution of the screen), as there will otherwise be some visual aliasing, such that very fast changing areas will look very smooth. `phaseplot` always calculates the phase of the full time/frequency plane (as opposed to `sgram`), and you therefore risk running out of memory for long signals.

`phaseplot` takes the following flags at the end of the line of input arguments:

'tfr',v	Set the ratio of frequency resolution to time resolution. A value $v = 1$ is the default. Setting $v > 1$ will give better frequency resolution at the expense of a worse time resolution. A value of $0 < v < 1$ will do the opposite.
'wlen',s	Window length. Specifies the length of the window measured in samples. See help of <code>pgauss</code> on the exact details of the window length.

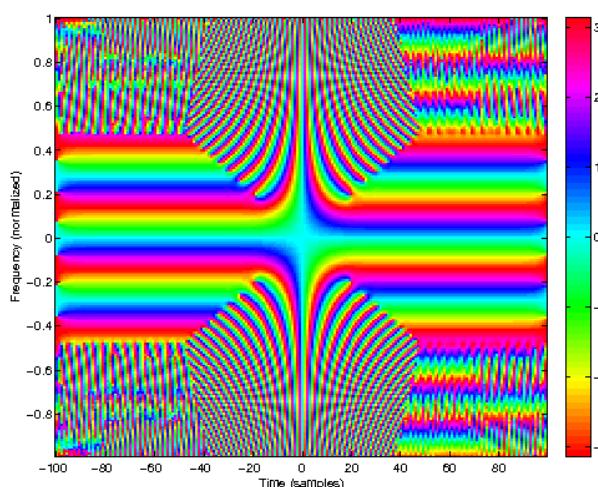
'nf'	Display negative frequencies, with the zero-frequency centered in the middle. For real signals, this will just mirror the upper half plane. This is standard for complex signals.
'tc'	Time centering. Move the beginning of the signal to the middle of the plot. This is useful for visualizing the window functions of the toolbox.
'thr',r	Keep the coefficients with a magnitude larger than r times the largest magnitude. Set the phase of the rest of the coefficients to zero. This is useful, because for small amplitude the phase values can be meaningless.
'timeinv'	Display the phase as computed by a time-invariant dgt. This is the default.
'freqinv'	Display the phase as computed by a frequency-invariant dgt.
'fmax',y	Display y as the highest frequency.
'colorbar'	Display the colorbar. This is the default.
'nocolorbar'	Do not display the colorbar.

For the best result when using `phaseplot`, use a circulant color map, for instance `hsv`.

Examples:

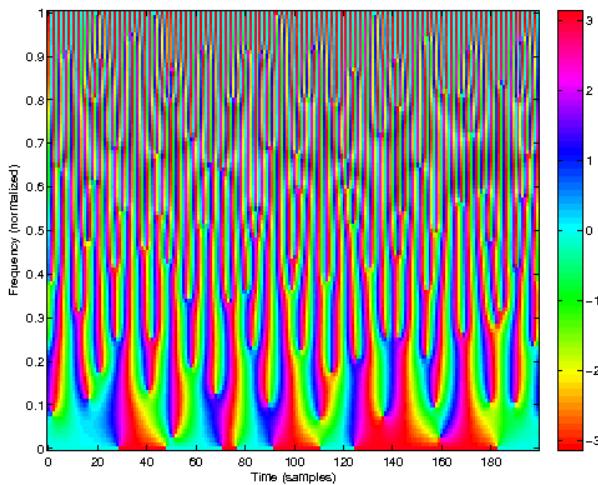
The following code shows the phaseplot of a periodic, hyperbolic secant visualized using the `hsv` colormap:

```
phaseplot(psech(200), 'tc', 'nf');
colormap(hsv);
```



The following phaseplot shows the phase of white, Gaussian noise:

```
phaseplot(randn(200,1));
colormap(hsv);
```



References: [20]

Chapter 3

LTFAT - Basic Fourier and DCT analysis.

3.1 Support routines

3.1.1 FFTINDEX - Frequency index of FFT modulations

Usage

```
n=fftindex(N);
```

Description

`fftindex(N)` returns the index of the frequencies of the standard FFT of length N as they are ordered in the output from the `fft` routine. The numbers returned are in the range `-ceil(N/2)+1:floor(N/2)`.
`fftindex(N, 0)` does as above, but sets the Nyquist frequency to zero.

3.1.2 MODCENT - Centered modulo

Usage

```
y=modcent(x, r);
```

Description

`modcent(x, r)` computes the modulo of x in the range $[-r/2, r/2[$.
As an example, to compute the modulo of x in the range $[-\pi, \pi[$ use the call:

```
y = modcent(x, 2*pi);
```

3.1.3 FLOOR23 - Previous number with only 2,3 factors

Usage

```
nceil=floor23(n);
```

Description

`floor23(n)` returns the first number less than or equal to n , which can be written as a product of powers of 2 and 3.

The algorithm will look up the best size in a table, which is computed the first time the function is run. If the input size is larger than the largest value in the table, the input size will be reduced by factors of 2, until it is in range.

`[nceil, table]=floor23(n)` additionally returns the table used for lookup.

Examples:

Return the first number smaller or equal to 26 that can be written solely as products of powers of 2 and 3:

```
floor23(26)
```

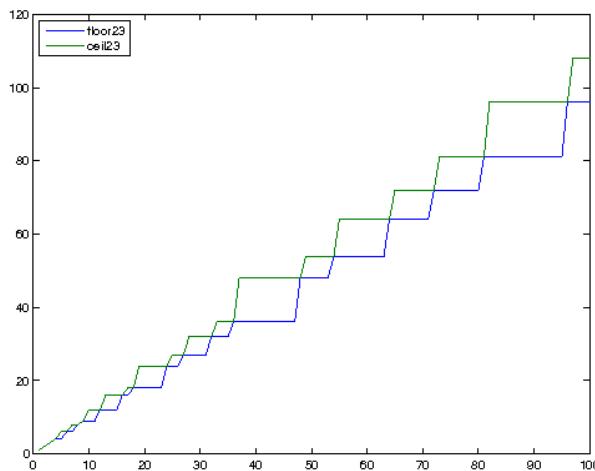
This code produces the following output:

```
ans =
```

```
24
```

This plot shows the behaviour of floor23 and ceil23 for numbers up to 100:

```
x=1:100;
plot(x,floor23(x),x,ceil23(x));
legend('floor23','ceil23','Location','Northwest');
```

**3.1.4 FLOOR235 - Previous number with only 2,3 and 5 factors****Usage**

```
nfloor=floor235(n);
```

Description

`floor235(n)` returns the next number greater than or equal to n , which can be written as a product of powers of 2, 3 and 5.

The algorithm will look up the best size in a table, which is computed the first time the function is run. If the input size is larger than the largest value in the table, the input size will be reduced by factors of 2, until it is in range.

`[nfloor,table]=floor235(n)` additionally returns the table used for lookup.

Examples:

Return the first number smaller or equal to 26 that can be written solely as products of powers of 2, 3 and 5:

```
floor235(26)
```

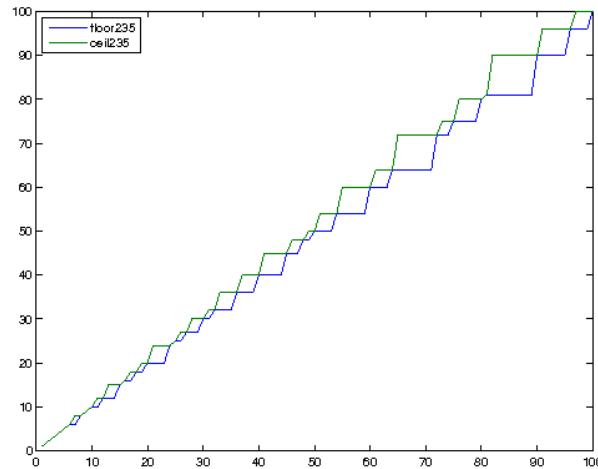
This code produces the following output:

```
ans =
```

```
25
```

This plot shows the behaviour of floor235 and ceil235 for numbers up to 100:

```
x=1:100;
plot(x,floor235(x),x,ceil235(x));
legend('floor235','ceil235','Location','Northwest');
```



3.1.5 CEIL23 - Next number with only 2,3 factors

Usage

```
nceil=ceil23(n);
```

Description

`ceil23(n)` returns the next number greater than or equal to n , which can be written as a product of powers of 2 and 3.

The algorithm will look up the best size in a table, which is computed the first time the function is run. If the input size is larger than the largest value in the table, the input size will be reduced by factors of 2, until it is in range.

`[nceil,table]=ceil23(n)` additionally returns the table used for lookup.

Examples:

Return the first number larger or equal to 19 that can be written solely as products of powers of 2 and 3:

```
ceil23(19)
```

This code produces the following output:

```
ans =
```

```
24
```

3.1.6 CEIL235 - Next number with only 2,3 and 5 factors

Usage

```
nceil=ceil235(n);
```

Description

`ceil235(n)` returns the next number greater than or equal to n , which can be written as a product of powers of 2, 3 and 5.

The algorithm will look up the best size in a table, which is computed the first time the function is run. If the input size is larger than the largest value in the table, the input size will be reduced by factors of 2, until it is in range.

`[nceil,table]=ceil235(n)` additionally returns the table used for lookup.

Examples:

Return the first number larger or equal to 19 that can be written solely as products of powers of 2, 3 and 5:

```
ceil235(19)
```

This code produces the following output:

```
ans =
```

```
20
```

3.1.7 NEXTFASTFFT - Next higher number with a fast FFT**Usage**

```
nfft=nextfastfft(n);
```

Description

`nextfastfft(n)` returns the next number greater than or equal to n , for which the computation of a FFT is fast. Such a number is solely comprised of small prime-factors of 2, 3, 5 and 7.

`nextfastfft` is intended as a replacement of `nextpow2`, which is often used for the same purpose. However, a modern FFT implementation (like FFTW) usually performs well for sizes which are powers of 2, 3, 5 and 7, and not only just for powers of 2.

The algorithm will look up the best size in a table, which is computed the first time the function is run. If the input size is larger than the largest value in the table, the input size will be reduced by factors of 2, until it is in range.

`[n,nfft]=nextfastfft(n)` additionally returns the table used for lookup.

References: [35], [23], [88]

3.2 Basic Fourier analysis**3.2.1 DFT - Normalized Discrete Fourier Transform****Usage**

```
f=dft(f);
f=dft(f,N,dim);
```

Description

`dft` computes a normalized or unitary discrete Fourier transform. The unitary discrete Fourier transform is computed by

$$c(k+1) = \frac{1}{\sqrt{L}} \sum_{l=0}^{L-1} f(l+1) e^{-2\pi i k l / L}$$

for $k = 0, \dots, L-1$.

The output of `dft` is a scaled version of the output from `fft`. The function takes exactly the same arguments as `fft`. See the help on `fft` for a thorough description.

3.2.2 IDFT - Inverse normalized Discrete Fourier Transform

Usage

```
f=idft(c);
f=idft(c,N,dim);
```

Description

`idft` computes a normalized or unitary inverse discrete Fourier transform. The unitary discrete Fourier transform is computed by

$$f(l+1) = \frac{1}{\sqrt{L}} \sum_{k=0}^{L-1} c(k+1) e^{2\pi i k l / L}$$

for $l = 0, \dots, L-1$.

The output of `idft` is a scaled version of the output from `ifft`. The function takes exactly the same arguments as `ifft`. See the help on `ifft` for a thorough description.

3.2.3 FFTREAL - FFT for real valued input data

Usage

```
f=fftreal(f);
f=fftreal(f,N,dim);
```

Description

`fftreal(f)` computes the coefficients corresponding to the positive frequencies of the FFT of the real valued input signal f .

The function takes exactly the same arguments as `fft`. See the help on `fft` for a thorough description.

3.2.4 IFFTREAL - Inverse FFT for real valued signals

Usage

```
f=ifftreal(c,N);
f=ifftreal(c,N,dim);
```

Description

`ifftreal(c,N)` computes an inverse FFT of the positive frequency Fourier coefficients c . The length N must always be specified, because the correct transform length cannot be determined from the size of c .

`ifftreal(c,N,dim)` does the same along dimension dim .

3.2.5 GGA - Generalized Goertzel algorithm

Usage

```
c = gga(x,fvec)
c = gga(x,fvec,fs)
```

Input parameters

x	Input data.
fvec	Indices to calculate.
fs	Sampling frequency.

Output parameters

c Coefficient vector.

Description

`c=gga(f,fvec)` computes the discrete-time fourier transform DTFT of f at frequencies in `fvec` as $c(k) = F(2\pi f_{vec}(k))$ where $F = DTFT(f)$, $k = 1, \dots, K$ and $K = \text{length}(fvec)$ using the generalized second-order Goertzel algorithm. Thanks to the generalization, values in `fvec` can be arbitrary numbers in range $0 - 1$ and not restricted to l/L_s , $l = 0, \dots, L_s - 1$ (usual DFT samples) as the original Goertzel algorithm is. L_s is the length of the first non-singleton dimension of f . If `fvec` is empty or omitted, `fvec` is assumed to be $(0:L_s-1)/L_s$ and results in the same output as `fft`.

`c=gga(f,fvec,fs)` computes the same with `fvec` in Hz relative to `fs`.

The input f is processed along the first non-singleton dimension or along dimension `dim` if specified.

Remark: Besides the generalization the algorithm is also shortened by one iteration compared to the conventional Goertzel.

Examples:

Calculating DTFT samples of interest:

```
% Generate input signal
fs = 8000;
L = 2^10;
k = (0:L-1).';
freq = [400,510,620,680,825];
phase = [pi/4,-pi/4,-pi/8,pi/4,-pi/3];
amp = [5,3,4,1,2];
f = arrayfun(@(a,f,p) a*sin(2*pi*k*f/fs+p),...
            amp,freq,phase,'UniformOutput',0);
f = sum(cell2mat(f),2);

% This is equal to fft(f)
ck = gga(f);

%GGA to FFT error:
norm(ck-fft(f))

% DTFT samples at 400,510,620,680,825 Hz
ckgga = gga(f,freq,fs);

% Plot modulus of coefficients
figure(1);clf;hold on;
stem(k/L*fs,2*abs(ck)/L,'k');
stem(freq,2*abs(ckgga)/L,'r:');
set(gca,'XLim',[freq(1)-50,freq(end)+50]);
set(gca,'YLim',[0 6]);
xlabel('f[Hz]');
ylabel('|c(k)|');
hold off;

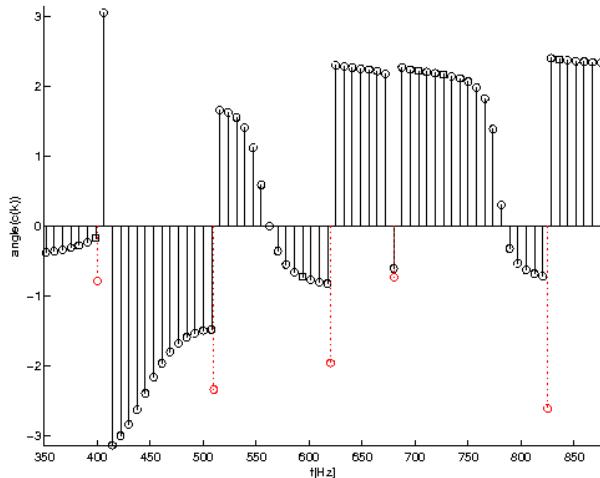
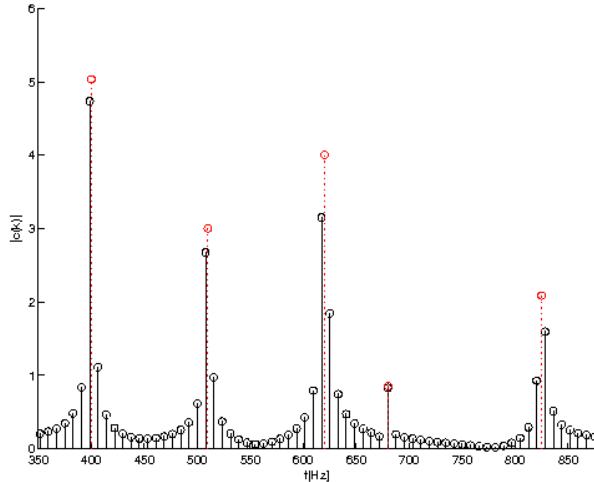
% Plot phase of coefficients
figure(2);clf;hold on;
stem(k/L*fs,angle(ck),'k');
stem(freq,angle(ckgga),'r:');
set(gca,'XLim',[freq(1)-50,freq(end)+50]);
set(gca,'YLim',[-pi pi]);
```

```
xlabel('f[Hz]');
ylabel('angle(c(k))');
hold off;
```

This code produces the following output:

ans =

1.604964106526122e-09



References: [90]

3.2.6 CHIRPZT - Chirped Z-transform

Usage

```
c = chirpz(f,K,fdiff)
c = chirpz(f,K,fdiff,foff)
c = chirpz(f,K,fdiff,foff,fs)
```

Input parameters

f Input data.

K Number of values.

fdiff	Frequency increment.
foff	Starting frequency.
fs	Sampling frequency.

Output parameters

c	Coefficient vector.
----------	---------------------

Description

`c = chirpzt(f, K, fdiff, foff)` computes K samples of the discrete-time fourier transform DTFT c of f at values $c(k+1) = F(2\pi(f_{off} + kf_{diff}))$ for $k = 0, \dots, K-1$ where $F = DTFT(f)$. Values `foff` and `fdiff` should be in range of 0–1. If `foff` is omitted or empty, it is considered to be 0. If `fdiff` is omitted or empty, K equidistant values $c(k+1) = F(2\pi k/K)$ are computed. If even K is omitted or empty, input length is used instead resulting in the same values as `fft` does.

`c = chirpzt(f, K, fdiff, foff, fs)` computes coefficients using frequency values relative to `fs` $c(k+1) = F(2\pi(f_{off} + kf_{diff})/fs)$ for $k = 0, \dots, K-1$.

The input f is processed along the first non-singleton dimension or along dimension `dim` if specified.

Examples:

Calculating DTFT samples of interest (aka zoom FFT):

```
% Generate input signal
fs = 8000;
L = 2^10;
k = (0:L-1).';
f1 = 400;
f2 = 825;
f = 5*sin(2*pi*k*f1/fs + pi/4) + 2*sin(2*pi*k*f2/fs - pi/3);

% This is equal to fft(f)
ck = chirpzt(f,L);

%chirpzt to FFT error:
norm(ck-fft(f))

% Frequency "resolution" in Hz
fdiff = 0.4;
% Frequency offset in Hz
foff = 803.9;
% Number of frequency values
K = 125;
% DTFT samples. The frequency range of interest is 803.9–853.5 Hz
ckchzt = chirpzt(f,K,fdiff,foff,fs);

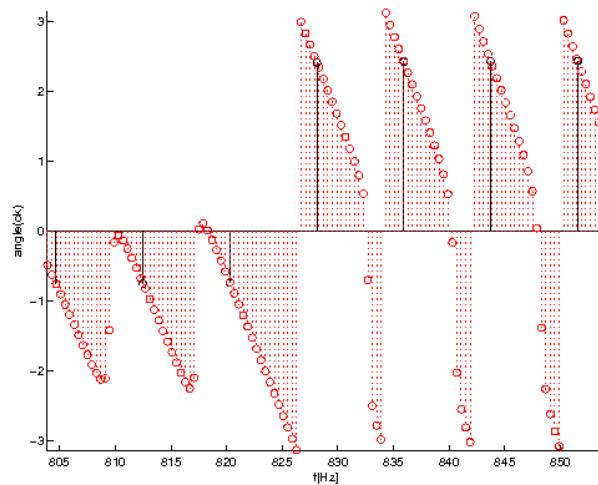
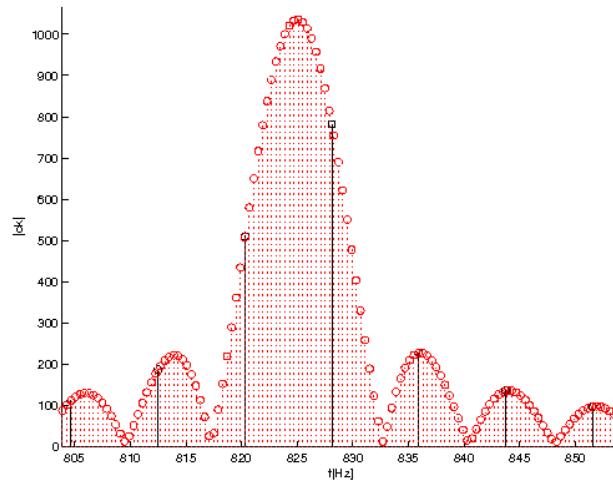
% Plot modulus of coefficients
figure(1);
fax=foff+fdiff.* (0:K-1);
hold on;
stem(k/L*fs,abs(ck),'k');
stem(fax,abs(ckchzt),'r:');
set(gca,'XLim',[foff,foff+K*fdiff]);
set(gca,'YLim',[0 1065]);
xlabel('f[Hz]');
ylabel('|ck|');
```

```
% Plot phase of coefficients
figure(2);
hold on;
stem(k/L*fs,angle(ck),'k');
stem(fax,angle(ckchzt),'r:');
set(gca,'XLim',[foff,foff+K*fdf]);
set(gca,'YLim',[-pi pi]);
xlabel('f[Hz]');
ylabel('angle(ck)');
```

This code produces the following output:

ans =

5.944006695631077e-10



References: [78]

3.2.7 FFTGRAM - Plot the energy of the discrete Fourier transform

Usage

`fftgram(f)`

```
fftgram(f, fs)
```

Description

`fftgram(f)` plots the energy of the discrete Fourier transform computed from the function `f`. The function forms a Fourier pair with the periodic autocorrelation function.

`fftgram(f, fs)` does the same for a signal sampled with a sampling frequency of `fs` Hz. If `fs` is no specified, the plot will display normalized frequencies.

`fftgram(f, fs, dynrange)` additionally specifies the dynamic range to display on the figure.

Additional arguments for `fftgram`:

'db' Plots the energy on a dB scale. This is the default.

'lin' Plots the energy on a linear scale.

In addition to these parameters, `fftgram` accepts any of the flags from `normalize`. The input signal will be normalized as specified.

3.2.8 PLOTFFT - Plot the output from FFT

Usage

```
plotfft(coef);
plotfft(coef, fs);
```

Description

`plotfft(coef)` plots the output from the `fft` function. The frequency axis will use normalized frequencies between 0 and 1 (the Nyquist frequency).

`plotfft(coef, fs)` does the same for the FFT of a signal sampled at a sampling rate of `fs` Hz.

`plotfft(coef, fs, dynrange)` additionally limits the dynamic range of the plot. See the description of the 'dynrange' parameter below.

`plotfft` accepts the following optional arguments:

'dynrange',r Limit the dynamical range to `r` by using a colormap in the interval `[chigh-r, chigh]`, where `chigh` is the highest value in the plot. The default value of [] means to not limit the dynamical range.

'db' Apply $20 \cdot \log_{10}$ to the coefficients. This makes it possible to see very weak phenomena, but it might show too much noise. This is the default.

'dbsq' Apply $10 \cdot \log_{10}$ to the coefficients. Same as the 'db' option, but assumes that the input is already squared.

'lin' Show the coefficients on a linear scale. This will display the raw input without any modifications. Only works for real-valued input.

'linsq' Show the square of the coefficients on a linear scale.

'linabs' Show the absolute value of the coefficients on a linear scale.

'nf' Display negative frequencies, with the zero-frequency centered in the middle. This is the default.

'posfreq' Display only the positive frequencies.

'dim',dim If `coef` is multidimensional, `dim` indicates the dimension along which are the individual channels oriented. Value 1 indicates columns, value 2 rows.

'flog'	Use logarithmic scale for the frequency axis. This flag is only valid in conjunction with the 'posfreq' flag.
---------------	---

In addition to these parameters, `plotfft` accepts any of the flags from `normalize`. The coefficients will be normalized as specified before plotting.

3.2.9 PLOTFFTREAL - Plot the output from FFTREAL

Usage

```
plotfftreal(coef);
plotfftreal(coef, fs);
```

Description

`plotfftreal(coef)` plots the output from the `fftreal` function. The frequency axis will use normalized frequencies between 0 and 1 (the Nyquist frequency). It is assumed that the length of the original transform was even.

`plotfftreal(coef, fs)` does the same for the `fftreal` of a signal sampled at a sampling rate of fs Hz.

`plotfftreal(coef, fs, dynrange)` additionally limits the dynamic range of the plot. See the description of the 'dynrange' parameter below.

`plotfftreal` accepts the following optional arguments:

'dynrange',r	Limit the dynamical range to r by using a colormap in the interval $[chigh-r, chigh]$, where $chigh$ is the highest value in the plot. The default value of [] means to not limit the dynamical range.
'db'	Apply $20 \cdot \log_{10}$ to the coefficients. This makes it possible to see very weak phenomena, but it might show too much noise. This is the default.
'dbsq'	Apply $10 \cdot \log_{10}$ to the coefficients. Same as the 'db' option, but assumes that the input is already squared.
'lin'	Show the coefficients on a linear scale. This will display the raw input without any modifications. Only works for real-valued input.
'linsq'	Show the square of the coefficients on a linear scale.
'linabs'	Show the absolute value of the coefficients on a linear scale.
'N',N	Specify the transform length N . Use this if you are unsure if the original input signal was of even length.
'dim',dim	If <code>coef</code> is multidimensional, <code>dim</code> indicates the dimension along which are the individual channels oriented. Value 1 indicates columns, value 2 rows.
'flog'	Use logarithmic scale for the frequency axis.

In addition to these parameters, `plotfftreal` accepts any of the flags from `normalize`. The coefficients will be normalized as specified before plotting.

3.3 Simple operations on periodic functions

3.3.1 INVOLUTE - Involution

Usage

```
finv=involute(f);
finv=involute(f, dim);
```

Description

`involute(f)` will return the involution of f .

`involute(f, dim)` will return the involution of f along dimension dim . This can for instance be used to calculate the 2D involution:

```
f=involute(f,1);
f=involute(f,2);
```

The involution f_{inv} of f is given by:

```
finv(l+1)=conj(f(mod(-l,L)+1));
```

for $l = 0, \dots, L - 1$.

The relation between conjugation, Fourier transformation and involution is expressed by:

```
conj(dft(f)) == dft(involute(f))
```

for all signals f . The inverse discrete Fourier transform can be expressed by:

```
idft(f) == conj(involute(dft(f)));
```

3.3.2 PEVEN - Even part of periodic function**Usage**

```
fe=peven(f);
fe=peven(f,dim);
```

Description

`peven(f)` returns the even part of the periodic sequence f .

`peven(f, dim)` does the same along dimension dim .

3.3.3 PODD - Odd part of periodic function**Usage**

```
fe=podd(f);
fe=podd(f,dim);
```

Description

`podd(f)` returns the odd part of the periodic sequence f .

`podd(f, dim)` does the same along dimension dim .

3.3.4 PCONV - Periodic convolution**Usage**

```
h=pconv(f,g)
h=pconv(f,g,fstype);
```

Description

`pconv(f, g)` computes the periodic convolution of f and g . The convolution is given by

$$h(l+1) = \sum_{k=0}^{L-1} f(k+1)g(l-k+1)$$

`pconv(f, g, 'r')` computes the convolution where g is reversed (involtuted) given by

$$h(l+1) = \sum_{k=0}^{L-1} f(k+1) \overline{g(k-l+1)}$$

This type of convolution is also known as cross-correlation.

`pconv(f, g, 'rr')` computes the alternative where both f and g are reversed given by

$$h(l+1) = \sum_{k=0}^{L-1} f(-k+1) g(l-k+1)$$

In the above formulas, $l-k$, $k-l$ and $-k$ are computed modulo L .

The input arrays f and g can be 1D vectors or one of them can be a multidimensional array. In either case, the convolution is performed along columns with row vectors transformed to columns.

3.3.5 PXCORR - Periodic cross correlation

Usage

```
h=pxcorr(f, g)
```

Description

`pxcorr(f, g)` computes the periodic cross correlation of the input signals f and g . The cross correlation is defined by

$$h(l+1) = \sum_{k=0}^{L-1} f(k+1) \overline{g(k-l+1)}$$

In the above formula, $k-l$ is computed modulo L .

`pxcorr(f, g, 'normalize')` does the same, but normalizes the output by the product of the l^2 -norm of f and g .

3.3.6 LCONV - Linear convolution

Usage

```
h=lconv(f, g);
```

Description

`lconv(f, g)` computes the linear convolution of f and g . The linear convolution is given by

$$h(l+1) = \sum_{k=0}^{L_h-1} f(k+1) g(l-k+1)$$

with $L_h = L_f + L_g - 1$ where L_f and L_g are the lengths of f and g , respectively.

`lconv(f, g, 'r')` computes the linear convolution of f and g where g is reversed. This type of convolution is also known as linear cross-correlation and is given by

$$h(l+1) = \sum_{k=0}^{L_h-1} f(k+1) \overline{g(k-l+1)}$$

`lconv(f, g, 'rr')` computes the alternative where both f and g are reversed given by

$$h(l+1) = \sum_{k=0}^{L_h-1} f(-k+1) g(l-k+1)$$

In the above formulas, $l-k$, $k-l$ and $-k$ are computed modulo L_h .

The input arrays f and g can be 1D vectors or one of them can be a multidimensional array. In either case, the convolution is performed along columns with row vectors transformed to columns.

3.3.7 LXCORR - Linear crosscorrelation

Usage

```
h=lxcorr(f, g)
```

Description

`lxcorr(f)` computes the linear crosscorrelation of the input signal f and g . The linear cross-correlation is computed by

$$h(l+1) = \sum_{k=0}^{L-1} f(k+1) \overline{g(k-l+1)}$$

with $L_h = L_f + L_g - 1$ where L_f and L_g are the lengths of f and g , respectively.

`lxcorr(f, 'normalize')` does the same, but normalizes the output by the product of the l^2 -norm of f and g .

3.3.8 ISEVENFUNCTION - True if function is even

Usage

```
t=isevenfunction(f);  
t=isevenfunction(f, tol);
```

Description

`isevenfunction(f)` returns 1 if f is whole point even. Otherwise it returns 0.

`isevenfunction(f, tol)` does the same, using the tolerance tol to measure how large the error between the two parts of the vector can be. Default is $1e-10$.

Adding the flag '`hp`' as the last argument does the same for half point even functions.

3.3.9 MIDDLEPAD - Symmetrically zero-extends or cuts a function

Usage

```
h=middlepad(f, L);  
h=middlepad(f, L, dim);  
h=middlepad(f, L, ...);
```

Description

`middlepad(f, L)` cuts or zero-extends f to length L by inserting zeros in the middle of the vector, or by cutting in the middle of the vector.

If f is whole-point even, `middlepad(f, L)` will also be whole-point even.

`middlepad(f, L, dim)` does the same along dimension dim .

If f has even length, then f will not be purely zero-extended, but the last element will be repeated once and multiplied by 1/2. That is, the support of f will increase by one!

Adding the flag '`wp`' as the last argument will cut or extend whole point even functions. Adding '`hp`' will do the same for half point even functions.

3.4 Periodic functions

3.4.1 EXPWAVE - Complex exponential wave

Usage

```
h=expwave(L, m);  
h=expwave(L, m, cent);
```

Description

`expwave(L, m)` returns an exponential wave revolving m times around the origin. The collection of all waves with wave number $m = 0, \dots, L - 1$ forms the basis of the discrete Fourier transform.

The wave has absolute value 1 everywhere. To get an exponential wave with unit l^2 -norm, divide the wave by \sqrt{L} . This is the normalization used in the `dft` function.

`expwave(L, m, cent)` makes it possible to shift the sampling points by the amount *cent*. Default is *cent* = 0.

3.4.2 PCHIRP - Periodic chirp**Usage**

```
g=pchirp(L, n);
```

Description

`pchirp(L, n)` returns a periodic, discrete chirp of length L that revolves n times around the time-frequency plane in frequency. n must be an integer number.

To get a chirp that revolves around the time-frequency plane in time, use

```
dft(pchirp(L, N));
```

The chirp is computed by:

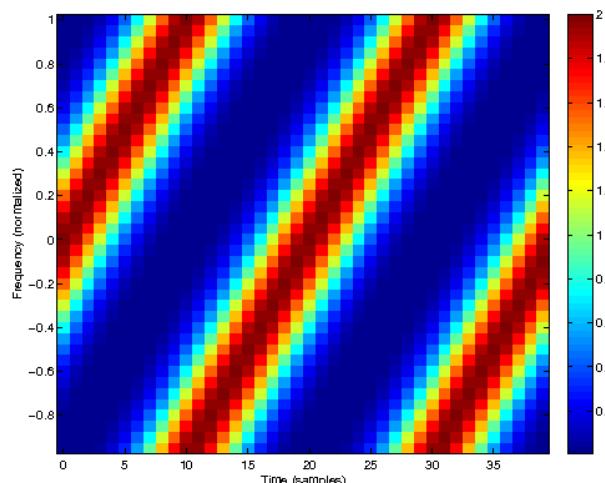
$$g(l+1) = e^{\pi i n(l-\lceil L/2 \rceil)^2(L+1)/L}, \quad l = 0, \dots, L-1$$

The chirp has absolute value 1 everywhere. To get a chirp with unit l^2 -norm, divide the chirp by \sqrt{L} .

Examples:

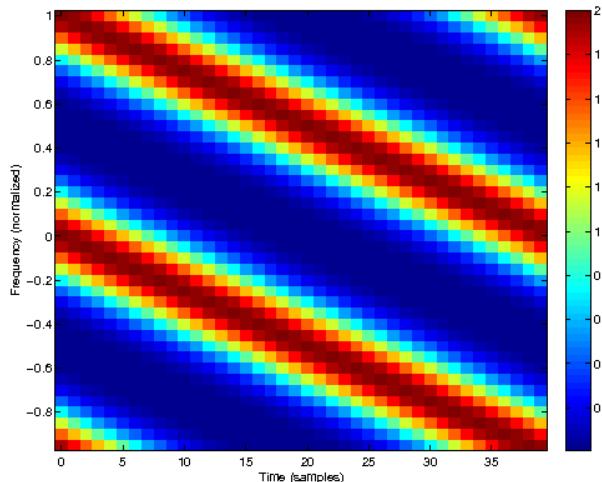
A spectrogram on a linear scale of an even length chirp:

```
sgram(pchirp(40, 2), 'lin');
```



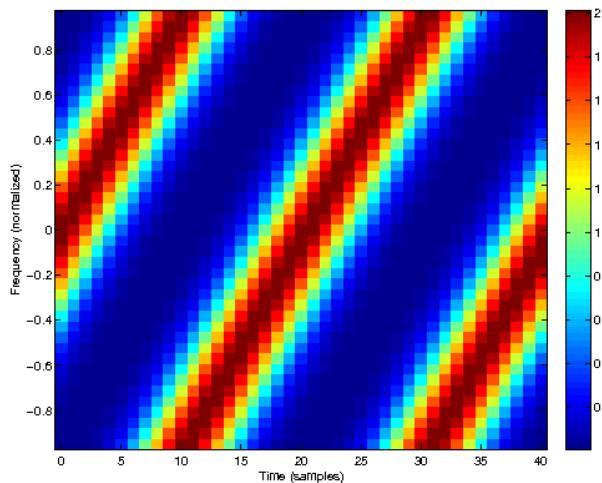
The DFT of the same chirp, now revolving around in time:

```
sgram(dft(pchirp(40, 2)), 'lin');
```



An odd-length chirp. Notice that the chirp starts at a frequency between two sampling points:

```
sgram(pchirp(41,2),'lin');
```



References: [31]

3.4.3 PGAUSS - Sampled, periodized Gaussian

Usage

```
g=pgauss(L);
g=pgauss(L,tfr);
g=pgauss(L,...);
[g,tfr]=pgauss( ... );
```

Input parameters

L Length of vector.

tfr ratio between time and frequency support.

Output parameters

g The periodized Gaussian.

Description

`pgauss(L,tfr)` computes samples of a periodized Gaussian. The function returns a regular sampling of the periodization of the function $\exp(-\pi i * (x^2/tfr))$.

The l^2 norm of the returned Gaussian is equal to 1.

The parameter `tfr` determines the ratio between the effective support of g and the effective support of the DFT of g . If $tfr > 1$ then g has a wider support than the DFT of g .

`pgauss(L)` does the same setting `tfr=1`.

`[g,tfr] = pgauss(...)` will additionally return the time-to-frequency support ratio. This is useful if you did not specify it (i.e. used the '`width`' or '`bw`' flag).

The function is whole-point even. This implies that `fft(pgauss(L,tfr))` is real for any L and tfr . The DFT of g is equal to `pgauss(L,1/tfr)`.

In addition to the '`width`' flag, `pgauss` understands the following flags at the end of the list of input parameters:

'fs',fs	Use a sampling rate of fs Hz as unit for specifying the width, bandwidth, centre frequency and delay of the Gaussian. Default is <code>fs=1</code> which indicates to measure everything in samples.
'width',s	Set the width of the Gaussian such that it has an effective support of s samples. This means that approx. 96% of the energy or 79% of the area under the graph is contained within s samples. This corresponds to -6dB or to width at the half of the height. This is equivalent to calling <code>pgauss(L,pi*s^2/4L*log(2))</code> .
'atheight',ah	Used only in conjunction with ' <code>width</code> '. Forces the Gaussian to width s at the ah fraction of the height.
'bw',bw	As for the ' <code>width</code> ' argument, but specifies the width in the frequency domain. The bandwidth is measured in normalized frequencies, unless the ' <code>fs</code> ' value is given.
'cf',cf	Set the centre frequency of the Gaussian to fc .
'wp'	Output is whole point even. This is the default.
'hp'	Output is half point even, as most Matlab filter routines.
'delay',d	Delay the output by d . Default is zero delay.

In addition to these parameters, `pgauss` accepts any of the flags from `normalize`. The output will be normalized as specified.

If this function is used to generate a window for a Gabor frame, then the window giving the smallest frame bound ratio is generated by `pgauss(L,a*M/L)`.

Examples:

This example creates a Gaussian function, and demonstrates that it is its own Discrete Fourier Transform:

```
g=pgauss(128);
% Test of DFT invariance: Should be close to zero.
norm(g-dft(g))
```

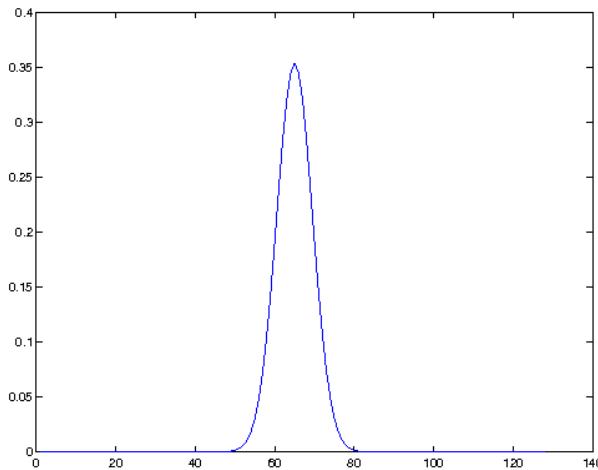
This code produces the following output:

```
ans =
```

```
2.481524897647706e-15
```

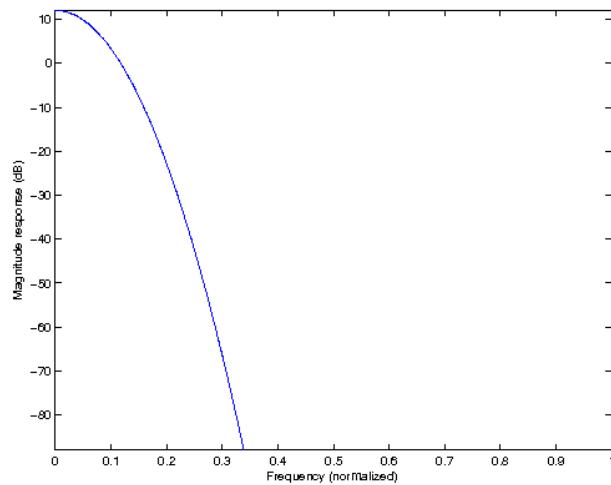
The next plot shows the Gaussian in the time domain:

```
plot(fftshift(pgauss(128)));
```



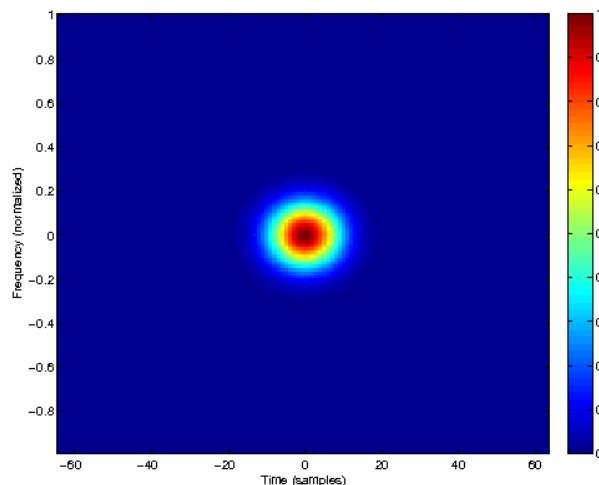
The next plot shows the Gaussian in the frequency domain on a log scale:

```
magresp(pgauss(128), 'dynrange', 100);
```



The next plot shows the Gaussian in the time-frequency plane:

```
sgram(pgauss(128), 'tc', 'nf', 'lin');
```



References: [62]

3.4.4 PSECH - Sampled, periodized hyperbolic secant

Usage

```
g=psech(L);
g=psech(L,tfr);
g=psech(L,s,'samples');
[g,tfr]=psech( ... );
```

Input parameters

L Length of vector.

tfr ratio between time and frequency support.

Output parameters

g The periodized hyperbolic cosine.

Description

`psech(L,tfr)` computes samples of a periodized hyperbolic secant. The function returns a regular sampling of the periodization of the function

The returned function has norm equal to 1.

The parameter *tfr* determines the ratio between the effective support of *g* and the effective support of the DFT of *g*. If *tfr* > 1 then *g* has a wider support than the DFT of *g*.

`psech(L)` does the same setting *tfr* = 1.

`psech(L,s,'samples')` returns a hyperbolic secant with an effective support of *s* samples. This means that approx. 96% of the energy or 74% of the area under the graph is contained within *s* samples. This is equivalent to `psech(L,s^2/L)`.

`[g,tfr] = psech(...)` additionally returns the time-to-frequency support ratio. This is useful if you did not specify it (i.e. used the '*'samples'* input format).

The function is whole-point even. This implies that `fft(psech(L,tfr))` is real for any *L* and *tfr*.

If this function is used to generate a window for a Gabor frame, then the window giving the smallest frame bound ratio is generated by `psech(L,a*M/L)`.

Examples:

This example creates a `psech` function, and demonstrates that it is its own Discrete Fourier Transform:

```
g=psech(128);

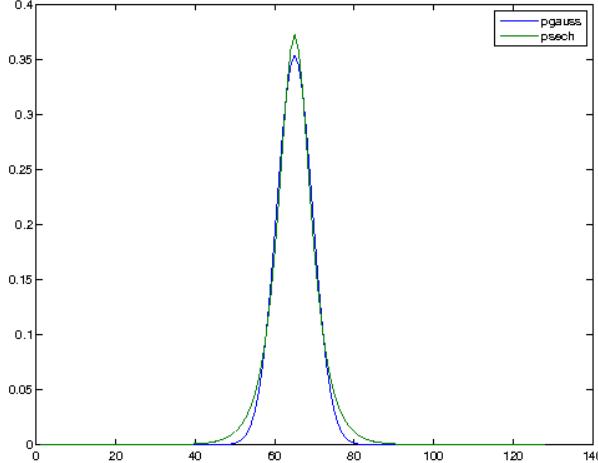
% Test of DFT invariance: Should be close to zero.
norm(g-dft(g))
```

This code produces the following output:

```
ans =
2.400287682482777e-15
```

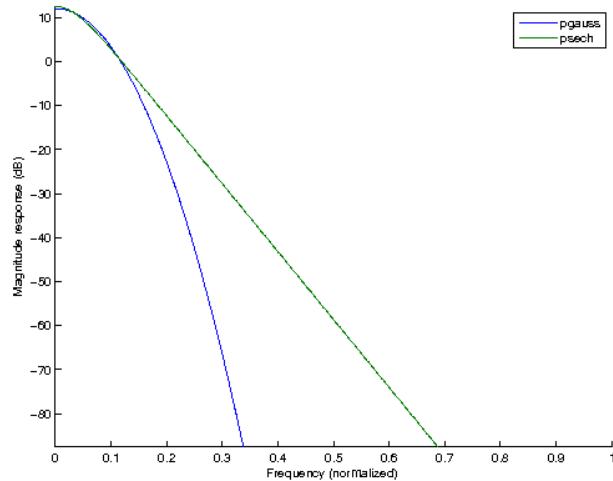
The next plot shows the `psech` in the time domain compared to the Gaussian:

```
plot((1:128)',fftshift(pgauss(128)),...
      (1:128)',fftshift(psech(128)));
legend('pgauss','psech');
```



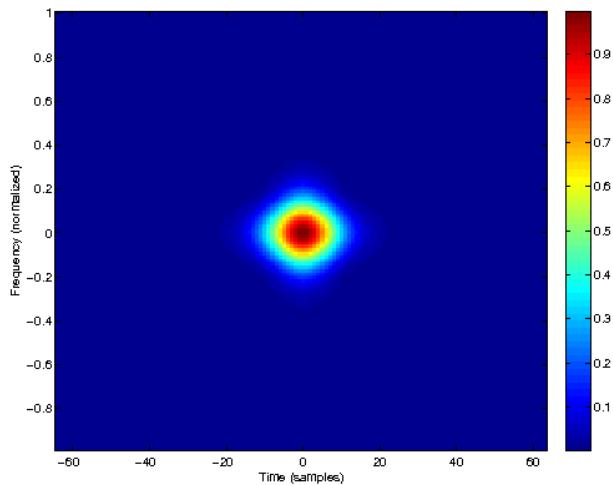
The next plot shows the `psech` in the frequency domain on a log scale compared to the Gaussian:

```
hold all;
magresp(pgauss(128),'dynrange',100);
magresp(psech(128),'dynrange',100);
legend('pgauss','psech');
```



The next plot shows psech in the time-frequency plane:

```
sgram(psech(128), 'tc', 'nf', 'lin');
```



References: [48]

3.4.5 PBSPLINE - Periodized B-spline

Usage

```
g=pbspline(L,order,a,...);
[g,nlen]=pbspline(L,order,a,...);
```

Input parameters

- L** Length of window.
- order** Order of B-spline.
- a** Time-shift parameter for partition of unity.

Output parameters

- g** Fractional B-spline.
- nlen** Number of non-zero elements in out.

Description

`pbspline(L, order, a)` computes a (slightly modified) B-spline of order *order* of total length *L*.

If shifted by the distance *a*, the returned function will form a partition of unity. The result is normalized such that the functions sum to $1/\sqrt{a}$.

`pbspline` takes the following flags at the end of the input arguments:

'ed'	Even discrete fractional spline. This is the default	
'xd'	'flat'	discrete fractional spline.
'stard'" pointy discrete fractional spline		
'ec"	Even fractional spline by sampling.	
'xc'	'flat'	fractional spline by sampling.
'starc'" pointy fractional spline by sampling.		
'wp'''	Generate whole point centered splines. This is the default.	
'hp'	Generate half point centered splines.	

The different types are accurately described in the referenced paper. Generally, the 'd' types of splines are very fast to compute, while the 'c' types are samplings of the continuous splines. The 'e' types coincides with the regular B-splines for integer orders. The 'x' types do not coincide, but generate Gabor frames with favorable frame bounds. The default type is 'ed' to guarantee fast computation and a familiar shape of the splines.

[out, nlen]=`pbspline`(...) will additionally compute the number of non-zero elements in *out*. If *nlen*=*L*, the function returned will be a periodization of a B-spline.

If *nlen*<*L*, you can choose to remove the additional zeros by calling `g=middlepad(g, nlen)`.

Additionally, `pbspline` accepts flags to normalize the output. Please see the help of `normalize`. Default is to use 'peak' normalization.

References: [87]

3.4.6 SHAH - Discrete Shah-distribution**Usage**

```
f=shah(L, a);
```

Description

`shah(L, a)` computes the discrete, normalized Shah-distribution of length *L* with a distance of *a* between the spikes.

The Shah distribution is defined by

$$f(n \cdot a + 1) = \frac{1}{\sqrt{(L/a)}}$$

for integer *n*, otherwise *f* is zero.

This is also known as an impulse train or as the comb function, because the shape of the function resembles a comb. It is the sum of unit impulses ('diracs') with the distance *a*.

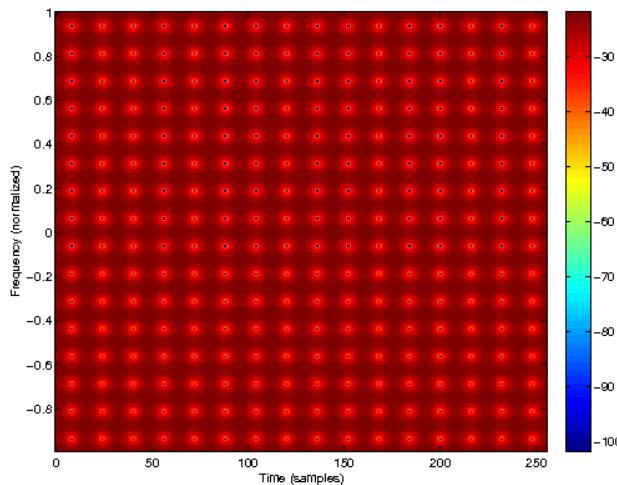
If *a* divides *L*, then the dft of `shah(L, a)` is `shah(L, L/a)`.

The Shah function has an extremely bad time-frequency localization. It does not generate a Gabor frame for any *L* and *a*.

Examples:

A simple spectrogram of the Shah function (includes the negative frequencies to display the whole TF-plane):

```
sgram(shah(256,16), 'dynrange', 80, 'nf')
```

**3.4.7 PHEAVISIDE - Periodic Heaviside function****Usage**

```
h=pheaviside(L);
```

Description

`pheaviside(L)` returns a periodic Heaviside function. The periodic Heaviside function takes on the value 1 for indices corresponding to positive frequencies, 0 corresponding to negative frequencies and the value .5 for the zero and Nyquist frequencies.

To get a function that weights the negative frequencies by 1 and the positive by 0, use `involute(pheaviside(L))`

As an example, the `pheaviside` function can be used to calculate the Hilbert transform for a column vector f :

```
h=2*ifft(fft(f).*pheaviside(length(f)));
```

3.4.8 PRECT - Periodic rectangle**Usage**

```
f=prect(L,n);
```

Description

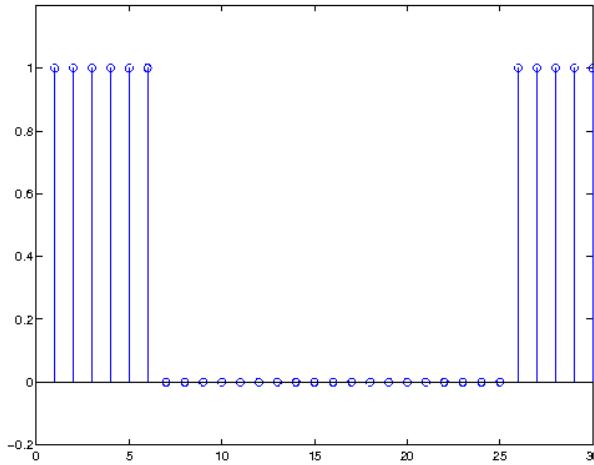
`psinc(L,n)` computes the periodic rectangle (or square) function of length L supported on n samples. The dft of the periodic rectangle function is the periodic sinc function, `psinc`.

- If n is odd, the output will be supported on exactly n samples centered around the first sample.
- If n is even, the output will be supported on exactly $n+1$ samples centered around the first sample. The function value on the two samples on the edge of the function will have half the magnitude of the other samples.

Examples:

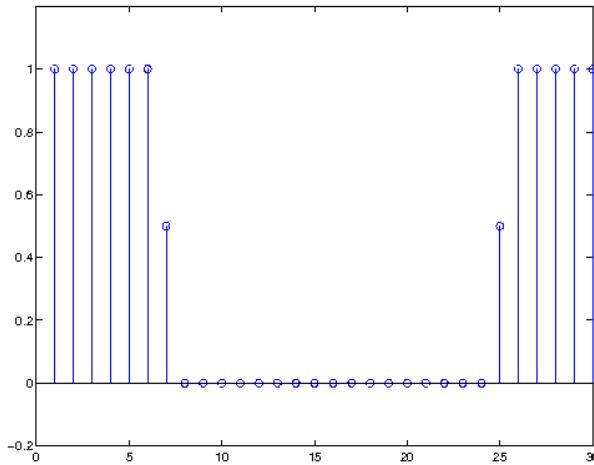
This figure displays an odd length periodic rectangle:

```
stem(prect(30,11));
ylim([- .2 1.2]);
```



This figure displays an even length periodic rectangle. Notice the border points:

```
stem(prect(30,12));
ylim([- .2 1.2]);
```



3.4.9 PSINC - Periodic Sinc function (Dirichlet function)

Usage

```
f=psinc(L,n);
```

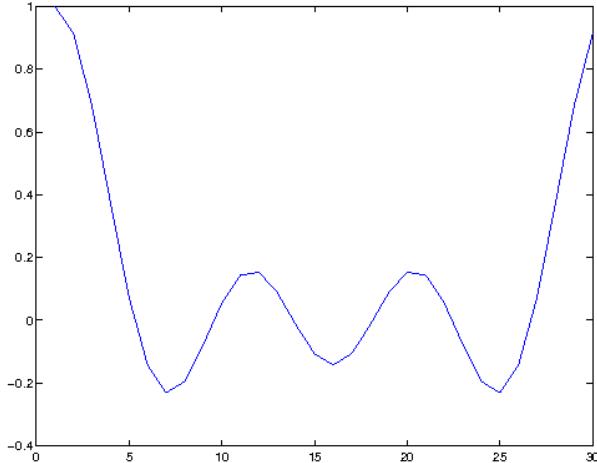
Description

`psinc(L,n)` computes the periodic Sinc function of length L with $n - 1$ local extrema. The dft of the periodic Sinc function is the periodic rectangle, `prect`, of length n .

Examples:

This figure displays a the periodic sinc function with 6 local extrema:

```
plot(psinc(30, 7));
```



3.5 Hermite functions and fractional Fourier transforms

3.5.1 PHERM - Periodized Hermite function

Usage

```
g=pherm(L,order);
g=pherm(L,order,tfr);
[g,D]=phermd(...);
```

Input parameters

L	Length of vector.
order	Order of Hermite function.
tfr	ratio between time and frequency support.

Output parameters

g	The periodized Hermite function
----------	---------------------------------

Description

`phermd(L,order,tfr)` computes samples of a periodized Hermite function of order *order*. *order* is counted from 0, so the zero'th order Hermite function is the Gaussian.

The parameter *tfr* determines the ratio between the effective support of *g* and the effective support of the DFT of *g*. If *tfr* > 1 then *g* has a wider support than the DFT of *g*.

`phermd(L,order)` does the same setting *tfr* = 1.

If *order* is a vector, `phermd` will return a matrix, where each column is a Hermite function with the corresponding *order*.

`[g,D]=phermd(...)` also returns the eigenvalues *D* of the Discrete Fourier Transform corresponding to the Hermite functions.

The returned functions are eigenvectors of the DFT. The Hermite functions are orthogonal to all other Hermite functions with a different eigenvalue, but eigenvectors with the same eigenvalue are not orthogonal (but see the flags below).

`pherm` takes the following flags at the end of the line of input arguments:

'accurate'	Use a numerically very accurate that computes each Hermite function individually. This is the default.
'fast'	Use a less accurate algorithm that calculates all the Hermite up to a given order at once.
'noorth'	No orthonormalization of the Hermite functions. This is the default.
'polar'	Orthonormalization of the Hermite functions using the polar decomposition orthonormalization method.
'qr'	Orthonormalization of the Hermite functions using the Gram-Schmidt orthonormalization method (using <code>qr</code>).

If you just need to compute a single Hermite function, there is no speed difference between the '`accurate`' and '`fast`' algorithm.

Examples:

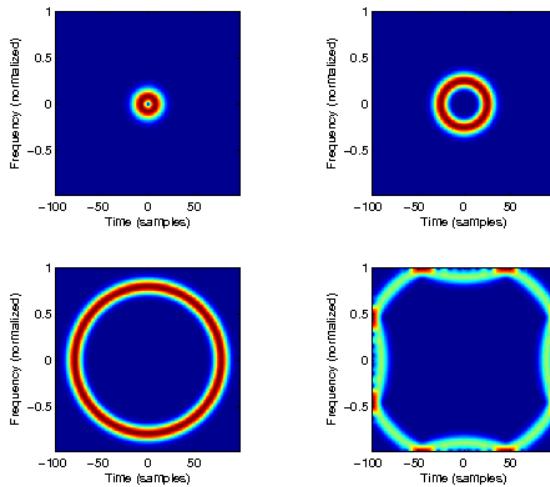
The following plot shows the spectrograms of 4 Hermite functions of length 200 with order 1, 10, 100, and 190:

```
subplot(2,2,1);
sgram(pherm(200,1),'nf','tc','lin','nocolorbar'); axis('square');

subplot(2,2,2);
sgram(pherm(200,10),'nf','tc','lin','nocolorbar'); axis('square');

subplot(2,2,3);
sgram(pherm(200,100),'nf','tc','lin','nocolorbar'); axis('square');

subplot(2,2,4);
sgram(pherm(200,190),'nf','tc','lin','nocolorbar'); axis('square');
```



3.5.2 HERMBASIS - Orthonormal basis of discrete Hermite functions

Usage

```
V=hermbasis(L,p);
V=hermbasis(L);
[V,D]=hermbasis(...);
```

Description

`hermbasis(L,p)` computes an orthonormal basis of discrete Hermite functions of length L . The vectors are returned as columns in the output. p is the order of approximation used to construct the position and difference operator.

All the vectors in the output are eigenvectors of the discrete Fourier transform, and resemble samplings of the continuous Hermite functions to some degree (for low orders).

`[V,D]=hermbasis(...)` also returns the eigenvalues D of the Discrete Fourier Transform corresponding to the Hermite functions.

Examples:

The following plot shows the spectrograms of 4 Hermite functions of length 200 with order 1, 10, 100, and 190:

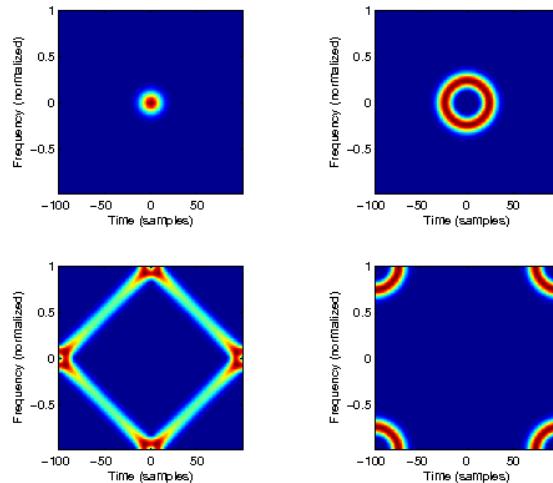
```
H=hermbasis(200);

subplot(2,2,1);
sgram(H(:,1),'nf','tc','lin','nocolorbar'); axis('square');

subplot(2,2,2);
sgram(H(:,10),'nf','tc','lin','nocolorbar'); axis('square');

subplot(2,2,3);
sgram(H(:,100),'nf','tc','lin','nocolorbar'); axis('square');

subplot(2,2,4);
sgram(H(:,190),'nf','tc','lin','nocolorbar'); axis('square');
```



References: [70, 19]

3.5.3 DFRACFT - Discrete Fractional Fourier transform

Usage

```
V=dfracft(f,a,p);
V=dfracft(f,a);
```

Description

`dfracft(f,a)` computes the discrete fractional Fourier Transform of the signal f to the power a . For $a=1$ it corresponds to the ordinary discrete Fourier Transform. If f is multi-dimensional, the transformation is applied along the first non-singleton dimension.

`dfracft(f,a,dim)` does the same along dimension dim .

`dfracft(f,a,[],p)` or `dfracft(f,a,dim,p)` allows to choose the order of approximation of the second difference operator (default: $p=2$).

References: [70, 19]

3.5.4 FFRACFT - Approximate fast fractional Fourier transform

Usage

```
frf=ffracft(f,a)
frf=ffracft(f,a,dim)
```

Description

`ffracft(f,a)` computes an approximation of the fractional Fourier transform of the signal f to the power a . If f is multi-dimensional, the transformation is applied along the first non-singleton dimension.

`ffracft(f,a,dim)` does the same along dimension dim .

`ffracft` takes the following flags at the end of the line of input arguments:

'origin'

Rotate around the origin of the signal. This is the same action as the `dft`, but the signal will split in the middle, which may not be the correct action for data signals. This is the default.

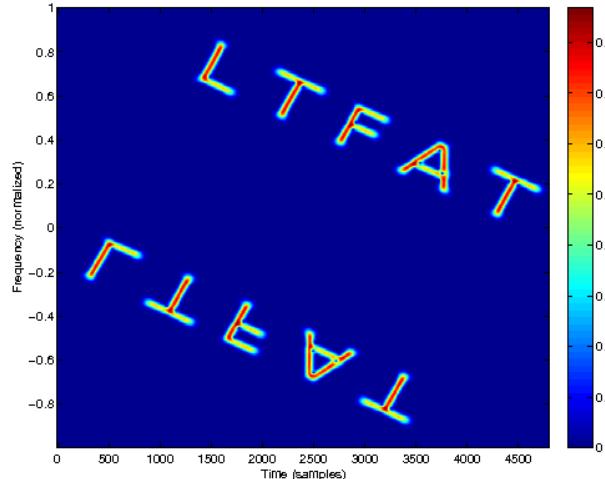
'middle'

Rotate around the middle of the signal. This will not break the signal in the middle, but the `dft` cannot be obtained in this way.

Examples:

The following example shows a rotation of the `ltfatlogo` test signal:

```
sgram(ffracft(ltfatlogo,.3,'middle'),'lin','nf');
```



References: [19]

3.6 Approximation of continuous functions

3.6.1 FFTRESAMPLE - Resample signal using Fourier interpolation

Usage

```
h=fftresample(f,L);
h=fftresample(f,L,dim);
```

Description

`fftresample(f,L)` returns a Fourier interpolation of the signal f to length L . If the function is applied to a matrix, it will apply to each column.

`fftresample(f,L,dim)` does the same along dimension dim .

If the input signal is **not** a periodic signal (or close to), the `dctresample` method gives much better results at the endpoints.

3.6.2 DCTRESAMPLE - Resample signal using Fourier interpolation

Usage

```
h=dctresample(f,L);
h=dctresample(f,L,dim);
```

Description

`dctresample(f,L)` returns a discrete cosine interpolation of the signal f to length L . If the function is applied to a matrix, it will apply to each column.

`dctresample(f,L,dim)` does the same along dimension dim .

If the input signal is not a periodic signal (or close to), this method will give much better results than `fftresample` at the endpoints, as this method assumes that the signal is even at the endpoints.

The algorithm uses a DCT type iii.

3.6.3 PDERIV - Derivative of smooth periodic function

Usage

```
fd=pderiv(f);
fd=pderiv(f,dim);
fd=pderiv(f,dim,difforder);
```

Description

`pderiv(f)` will compute the derivative of f using a using a 4th order centered finite difference scheme. f must have been obtained by a regular sampling. If f is a matrix, the derivative along the columns will be found.

`pderiv(f,dim)` will do the same along dimension dim .

`pderiv(f,dim,difforder)` uses a centered finite difference scheme of order $difforder$ instead of the default.

`pderiv(f,dim,Inf)` will compute the spectral derivative using a DFT.

`pderiv` assumes that f is a regular sampling of a function on the torus $[0, 1]$. The derivative of a function on a general torus $[0, T]$ can be found by scaling the output by $1/T$.

3.6.4 FFTANALYTIC - Compute analytic representation

Usage

```
z = fftanalytic(f);
z = fftanalytic(f,L);
z = fftanalytic(f,L,dim);
```

Input parameters

f	Input data.
L	Extend or truncate f to this length.
dim	Dimension to along which to apply the computations.

Output parameters

z	Analytic signal.
----------	------------------

Description

`fftanalytic(f)` computes the analytic representation of a real-valued signal f . The analytic representation is computed through the FFT of f . The computations are done along the first non-singleton dimension.

`fftanalytic(f, L)` acts as before but f is padded with zeros or truncated to length L .

`fftanalytic(f, L, dim)` in addition allows specifying the dimension along which the computation should be done.

The real part of the analytic representation z equals the signal f and the imaginary part is the Hilbert transform of f .

The instantaneous amplitude (a Hilbert envelope) of the signal f can be computed as:

```
abs(fftanalytic(f));
```

The instantaneous phase of the function f can be computed as:

```
angle(fftanalytic(f));
```

3.7 Cosine and Sine transforms.

3.7.1 DCTI - Discrete Cosine Transform type I

Usage

```
c=dcti(f);
c=dcti(f,L);
c=dcti(f,[],dim);
c=dcti(f,L,dim);
```

Description

`dcti(f)` computes the discrete cosine transform of type I of the input signal f . If f is a matrix then the transformation is applied to each column. For N-D arrays, the transformation is applied to the first non-singleton dimension.

`dcti(f, L)` zero-pads or truncates f to length L before doing the transformation.

`dcti(f, [], dim)` or `dcti(f, L, dim)` applies the transformation along dimension dim .

The transform is real (output is real if input is real) and it is orthonormal.

This transform is its own inverse.

Let f be a signal of length L , let $c = \text{dcti}(f)$ and define the vector w of length L by

$$w(n) = \begin{cases} \frac{1}{\sqrt{2}} & \text{if } n = 0 \text{ or } n = L - 1 \\ 1 & \text{otherwise} \end{cases}$$

Then

$$c(n+1) = \sqrt{\frac{2}{L-1}} \sum_{m=0}^{L-1} w(n) w(m) f(m+1) \cos\left(\frac{\pi nm}{L-1}\right)$$

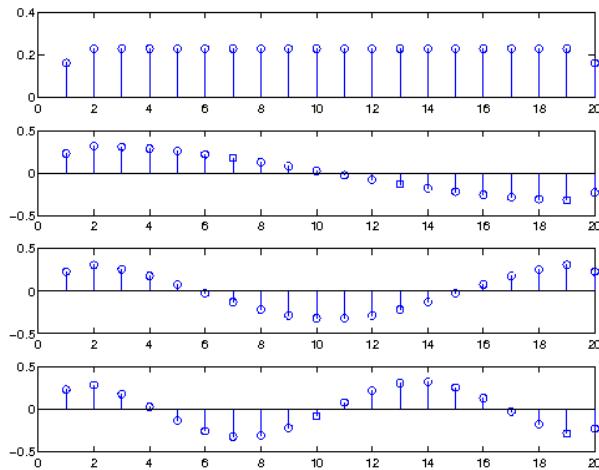
The implementation of this function uses a simple algorithm that requires an FFT of length $2L-2$, which might potentially be the product of a large prime number. This may cause the function to sometimes execute slowly. If guaranteed high speed is a concern, please consider using one of the other DCT transforms.

Examples:

The following figures show the first 4 basis functions of the DCTI of length 20:

```
% The dcti is its own adjoint.
F=dcti(eye(20));

for ii=1:4
    subplot(4,1,ii);
    stem(F(:,ii));
end;
```



References: [79], [97]

3.7.2 DCTII - Discrete Cosine Transform type II

Usage

```
c=dctii(f);
c=dctii(f,L);
c=dctii(f,[],dim);
c=dctii(f,L,dim);
```

Description

`dctii(f)` computes the discrete cosine transform of type II of the input signal f . If f is multi-dimensional, the transformation is applied along the first non-singleton dimension.

`dctii(f,L)` zero-pads or truncates f to length L before doing the transformation.

`dctii(f, [], dim)` or `dctii(f, L, dim)` applies the transformation along dimension dim .

The transform is real (output is real if input is real) and orthonormal.

This is the inverse of `dctiii`.

Let f be a signal of length L , let $c = \text{dctii}(f)$ and define the vector w of length L by

$$w(n) = \begin{cases} \frac{1}{\sqrt{2}} & \text{if } n = 0 \\ 1 & \text{otherwise} \end{cases}$$

Then

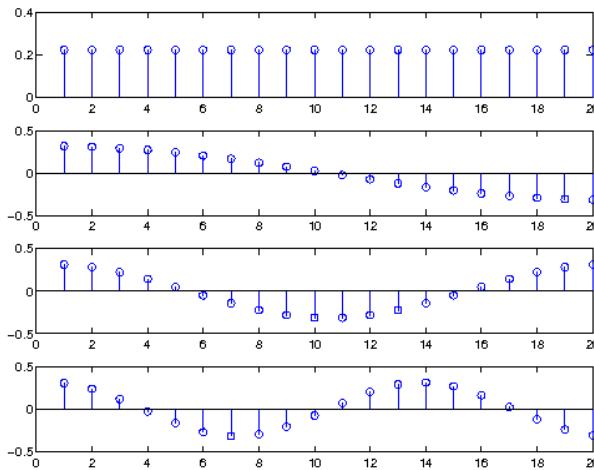
$$c(n+1) = \sqrt{\frac{2}{L}} \sum_{m=0}^{L-1} w(m) f(m+1) \cos\left(\frac{\pi}{L} m \left(n + \frac{1}{2}\right)\right)$$

Examples:

The following figures show the first 4 basis functions of the DCTII of length 20:

```
% The dctiii is the adjoint of dctii.
F=dctiii(eye(20));

for ii=1:4
    subplot(4,1,ii);
    stem(F(:,ii));
end;
```



References: [79], [97]

3.7.3 DCTIII - Discrete Cosine Transform type III

Usage

```
c=dctiii(f);
c=dctiii(f,L);
c=dctiii(f,[],dim);
c=dctiii(f,L,dim);
```

Description

`dctiii(f)` computes the discrete cosine transform of type III of the input signal f . If f is multi-dimensional, the transformation is applied along the first non-singleton dimension.

`dctiii(f, L)` zero-pads or truncates f to length L before doing the transformation.

`dctiii(f, [], dim)` or `dctiii(f, L, dim)` applies the transformation along dimension dim .

The transform is real (output is real if input is real) and orthonormal.

This is the inverse of `dctii`.

Let f be a signal of length L , let $c = \text{dctiii}(f)$ and define the vector w of length L by

$$w(n) = \begin{cases} \frac{1}{\sqrt{2}} & \text{if } n = 0 \\ 1 & \text{otherwise} \end{cases}$$

Then

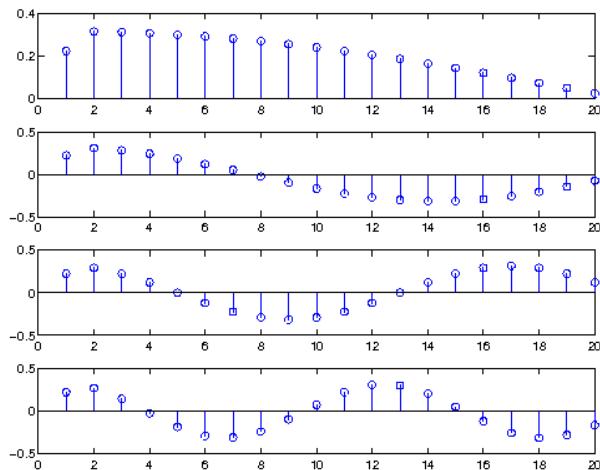
$$c(n+1) = \sqrt{\frac{2}{L}} \sum_{m=0}^{L-1} w(m) f(m+1) \cos\left(\frac{\pi}{L}\left(n + \frac{1}{2}\right)m\right)$$

Examples:

The following figures show the first 4 basis functions of the DCTIII of length 20:

```
% The dctii is the adjoint of dctiii.
F=dctii(eye(20));

for ii=1:4
    subplot(4,1,ii);
    stem(F(:,ii));
end;
```



References: [79], [97]

3.7.4 DCTIV - Discrete Cosine Transform type IV

Usage

```
c=dctiv(f);
```

Description

`dctiv(f)` computes the discrete cosine transform of type IV of the input signal f . If f is multi-dimensional, the transformation is applied along the first non-singleton dimension.

`dctiv(f, L)` zero-pads or truncates f to length L before doing the transformation.

`dctiv(f, [], dim)` or `dctiv(f, L, dim)` applies the transformation along dimension dim .

The transform is real (output is real if input is real) and orthonormal. It is its own inverse.

Let f be a signal of length L and let $c = \text{dctiv}(f)$. Then

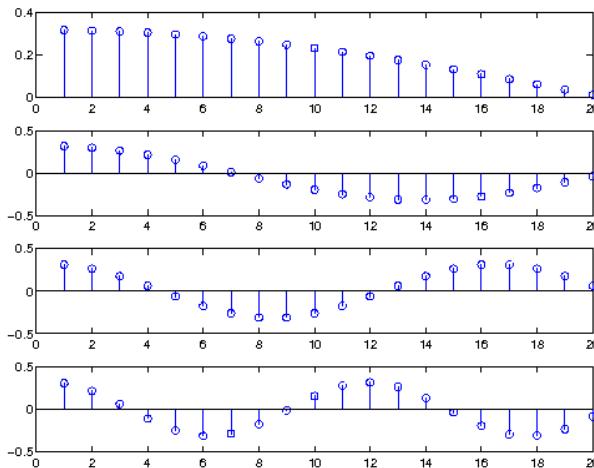
$$c(n+1) = \sqrt{\frac{2}{L}} \sum_{m=0}^{L-1} f(m+1) \cos\left(\frac{\pi}{L}\left(n+\frac{1}{2}\right)\left(m+\frac{1}{2}\right)\right)$$

Examples:

The following figures show the first 4 basis functions of the DCTIV of length 20:

```
% The dctiv is its own adjoint.
F=dctiv(eye(20));

for ii=1:4
    subplot(4,1,ii);
    stem(F(:,ii));
end;
```



References: [79], [97]

3.7.5 DSTI - Discrete Sine Transform type I

Usage

```
c=dsti(f);
c=dsti(f,L);
c=dsti(f,[],dim);
c=dsti(f,L,dim);
```

Description

`dsti(f)` computes the discrete sine transform of type I of the input signal f . If f is multi-dimensional, the transformation is applied along the first non-singleton dimension.

`dsti(f, L)` zero-pads or truncates f to length L before doing the transformation.

`dsti(f, [], dim)` or `dsti(f, L, dim)` applies the transformation along dimension dim .

The transform is real (output is real if input is real) and orthonormal.

This transform is its own inverse.

Let f be a signal of length L and let $c = \text{dsti}(f)$. Then

$$c(n+1) = \sqrt{\frac{2}{L+1}} \sum_{m=0}^{L-1} f(m+1) \sin\left(\frac{\pi(n+1)(m+1)}{L+1}\right)$$

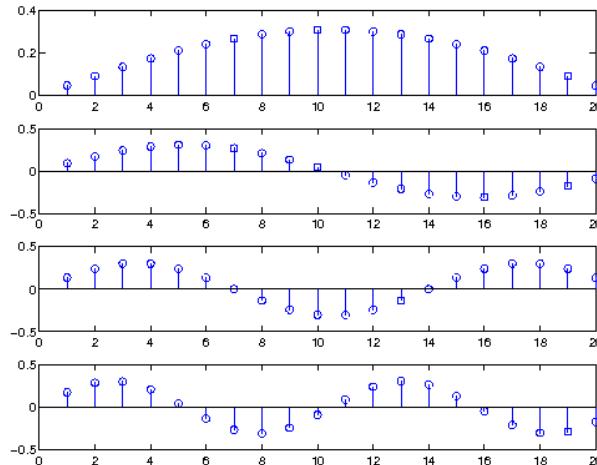
The implementation of this function uses a simple algorithm that requires an FFT of length $2N+2$, which might potentially be the product of a large prime number. This may cause the function to sometimes execute slowly. If guaranteed high speed is a concern, please consider using one of the other DST transforms.

Examples:

The following figures show the first 4 basis functions of the DSTI of length 20:

```
% The dsti is its own adjoint.
F=dsti(eye(20));

for ii=1:4
    subplot(4,1,ii);
    stem(F(:,ii));
end;
```



References: [79], [97]

3.7.6 DSTII - Discrete Sine Transform type II

Usage

```
c=dstii(f);
c=dstii(f,L);
c=dstii(f,[],dim);
c=dstii(f,L,dim);
```

Description

`dstii(f)` computes the discrete sine transform of type II of the input signal f . If f is multi-dimensional, the transformation is applied along the first non-singleton dimension.

`dstii(f,L)` zero-pads or truncates f to length L before doing the transformation.

`dstii(f,[],dim)` or `dstii(f,L,dim)` applies the transformation along dimension dim .

The transform is real (output is real if input is real) and orthonormal.

The inverse transform of dstii is dstiii.

Let f be a signal of length L , let $c = \text{dstiii}(f)$ and define the vector w of length L by

$$w(n) = \begin{cases} \frac{1}{\sqrt{2}} & \text{if } n = L - 1 \\ 1 & \text{otherwise} \end{cases}$$

Then

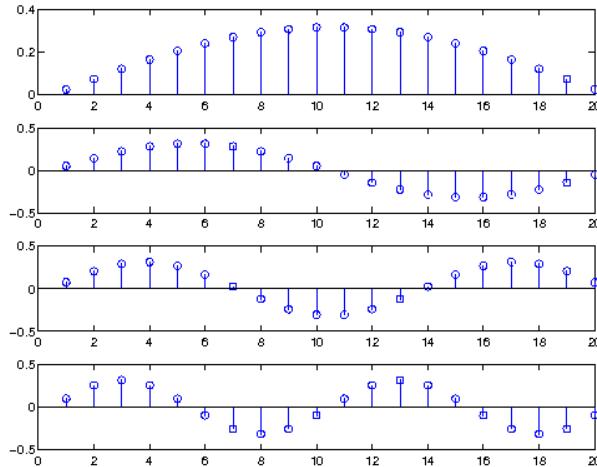
$$c(n+1) = \sqrt{\frac{2}{L}} \sum_{m=0}^{L-1} w(m) f(m+1) \sin\left(\frac{\pi}{L}n\left(m+\frac{1}{2}\right)\right)$$

Examples:

The following figures show the first 4 basis functions of the DSTII of length 20:

```
% The dstiii is the adjoint of dstii.
F=dstiii(eye(20));

for ii=1:4
    subplot(4,1,ii);
    stem(F(:,ii));
end;
```



References: [79], [97]

3.7.7 DSTIII - Discrete sine transform type III

Usage

```
c=dstiii(f);
c=dstiii(f,L);
c=dstiii(f,[],dim);
c=dstiii(f,L,dim);
```

Description

`dstiii(f)` computes the discrete sine transform of type III of the input signal f . If f is multi-dimensional, the transformation is applied along the first non-singleton dimension.

`dstiii(f,L)` zero-pads or truncates f to length L before doing the transformation.

`dstiii(f,[],dim)` or `dstiii(f,L,dim)` applies the transformation along dimension dim .

The transform is real (output is real if input is real) and orthonormal.

This is the inverse of dstii.

Let f be a signal of length L , let $c = \text{dstiii}(f)$ and define the vector w of length L by

$$w(n) = \begin{cases} \frac{1}{\sqrt{2}} & \text{if } n = L - 1 \\ 1 & \text{otherwise} \end{cases}$$

Then

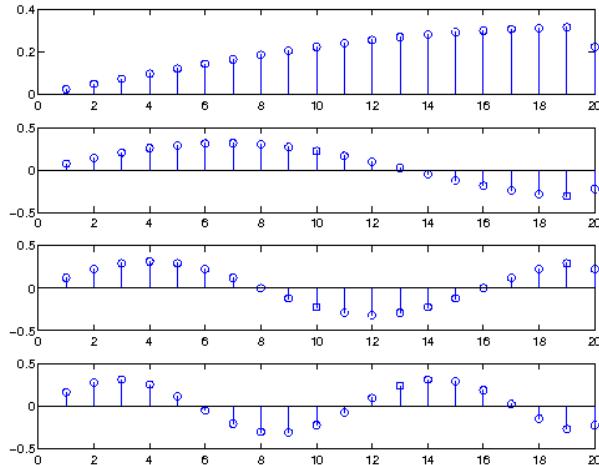
$$c(n+1) = \sqrt{\frac{2}{L}} \sum_{m=0}^{L-1} w(m) f(m+1) \sin\left(\frac{\pi}{L}\left(n + \frac{1}{2}\right)m\right)$$

Examples:

The following figures show the first 4 basis functions of the DSTIII of length 20:

```
% The dstii is the adjoint of dstiii.
F=dstii(eye(20));

for ii=1:4
    subplot(4,1,ii);
    stem(F(:,ii));
end;
```



References: [79], [97]

3.7.8 DSTIV - Discrete Sine Transform type IV

Usage

```
c=dstiv(f);
c=dstiv(f,L);
c=dstiv(f,[],dim);
c=dstiv(f,L,dim);
```

Description

`dstiv(f)` computes the discrete sine transform of type IV of the input signal f . If f is a matrix, then the transformation is applied to each column. For N-D arrays, the transformation is applied to the first non-singleton dimension.

`dstiv(f, L)` zero-pads or truncates f to length L before doing the transformation.

`dstiv(f, [], dim)` applies the transformation along dimension dim . `dstiv(f, L, dim)` does the same, but pads or truncates to length L .

The transform is real (output is real if input is real) and it is orthonormal. It is its own inverse.

Let f be a signal of length L and let $c = \text{dstiv}(f)$. Then

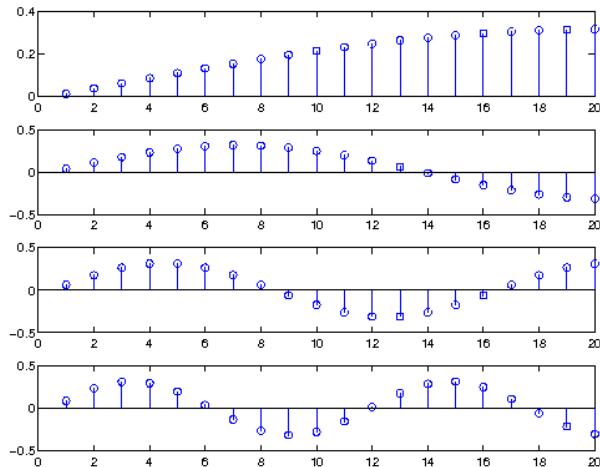
$$c(n+1) = \sqrt{\frac{2}{L}} \sum_{m=0}^{L-1} f(m+1) \sin\left(\frac{\pi}{L}\left(n+\frac{1}{2}\right)\left(m+\frac{1}{2}\right)\right)$$

Examples:

The following figures show the first 4 basis functions of the DSTIV of length 20:

```
% The dstiv is its own adjoint.
F=dstiv(eye(20));
```

```
for ii=1:4
    subplot(4,1,ii);
    stem(F(:,ii));
end;
```



References: [79], [97]

Chapter 4

LTFAT - Wavelets

4.1 Basic analysis/synthesis

4.1.1 FWT - Fast Wavelet Transform

Usage

```
c = fwt(f,w,J);  
c = fwt(f,w,J,dim);  
[c,info] = fwt(...);
```

Input parameters

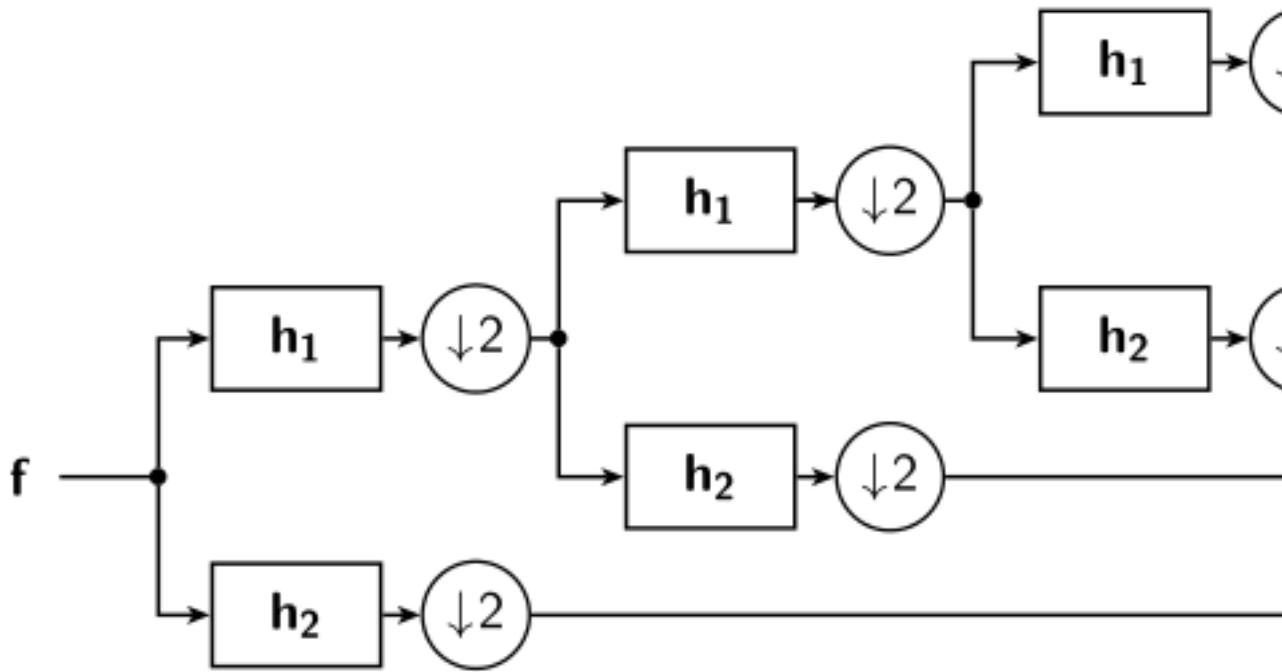
f	Input data.
w	Wavelet definition.
J	Number of filterbank iterations.
dim	Dimension to along which to apply the transform.

Output parameters

c	Coefficient vector.
info	Transform parameters struct.

Description

`fwt(f,w,J)` returns discrete wavelet coefficients of the input signal f using J iterations of the basic wavelet filterbank defined by w using the fast wavelet transform algorithm (Mallat's algorithm). The coefficients are the Discrete Wavelet transform (DWT) of the input signal f , if w defines two-channel wavelet filterbank. The following figure shows DWT with $J=3$.



The function can apply the Mallat's algorithm using basic filterbanks with any number of the channels. In such case, the transform have a different name.

Several formats of the basic filterbank definition w are recognized. One of them is a text string formed by a concatenation of a function name with the `wfilt_` prefix followed by a list of numerical arguments delimited by `:`. For example '`db10`' will result in a call to `wfilt_db(10)` or '`spline4:4`' in call to `wfilt_spline(4, 4)` etc. All filter defining functions can be listed by running `dir([ltfatbasepath, filesep, 'wavelets'])`. Please see help of the respective functions and follow references therein.

For other recognized formats of w please see `fwtinit`.

`[c, info] = fwt(f, w, J)` additionally returns struct. `info` containing transform parameters. It can be conveniently used for the inverse transform `ifwt` e.g. as `fhat = ifwt(c, info)`. It is also required by the `plotwavelets` function.

If f is row/column vector, the subbands c are stored in a single row/column in a consecutive order with respect to the increasing central frequency. The lengths of subbands are stored in `info.Lc` so the subbands can be easily extracted using `wavpack2cell`. Moreover, one can pass an additional flag '`'cell'`' to obtain the coefficient directly in a cell array. The cell array can be again converted to a packed format using `wavcell2pack`.

If the input f is a matrix, the transform is applied to each column if `dim==1` (default) and `[Ls, W]=size(f)`. If `dim==2` the transform is applied to each row `[W, Ls]=size(f)`. The output is then a matrix and the input orientation is preserved in the orientation of the output coefficients. The `dim` parameter has to be passed to the `wavpack2cell` and `wavcell2pack` when used.

Boundary handling:

`fwt(f, w, J, 'per')` (default) uses the periodic extension which considers the input signal as it was a one period of some infinite periodic signal as is natural for transforms based on the FFT. The resulting wavelet representation is non-expansive, that is if the input signal length is a multiple of a J -th power of the subsampling factor and the filterbank is critically subsampled, the total number of coefficients is equal to the input signal length. The input signal is padded with zeros to the next legal length L internally.

The default periodic extension can result in "false" high wavelet coefficients near the boundaries due to the possible discontinuity introduced by the zero padding and periodic boundary treatment.

`fwt(f, w, J, ext)` with `ext` other than '`per`' computes a slightly redundant wavelet representation

of the input signal f with the chosen boundary extension ext . The redundancy (expansivity) of the representation is the price to pay for using general filterbank and custom boundary treatment. The extensions are done at each level of the transform internally rather than doing the prior explicit padding.

The supported possibilities are:

'zero' Zeros are considered outside of the signal (coefficient) support.

'even' Even symmetric extension.

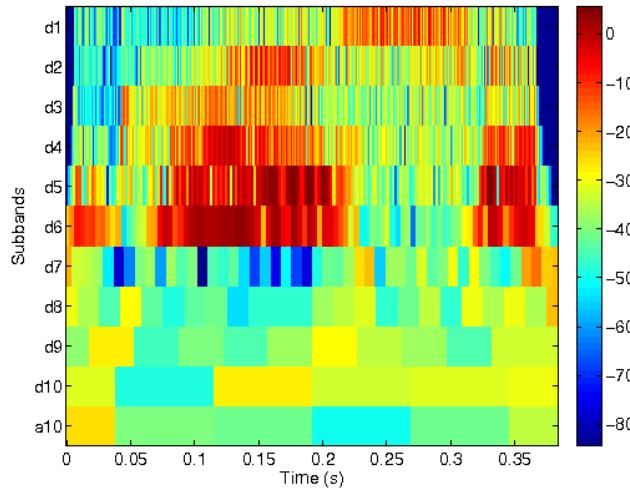
'odd' Odd symmetric extension.

Note that the same flag has to be used in the call of the inverse transform function ifwt if the `info` struct is not used.

Examples:

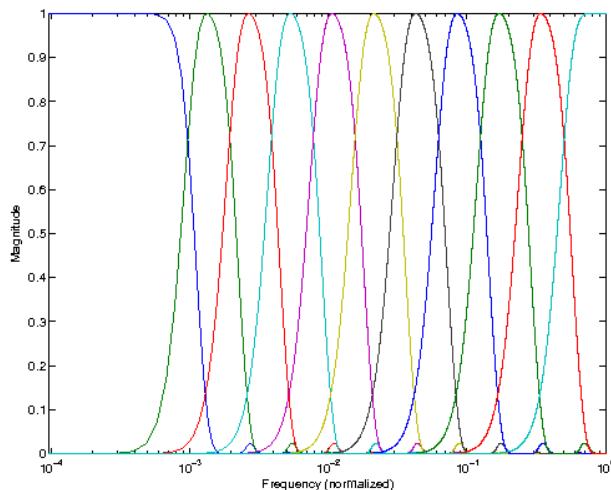
A simple example of calling the fwt function using 'db8' wavelet filters.:

```
[f, fs] = greasy;
J = 10;
[c, info] = fwt(f, 'db8', J);
plotwavelets(c, info, fs, 'dynrange', 90);
```



Frequency bands of the transform with x-axis in a log scale and band peaks normalized to 1. Only positive frequency band is shown.

```
[g, a] = wfbt2filterbank({'db8', 10, 'dwt'});
filterbankfreqz(g, a, 20*1024, 'linabs', 'posfreq', 'plot', 'inf', 'flog');
```



References: [60]

4.1.2 IFWT - Inverse Fast Wavelet Transform

Usage

```
f = ifwt(c, info)
f = ifwt(c, w, J, Ls)
f = ifwt(c, w, J, Ls, dim)
```

Input parameters

c	Wavelet coefficients.
info, w	Transform parameters struct/Wavelet filters definition.
J	Number of filterbank iterations.
Ls	Length of the reconstructed signal.
dim	Dimension to along which to apply the transform.

Output parameters

f	Reconstructed data.
----------	---------------------

Description

`f = ifwt(c, info)` reconstructs signal f from the wavelet coefficients c using parameters from `info` struct. both returned by `fwt` function.

`f = ifwt(c, w, J, Ls)` reconstructs signal f from the wavelet coefficients c using J -iteration synthesis filterbank build from the basic filterbank defined by w . The `Ls` parameter is mandatory due to the ambiguity of lengths introduced by the subsampling operation and by boundary treatment methods. Note that the same flag as in the `fwt` function have to be used, otherwise perfect reconstruction cannot be obtained.

In both cases, the fast wavelet transform algorithm (Mallat's algorithm) is employed. The format of c can be either packed, as returned by the `fwt` function or cell-array as returned by `wavpack2cell` function.

Please see the help on `fwt` for a detailed description of the parameters.

Examples:

A simple example showing perfect reconstruction:

```
f = gspi;
J = 8;
c = fwt(f,'db8',J);
fhat = ifwt(c,'db8',J,length(f));
% The following should give (almost) zero
norm(f-fhat)
```

This code produces the following output:

```
ans =
3.542640048596510e-14
```

References: [60]

4.1.3 FWT2 - Fast Wavelet Transform 2D

Usage

```
c = fwt2(f,w,J);
c = fwt2(f,w,J,...);
```

Input parameters

f	Input data.
w	Wavelet filter bank definition.
J	Number of filter bank iterations.

Output parameters

c	Coefficients stored in a matrix.
----------	----------------------------------

Description

`c=fwt2(f,w,J)` returns wavelet coefficients c of the input matrix f using J iterations of the basic wavelet filter bank defined by w . Please see `fwt` for description of w and J .

`fwt2` supports just the non-expansive boundary condition 'per' and critically subsampled filter banks in order to be able to pack the coefficients in a matrix. Also the J is limited to some maximum value for the same reason.

Additional flags make it possible to specify how the algorithm should subdivide the matrix:

'standard' Standard behaviour of the JPEG 2000 standard. This is the default.

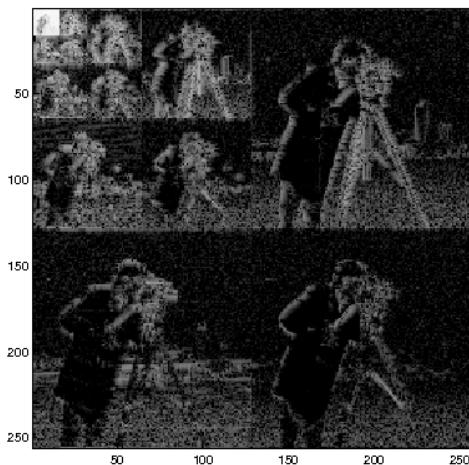
'tensor' This corresponds to doing a full `fwt` along each dimension of the matrix.

Examples:

Some simple example of calling the `fwt2` function, compare with the cameraman image. Only the 70 dB largest coefficients are shown, to make the structures more visible.

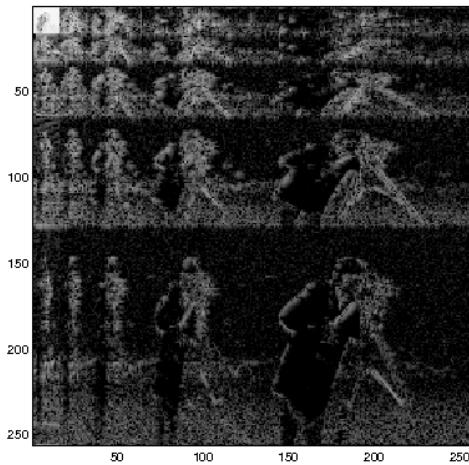
The first example uses the standard layout:

```
c = fwt2(cameraman,'db8',4);
imagesc(dynlimit(20*log10(abs(c)),70));
axis('image'); colormap(gray);
```



The second example uses the tensor product layout:

```
c = fwt2(cameraman,'db8',4,'tensor');
imagesc(dynlimit(20*log10(abs(c)),70));
axis('image'); colormap(gray);
```



References: [60]

4.1.4 IFWT2 - Inverse Fast Wavelet Transform

Usage

```
f = ifwt2(c,w,J)
f = ifwt2(c,w,J,Ls,...)
```

Input parameters

c	Coefficients stored in a matrix.
w	Wavelet filters definition.
J	Number of filterbank iterations.
Ls	Size of the reconstructed signal.

Output parameters

f	Reconstructed data.
----------	---------------------

Description

`f = ifwt2(c, w, J)` reconstructs signal f from the wavelet coefficients c using a J -iteration synthesis filterbank build from the basic synthesis filterbank defined by w . f is a matrix with `size(f) == size(c)`.

`f = ifwt2(c, w, J, Ls)` works as above but the result f is cut or extended to size Ls if Ls is a two-element vector or to $[Ls, Ls]$ if Ls is a scalar.

This function takes the same optional parameters as `fwt2`. Please see the help on `fwt2` for a description of the parameters.

References: [60]

4.1.5 UFWT - Undecimated Fast Wavelet Transform**Usage**

```
c = ufw(f, w, J);
[c, info] = ufw(...);
```

Input parameters

f	Input data.
w	Wavelet Filterbank.
J	Number of filterbank iterations.

Output parameters

c	Coefficients stored in $L \times J + 1$ matrix.
info	Transform paramaters struct.

Description

`ufwt(f, w, J)` computes redundant time (or shift) invariant wavelet representation of the input signal f using wavelet filters defined by w in the "a-trous" algorithm.

For all accepted formats of the parameter w see the `fwtinit` function.

`[c, info]=ufwt(f, w, J)` additionally returns the `info` struct. containing the transform parameters. It can be conviniently used for the inverse transform `iufwt` e.g. `fhat = iufwt(c, info)`. It is also required by the `plotwavelets` function.

The coefficents c are so called undecimated Discrete Wavelet transform of the input signal f , if w defines two-channel wavelet filterbank. Other names for this version of the wavelet transform are: the time-invariant wavelet transform, the stationary wavelet transform, maximal overlap discrete wavelet transform or even the "continuous" wavelet transform (as the time step is one sample). However, the function accepts any number filters (referred to as M) in the basic wavelet filterbank and the number of columns of c is then $J(M - 1) + 1$.

For one-dimensional input f of length L , the coefficients c are stored as columns of a matrix. The columns are ordered with increasing central frequency of the respective subbands.

If the input f is $L \times W$ matrix, the transform is applied to each column and the outputs are stacked along third dimension in the $L \times J(M - 1) + 1 \times W$ data cube.

Filter scaling

When compared to fwt, ufw subbands are gradually more and more redundant with increasing level of the subband. If no scaling of the filters is introduced, the energy of subbands tends to grow with increasing level. There are 3 flags defining filter scaling:

'sqrt' Each filter is scaled by $1/\sqrt{a}$, where a is the hop factor associated with it. If the original filterbank is orthonormal, the overall undecimated transform is a tight frame. This is the default.

'noscale' Uses filters without scaling.

'scale' Each filter is scaled by $1/a$.

If 'noscale' is used, 'scale' has to be used in iufwt (and vice versa) in order to obtain a perfect reconstruction.

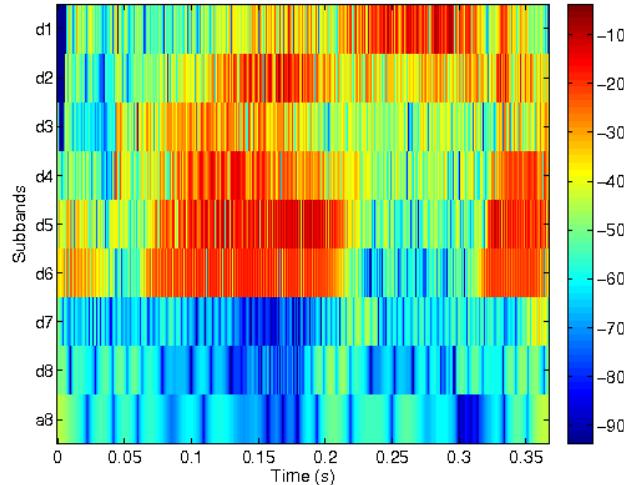
Boundary handling:

c=ufwt (f, w, J) uses periodic boundary extension. The extensions are done internally at each level of the transform, rather than doing the prior explicit padding.

Examples:

A simple example of calling the ufw function:

```
[f, fs] = greasy;
J = 8;
[c, info] = ufw(f, 'db8', J);
plotwavelets(c, info, fs, 'dynrange', 90);
```



References: [46]

4.1.6 IUFWT - Inverse Undecimated Fast Wavelet Transform

Usage

```
f = iufwt(c, info)
f = iufwt(c, w, J);
```

Input parameters

c	Coefficients stored in $L \times J + 1$ matrix.
info, w	Transform parameters struct/Wavelet filters definition.
J	Number of filterbank iterations.

Output parameters

f	Reconstructed data.
----------	---------------------

Description

`f = iufwt(c, info)` reconstructs signal f from the wavelet coefficients c using parameters from `info` struct. both returned by `ufwt` function.

`f = iufwt(c, w, J)` reconstructs signal f from the wavelet coefficients c using the wavelet filterbank consisting of the J levels of the basic synthesis filterbank defined by `w` using the "a-trous" algorithm. Node that the same flag as in the `ufwt` function have to be used.

Please see the help on `ufwt` for a description of the parameters.

Filter scaling

As in `ufwt`, 3 flags defining scaling of filters are recognized:

- **'sqrt'** Each filter is scaled by $1/\sqrt{a}$, there a is the hop factor associated with it. If the original filterbank is orthonormal, the overall undecimated transform is a tight frame. This is the default.
- **'noscale'** Uses filters without scaling.
- **'scale'** Each filter is scaled by $1/a$.

If '`noscale`' is used, '`scale`' must have been used in `ufwt` (and vice versa) in order to obtain a perfect reconstruction.

Examples:

A simple example showing perfect reconstruction:

```
f = gspi;
J = 8;
c = ufw(f,'db8',J);
fhat = iufwt(c,'db8',J);
% The following should give (almost) zero
norm(f-fhat)
```

This code produces the following output:

```
ans =
6.468488239052756e-14
```

References: [60]

4.1.7 FWTLENGTH - FWT length from signal**Usage**

```
L=ftlength(Ls,w,J);
```

Description

`fwtlength(Ls, w, J)` returns the length of a Wavelet system that is long enough to expand a signal of length L_s . Please see the help on `fwt` for an explanation of the parameters w and J .

If the returned length is longer than the signal length, the signal will be zero-padded by `fwt` to length L .

In addition, the function accepts flags defining boundary extension technique as in `fwt`. The returned length can be longer than the signal length only in case of '`per`' (periodic extension).

4.1.8 FWTCLLENGTH - FWT subbands lengths from a signal length**Usage**

```
Lc=fwtclength(Ls,w,J);
[Lc,L]=fwtclength(...);
```

Description

`Lc=fwtclength(Ls, w, J)` returns the lengths of the wavelet coefficient subbands for a signal of length L_s . Please see the help on `fwt` for an explanation of the parameters w and J .

`[Lc, L]=fwtclength(...)` additianally the function returns the next legal length of the input signal for the given extension type.

The function support the same boundary-handling flags as the `fwt` does.

4.2 Advanced analysis/synthesis**4.2.1 WFBT - Wavelet FilterBank Tree****Usage**

```
c=wfbt(f,wt);
c=wfbt(f,wt,ext);
[c,info]=wfbt(...);
```

Input parameters

f	Input data.
wt	Wavelet filterbank tree definition.

Output parameters

c	Coefficients stored in a cell-array.
info	Transform parameters struct.

Description

`wfbt(f,wt)` returns coefficients c obtained by applying a wavelet filterbank tree defined by `wt` to the input data f .

`[c,info]=wfbt(f,wt)` additionally returns struct. `info` containing transform parameters. It can be conviniently used for the inverse transform `iwfbt` e.g. `fhat = iwfbt(c,info)`. It is also required by the `plotwavelets` function.

`wt` defines a tree shaped filterbank structure build from the elementary two (or more) channel wavelet filters. The tree can have any shape and thus provide a flexible frequency covering. The outputs of the tree leaves are stored in c .

The `wt` parameter can have two formats:

- 1) Cell array containing 3 elements $\{w, J, \text{treetype}\}$, where w is the basic wavelet filterbank definition as in fwt function, J stands for the depth of the tree and the flag `treetype` defines the type of the tree to be used. Supported options are:

'dwt' Plain DWT tree (default). This gives one band per octave freq. resolution when using 2 channel basic wavelet filterbank and produces coefficients identical to the ones in fwt.

'full' Full filterbank tree. Both (all) basic filterbank outputs are decomposed further up to depth J achieving linear frequency band division.

'doubleband', 'quadband', 'octaband' The filterbank is designed such that it mimics 4-band, 8-band or 16-band complex wavelet transform provided the basic filterbank is 2 channel. In this case, J is treated such that it defines number of levels of 4-band, 8-band or 16-band transform.

- 2) Structure returned by the wfbtinit function and possibly modified by wfbtput and wfbtremove.

Please see wfbtinit for a detailed description and more options.

If f is row/column vector, the coefficient vectors $c\{j\}$ are columns.

If f is a matrix, the transformation is by default applied to each of W columns $[L_s, W] = \text{size}(f)$. In addition, the following flag groups are supported:

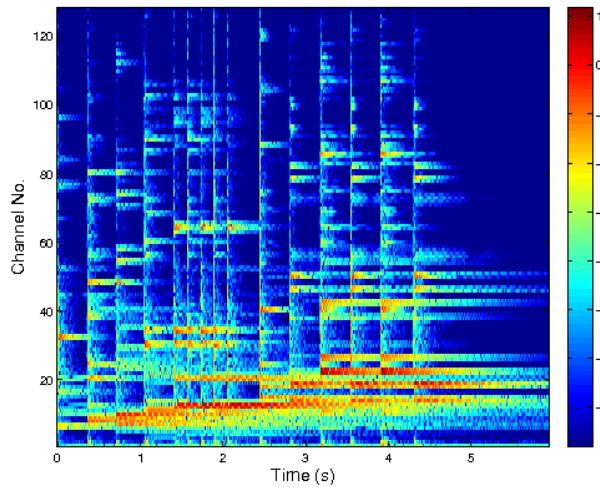
'per' **(default)**, **'zero'**, **'odd'**, **'even'** Type of the boundary handling. Please see the help on fwt for a description of the boundary condition flags.

'freq' **(default)**, **'nat'** Frequency or natural ordering of the coefficient subbands. The direct usage of the wavelet tree ('nat' option) does not produce coefficient subbands ordered according to the frequency. To achieve that, some filter shuffling has to be done ('freq' option).

Examples:

A simple example of calling the wfbt function using the "full decomposition" wavelet tree:

```
f = gspi;
J = 7;
[c,info] = wfbt(f,'sym10',J,'full');
plotwavelets(c,info,44100,'dynrange',90);
```



4.2.2 IWFBT - Inverse Wavelet Filterbank Tree

Usage

```
f=iwfbt(c,info);
f=iwfbt(c,wt,Ls);
```

Input parameters

c	Coefficients stored in a cell-array.
info, wt	Transform parameters struct/Wavelet Filterbank tree
Ls	Length of the reconstructed signal.

Output parameters

f	Reconstructed data.
----------	---------------------

Description

`f = iwfbt(c, info)` reconstructs signal f from the coefficients c using parameters from `info` struct, both returned by `wfbt` function.

`f = iwfbt(c, wt, Ls)` reconstructs signal f from the coefficients c using filterbank tree defined by `wt`. Please see `wfbt` function for possible formats of `wt`. The Ls parameter is mandatory due to the ambiguity of reconstruction lengths introduced by the subsampling operation and by boundary treatment methods. Note that the same flag as in the `wfbt` function have to be used, otherwise perfect reconstruction cannot be obtained. Please see help for `wfbt` for description of the flags.

Examples:

A simple example showing perfect reconstruction using `idtwfb`:

```
f = gspi;
J = 7;
wt = {'db6', J};
c = wfbt(f, wt);
fhat = iwfbt(c, wt, length(f));
% The following should give (almost) zero
norm(f-fhat)
```

This code produces the following output:

```
ans =
1.392090799573605e-13
```

4.2.3 UWFBT - Undecimated Wavelet FilterBank Tree**Usage**

```
c=uwfbt(f,wt);
[c,info]=uwfbt(...);
```

Input parameters

f	Input data.
wt	Wavelet Filterbank tree

Output parameters

c	Coefficients stored in $L \times M$ matrix.
----------	---

Description

`uwfbt(f,wt)` computes redundant time (or shift) invariant representation of the input signal f using the filterbank tree definition in wt and using the "a-trous" algorithm. Number of columns in c (M) is defined by the total number of outputs of nodes of the tree.

`[c,info]=uwfbt(f,wt)` additionally returns struct. `info` containing the transform parameters. It can be conveniently used for the inverse transform `iuwfbt` e.g. `fhat = iuwfbt(c,info)`. It is also required by the `plotwavelets` function.

If f is a matrix, the transformation is applied to each of W columns and the coefficients in c are stacked along the third dimension.

Please see help for `wfbt` description of possible formats of wt and description of frequency and natural ordering of the coefficient subbands.

Filter scaling

When compared to `wfbt`, the subbands produced by `uwfbt` are gradually more and more redundant with increasing depth in the tree. This results in energy grow of the coefficients. There are 3 flags defining filter scaling:

'sqrt' Each filter is scaled by $1/\sqrt{a}$, where a is the hop factor associated with it. If the original filterbank is orthonormal, the overall undecimated transform is a tight frame.

This is the default.

'noscale' Uses filters without scaling.

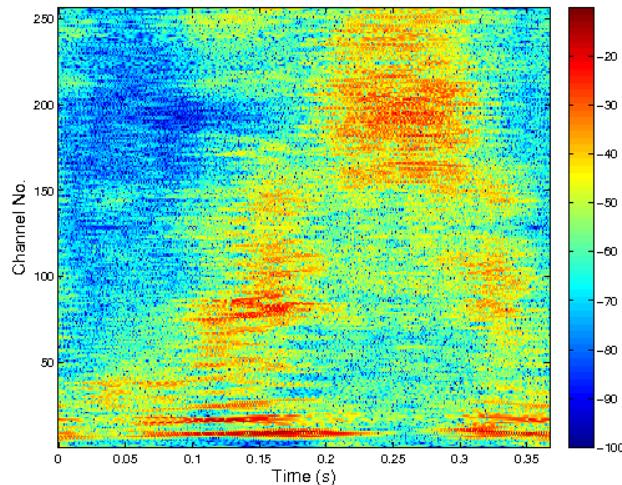
'scale' Each filter is scaled by $1/a$.

If '`noscale`' is used, '`scale`' has to be used in `iuwfbt` (and vice versa) in order to obtain a perfect reconstruction.

Examples:

A simple example of calling the `uwfbt` function using the "full decomposition" wavelet tree:

```
f = greasy;
J = 8;
[c,info] = uwfbt(f,{'sym10',J,'full'});
plotwavelets(c,info,16000,'dynrange',90);
```



4.2.4 IUWFBT - Inverse Undecimated Wavelet Filterbank Tree

Usage

```
f = iuwfbt(c, info)
f = iuwfbt(c, wt)
```

Input parameters

c	Coefficients stored in $L \times M$ matrix.
info, wt	Transform parameters struct/Wavelet tree definition.

Output parameters

f	Reconstructed data.
----------	---------------------

Description

`f = iuwfbt(c, info)` reconstructs signal f from the coefficients c using parameters from `info` struct, both returned by the `uwfbt` function.

`f = iuwfbt(c, wt)` reconstructs signal f from the wavelet coefficients c using the undecimated wavelet filterbank tree described by `wt`.

Please see help for `wfbt` description of possible formats of `wt`.

Filter scaling:

As in `uwfbt`, the function recognizes three flags controlling scaling of the filters:

- **'sqrt'** Each filter is scaled by $1/\sqrt{a}$, where a is the hop factor associated with it. If the original filterbank is orthonormal, the overall undecimated transform is a tight frame. This is the default.
- **'noscale'** Uses filters without scaling.
- **'scale'** Each filter is scaled by $1/a$.

If '`noscale`' is used, '`scale`' must have been used in `uwfbt` (and vice versa) in order to obtain a perfect reconstruction.

Examples:

A simple example showing perfect reconstruction using the "full decomposition" wavelet tree:

```
f = greasy;
J = 6;
c = uwfbt(f, {'db8', J, 'full'});
fhat = iuwfbt(c, {'db8', J, 'full'});
% The following should give (almost) zero
norm(f-fhat)
```

This code produces the following output:

```
ans =
2.647294733053261e-14
```

4.2.5 WPFBT - Wavelet Packet FilterBank Tree

Usage

```
c=wpfbt(f,wt);
[c,info]=wpfbt(...);
```

Input parameters

f	Input data.
wt	Wavelet Filterbank tree definition.

Output parameters

c	Coefficients stored in a cell-array.
info	Transform parameters struct.

Description

`c=wpfbt(f,wt)` returns wavelet packet coefficients c obtained by applying a wavelet filterbank tree defined by wt to the input data f .

`[c,info]=wpfbt(f,wt)` additionally returns struct. `info` containing transform parameters. It can be conveniently used for the inverse transform `iwpfbt` e.g. `fhat = iwpfbt(c,info)`. It is also required by the `plotwavelets` function.

In contrast to `wfbt`, the cell array c contain every intermediate output of each node in the tree. $c\{jj\}$ are ordered according to nodes taken in the breadth-first order.

If f is row/column vector, the coefficient vectors $c\{jj\}$ are columns. If f is a matrix, the transformation is applied to each of column of the matrix.

Scaling of intermediate outputs:

The following flags control scaling of intermediate outputs and therefore the energy relations between coefficient subbands. An intermediate output is an output of a node which is further used as an input to a descendant node.

'intsqrt' Each intermediate output is scaled by $1/\sqrt{2}$. If the filterbank in each node is orthonormal, the overall undecimated transform is a tight frame. This is the default.

'intnoscale' No scaling of intermediate results is used. This is necessary for the `wpbest` function to correctly work with the cost measures.

'intscale' Each intermediate output is scaled by $1/2$.

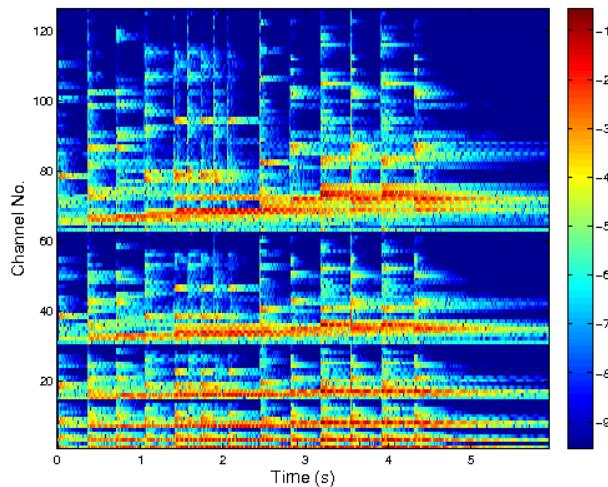
If '`intnoscale`' is used, '`intscale`' must be used in `iwpfbt` (and vice versa) in order to obtain a perfect reconstruction.

Please see help for `wfbt` description of possible formats of `wt` and of the additional flags defining boundary handling.

Examples:

A simple example of calling the `wpfbt` function using the "full decomposition" wavelet tree:

```
f = gspi;
J = 6;
[c,info] = wpfbt(f,'sym10',J,'full');
plotwavelets(c,info,44100,'dynrange',90);
```



4.2.6 IWPFBT - Inverse Wavelet Packet Filterbank Tree

Usage

```
f=iwpfbt(c,info);
f=iwpfbt(c,wt,Ls);
```

Input parameters

c	Coefficients stored in a cell-array.
info, wt	Transform parameters struct/Wavelet Filterbank tree.
Ls	Length of the reconstructed signal.

Output parameters

f	Reconstructed data.
----------	---------------------

Description

`f = iwpfbt(c,info)` reconstructs signal f from the coefficients c using parameters from `info` struct. both returned by `wfbt` function.

`f = iwpfbt(c,wt,Ls)` reconstructs signal f from the coefficients c using filter bank tree defined by `wt`. Please see `wfbt` function for possible formats of `wt`. The `Ls` parameter is mandatory due to the ambiguity of reconstruction lengths introduced by the subsampling operation and by boundary treatment methods.

Please see help for `wfbt` description of possible formats of `wt` and of the additional flags.

Scaling of intermediate outputs:

The following flags control scaling of the intermediate coefficients. The intermediate coefficients are outputs of nodes which are also inputs to nodes further in the tree.

- **'intsqrt'** Each intermediate output is scaled by $1/\sqrt{2}$. If the filterbank in each node is orthonormal, the overall undecimated transform is a tight frame. This is the default.
- **'intnoscale'** No scaling of intermediate results is used.
- **'intscale'** Each intermediate output is scaled by $1/2$.

If '`intnoscale`' is used, '`intscale`' must have been used in `wfbt` (and vice versa) in order to obtain a perfect reconstruction.

Examples:

A simple example showing perfect reconstruction using the "full decomposition" wavelet tree:

```
f = gspi;
J = 7;
wt = {'db10',J,'full'};
c = wpfbt(f,wt);
fhat = iwpfbt(c,wt,length(f));
% The following should give (almost) zero
norm(f-fhat)
```

This code produces the following output:

```
ans =
8.015867730167378e-14
```

4.2.7 UWPFBT - Undecimated Wavelet Packet FilterBank Tree

Usage

```
c=uwpfbt(f,wt);
[c,info]=uwpfbt(...);
```

Input parameters

f	Input data.
wt	Wavelet Filterbank tree

Output parameters

c	Coefficients in a $L \times M$ matrix.
----------	--

Description

`c=uwpfbt(f,wt)` returns coefficients c obtained by applying the undecimated wavelet filterbank tree defined by `wt` to the input data `f` using the "a-trous" algorithm. Number of columns in c (M) is defined by the total number of outputs of each node. The outputs $c(:, j)$ are ordered in the breadth-first node order manner.

`[c,info]=uwpfbt(f,wt)` additionally returns struct. `info` containing the transform parameters. It can be conveniently used for the inverse transform `iuwpfbt` e.g. `fhat = iuwpfbt(c,info)`. It is also required by the `plotwavelets` function.

If f is a matrix, the transformation is applied to each of W columns and the coefficients in c are stacked along the third dimension.

Please see help for `wfbt` description of possible formats of `wt`.

Scaling of intermediate outputs:

The following flags control scaling of intermediate outputs and therefore the energy relations between coefficient subbands. An intermediate output is an output of a node which is further used as an input to a descendant node.

'intsqrt' Each intermediate output is scaled by $1/\sqrt{2}$. If the filterbank in each node is orthonormal, the overall undecimated transform is a tight frame. This is the default.

'intnoscale' No scaling of intermediate results is used. This is necessary for the `wpbest` function to correctly work with the cost measures.

'intscale' Each intermediate output is scaled by $1/2$.

If '`intnoscale`' is used, '`intscale`' must be used in `iuwpfbt` (and vice versa) in order to obtain a perfect reconstruction.

Scaling of filters:

When compared to wpfbt, the subbands produced by uwpfbt are gradually more and more redundant with increasing depth in the tree. This results in energy grow of the coefficients. There are 3 flags defining filter scaling:

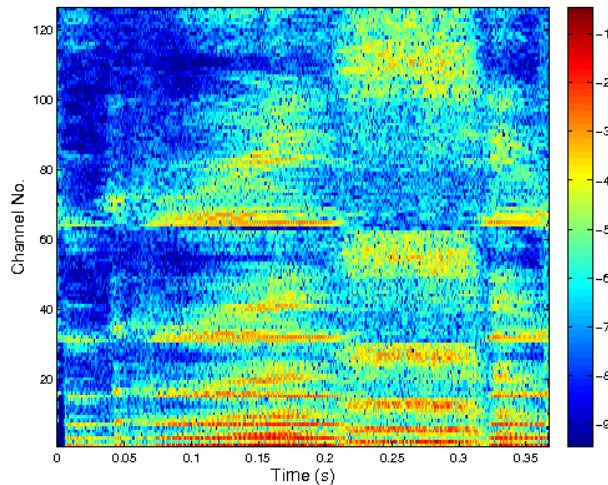
- **'sqrt'** Each filter is scaled by $1/\sqrt{a}$, where a is the hop factor associated with it. If the original filterbank is orthonormal, the overall undecimated transform is a tight frame. This is the default.
- **'noscale'** Uses filters without scaling.
- **'scale'** Each filter is scaled by $1/a$.

If 'noscale' is used, 'scale' must be used in iuwpfbt (and vice versa) in order to obtain a perfect reconstruction.

Examples:

A simple example of calling the uwpfbt function using the "full decomposition" wavelet tree:

```
[f,fs] = greasy;
J = 6;
[c,info] = uwpfbt(f,{'db10',J,'full'});
plotwavelets(c,info,fs,'dynrange',90);
```



4.2.8 IUWPFBT - Inverse Undecimated Wavelet Packet Filterbank Tree

Usage

```
f=iuwpfbt(c,info);
f=iuwpfbt(c,wt);
```

Input parameters

c Coefficients stored in $L \times M$ matrix.

info, wt Transform parameters struct/Wavelet tree definition.

Output parameters

f Reconstructed data.

Description

`f = iuwpfbt(c, info)` reconstructs signal f from the wavelet packet coefficients c using parameters from `info` struct. both returned by the `uwpfbt` function.

`f = iuwpfbt(c, wt)` reconstructs signal f from the wavelet packet coefficients c using the undecimated wavelet filterbank tree described by `wt`.

Please see help for `wfbt` description of possible formats of `wt`.

Filter scaling:

As in `uwpfbt`, the function recognizes three flags controlling scaling of filters:

'sqrt' Each filter is scaled by $1/\sqrt{a}$, there a is the hop factor associated with it. If the original filterbank is orthonormal, the overall undecimated transform is a tight frame. This is the default.

'noscale' Uses filters without scaling.

'scale' Each filter is scaled by $1/a$.

If '`noscale`' is used, '`scale`' must have been used in `uwpfbt` (and vice versa) in order to obtain a perfect reconstruction.

Scaling of intermediate outputs:

The following flags control scaling of the intermediate coefficients. The intermediate coefficients are outputs of nodes which are also inputs to nodes further in the tree.

'intsqrt' Each intermediate output is scaled by $1/\sqrt{2}$. If the filterbank in each node is orthonormal, the overall undecimated transform is a tight frame. This is the default.

'intnoscale' No scaling of intermediate results is used.

'intscale' Each intermediate output is scaled by $1/2$.

If '`intnoscale`' is used, '`intscale`' must have been used in `uwpfbt` (and vice versa) in order to obtain a perfect reconstruction.

Examples:

A simple example showing perfect reconstruction using the "full decomposition" wavelet tree:

```
f = greasy;
J = 7;
wtdef = {'db10', J, 'full'};
c = uwpfbt(f, wtdef);
fhat = iuwpfbt(c, wtdef);
% The following should give (almost) zero
norm(f-fhat)
```

This code produces the following output:

```
ans =
```

```
3.279270462493657e-14
```

4.2.9 WPBEST - Best Tree selection

Usage

```
c = wpbest(f, w, J, cost);
[c, info] = wpbest(...);
```

Input parameters

f	Input data.
w	Wavelet Filterbank.
J	Maximum depth of the tree.

Output parameters

c	Coefficients stored in a cell-array.
info	Transform parameters struct.

Description

[**c**, **info**] = wpbest(**f**, **w**, **J**, **cost**) selects the best sub-tree **info.wt** from the full tree with max. depth **J**, which minimizes the cost function.

Only one-dimensional input **f** is accepted. The supported formats of the parameter **w** can be found in help for fwt. The format of the coefficients **c** and the **info** struct is the same as in wfbt.

Please note that **w** should define orthonormal wavelet filters.

First, the depth **J** wavelet packet decomposition is performed using wpfbt. Then the nodes are traversed in the breadth-first and bottom-up order and the value of the cost function of the node input and cost of the combined node outputs is compared. If the node input cost function value is less than the combined output cost, the current node and all possible descendant nodes are marked to be deleted, if not, the input is assigned the combined output cost. At the end, the marked nodes are removed and the resulting tree is considered to be a best basis (or near-best basis) in the chosen cost function sense.

The **cost** parameter can be a cell array or an user-defined function handle. accepting a single column vector. The cell array should consist of a string, followed by a numerical arguments. The possible formats are listed in the following text.

Additive costs:

The additive cost **E** of a vector **x** is a real valued cost function such that:

$$E(x) = \sum_k E(x(k))$$

and $E(0) = 0$. Given a collection of vectors x_i being coefficients in orthonormal bases B_i , the best basis relative to **E** is the one for which the $E(x_i)$ is minimal.

Additive cost functions allows using the fast best-basis search algorithm since the costs can be precomputed and combined cost of two vectors is just a sum of their costs.

{'shannon'} A cost function derived from the Shannon entropy:

$$E_{sh}(x) = - \sum_{k:x(k) \neq 0} |x(k)|^2 \log |x(k)|^2$$

{'log'} A logarithm of energy:

$$E_{log}(x) = \sum_{k:x(k) \neq 0} \ln |x(k)|^2$$

{'lpnorm', p} Concentration in l^p norm:

$$E_{lp}(x) = \left(\sum_k |x(k)|^p \right)$$

{'thre', th} Number of coefficients above a threshold **th**.

Non-additive costs:

Cost function, which is not additive cost but which is used for the basis selection is called a non-additive cost. The resulting basis for which the cost is minimal is called near-best, because the non-additive cost cannot guarantee the selection of a best basis relative to the cost function.

{'wlpnorm', p} The weak- l^p norm cost function:

$$E_{wlp}(x) = \max k^{1/p} v_k(x),$$

where $0 < p \leq 2$ and $v_k(x)$ denotes the k -th largest absolute value of x .

{'compn', p, f} Compression number cost:

$$E_{cn}(x) = \arg \min_k |w_k(x, p) - f|,$$

where $0 < p \leq 2$, $0 < f < 1$ and $w_k(u, p)$ denotes decreasingly sorted, powered, cumulatively summed and renormalized vector:

$$w_k(x, p) = \frac{\sum_{j=1}^k v_j^p(x)}{\sum_{j=1}^N v_j^p(x)}$$

where $v_k(x)$ denotes the k -th largest absolute value of x and N is the number of elements of x .

{'compa', p} Compression area cost:

$$E_{ca}(x) = N - \sum_k w_k(x, p),$$

where $0 < p \leq 2$ and $w_k(u, p)$ and N as in the previous case.

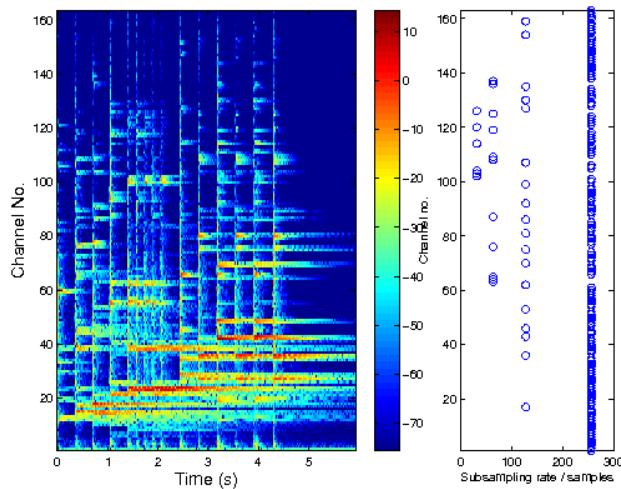
Examples:

A simple example of calling wpbest

```
f = gspi;
J = 8;
[c,info] = wpbest(f,'sym10',J,'cost','shannon');

% Use 2/3 of the space for the first plot, 1/3 for the second.
subplot(3,3,[1 2 4 5 7 8]);
plotwavelets(c,info,44100,90);

subplot(3,3,[3 6 9]);
N=cellfun(@numel,c); L=sum(N); a=L./N;
plot(a,'o');
xlim([1,numel(N)]);
view(90,-90);
xlabel('Channel no.');
ylabel('Subsampling rate / samples');
```



References: [96], [91]

4.2.10 WFBTLENGTH - WFBT length from signal

Usage

```
L=wfbtlength(Ls,wt);
```

Description

`wfbtlength(Ls,wt)` returns the length of a Wavelet system that is long enough to expand a signal of length L_s . Please see the help on `wfbt` for an explanation of the parameter `wt`.

If the returned length is longer than the signal length, the signal will be zero-padded by `wfbt` to length L .

In addition, the function accepts flags defining boundary extension technique as in `wfbt`. The returned length can be longer than the signal length only in case of 'per' (periodic extension).

4.2.11 WFBTCLENGTH - WFBT subband lengths from a signal length

Usage

```
Lc=wfbtclength(Ls,wt);
[Lc,L]=wfbtclength(...);
```

Description

`Lc=wfbtclength(Ls,wt)` returns the lengths of coefficient subbands obtained from `wfbt` for a signal of length L_s . Please see the help on `wfbt` for an explanation of the parameters `wt`.

`[Lc,L]=wfbtclength(...)` additionally returns the next legal length of the input signal for the given extension type.

The function support the same boundary-handling flags as the `fwt` does.

4.2.12 WPFBTCLENGTH - WPFBT subband length from a signal length

Usage

```
Lc=wpfbtclength(Ls,wt);
[Lc,L]=wpfbtclength(Ls,wt);
```

Description

`Lc=wpfbtclength(Ls,wt)` returns the lengths of coefficient subbands obtained from wpfbt for a signal of length L_s . Please see the help on wpfbt for an explanation of the parameter wt .

`[Lc,L]=wpfbtclength(...)` additionally returns the next legal length of the input signal for the given extension type.

The function support the same boundary-handling flags as the fwt does.

4.3 Dual-tree complex wavelet transform

4.3.1 DTWFB - Dual-Tree Wavelet Filter Bank

Usage

```
c=dtwfb(f,dualwt);
c=dtwfb(f,{dualw,J});
[c,info]=dtwfb(...);
```

Input parameters

f	Input data.
dualwt	Dual-tree Wavelet Filter bank definition.

Output parameters

c	Coefficients stored in a cell-array.
info	Additional transform parameters struct.

Description

`c=dtwfb(f,dualwt)` computes dual-tree complex wavelet coefficients of the signal f . The representation is approximately time-invariant and provides analytic behaviour. Due to these facts, the resulting subbands are nearly aliasing free making them suitable for severe coefficient modifications. The representation is two times redundant, provided critical subsampling of all involved filter banks.

The shape of the filterbank tree and filters used is controlled by `dualwt` (for possible formats see below). The output c is a cell-array with each element containing a single subband. The subbands are ordered with the increasing subband centre frequency.

In addition, the function returns struct. `info` containing transform parameters. It can be conveniently used for the inverse transform `idtwfb` e.g. `fhat = idtwfb(c,info)`. It is also required by the `plot-wavelets` function.

If f is a matrix, the transform is applied to each column.

Two formats of `dualwt` are accepted:

- 1) Cell array of parameters. First two elements of the array are mandatory `{dualw,J}`.

dualw Basic dual-tree filters

J Number of levels of the filter bank tree

Possible formats of `dualw` are the same as in `fwtinit` except the `wfilt dt_` prefix is used when searching for function specifying the actual impulse responses. These filters were designed specially for the dual-tree filter bank to achieve the half-sample shift ultimately resulting in analytic (complex) behaviour of the transform.

The default shape of the filter bank tree is DWT i.e. only low-pass output is decomposed further (J times in total).

Different filter bank tree shapes can be obtained by passing additional flag in the cell array. Supported flags (mutually exclusive) are:

'dwt' Plain DWT tree (default). This gives one band per octave freq. resolution when using 2 channel basic wavelet filter bank.

'full' Full filter bank tree. Both (all) basic filter bank outputs are decomposed further up to depth J achieving linear frequency band division.

'doubleband','quadband', 'octaband' The filter bank is designed such that it mimics 4-band, 8-band or 16-band complex wavelet transform provided the basic filter bank is 2 channel. In this case, J is treated such that it defines number of levels of 4-band, 8-band or 16-band transform.

The dual-tree wavelet filter bank can use any basic wavelet filter bank in the first stage of both trees, provided they are shifted by 1 sample (done internally). A custom first stage filter bank can be defined by passing the following key-value pair in the cell array:

'first','w' w defines a regular basic filter bank. Accepted formats are the same as in fwtinit assuming the wfilt_ prefix.

Similarly, when working with a filter bank tree containing decomposition of high-pass outputs, some filters in both trees must be replaced by a regular basic filter bank in order to achieve the approximately analytic behaviour. A custom filter bank can be specified by passing another key-value pair in the cell array:

'leaf','w' w defines a regular basic filter bank. Accepted formats are the same as in fwtinit assuming the wfilt_ prefix.

- 2) Another possibility is to pass directly a struct. returned by dtwfbinit and possibly modified by wfbtremove.

Optional args.:

In addition, the following flag groups are supported:

'freq','nat' Frequency or natural (Paley) ordering of coefficient subbands. By default, subbands are ordered according to frequency. The natural ordering is how the subbands are obtained from the filter bank tree without modifications. The ordering differs only in non-plain DWT case.

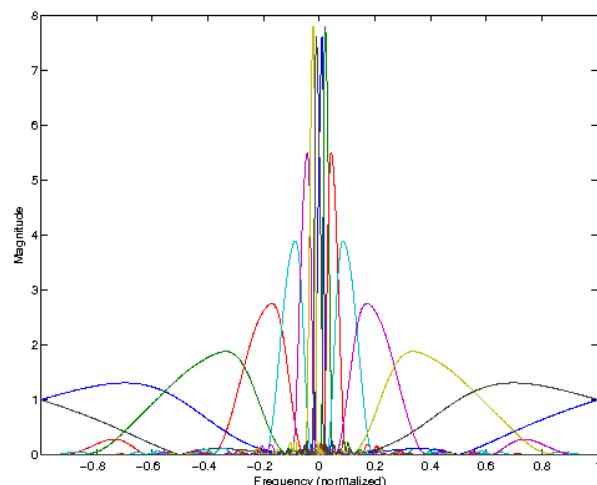
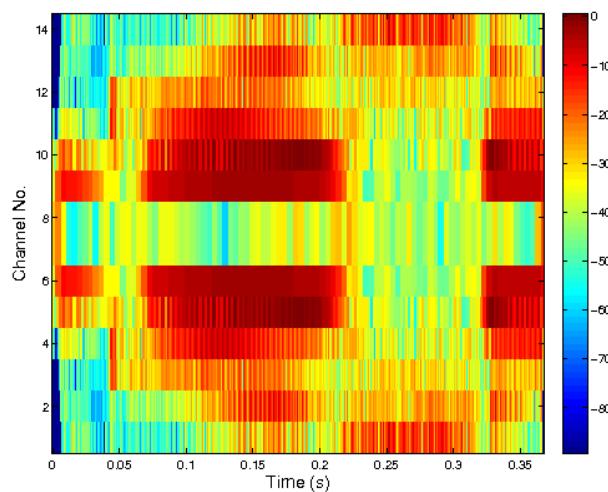
Boundary handling:

In contrast with fwt, wfbt and wpfbt, this function supports periodic boundary handling only.

Examples:

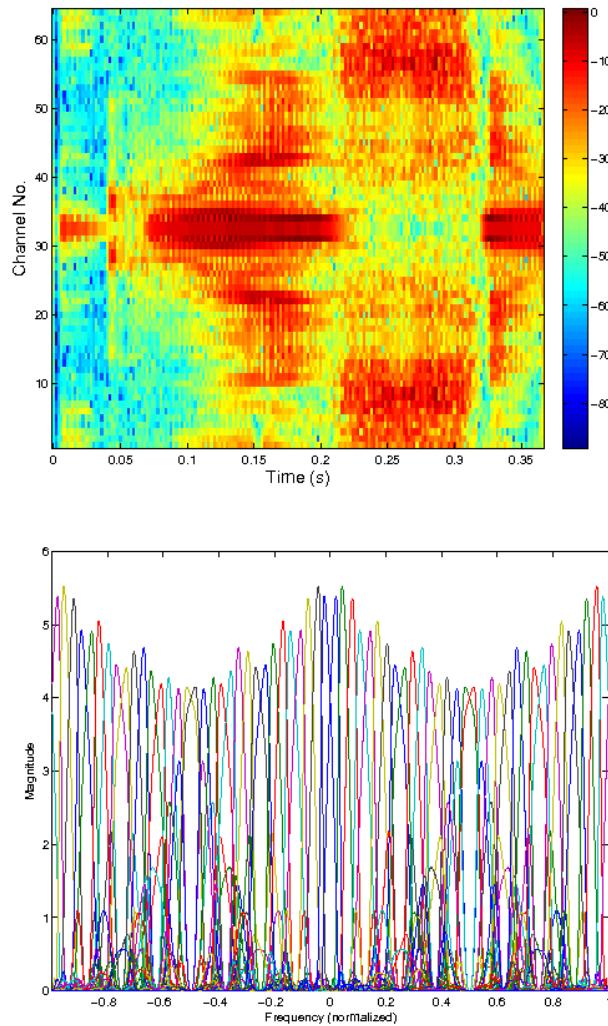
A simple example of calling the dtwfb function using the regular DWT iterated filter bank. The second figure shows a magnitude frequency response of an identical filter bank.:

```
[f,fs] = greasy;
J = 6;
[c,info] = dtwfb(f,{'qshift3',J});
figure(1);
plotwavelets(c,info,fs,'dynrange',90);
figure(2);
[g,a] = dtwfb2filterbank({'qshift3',J});
filterbankfreqz(g,a,1024,'plot','linabs');
```



The second example shows a decomposition using a full filter bank tree of depth J :

```
[f,fs] = greasy;
J = 5;
[c,info] = dtwfb(f,'qshift4',J,'full');
figure(1);
plotwavelets(c,info,fs,'dynrange',90);
figure(2);
[g,a] = dtwfb2filterbank({'qshift4',J,'full'});
filterbankfreqz(g,a,1024,'plot','linabs');
```



References: [50], [86], [12]

4.3.2 IDTWFB - Inverse Dual-tree Filterbank

Usage

```
f=idtwfb(c,info);
f=idtwfb(c,dualwt,Ls);
```

Input parameters

c	Input coefficients.
info	Transform params. struct
dualwt	Dual-tree Wavelet Filterbank definition
Ls	Length of the reconstructed signal.

Output parameters

f	Reconstructed data.
----------	---------------------

Description

`f = idtwfb(c, info)` reconstructs signal f from the coefficients c using parameters from `info` struct, both returned by `dwtfb` function.

`f = idtwfb(c, dualwt, Ls)` reconstructs signal f from the coefficients c using dual-tree filterbank defined by `dualwt`. Please see `dwtfb` for supported formats. The Ls parameter is mandatory due to the ambiguity of reconstruction lengths introduced by the subsampling operation. Note that the same flag as in the `dwtfb` function have to be used, otherwise perfect reconstruction cannot be obtained. Please see help for `dwtfb` for description of the flags.

Examples:

A simple example showing perfect reconstruction using `idtwfb`:

```
f = gspi;
J = 7;
wtdef = {'qshift3', J};
c = dwtfb(f, wtdef);
fhat = idtwfb(c, wtdef, length(f));
% The following should give (almost) zero
norm(f-fhat)
```

This code produces the following output:

```
ans =
9.605850455567395e-14
```

4.3.3 DTWFBREAL - Dual-Tree Wavelet FilterBank for real-valued signals

Usage

```
c=dtwfbreal(f, dualwt);
c=dtwfbreal(f, {dualwt, J});
[c, info]=dtwfbreal(...);
```

Input parameters

f	Input data.
dualwt	Dual-tree Wavelet Filterbank definition.

Output parameters

c	Coefficients stored in a cell-array.
info	Additional transform parameters struct.

Description

`c=dtwfbtreal(f, dualwt)` computes dual-tree complex wavelet coefficients of the real-valued signal f . The representation is approximately time-invariant and provides analytic behavior. Due to these facts, the resulting subbands are nearly aliasing free making them suitable for severe coefficient modifications. The representation is two times redundant, provided critical subsampling of all involved filterbanks, but one half of the coefficients is complex conjugate of the other.

The shape of the filterbank tree and filters used is controlled by `dualwt` (for possible formats see below). The output c is a cell-array with each element containing a single subband. The subbands are ordered with increasing subband center frequency.

In addition, the function returns struct. `info` containing transform parameters. It can be conveniently used for the inverse transform `idtwfbreal` e.g. `fhat = idtwfbreal(c, info)`. It is also required by the `plotwavelets` function.

If f is a matrix, the transform is applied to each column.

Two formats of `dualwt` are accepted:

- 1) Cell array of parameters. First two elements of the array are mandatory `{dualw, J}`.

dualw Basic dual-tree filters

J Number of levels of the filterbank tree

Possible formats of `dualw` are the same as in `fwtnit` except the `wfilt_dt_` prefix is used when searching for function specifying the actual impulse responses. These filters were designed specially for the dual-tree filterbank to achieve the half-sample shift ultimately resulting in analytic (complex) behavior of the transform.

The default shape of the filterbank tree is DWT i.e. only low-pass output is decomposed further (J times in total).

Different filterbank tree shapes can be obtained by passing additional flag in the cell array. Supported flags (mutually exclusive) are:

- '**dwt**' Plain DWT tree (default). This gives one band per octave freq. resolution when using 2 channel basic wavelet filterbank.
- '**full**' Full filterbank tree. Both (all) basic filterbank outputs are decomposed further up to depth J achieving linear frequency band division.
- '**doubleband**', '**quadband**', '**octaband**' The filterbank is designed such that it mimics 4-band, 8-band or 16-band complex wavelet transform provided the basic filterbank is 2 channel. In this case, J is treated such that it defines number of levels of 4-band, 8-band or 16-band transform.

The dual-tree wavelet filterbank can use any basic wavelet filterbank in the first stage of both trees, provided they are shifted by 1 sample (done internally). A custom first stage filterbank can be defined by passing the following key-value pair in the cell array:

- '**first**', '**w**' w defines a regular basic filterbank. Accepted formats are the same as in `fwtnit` assuming the `wfilt_` prefix.

Similarly, when working with a filterbank tree containing decomposition of high-pass outputs, some filters in both trees must be replaced by a regular basic filterbank in order to achieve the approximately analytic behavior. A custom filterbank can be specified by passing another key-value pair in the cell array:

- '**leaf**', '**w**' w defines a regular basic filterbank. Accepted formats are the same as in `fwtnit` assuming the `wfilt_` prefix.

- 2) Another possibility is to pass directly a struct. returned by `dtwfbinit` and possibly modified by `wfbtremove`.

Optional args.:

In addition, the following flag groups are supported:

- '**freq**', '**nat**' Frequency or natural (Paley) ordering of coefficient subbands. By default, subbands are ordered according to frequency. The natural ordering is how the subbands are obtained from the filterbank tree without modifications. The ordering differ only in non-plain DWT case.

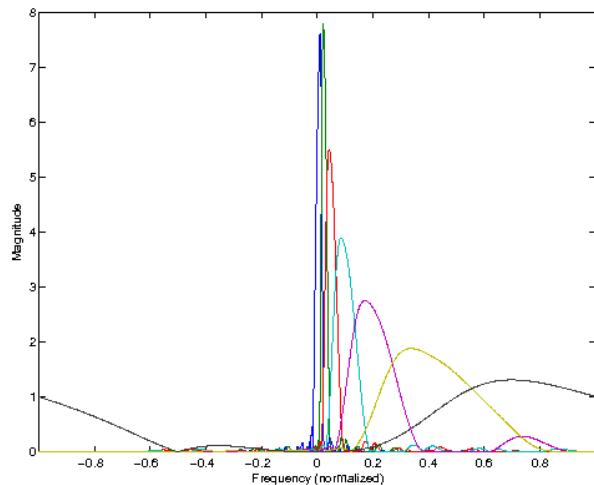
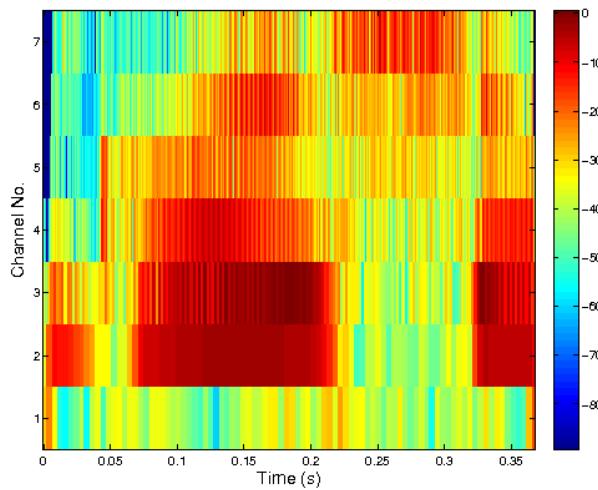
Boundary handling:

In contrast with `fwt`, `wfbt` and `wpfbt`, this function supports periodic boundary handling only.

Examples:

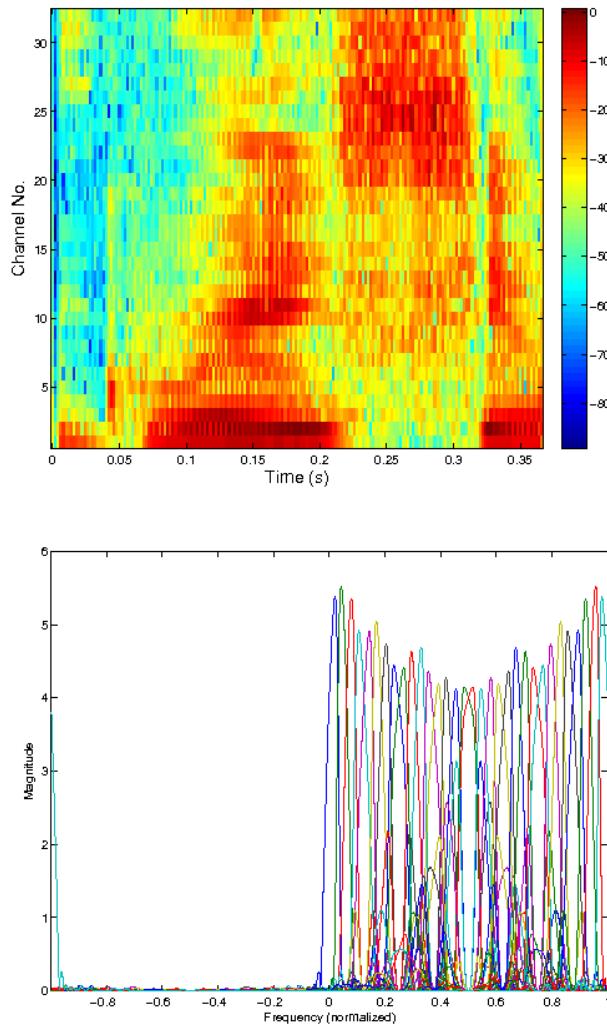
A simple example of calling the dtwfbleal function using the regular DWT iterated filterbank. The second figure shows a magnitude frequency response of an identical filterbank.:

```
[f,fs] = greasy;
J = 6;
[c,info] = dtwfbleal(f,'qshift3',J);
figure(1);
plotwavelets(c,info,fs,'dynrange',90);
figure(2);
[g,a] = dtwfb2filterbank({'qshift3',J},'real');
filterbankfreqz(g,a,1024,'plot','linabs');
```



The second example shows a decomposition using a full filterbank tree of depth J :

```
[f,fs] = greasy;
J = 5;
[c,info] = dtwfbleal(f,'qshift4',J,'full');
figure(1);
plotwavelets(c,info,fs,'dynrange',90);
figure(2);
[g,a] = dtwfb2filterbank({'qshift4',J,'full'},'real');
filterbankfreqz(g,a,1024,'plot','linabs');
```



References: [50], [86], [12]

4.3.4 IDTWFBRREAL - Inverse Dual-tree Filterbank for real-valued signals

Usage

```
f=idtwfbreal(c,info);
f=idtwfbreal(c,dualwt,Ls);
```

Input parameters

c	Input coefficients.
info	Transform params. struct
dualwt	Dual-tree Wavelet Filterbank definition
Ls	Length of the reconstructed signal.

Output parameters

f	Reconstructed data.
----------	---------------------

Description

`f = idtwfbreal(c, info)` reconstructs real-valued signal f from the coefficients c using parameters from `info` struct. both returned by `dwtfbreal` function.

`f = idtwfbreal(c, dualwt, Ls)` reconstructs real-valued signal f from the coefficients c using dual-tree filterbank defined by `dualwt`. Please see `dwtfbreal` for supported formats. The `Ls` parameter is mandatory due to the ambiguity of reconstruction lengths introduced by the subsampling operation. Note that the same flag as in the `dwtfbreal` function have to be used, otherwise perfect reconstruction cannot be obtained. Please see help for `dwtfbreal` for description of the flags.

Examples:

A simple example showing perfect reconstruction using `idtwfbreal`:

```
f = gspi;
J = 7;
wtdef = {'qshift3', J};
c = dwtfbreal(f, wtdef);
fhat = idtwfbreal(c, wtdef, length(f));
% The following should give (almost) zero
norm(f-fhat)
```

This code produces the following output:

```
ans =
9.605850455567395e-14
```

4.4 Wavelet Filterbank trees manipulation

4.4.1 WFBTINIT - Initialize Filterbank Tree

Usage

```
wt = wfbtinit(wtdef);
```

Input parameters

wtdef	Filterbank tree definition.
--------------	-----------------------------

Output parameters

wt	Structure describing the filter tree.
-----------	---------------------------------------

Description

`wfbtinit({w, J, flag})` creates a filterbank tree of depth J . The parameter w defines a basic wavelet filterbank. For all possible formats see `fwt`. The following optional flags (still inside of the cell-array) are recognized:

'dwt','full','doubleband','quadband','octaband' Type of the tree to be created.

`wfbtinit({w, J, flag, 'mod', mod})` creates a filterbank tree as before, but modified according to the value of `mod`. Recognized options:

'powshiftable' Changes subsampling factors of the root to 1. This results in redundant near-shift invariant representation.

The returned structure `wt` has the following fields:

.nodes Cell-array of structures obtained from `fwtinit`. Each element define a basic wavelet filterbank.

- .children** Indexes of children nodes
- .parents** Indexes of a parent node
- .forder** Frequency ordering of the resultant frequency bands.

The structure together with functions from the wfbtmanip subdirectory acts as an abstract data structure tree.

Regular wfbtinit flags:

- 'freq','nat'** Frequency or natural ordering of the coefficient subbands. The direct usage of the wavelet tree (' nat' option) does not produce coefficient subbands ordered according to the frequency. To achieve that, some filter shuffling has to be done (' freq' option).

4.4.2 DTWFBINIT - Dual-Tree Wavelet Filterbank Initialization

Usage

```
dualwt=dtwfbinit (dualwtdef);
```

Input parameters

dualwtdef	Dual-tree filterbank definition.
------------------	----------------------------------

Output parameters

dualwt	Dual-tree filterbank structure.
---------------	---------------------------------

Description

dtwfbinit () (a call without arguments) creates an empty structure. It has the same fields as the struct returned from wfbtinit plus a field to hold nodes from the second tree:

- .nodes Filterbank nodes of the first tree
- .dualnodes Filterbank nodes of the second tree
- .children Indexes of children nodes
- .parents Indexes of a parent node
- .forder Frequency ordering of the resultant frequency bands.

dtwfbinit ({dualw, J, flag}) creates a structure representing a dual-tree wavelet filterbank of depth J , using dual-tree wavelet filters specified by `dualw`. The shape of the tree is controlled by `flag`. Please see help on `dtwfb` or `dtwfbreal` for description of the parameters.

[`dualwt, info`]=dtwfbinit (...) additionally returns `info` struct which provides some information about the computed window:

info.tight True if the overall tree construct a tight frame.

info.dw A structure containing basic dual-tree filters as returned from `fwtinit (dualwtdef, 'wfiltdt_')`.

Additional optional flags

- 'freq','nat'** Frequency or natural ordering of the resulting coefficient subbands. Default ordering is ' freq'.

4.4.3 WFBTPUT - Put node to the filterbank tree

Usage

```
wt = wfbtput (d, k, w, wt);
wt = wfbtput (d, k, w, wt, 'force');
```

Input parameters

d	Level in the tree (0 - root).
k	Index (array of indexes) of the node at level <i>d</i> (starting at 0).
w	Node, basic wavelet filterbank.
wt	Wavelet filterbank tree structure (as returned from wfbtinit).

Output parameters

wt	Modified filterbank structure.
-----------	--------------------------------

Description

`wfbtput(d, k, w, wt)` puts the basic filterbank *w* to the filter tree structure *wt* at level *d* and index(es) *k*. The output is a modified tree structure. *d* and *k* have to specify unconnected output of the leaf node. Error is issued if *d* and *k* points to already existing node. For possible formats of parameter *w* see help of `fwt`. Parameter *wt* has to be a structure returned by `wfbtinit`.

`wfbtput(d, k, w, wt, 'force')` does the same but replaces node at *d* and *k* if it already exists. If the node to be replaced has any children, the number of outputs of the replacing node have to be equal to number of outputs of the node beeing replaced.

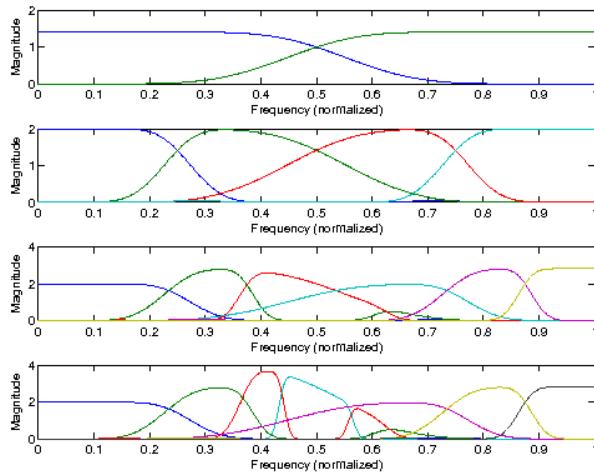
Examples:

This example shows magnitude frequency responses of a tree build from the root:

```
% Initialize empty struct
wt = wfbtinit();
% Put root node to the empty struct
wt1 = wfbtput(0,0,'db8',wt);
% Connect a different nodes to both outputs of the root
wt2 = wfbtput(1,[0,1],'db10',wt1);
% Connect another nodes just to high-pass outputs of nodes just added
wt3 = wfbtput(2,[1,3],'db10',wt2);
% Add another node at level 3
wt4 = wfbtput(3,1,'db16',wt3);

% Create identical filterbanks
[g1,a1] = wfbt2filterbank(wt1,'freq');
[g2,a2] = wfbt2filterbank(wt2,'freq');
[g3,a3] = wfbt2filterbank(wt3,'freq');
[g4,a4] = wfbt2filterbank(wt4,'freq');

% Plot frequency responses of the growing tree. Linear scale
% (both axis) is used and positive frequencies only are shown.
subplot(4,1,1);
filterbankfreqz(g1,a1,1024,'plot','linabs','posfreq');
subplot(4,1,2);
filterbankfreqz(g2,a2,1024,'plot','linabs','posfreq');
subplot(4,1,3);
filterbankfreqz(g3,a3,1024,'plot','linabs','posfreq');
subplot(4,1,4);
filterbankfreqz(g4,a4,1024,'plot','linabs','posfreq');
```



4.4.4 WFBTREMOVE - Remove node(s) from the filterbank tree

Usage

```
wt = wbftremove(d,kk,wt);
wt = wfbtremove(d,kk,wt,'force');
```

Input parameters

- | | |
|-----------|--|
| d | Level in the tree (0 - root). |
| kk | Index of the node at level <i>d</i> (starting at 0) or array of indexes. |
| wt | Wavelet filterbank tree structure (as returned from wfbtinit). |

Output parameters

- | | |
|-----------|--------------------------------|
| wt | Modified filterbank structure. |
|-----------|--------------------------------|

Description

wfbtremove (*d*, *kk*, *wt*) removes existing node at level *d* and index *kk* from the filterbank tree structure *wt*. The function fails if the node has any children (it is not a leaf node).

wfbtremove (*d*, *k*, *wt*, 'force') does the same, but any children of the node are removed too.

Examples:

The following example shows magnitude frequency responses of filterbank tree before and after pruning.:

```
% Create a full filterbank tree usinf 'db10' basic filterbank.
wt1 = wfbtinit({'db10',4,'full'});
% Remove a subtree starting by root's high-pass filter. Force flag
% is used because we are removing a non-leaf node.
wt2 = wfbtremove(1,1,wt1,'force');

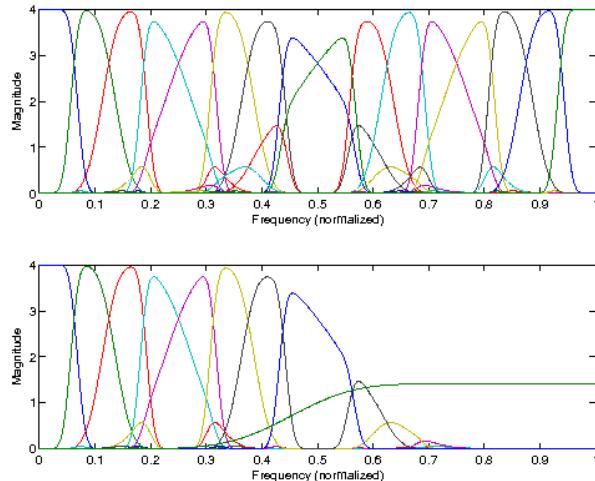
% Create identical filterbanks
[g1,a1] = wfbt2filterbank(wt1,'freq');
[g2,a2] = wfbt2filterbank(wt2,'freq');

% Plot the frequency responses
```

```

subplot(2,1,1);
filterbankfreqz(g1,a1,1024,'plot','posfreq','linabs');
subplot(2,1,2);
filterbankfreqz(g2,a2,1024,'plot','posfreq','linabs');

```



4.4.5 WFBT2FILTERBANK - WFBT equivalent non-iterated filterbank

Usage

```
[g, a] = wfbt2filterbank(wt)
```

Input parameters

wt Wavelet filter tree definition

Output parameters

g Cell array containing filters

a Vector of sub-/upsampling factors

Description

[**g, a**]=wfbt2filterbank(**wt**) calculates the impulse responses **g** and the subsampling factors **a** of non-iterated filterbank, which is equivalent to the wavelet filterbank tree described by **wt** used in wfbt. The returned parameters can be used directly in filterbank and other routines.

[**g, a**]=wfbt2filterbank({**w, J, 'dwt'**}) does the same for the DWT (fwt) filterbank tree.

Please see help on wfbt for description of **wt** and help on fwt for description of **w** and **J**.

The function additionally support the following flags:

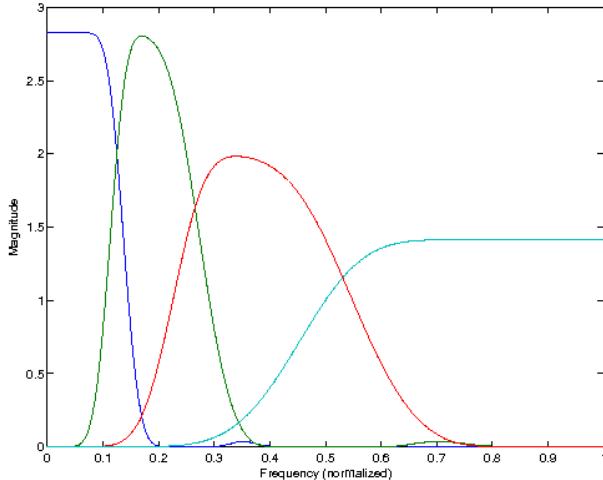
'freq' '(default)', 'nat' The filters are ordered to produce subbands in the same order as wfbt with the same flag.

'scaling_notset' '(default)', 'noscale', 'scale', 'sqrt' Support for scaling flags as described in uwfb. By default, the returned filterbank **g** and **a** is equivalent to wfbt, passing any of the non-default flags results in a filterbank equivalent to uwfb i.e. scaled and with **a (:)=1**.

Examples:

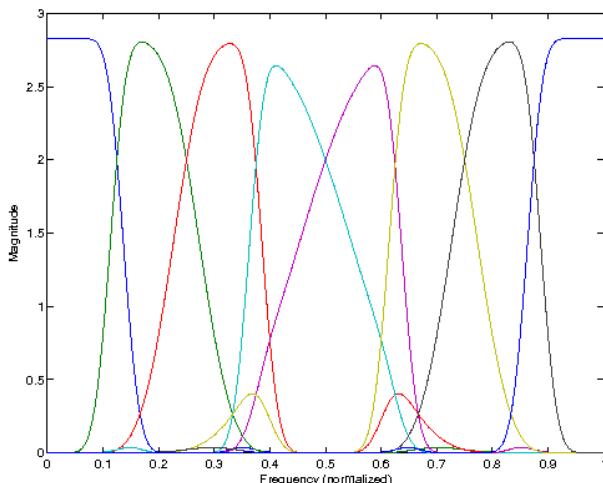
The following two examples create a multirate identity filterbank using a tree of depth 3. In the first example, the filterbank is identical to the DWT tree:

```
[g,a] = wfbt2filterbank({'db10',3,'dwt'});
filterbankfreqz(g,a,1024,'plot','linabs','posfreq');
```



In the second example, the filterbank is identical to the full wavelet tree:

```
[g,a] = wf2fbt2filterbank({'db10',3,'full'});
filterbankfreqz(g,a,1024,'plot','linabs','posfreq');
```



4.4.6 WPFBT2FILTERBANK - WPFBT equivalent non-iterated filterbank

Usage

```
[g,a] = wpfbt2filterbank(wt)
```

Input parameters

wt	Wavelet filter tree definition
-----------	--------------------------------

Output parameters

g	Cell array containing filters
a	Vector of sub-/upsampling factors

Description

`wpfbt2filterbank(wt)` calculates the impulse responses g and the subsampling factors a of non-iterated filterbank, which is equivalent to the wavelet packet filterbank tree described by wt . The returned parameters can be used directly in filterbank, ufilterbank or filterbank.

Please see help on `wfbt` for description of wt . The function additionally support the following flags:

'freq' **(default)**, **'nat'** The filters are ordered to produce subbands in the same order as `wpfbt` with the same flag.

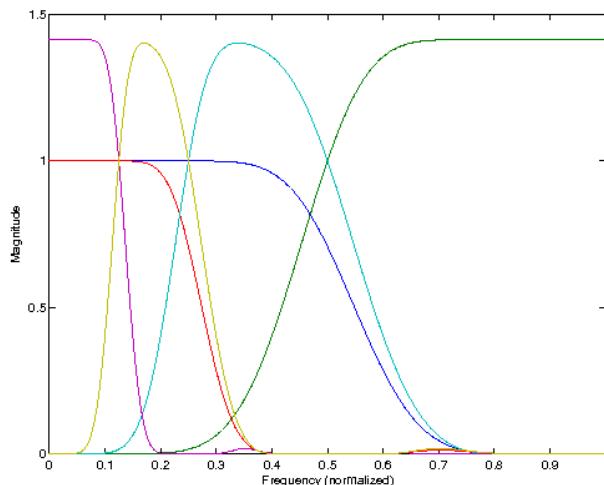
'intsqrt' **(default)**, **'intnoscale'**, **'intsquare'** The filters in the filterbank tree are scaled to reflect the behavior of `wpfbt` and `iwpfbt` with the same flags.

'scaling_notset' **(default)**, **'noscale'**, **'scale'**, **'sqrt'** Support for scaling flags as described in `uwpfbt`. By default, the returned filterbank g and a is equivalent to `wpfbt`, passing any of the non-default flags results in a filterbank equivalent to `uwpfbt` i.e. scaled and with $a(:)=1$.

Examples:

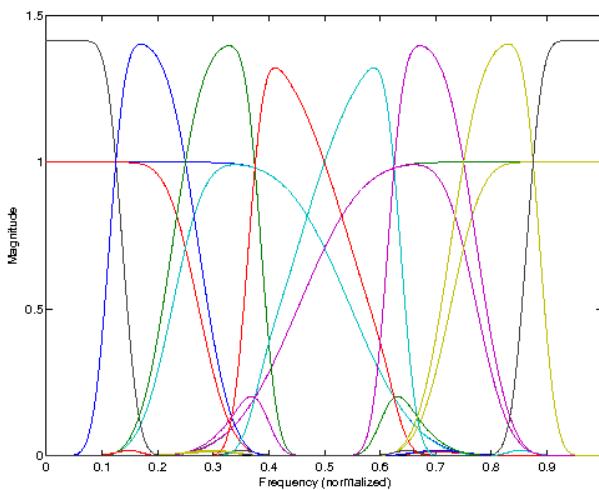
The following two examples create a multirate identity filterbank using a tree of depth 3. In the first example, the filterbank is identical to the DWT tree:

```
[g,a] = wpfbt2filterbank({'db10',3,'dwt'});
filterbankfreqz(g,a,1024,'plot','linabs','posfreq');
```



In the second example, the filterbank is identical to the full wavelet tree:

```
[g,a] = wpfbt2filterbank({'db10',3,'full'});
filterbankfreqz(g,a,1024,'plot','linabs','posfreq');
```



4.4.7 DTWFB2FILTERBANK - DTWFB equivalent non-iterated filterbank

Usage

```
[g, a] = dtwfb2filterbank(dualwt)
[g, a, info] = dtwfb2filterbank(...)
```

Input parameters

dualwt Dual-tree wavelet filterbank specification.

Output parameters

g Cell array of filters.

a Downsampling rate for each channel.

info Additional information.

Description

`[g, a] = dtwfb2filterbank(dualwt)` constructs a set of filters *g* and subsampling factors *a* of a non-iterated filterbank, which is equivalent to the dual-tree wavelet filterbank defined by *dualwt*. The returned parameters can be used directly in filterbank and other routines. The format of *dualwt* is the same as in dtwfb and dtwfbreal.

The function internally calls dtwfbinit and passes *dualwt* and all additional parameters to it.

`[g, a, info] = dtwfb2filterbank(...)` additionally outputs a *info* struct containing equivalent filterbanks of individual real-valued trees as fields *info.g1* and *info.g2*.

Additional parameters:

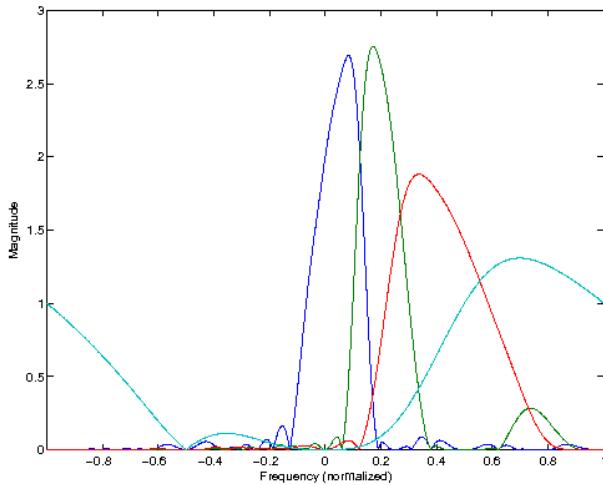
'real' By default, the function returns a filtebank equivalent to dtwfb. The filters can be restricted to cover only the positive frequencies and to be equivalent to dtwfbreal by passing a '**real**' flag.

'freq' '(default)', 'nat' The filters are ordered to produce subbands in the same order as dtwfb or dtwfbreal with the same flag.

Examples:

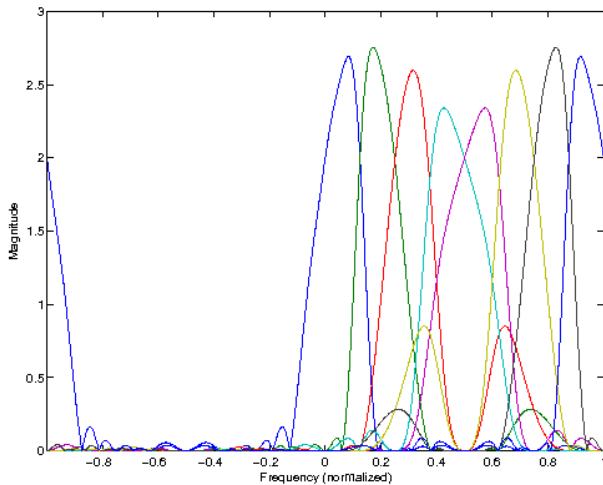
The following two examples create a multirate identity filterbank using a dual-tree of depth 3:

```
[g,a] = dtwfb2filterbank({'qshift3',3},'real');
filterbankfreqz(g,a,1024,'plot','linabs');
```



In the second example, the filterbank is identical to the full wavelet tree:

```
[g,a] = dtwfb2filterbank({'qshift3',3,'full'},'real');
filterbankfreqz(g,a,1024,'plot','linabs');
```



4.4.8 FWTINIT - Wavelet Filterbank Structure Initialization

Usage

```
w = fwtinit(wdef);
w = fwtinit(wdef,prefix);
[w,info]=fwtinit(...)
```

Input parameters

wdef	Wavelet filters specification.
prefix	Function name prefix

Output parameters

w Structure defining the filterbank.

Description

fwtinit(wdef) produces a structure describing the analysis (field w.h) and synthesis (field w.g) filterbanks and a hop factors (field w.a) of a basic wavelet-type filterbank defined by wdef.

The analysis filterbank w.h is by default used in fwt and the synthesis filterbank w.g in ifwt.

Both w.h and w.g are cell arrays of structs defining FIR filters compatible with filterbank, ifilterbank and related functions. More precisely, each element of either cell array is a struct with fields .h and .offset defining impulse response and the initial shift respectively.

[w,info]=fwtinit(...) additionally returns a info struct which provides some information about the wavelet filterbank:

info.istight Wavelet filterbank forms a tight frame. In such case, w.h and w.g are identical.

The function is a wrapper for calling all the functions with the wfilt_ prefix defined in the LTFAT wavelets directory.

The possible formats of the wdef are the following:

1) Cell array with first element being the name of the function defining the basic wavelet filters (wfilt_ prefix) and the other elements are the parameters of the function.

2) Character string as concatenation of the name of the wavelet filters defining function (as above) and the numeric parameters delimited by ':' character. Examples:

{'db',10} or 'db10' Daubechies with 10 vanishing moments. It calls wfilt_db(10) internally.

{'spline',4,4} or 'spline4:4' Biorthonal spline wavelet filters with 4 vanishing moments. Calls wfilt_spline(4,4) internally.

{'dden',1} or 'dden1' Double density wavelet filters. Calls wfilt_dden(1) where the filters are stored as numerical vectors.

3) Cell array of one dimensional numerical vectors directly defining the wavelet filter impulse responses. By default, outputs of the filters are subsampled by a factor equal to the number of the filters. Pass additional key-value pair 'a',a (still inside of the cell array) to define the custom subsampling factors, e.g.: {h1,h2,'a',[2,2]}.

4) The fourth option is to pass again the structure obtained from the fwtinit function. The structure is checked whether it has a valid format.

5) Two element cell array. First element is the string 'dual' and the second one is in format 1), 2) or 4). This returns a dual of whatever is passed as the second argument.

6) Two element cell array. First element is the string 'strict' and the second one is in format 1), 2), 4) or 5). This in the non tight case the filters has to be defined explicitly using 'ana' and 'syn' identifiers. See below.

7) Two element cell array. First element is a cell array of structures defining FIR filterbank (.h and .offset fields) as in filterbankwin and the second element is a numeric vector of subsampling factors.

One can interchange the filter in w.h and w.g and use the filterbank intended for synthesis in fwt and vice versa by re-using the items 1) and 2) in the following way:

1) Add 'ana' or 'syn' as the first element in the cell array e.g. {'ana','spline',4,4} or {'syn','spline',4,4}.

2) Add 'ana:' or 'syn:' to the beginning of the string e.g. 'ana:spline4:4' or 'syn:spline4:4'.

This only makes difference if the filterbanks are biorthogonal (e.g. `wfilt_spline`) or a general frame (e.g. `'symds2'`), in other cases, the analysis and synthesis filters are identical.

Please note that using e.g. `c=fwt(f,'ana:spline4:4',J)` and `fhat=ifwt(c,'ana:spline4:4',J,size(f,1))` will not give a perfect reconstruction.

The output structure has the following additional field:

w.origArgs Original parameters in format 1).

References: [61]

4.5 Frame properties of wavelet filter banks:

4.5.1 WFBTBOUNDS - Frame bounds of WFBT

Usage

```
fcond=wfbtbounds(wt,L);
[A,B]=wfbtbounds(wt,L);
[...]=wfbtbounds(wt);
```

Description

`wfbtbounds(wt,L)` calculates the ratio B/A of the frame bounds of the filterbank tree specified by `wt` for a system of length L . The ratio is a measure of the stability of the system.

`wfbtbounds({w,J,'dwt'},L)` calculates the ratio B/A of the frame bounds of the DWT (fwt) filterbank specified by `w` and `J` for a system of length L .

`wfbtbounds(wt)` does the same thing, but L is assumed to be the next compatible length bigger than the longest filter in the identical filterbank.

`[A,B]=wfbtbounds(...)` returns the lower and upper frame bounds explicitly.

See `wfbt` for explanation of parameter `wt` and `fwt` for explanation of parameters `w` and `J`.

The function supports the following flag groups:

'scaling_notset' '(default)', 'noscale', 'scale', 'sqrt' Support for scaling flags as described in `uwfbt`. By default, the bounds are computed for `wfbt`, passing any of the non-default flags results in framebounds for `uwfbt`.

4.5.2 WPFBTBOUNDS - Frame bounds of WPFBT

Usage

```
fcond=wpfbtbounds(wt,L);
[A,B]=wpfbtbounds(wt,L);
[...]=wpfbtbounds(wt);
```

Description

`wpfbtbounds(wt,L)` calculates the ratio B/A of the frame bounds of the wavelet packet filterbank specified by `wt` for a system of length L . The ratio is a measure of the stability of the system.

`wpfbtbounds(wt)` does the same, except L is chosen to be the next compatible length bigger than the longest filter from the identical filterbank.

`[A,B]=wpfbtbounds(...)` returns the lower and upper frame bounds explicitly.

See `wfbt` for explanation of parameter `wt`.

Additionally, the function accepts the following flags:

'intsqrt' '(default)', 'intnoscale', ''intscale' The filters in the filterbank tree are scaled to reflect the behavior of `wpfbt` and `iwpfbt` with the same flags.

'scaling_notset' '(default)', 'noscale', 'scale', 'sqrt' Support for scaling flags as described in `uwfpbt`. By default, the bounds are calculated for `wpfbt`, passing any of the non-default flags results in bounds for `uwfpbt`.

4.5.3 DTWFBBOUNDS - Frame bounds of DTWFB

Usage

```
fcond=dtwfbbounds(dualwt,L);
[A,B]=dtwfbbounds(dualwt,L);
[...]=dtwfbbounds(dualwt);
```

Description

`dtwfbbounds(dualwt,L)` calculates the ratio B/A of the frame bounds of the dual-tree filterbank specified by `dualwt` for a system of length L . The ratio is a measure of the stability of the system.

`dtwfbbounds(dualwt)` does the same thing, but L is the next compatible length bigger than the longest filter in the identical filterbank.

`[A,B]=dtwfbbounds(...)` returns the lower and upper frame bounds explicitly.

See `dtwfb` for explanation of parameter `dualwt`.

4.6 Plots

4.6.1 PLOTWAVELETS - Plot wavelet coefficients

Usage

```
plotwavelets(c,info,fs)
plotwavelets(c,info,fs,'dynrange',dynrange,...)
```

Description

`plotwavelets(c,info)` plots the wavelet coefficients `c` using additional parameters from struct. `info`. Both parameters are returned by any forward transform function in the wavelets directory.

`plotwavelets(c,info,fs)` does the same plot assuming a sampling rate `fs` Hz of the original signal.

`plotwavelets(c,info,fs,'dynrange',dynrange)` additionally limits the dynamic range.

`C=plotwavelets(...)` returns the processed image data used in the plotting. Inputting this data directly to `imagesc` or similar functions will create the plot. This is usefull for custom post-processing of the image data.

`plotwavelets` supports optional parameters of `tfplot`. Please see the help of `tfplot` for an exhaustive list.

4.6.2 WFILTINFO - Plots filters info

Usage

```
wfiltinfo(w);
```

Input parameters

<code>w</code>	Basic wavelet filterbank.
----------------	---------------------------

Description

`wfiltinfo(w)` plots impulse responses, frequency responses and approximation of the scaling and of the wavelet function(s) associated with the wavelet filters defined by `w` in a single figure. Format of `w` is the same as in `fwt`.

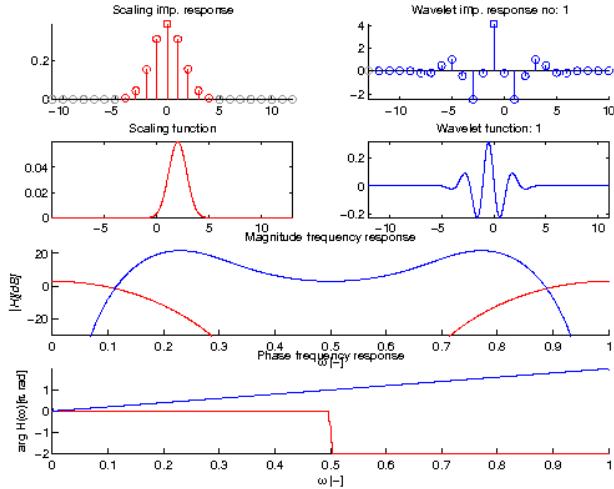
Optionally it is possible to define scaling of the y axis of the frequency seponses. Supported are:

'`db`', '`lin`' dB or linear scale respectively. By deault a dB scale is used.

Examples:

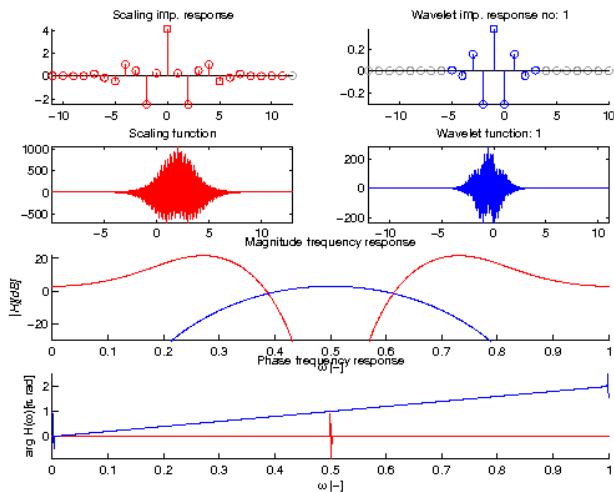
Details of the 'syn:spline8:8' wavelet filters (see wfilt_spline):

```
wfiltinfo('syn:spline8:8');
```



Details of the 'ana:spline8:8' wavelet filters:

```
wfiltinfo('ana:spline8:8');
```



4.6.3 WFILTDTINFO - Plots dual-tree filters info

Usage

```
wfiltdtinfo(dw);
```

Input parameters

dw	Wavelet dual-tree filterbank
-----------	------------------------------

Description

wfiltdtinfo (w) plots impulse responses, frequency responses and approximation of the scaling and of the wavelet function(s) associated with the dual-tree wavelet filters defined by w in a single figure. Format of dw is the same as in dtwfb.

The figure is organized as follows:

First row shows impulse responses of the first (real) tree.

Second row shows impulse responses of the second (imag) tree.

Third row contains plots of real (green), imaginary (red) and absolute (blue) parts of approximation of scaling and wavelet function(s).

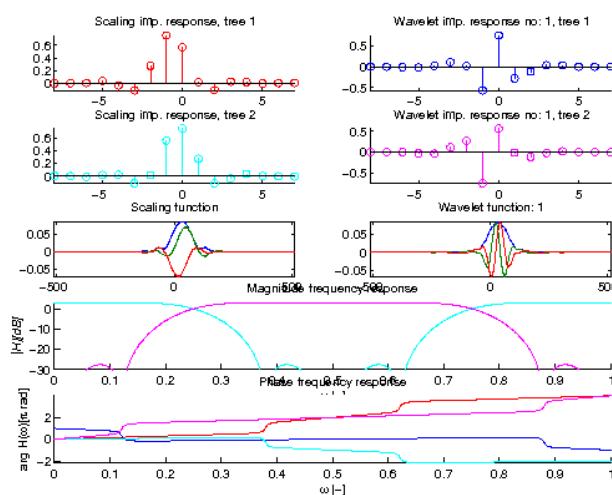
Fourth and fifth row show magnitude and phase frequency responses respectively of filters from rows 1 and 2 with matching colors.

Optionally it is possible to define scaling of the y axis of the frequency seponses. Supported are:

'db','lin' dB or linear scale respectively. By deault a dB scale is used.

Examples:

```
wfilt dtinfo ('qshift4');
```



4.7 Auxilary

4.7.1 WAVFUN - Wavelet Function

Usage

```
[w,s,xvals] = wavfun(g)
[w,s,xvals] = wavfun(g,N)
```

Input parameters

w Wavelet filterbank

N Number of iterations

Output parameters

wfunc Approximation of wavelet function(s)

sfunc Approximation of the scaling function

xvals Correct x-axis values

Description

Iteratively generate (N iterations) a discrete approximation of wavelet and scaling functions using filters obtained from w . The possible formats of w are the same as for the fwt function. The algorithm is equal to the DWT reconstruction of a single coefficient at level $N + 1$ set to 1. $xvals$ contains correct x-axis values. All but last columns belong to the $wfunc$, last one to the $sfunc$.

The following flags are supported (first is default):

'fft', **'conv'** How to do the computations. Whatever is faster depends on the speed of the `conv2` function.

WARNING: The output array lengths L depend on N exponentially like:

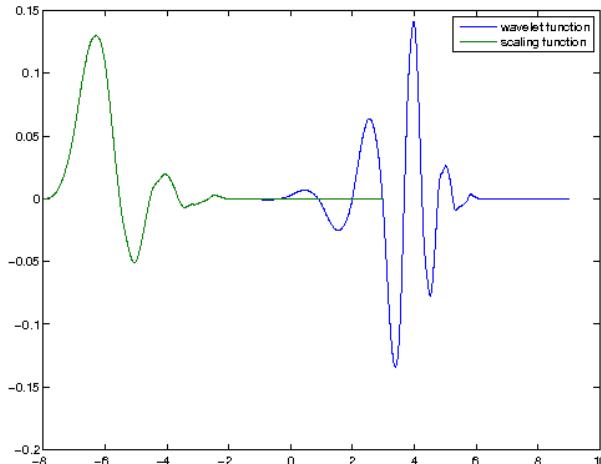
$$L = \frac{a^N - 1}{a - 1} (m - 1) + 1$$

where a is subsampling factor after the lowpass filter in the wavelet filterbank and m is length of the filters. Expect issues for high N e.g. '`db10`' ($m = 20$) and $N = 20$ yields a ~150MB array.

Examples:

Approximation of a Daubechies wavelet and scaling functions from the 12 tap filters:

```
[wfn,sfn,xvals] = wavfun('db6');
plot(xvals,[wfn,sfn]);
legend('wavelet function','scaling function');
```



4.7.2 WAVCELL2PACK - Changes wavelet coefficients storing format

Usage

```
[cvec,Lc] = wavcell2pack(ccell);
[cvec,Lc] = wavcell2pack(ccell,dim);
```

Input parameters

ccell Coefficients stored in a column cell-array.

dim Dimension along which the data were transformed.

Output parameters

cvec	Coefficients in packed format.
Lc	Vector containing coefficients lengths.

Description

[cvec, Lc] = wavcell2pack(ccell) assembles a column vector or a matrix *cvec* using elements of the cell-array *ccell* in the following manner:

```
cvec(1+sum(Lc(1:j-1)) : sum(Lc(1:j), :) ) = ccell{j};
```

where *Lc* is a vector of length *numel(ccell)* containing number of rows of each element of *ccell*.

[cvec, Lc] = wavcell2pack(ccell, dim) with *dim*=2 returns a transposition of the previous.

4.7.3 WAVPACK2CELL - Changes wavelet coefficients storing format**Usage**

```
ccell = wavpack2cell(cvec, Lc);
ccell = wavpack2cell(cvec, Lc, dim);
```

Input parameters

cvec	Coefficients in packed format.
Lc	Vector containing coefficients lengths.
dim	Dimension along which the data were transformed.

Output parameters

ccell	Coefficients stored in a cell-array. Each element is a column vector or a matrix.
dim	Return used dim. Usefull as an input of the complementary function wavcell2pack.

Description

ccell = wavpack2cell(cvec, Lc) copies coefficients from a single column vector or columns of a matrix *cvec* of size [sum(*Lc*), *W*] to the cell array *ccell* of length *length(Lc)*. Size of *j*-th element of *ccell* is [*Lc(j)*, *W*] and it is obtained by:

```
ccell{j}=cvec(1+sum(Lc(1:j-1)) : sum(Lc(1:j), :) );
```

ccell = wavpack2cell(cvec, Lc, dim) allows specifying along which dimension the coefficients are stored in *cvec*. *dim*=1 (default) considers columns (as above) and *dim*=2 rows to be coefficients belonging to separate channels. Other values are not supported. For *dim*=2, *cvec* size is [*W*, sum(*Lc*)], Size of *j*-th element of *ccell* is [*Lc(j)*, *W*] and it is obtained by:

```
ccell{j}=cvec(:, 1+sum(Lc(1:j-1)) : sum(Lc(1:j)) .';
```

4.8 Wavelet Filters defined in the time-domain**4.8.1 WFILT_ALGMBAND - An ALgebraic construction of orthonormal M-BAND wavelets with perfect reconstruction****Usage**

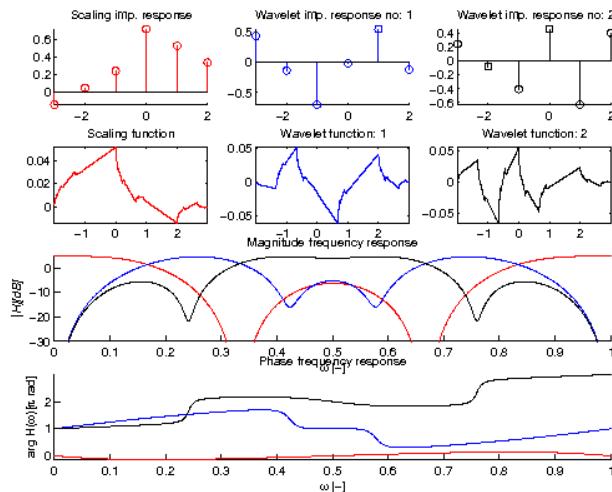
```
[h, g, a] = wfilt_algmband(K);
```

Description

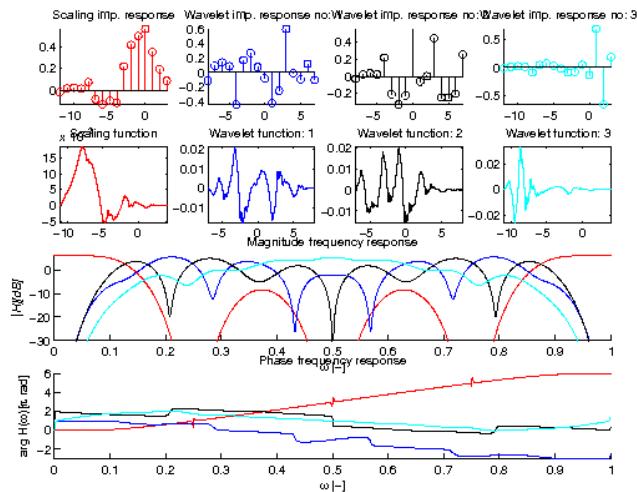
`[h, g, a]=wfilt_algmband(K)` with $K \in 1, 2$ returns wavelet filters from the reference paper. The filters are 3-band ($K == 1$) and 4-band ($K == 2$) with critical subsampling.

Examples:

```
wfiltinfo('algmband1');
```



```
wfiltinfo('algmband2');
```



References: [57]

4.8.2 WFILT_CMBAND - Generates M-Band cosine modulated wavelet filters

Usage

```
[h, g, a] = wfilt_cmband(M);
```

Input parameters

M

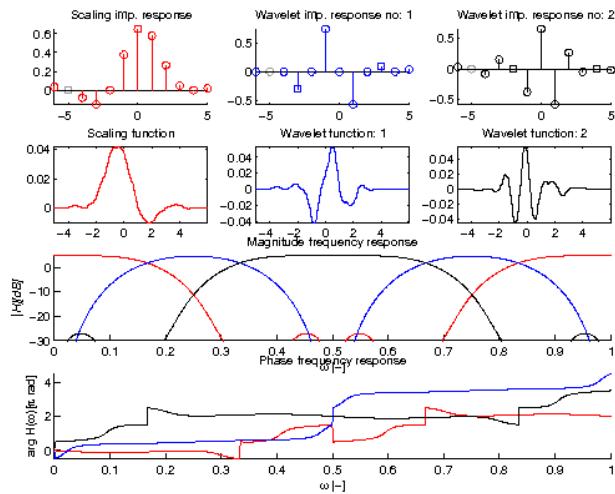
Number of channels.

Description

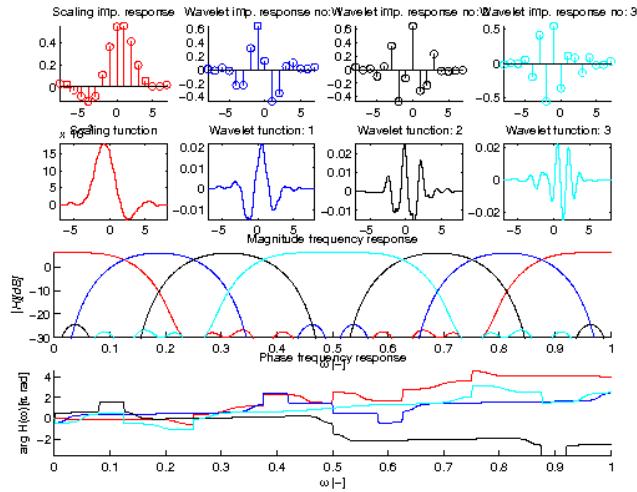
`[h, g, a]=wfilt_cmband(M)` with $M \in 2, 3, \dots$ returns smooth, 1-regular cosine modulated M -band wavelet filters according to the reference paper. The length of the filters is $4M$.

Examples:

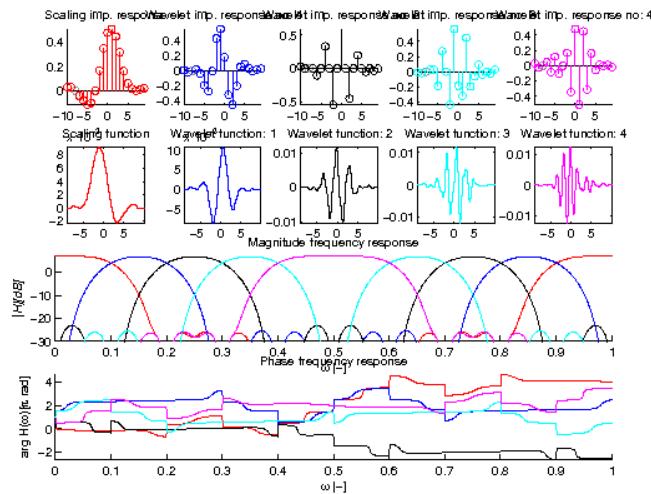
```
wfiltinfo('cmband3');
```



```
wfiltinfo('cmband4');
```



```
wfiltinfo('cmband5');
```



References: [38]

4.8.3 WFILT_COIF - Coiflets

Usage

```
[h, g, a] = wfilt_coif(K);
```

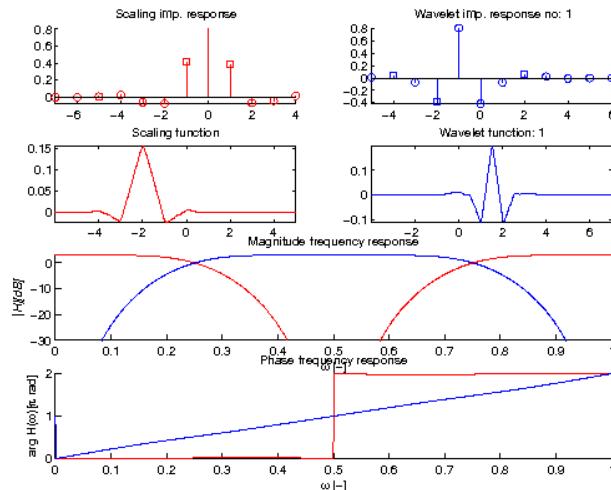
Description

[h, g, a]=wfilt_coif(K) with $K \in 1, 2, 3, 4, 5$ returns a Coiflet filters of order $2K$ the number of vanishing moments of both the scaling and the wavelet functions.

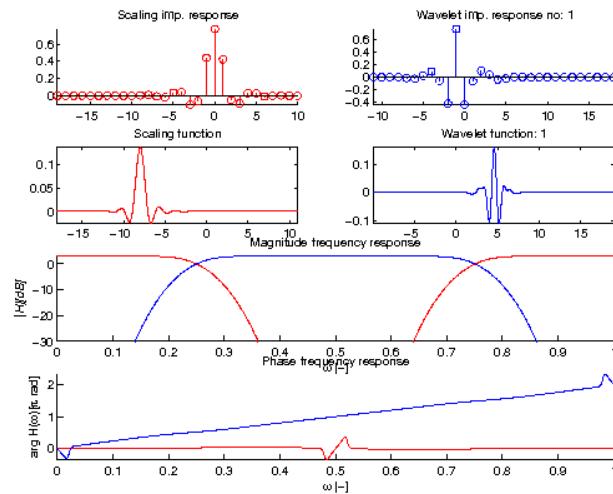
Values are taken from table 8.1 from the reference. REMARK: There is a typo in 2nd element for $K == 1$.

Examples:

```
wfiltinfo('coif2');
```



```
wfiltinfo('coif5');
```



References: [24]

4.8.4 WFILT_DB - Daubechies FIR filterbank

Usage

```
[h, g] = wfilt_db(N);
```

Input parameters

N Order of Daubechies filters.

Output parameters

H cell array of analysing filters impulse responses

G cell array of synthetizing filters impulse responses

a array of subsampling (or hop) factors accociated with corresponding filters

Description

$[H, G] = \text{dbfilt}(N)$ computes a two-channel Daubechies FIR filterbank from prototype maximum-phase analysing lowpass filter obtained by spectral factorization of the Lagrange interpolator filter. N also denotes the number of zeros at $z = -1$ of the lowpass filters of length $2N$. The prototype lowpass filter has the following form (all roots of $R(z)$ are outside of the unit circle):

$$H_l(z) = (1 + z^{-1})^N R(z),$$

where $R(z)$ is a spectral factor of the Lagrange interpolator $P(z) = 2R(z) * R(z^{-1})$. All subsequent filters of the two-channel filterbank are derived as follows:

$$H_h(z) = H_l((-z)^{-1})$$

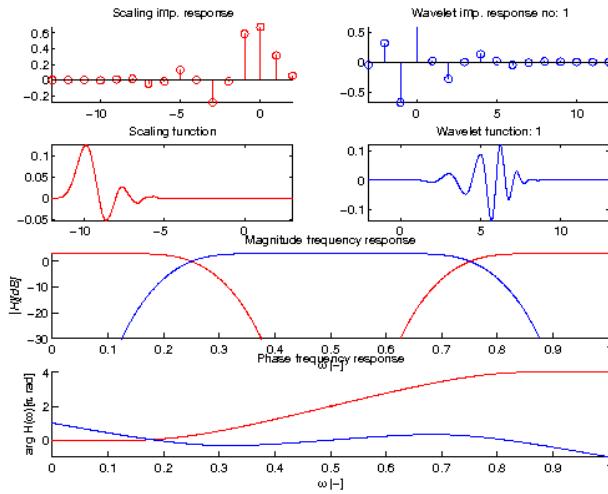
$$G_l(z) = H_l(z^{-1})$$

$$G_h(z) = -H_l(-z)$$

making them an orthogonal perfect-reconstruction QMF.

Examples:

```
wfiltinfo('db8');
```



References: [24]

4.8.5 WFILT_DDEN - Double-DENsity DWT filters (tight frame)**Usage**

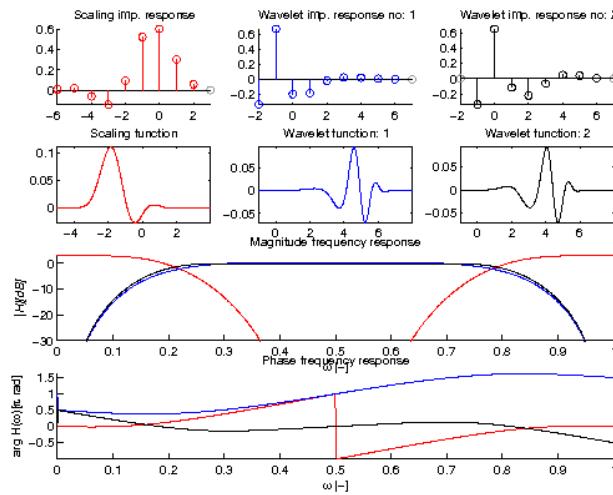
```
[h, g, a] = wfilt_dden(N);
```

Description

$[h, g, a] = \text{wfilt_dden}(N)$ computes oversampled dyadic double-density DWT filters. The redundancy of the basic filterbank is equal to 1.5.

Examples:

```
wfiltinfo('dden5');
```



References: [72]

4.8.6 WFILT_DGRID - Dense GRID framelets (tight frame, symmetric)

Usage

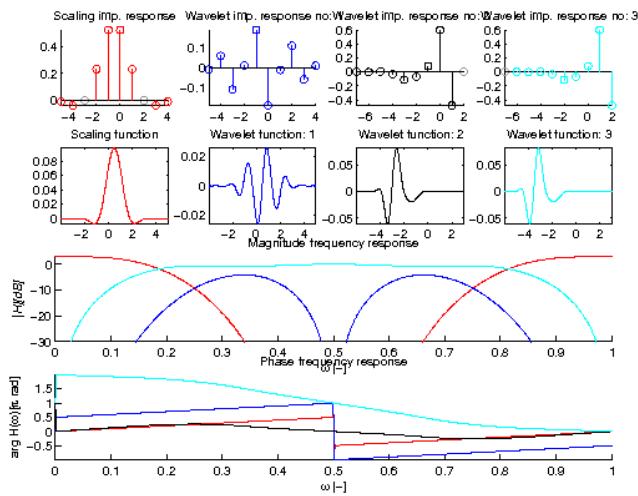
```
[h, g, a] = wfilt_dgrid(N);
```

Description

[h, g, a]=wfilt_dgrid(N) computes Dense GRID framelets. Redundancy equal to 2.

Examples:

```
wfiltinfo('dgrid3');
```



References: [1]

4.8.7 WFILT_HDEN - Higher DENsity dwt filters (tight frame, frame)

Usage

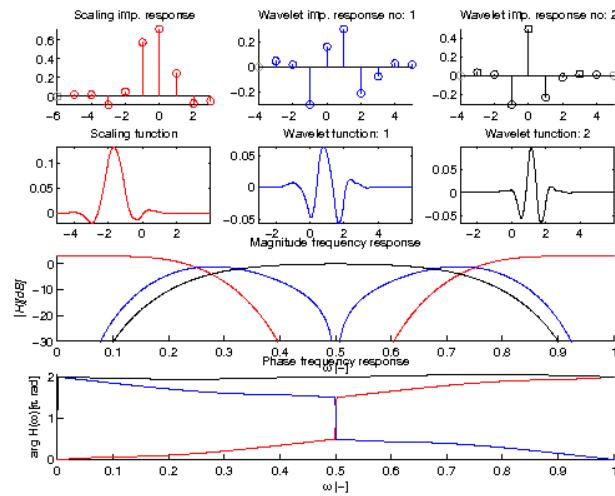
```
[h, g, a] = wfilt_hden(K);
```

Description

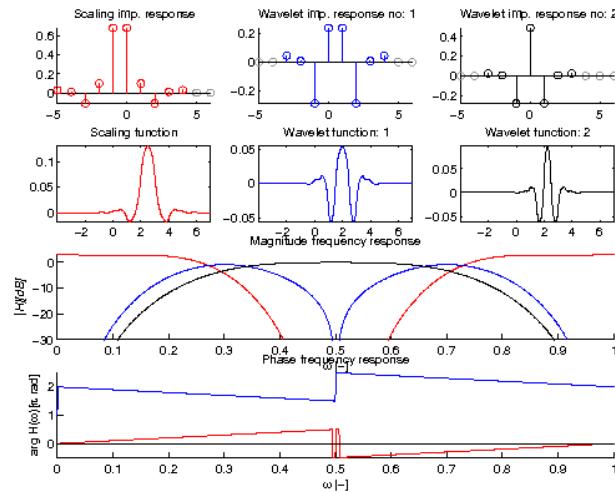
[h, g, a]=wfilt_hden(K) with $K \in \{1, 2, 3, 4\}$ returns Higher DENsity dwt filters (tight frame, frame) from the reference. The filterbanks have 3 channels and unusual non-uniform subsampling factors $[2, 2, 1]$.

Examples:

```
wfiltinfo('hdens');
```



```
wfiltinfo('ana:hden4');
```



References: [83]

4.8.8 WFILT_LEMARIE - Battle and Lemarie filters

Usage

```
[h, g, a]=wfilt_lemarie(N)
```

Input parameters

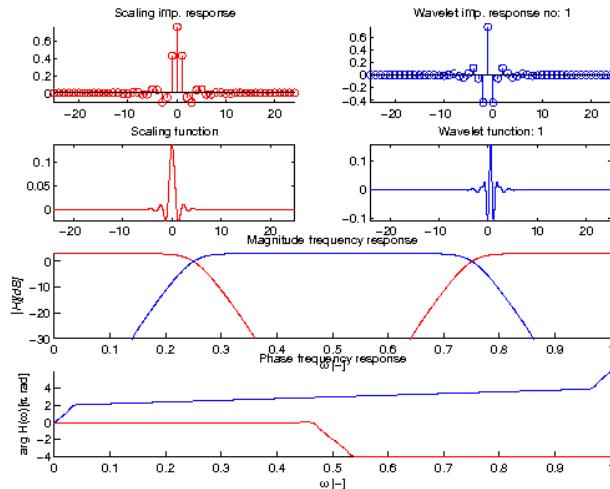
N Filter length, must be even.

Description

`[h, g, a]=wfilt_lemarie(N)` calculates N (even) truncated coefficients of orthonormal Battle-Lemarie wavelets. Filter coefficients are obtained by frequency domain sampling and truncating the impulse response. Due to the truncation, the filterbank might not achieve a perfect reconstruction. The filters are included nevertheless since they were the original ones used in the first MRA paper.

Examples:

```
wfiltinfo('lemarie50');
```



References: [63]

4.8.9 WFILT_MATLABWRAPPER - Wrapper of the Matlab Wavelet Toolbox wfilters function

Usage

```
[h, g, a] = wfilt_matlabwrapper(wname);
```

Description

`[h, g, a]=wfilt_matlabwrapper(wname)` calls Matlab Wavelet Toolbox function `wfilters` and passes the parameter `wname` to it.

This function requires the Matlab Wavelet Toolbox.

4.8.10 WFILT_MBAND - Generates 4-band coder

Usage

```
[h, g, a] = wfilt_mband(N);
```

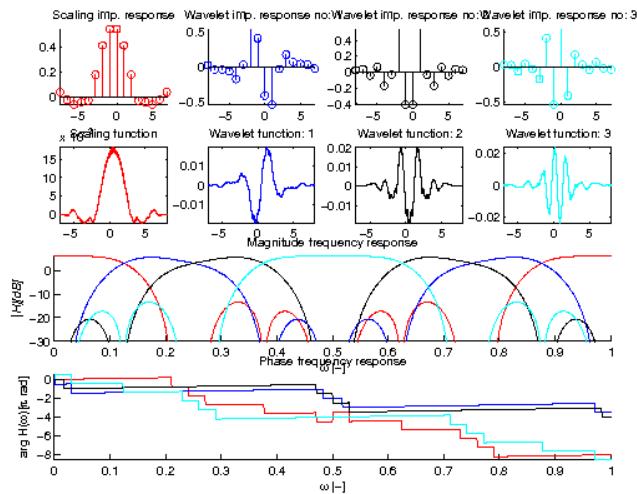
Description

`[h, g, a]=wfilt_mband(1)` returns linear-phase 4-band filters from the reference.

The filters are not actually proper wavelet filters, because the scaling filter is not regular, therefore it is not stable under iterations (does not converge to a scaling function).

Examples:

```
wfiltinfo('mband1');
```



References: [7]

4.8.11 WFILT_REMEZ - Filters designed using Remez exchange algorithm

Usage

```
[h, g, a]=wfilt_remez(L, K, B)
```

Input parameters

L Length of the filters.

K Degree of flatness (regularity) at $z = -1$.

B Normalized transition bandwidth.

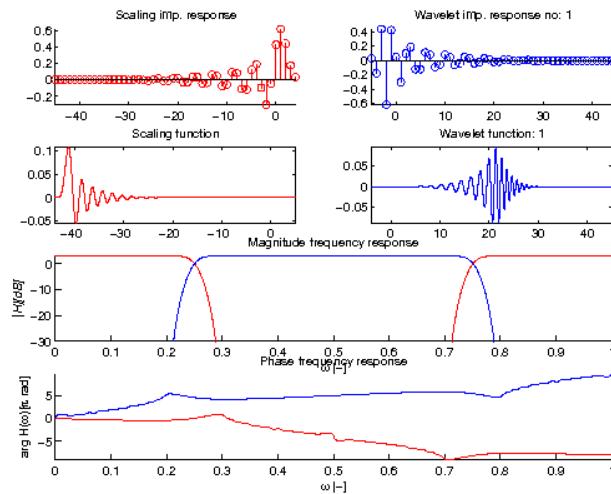
Description

[h, g, a]=wfilt_remez(L, K, B) calculates a set of wavelet filters. Regularity, frequency selectivity, and length of the filters can be controlled by K , B and L parameters respectively.

The filter design algorithm is based on a Remez algorithm and a factorization of the complex cepstrum of the polynomial.

Examples:

```
wfiltinfo('remez50:2:0.1');
```



References: [80]

4.8.12 WFILT_SYMDS - Symmetric wavelets dyadic sibling

Usage

```
[h, g, a] = wfilt_symds (K);
```

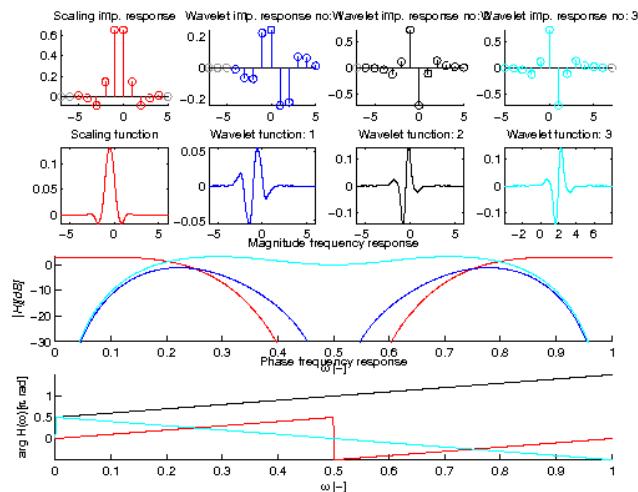
Description

[h, g, a]=wfilt_symds (K) with $K \in 1, 2, 3, 4, 5$ returns symmetric dyadic sibling wavelet frame filters from the reference.

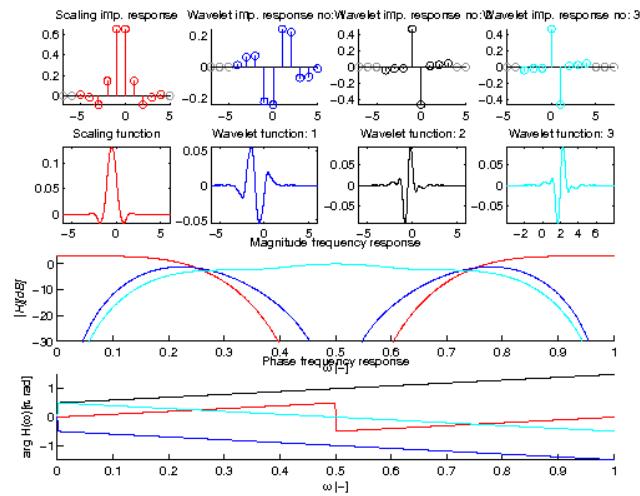
The returned filterbank has redundancy equal to 2 and it does not form a tight frame.

Examples:

```
wfiltinfo('ana:symds3');
```



```
wfiltinfo('syn:symds3');
```



References: [3]

4.8.13 WFILT_SPLINE - Biorthogonal spline wavelets

Usage

```
[h, g, a]=wfilt_spline(m, n);
```

Input parameters

m Number of zeros at $z = -1$ of the lowpass filter in $g \{ 1 \}$

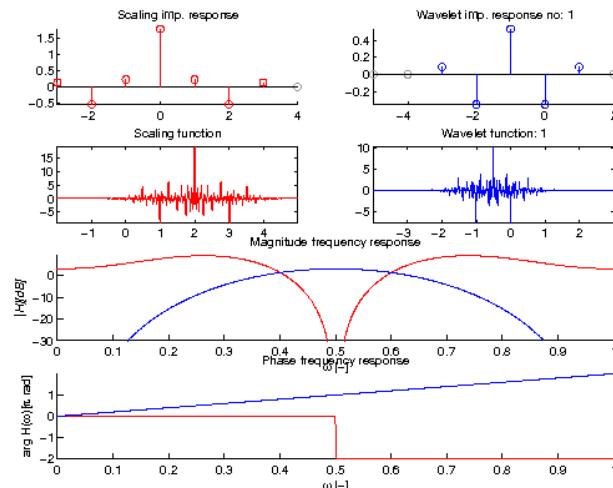
n Number of zeros at $z = -1$ of the lowpass filter in $h \{ 1 \}$

Description

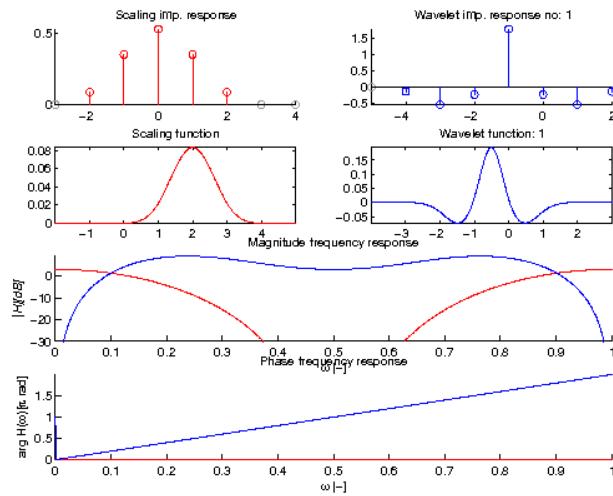
`[h, g, a]=wfilt_spline(m, n)` with $m+n$ being even returns biorthogonal spline wavelet filters.

Examples:

```
wfiltinfo('ana:spline4:2');
```



```
wfiltinfo('syn:spline4:2');
```



4.8.14 WFILT_SYM - Symlet filters

Usage

```
[h, g, a] = wfilt_sym(N);
```

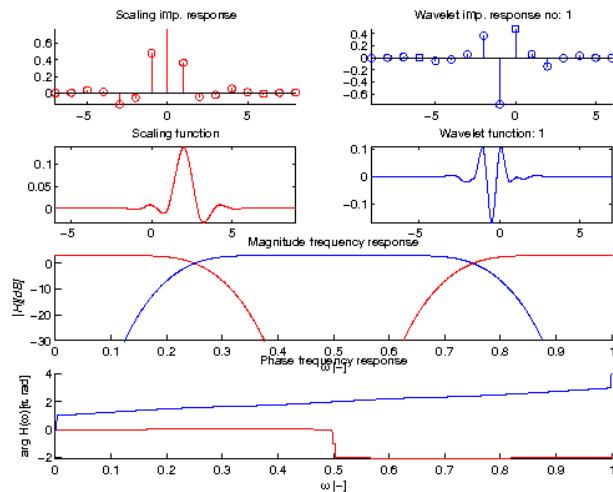
Description

`[h, g, a] = wfilt_sym(N)` generates the "least asymmetric" Daubechies' orthogonal wavelets or "symlets" with N vanishing moments and length $2N$. Zeros of the trigonometrical polynomial the filters consist of in the Z-plane are selected alternatingly inside and outside the unit circle.

Remark: Filters generated by this routine differ slightly from the ones in the reference (table 6.3, figure. 6.4) because of the ambiguity in the algorithm.

Examples:

```
wfiltinfo('sym8');
```



References: [24]

4.8.15 WFILT_SYMDDEN - Symmetric Double-Density DWT filters (tight frame)

Usage

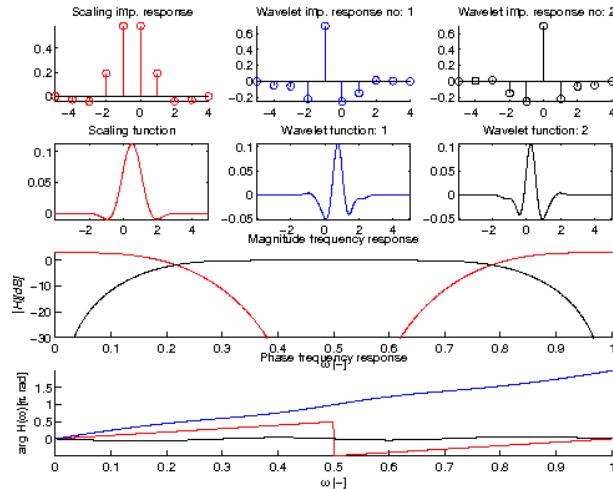
```
[h, g, a] = wfilt_symdden(K);
```

Description

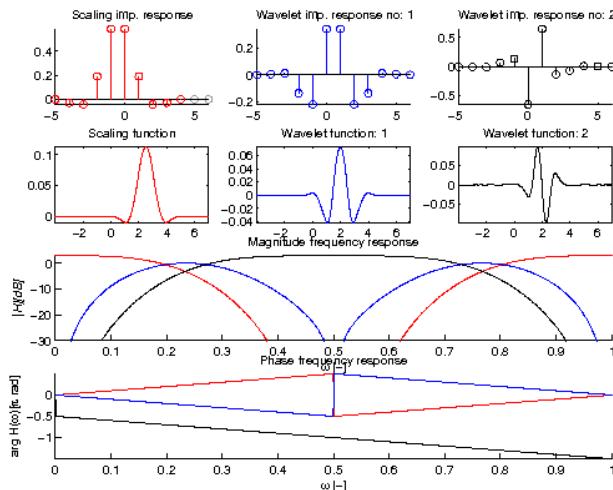
$[h, g, a] = \text{wfilt_symdden}(K)$ with $K \in 1, 2$ returns oversampled symmetric double-density DWT filters. The redundancy of the basic filterbank is equal to 1.5.

Examples:

```
wfiltinfo('symdden1');
```



```
wfiltinfo('symdden2');
```



References: [85]

4.8.16 WFILT_SYMORTH - Symmetric nearly-orthogonal and orthogonal nearly-symmetric

Usage

```
[h, g, a] = wfilt_symorth(N);
```

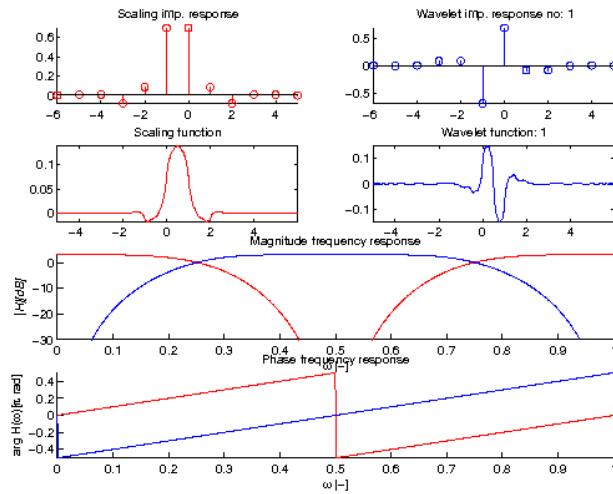
Description

[h, g, a]=wfilt_symorth(N) with $N \in 1, 2, 3$ returns orthogonal near-symmetric ($N == 1$) and symmetric near-orthogonal ($N == [2, 3]$) wavelet filters from the reference.

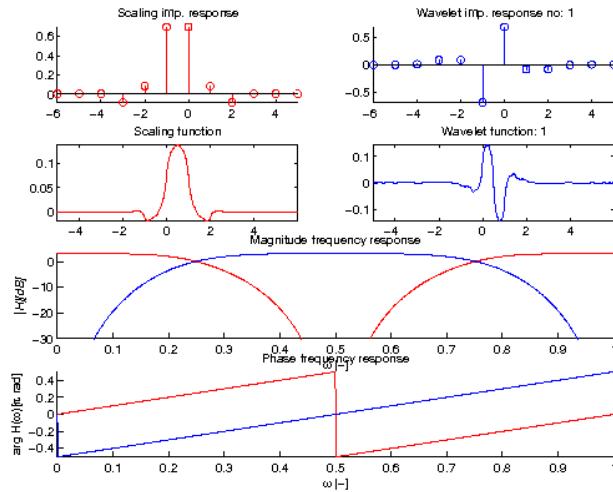
The filters exhibit a coiflet-like behavior i.e. the scaling filter has vanishing moments too.

Examples:

```
wfiltinfo('ana:symorth2');
```



```
wfiltinfo('syn:symorth2');
```



References: [4]

4.8.17 WFILT_SYMTIGHT - Symmetric Nearly Shift-Invariant Tight Frame Wavelets

Usage

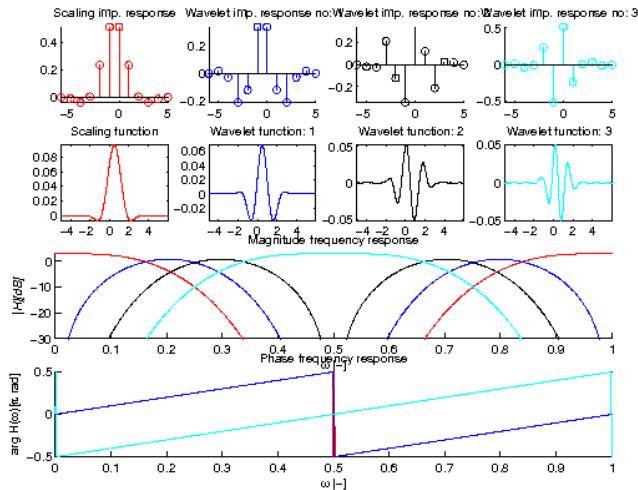
```
[h, g, a] = wfilt_symtight(K);
```

Description

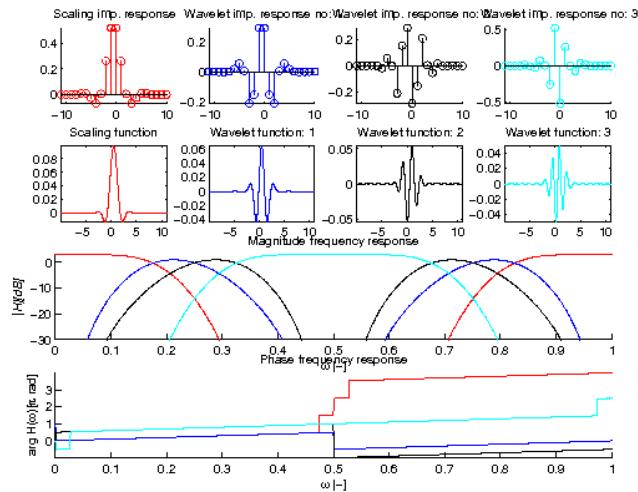
[h, g, a]=wfilt_symtight(K) with $K \in 1, 2$ returns 4-band symmetric nearly shift-invariant tight framelets.

Examples:

```
wfiltinfo('symtight1');
```



```
wfiltinfo('symtight2');
```



References: [2]

4.8.18 WFILT_QSHIFTA - Improved Orthogonality and Symmetry properties

Usage

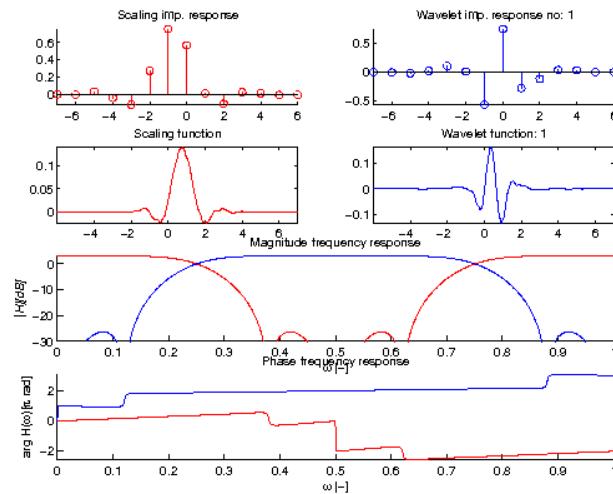
```
[h, g, a] = wfilt_qshifta(N);
```

Description

`[h, g, a]=wfilt_qshift(N)` with $N \in 1, 2, 3, 4, 5, 6, 7$ returns Kingsbury's Q-shift wavelet filters for tree A.

Examples:

```
figure(1);
wfiltinfo('qshifta3');
```



References: [52], [51]

4.8.19 WFILT_QSHIFTB - Improved Orthogonality and Symmetry properties

Usage

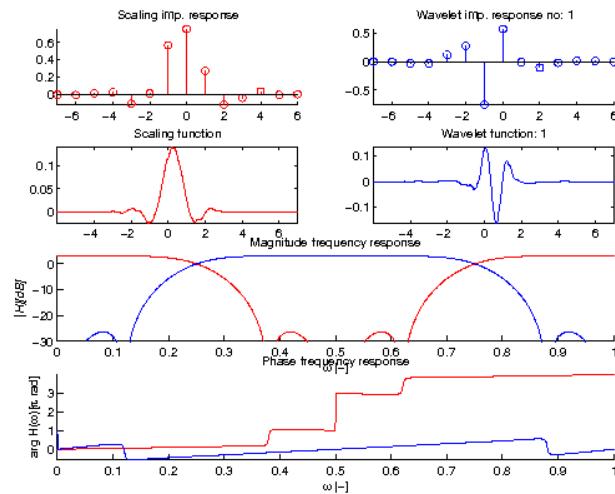
```
[h, g, a] = wfilt_qshiftb(N);
```

Description

`[h, g, a]=wfilt_qshiftb(N)` with $N \in 1, 2, 3, 4, 5, 6, 7$ returns Kingsbury's Q-shift wavelet filters for tree B.

Examples:

```
figure(1);
wfiltinfo('qshiftb3');
```



References: [52], [51]

4.8.20 WFILT_ODDEVENA - Kingsbury's symmetric even filters

Usage

```
[h, g, a] = wfilt_oddevena (N);
```

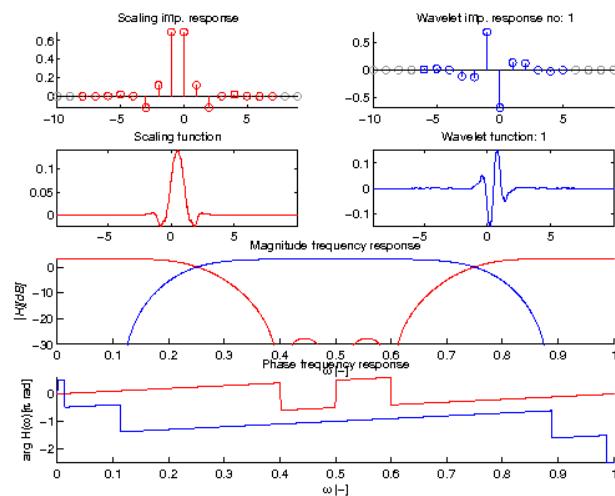
Description

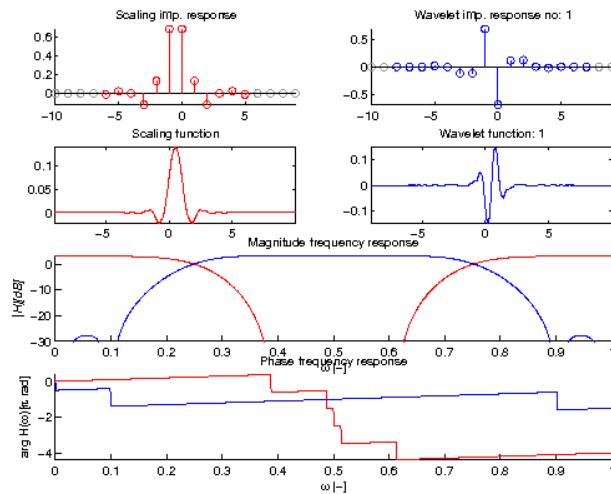
[h, g, a]=wfilt_oddevena (N) with $N \in 1$ returns Kingsbury's even filters.

Examples:

```
figure(1);
wfiltinfo('ana:oddevena1');

figure(2);
wfiltinfo('syn:oddevena1');
```





References: [50]

4.8.21 WFILT_ODDEVENB - Kingsbury's symmetric odd filters

Usage

```
[h, g, a] = wfilt_oddevenb(N);
```

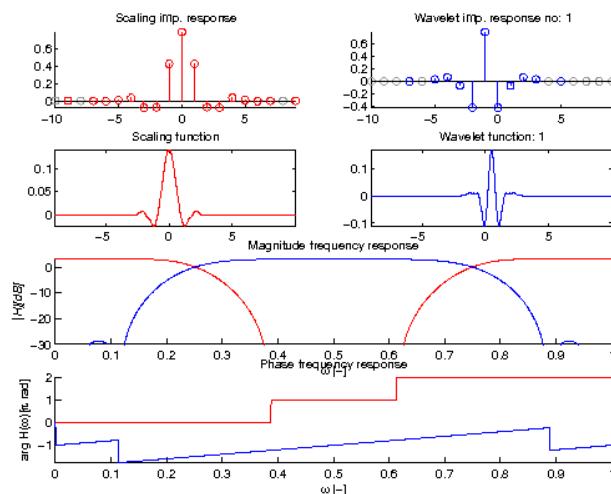
Description

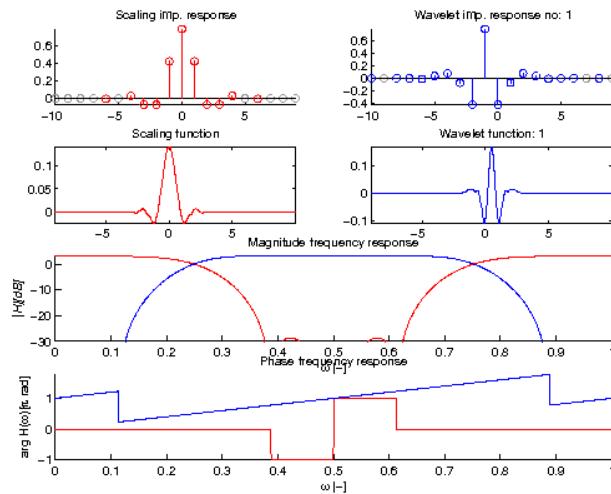
[h, g, a]=wfilt_oddevenb(N) with $N \in \mathbb{N}$ returns Kingsbury's odd filters.

Examples:

```
figure(1);
wfiltinfo('ana:oddevenb1');

figure(2);
wfiltinfo('syn:oddevenb1');
```





References: [50]

4.8.22 WFILT_OPTSYMA - Optimizatized Symmetric Self-Hilbertian Filters

Usage

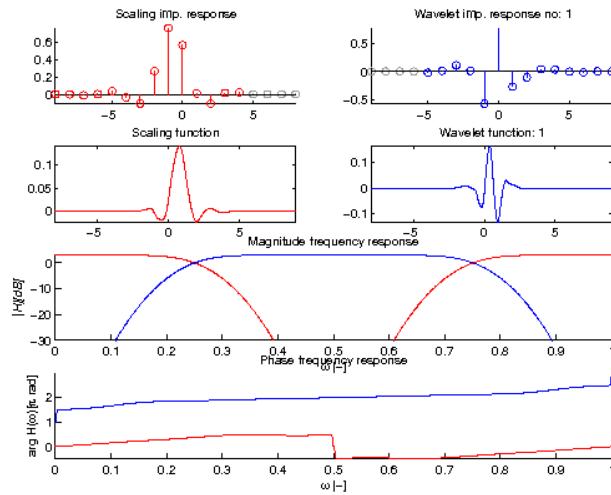
```
[h, g, a] = wfilt_optsyma(N);
```

Description

$[h, g, a] = \text{wfilt_dft_optsyma}(N)$ with $N \in 1, 2, 3$ returns filters suitable with optimized symmetry suitable for dual-tree complex wavelet transform tree A.

Examples:

```
wfiltinfo('optsyma3');
```



References: [29]

4.8.23 WFILT_OPTSYMB - Optimizatized Symmetric Self-Hilbertian Filters

Usage

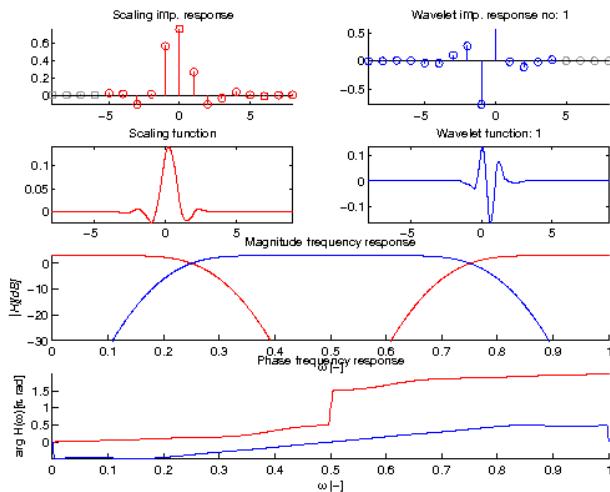
```
[h, g, a] = wfilt_optsymb(N);
```

Description

`[h, g, a]=wfiltddt_opt symb(N)` with $N \in 1, 2, 3$ returns filters suitable with optimized symmetry suitable for dual-tree complex wavelet transform tree B.

Examples:

```
wfiltinfo('opt symb3');
```



References: [29]

4.8.24 WFILT_DDENA - Double-Density Dual-Tree DWT filters

Usage

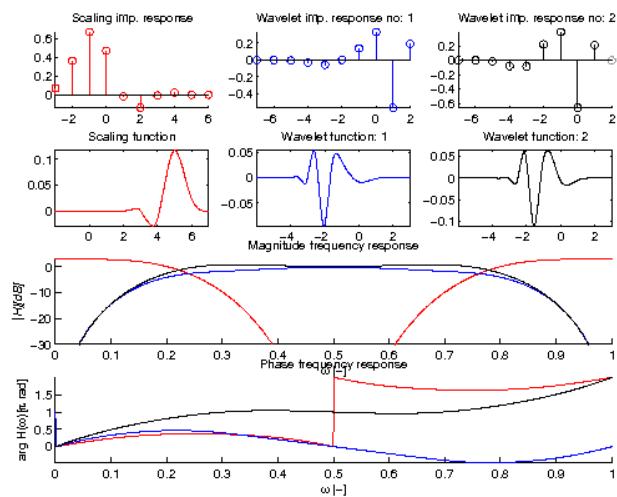
```
[h, g, a] = wfilt_ddena(N);
```

Description

`[h, g, a]=wfilt_ddena(N)` with $N \in 1, 2$ returns filters suitable for dual-tree double density complex wavelet transform tree A.

Examples:

```
wfiltinfo('ddena1');
```



References: [82]

4.8.25 WFILT_DDENB - Double-Density Dual-Tree DWT filters

Usage

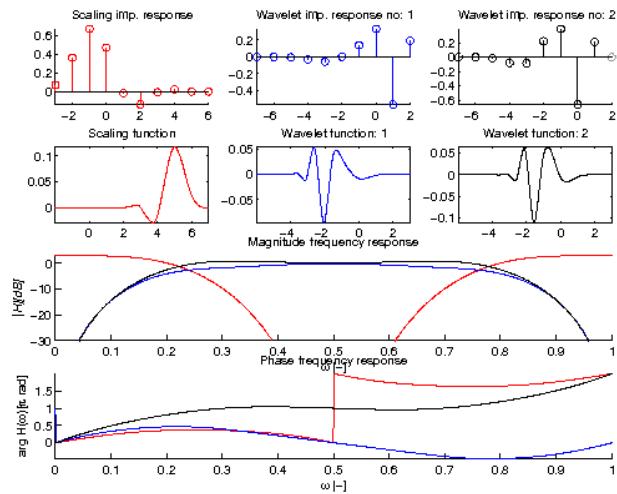
```
[h, g, a] = wfilt_ddenb(N);
```

Description

[h, g, a]=wfilt_ddenb(N) with $N \in 1, 2$ returns filters suitable for dual-tree double density complex wavelet transform tree A.

Examples:

```
wfiltinfo('ddenb');
```



References: [82]

4.9 Dual-Tree Filters

4.9.1 WFILTDT_QSHIFT - Improved Orthogonality and Symmetry properties

Usage

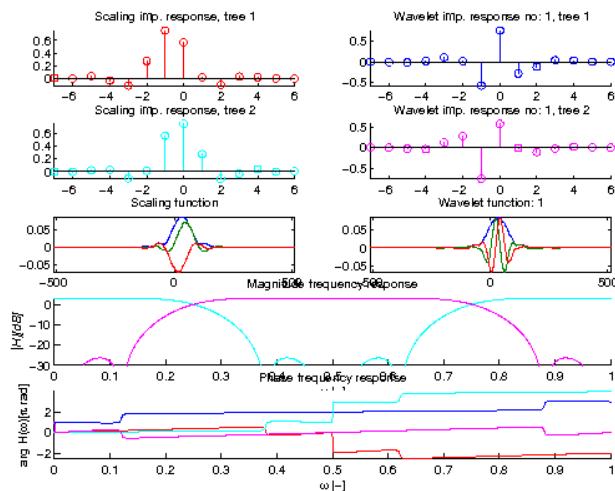
```
[h, g, a] = wfilt dt_qshift(N);
```

Description

[h, g, a]=wfilt dt_qshift(N) with $N \in 1, 2, 3, 4, 5, 6, 7$ returns Kingsbury's Q-shift filters suitable for dual-tree complex wavelet transform. Filters in both trees are orthogonal and based on a single prototype low-pass filter with a quarter sample delay. Other filters are derived by modulation and time reversal such that they fulfil the half-sample delay difference between the trees.

Examples:

```
wfilt dtinfo('qshift3');
```



References: [52], [51]

4.9.2 WFILTDT_OPTSYM - Optimized Symmetric Self-Hilbertian Filters

Usage

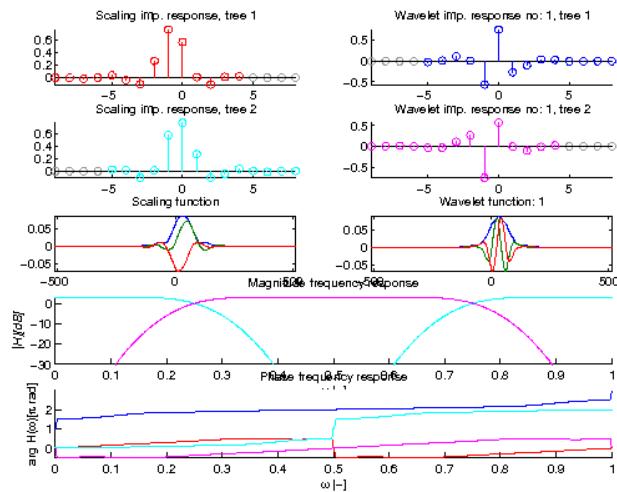
```
[h, g, a] = wfilt dt_optsym(N);
```

Description

[h, g, a]=wfilt dt_optsym(N) with $N \in 1, 2, 3$ returns filters suitable for dual-tree complex wavelet transform with optimized symmetry.

Examples:

```
wfilt dtinfo('optsym3');
```



References: [29]

4.9.3 WFILTDT_ODDEVEN - Kingsbury's symmetric odd and even filters

Usage

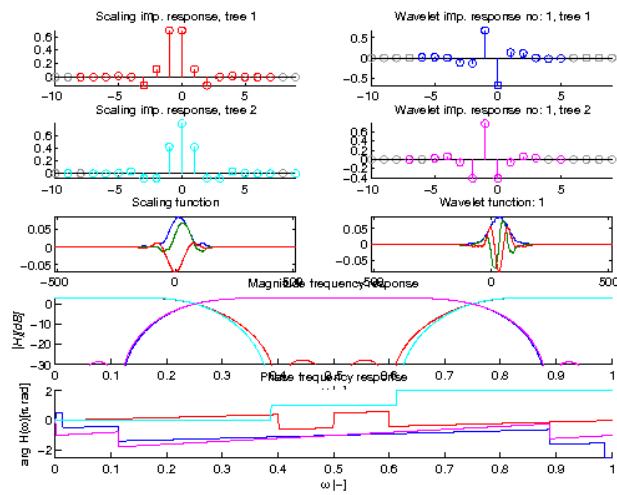
```
[h, g, a] = wfiltddt_oddeven(N);
```

Description

[h, g, a]=wfilt_ddeven(N) with $N \in 1$ returns the original odd and even symmetric filters suitable for dual-tree complex wavelet transform. The filters in individual trees are biorthogonal.

Examples:

```
wfiltddtinfo('ana:oddeven1');
```



References: [50]

4.9.4 WFILTDT_DDEN - Double-Density Dual-Tree DWT filters

Usage

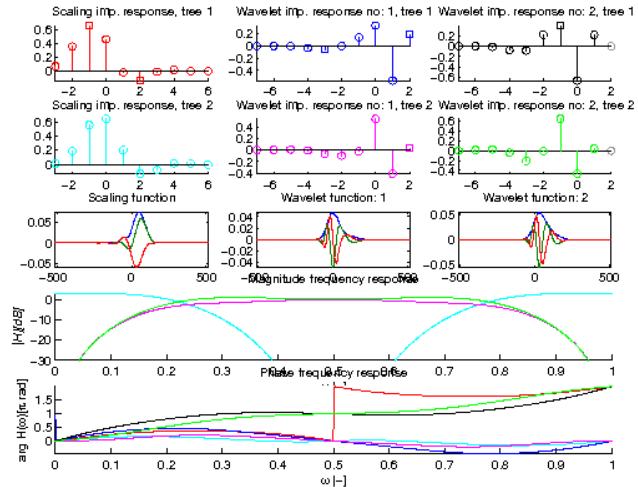
```
[h, g, a] = wfiltddt_dden(N);
```

Description

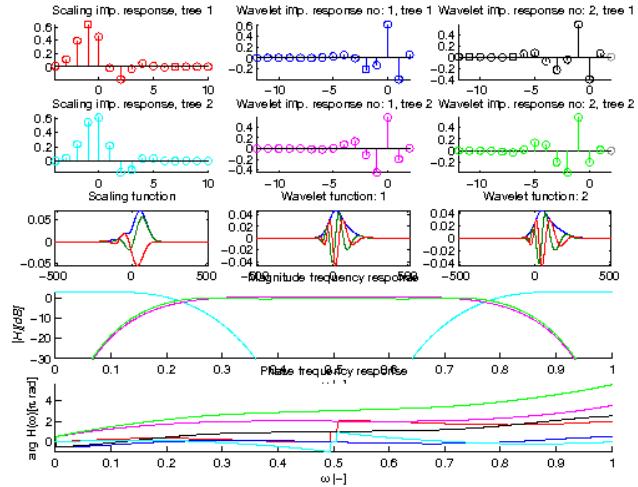
`[h, g, a]=wfiltddt_dden(N)` with $N \in 1, 2$ returns filters suitable for dual-tree double density complex wavelet transform.

Examples:

```
wfiltddtinfo('dden1');
```



```
wfiltddtinfo('dden2');
```



References: [82]

Chapter 5

LTFAT - Filterbanks

5.1 Transforms and basic routines

5.1.1 FILTERBANK - Apply filterbank

Usage

```
c=filterbank(f,g,a);
```

Description

`filterbank(f,g,a)` applies the filters given in g to the signal f . Each subband will be subsampled by a factor of a (the hop-size). In contrast to `ufilterbank`, a can be a vector so the hop-size can be channel-dependant. If f is a matrix, the transformation is applied to each column.

The filters g must be a cell-array, where each entry in the cell array corresponds to an FIR filter.

The output coefficients are stored a cell array. More precisely, the n 'th cell of c , $c\{m\}$, is a 2D matrix of size $M(n) \times W$ and containing the output from the m 'th channel subsampled at a rate of $a(m)$. $c\{m\}(n,l)$ is thus the value of the coefficient for time index n , frequency index m and signal channel l .

The coefficients c computed from the signal f and the filterbank with windows g_m are defined by

$$c_m(n+1) = \sum_{l=0}^{L-1} f(l+1) g(a_m n - l + 1)$$

where $an - l$ is computed modulo L .

References: [17]

5.1.2 UFILTERBANK - Apply Uniform filterbank

Usage

```
c=ufilterbank(f,g,a);
```

Description

`ufilterbank(f,g,a)` applies the filter given in g to the signal f . Each subband will be subsampled by a factor of a (the hop-size). If f is a matrix, the transformation is applied to each column.

The filters g must be a cell-array, where each entry in the cell array corresponds to a filter.

If f is a single vector, then the output will be a matrix, where each column in f is filtered by the corresponding filter in g . If f is a matrix, the output will be 3-dimensional, and the third dimension will correspond to the columns of the input signal.

The coefficients c computed from the signal f and the filterbank with windows g_m are defined by

$$c(n+1,m+1) = \sum_{l=0}^{L-1} f(l+1) g(an - l + 1)$$

5.1.3 IFILTERBANK - Filter bank inversion

Usage

```
f = ifilterbank(c, g, a);
```

Description

`ifilterbank(c, g, a)` synthesizes a signal f from the coefficients c using the filters stored in g for a channel subsampling rate of a (the hop-size). The coefficients has to be in the format returned by either filterbank or ufilterbank.

The filter format for g is the same as for filterbank.

If perfect reconstruction is desired, the filters must be the duals of the filters used to generate the coefficients. See the help on filterbankdual.

References: [17]

5.1.4 FILTERBANKWIN - Compute set of filter bank windows from text or cell array

Usage

```
[g, info] = filterbankwin(g, a, L);
```

Description

`[g, info] = filterbankwin(g, a, L)` computes a window that fits well with time shift a and transform length L . The window itself is as a cell array containing additional parameters.

The window can be specified directly as a cell array of vectors of numerical values. In this case, filterbankwin only checks assumptions about transform sizes etc.

`[g, info] = filterbankwin(g, a)` does the same, but the windows must be FIR windows, as the transform length is unspecified.

`filterbankwin(..., 'normal')` computes a window for regular filterbanks, while `filterbankwin(..., 'real')` does the same for the positive-frequency only filterbanks.

The window can also be specified as cell array. The possibilities are:

- {'dual', ...} Canonical dual window of whatever follows. See the examples below.
- {'realdual', ...} Canonical dual window for a positive-frequency filterbank of whatever follows. See the examples below.
- {'tight', ...} Canonical tight window of whatever follows. See the examples below.
- {'realtight', ...} Canonical tight window for a real-valued for a positive frequency filterbank of whatever follows.

The structure `info` provides some information about the computed window:

- `info.M` Number of windows (equal to the number of channels)
- `info.longestfilter` Length of the longest filter
- `info.gauss` True if the windows are Gaussian.
- `info.tfr` Time/frequency support ratios of the window. Set whenever it makes sense.
- `info.isfir` Input is an FIR window
- `info.isdual` Output is the dual window of the auxiliary window.
- `info.istight` Output is known to be a tight window.
- `info.auxinfo` Info about auxiliary window.
- `info.g1` Length of windows.
- `info.isfac` True if the frame generated by the window has a fast factorization.

5.1.5 FILTERBANKLENGTH - Filterbank length from signal

Usage

```
L=filterbanklength(Ls,a);
```

Description

`filterbanklength(Ls,a)` returns the length of a filterbank with time shifts a , such that it is long enough to expand a signal of length L_s .

If the filterbank length is longer than the signal length, the signal will be zero-padded by filterbank or `ufilterbank`.

If instead a set of coefficients are given, call `filterbanklengthcoef`.

5.1.6 FILTERBANKLENGTHCOEF - Filterbank length from coefficients

Usage

```
L=filterbanklengthcoef(coef,a);
```

Description

`filterbanklengthcoef(coef,a)` returns the length of a filterbank with time-shifts a , such that the filterbank is long enough to expand the coefficients $coef$.

If instead a signal is given, call `filterbanklength`.

5.2 Auditory inspired filter banks

5.2.1 CQT - Constant-Q non-stationary Gabor filterbank

Usage

```
[c,Ls,g,shift,M] = cqt(f,fmin,fmax,bins,fs,M)
[c,Ls,g,shift,M] = cqt(f,fmin,fmax,bins,fs)
[c,Ls,g,shift] = cqt(...)
[c,Ls] = cqt(...)
c = cqt(...)
```

Input parameters

f The signal to be analyzed (For multichannel signals, input should be a matrix which each column storing a channel of the signal).

fmin Minimum frequency (in Hz)

fmax Maximum frequency (in Hz)

bins Vector consisting of the number of bins per octave

fs Sampling rate (in Hz)

M Number of time channels (optional) If M is constant, the output is converted to a matrix

Output parameters

c	Transform coefficients (matrix or cell array)
Ls	Original signal length (in samples)
g	Cell array of Fourier transforms of the analysis windows
shift	Vector of frequency shifts
M	Number of time channels

Description

This function computes a constant-Q transform via non-stationary Gabor filterbanks. Given the signal f , the constant-Q parameters f_{min} , f_{max} and $bins$, as well as the sampling rate fs of f , the corresponding constant-Q coefficients c are given as output. For reconstruction, the length of f and the filterbank parameters can be returned also.

The transform produces phase-locked coefficients in the sense that each filter is considered to be centered at 0 and the signal itself is modulated accordingly.

Optional input arguments can be supplied like this:

```
cqt(f,fmin,fmax,bins,fs,'min_win',min_win)
```

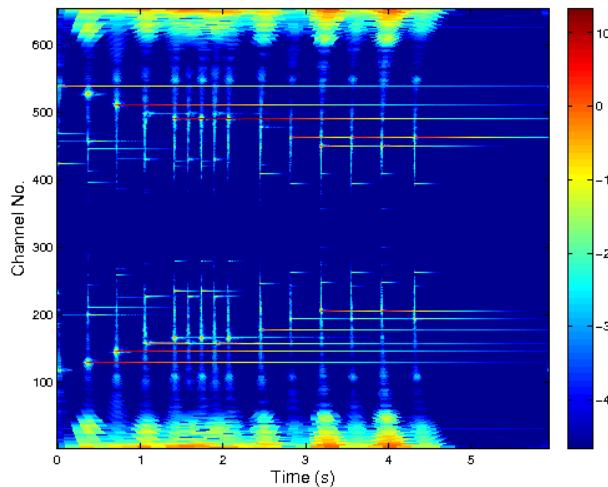
The arguments must be character strings followed by an argument:

'min_win' , min_win	Minimum admissible window length (in samples)
'Qvar' , Qvar	Bandwidth variation factor
'M_fac' , M_fac	Number of time channels are rounded to multiples of this
'wfun' , wfun	Filter prototype (see firwin for available filters)
'fractional'	Allow fractional shifts and bandwidths

Example:

The following example shows analysis and synthesis with cqt and icqt:

```
[f,fs] = gspi;
fmin = 200;
fmax = fs/2;
[c,Ls,g,shift,M] = cqt(f,fmin,fmax,48,fs);
fr = icqt(c,g,shift,Ls);
rel_err = norm(f-fr)/norm(f);
plotfilterbank(c,Ls./M,[],fs,'dynrange',60);
```



References: [93], [43]

5.2.2 ICQT - Constant-Q non-stationary Gabor synthesis

Usage

```
fr = icqt(c,g,shift,Ls,dual)
fr = icqt(c,g,shift,Ls)
fr = icqt(c,g,shift)
```

Input parameters

c	Transform coefficients (matrix or cell array)
g	Cell array of Fourier transforms of the analysis windows
shift	Vector of frequency shifts
Ls	Original signal length (in samples)
dual	Synthesize with the dual frame

Output parameters

fr	Synthesized signal (Channels are stored in the columns)
-----------	---

Description

Given the cell array *c* of non-stationary Gabor coefficients, and a set of filters *g* and frequency shifts *shift* this function computes the corresponding constant-Q synthesis.

If *dual* is set to 1 (default), an attempt is made to compute the canonical dual frame for the system given by *g*, *shift* and the size of the vectors in *c*. This provides perfect reconstruction in the painless case, see the references for more information.

References: [93], [43]

5.2.3 ERBLETT - ERBlet non-stationary Gabor filterbank

Usage

```
[c,Ls,g,shift,M] = erblett(f,bins,fs,varargin)
[c,Ls,g,shift] = erblett(...)
[c,Ls] = erblett(...)
c = erblett(...)
```

Input parameters

f	The signal to be analyzed (For multichannel signals, input should be a matrix which each column storing a channel of the signal)
bins	Desired bins per ERB
fs	Sampling rate of f (in Hz)
varargin	Optional input pairs (see table below)

Output parameters

c	Transform coefficients (matrix or cell array)
Ls	Original signal length (in samples)
g	Cell array of Fourier transforms of the analysis windows
shift	Vector of frequency shifts
M	Number of time channels

Description

This function computes an ERBlet constant-Q transform via non-stationary Gabor filterbanks. Given the signal f , the ERBlet parameter $bins$, as well as the sampling rate fs of f , the corresponding ERBlet coefficients c are given as output. For reconstruction, the length of f and the filterbank parameters can be returned also.

The transform produces phase-locked coefficients in the sense that each filter is considered to be centered at 0 and the signal itself is modulated accordingly.

Optional input arguments can be supplied like this:

```
erblett(f,bins,fs,'Qvar',Qvar)
```

The arguments must be character strings followed by an argument:

'Qvar',Qvar	Bandwidth variation factor
'M_fac',M_fac	Number of time channels are rounded to multiples of this
'wifun',wifun	Filter prototype (see firwin for available filters)

Examples:

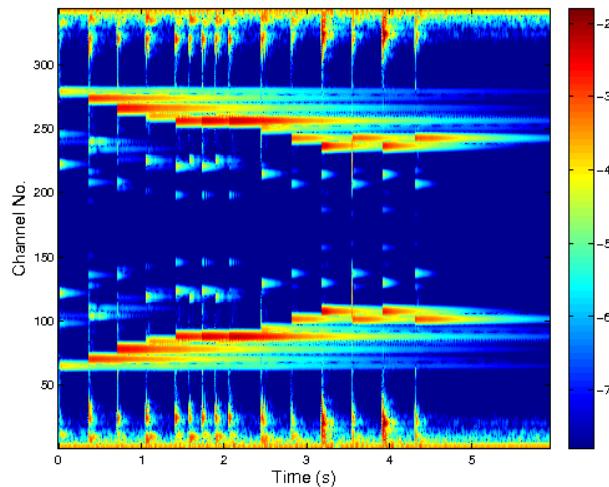
The following example shows analysis and synthesis with erblett and ierblett:

```
[f,fs] = gspi;
binsPerERB = 4;
[c,Ls,g,shift,M] = erblett(f,binsPerERB,fs);
fr = ierblett(c,g,shift,Ls);
rel_err = norm(f-fr)/norm(f)
plotfilterbank(c,Ls./M,[],fs,'dynrange',60);
```

This code produces the following output:

```
rel_err =
```

```
5.133913799759292e-16
```



References: [66]

5.2.4 IERBLETT - ERBlet non-stationary Gabor synthesis

Usage

```
fr = ierblett(c,g,shift,Ls,dual)
fr = ierblett(c,g,shift,Ls)
fr = ierblett(c,g,shift)
```

Input parameters

c	Transform coefficients (matrix or cell array)
g	Cell array of Fourier transforms of the analysis windows
shift	Vector of frequency shifts
Ls	Original signal length (in samples)
dual	Synthesize with the dual frame

Output parameters

fr	Synthesized signal (Channels are stored in the columns)
-----------	---

Description

Given the cell array *c* of non-stationary Gabor coefficients, and a set of filters *g* and frequency shifts *shift* this function computes the corresponding ERBlet synthesis.

If *dual* is set to 1 (default), an attempt is made to compute the canonical dual frame for the system given by *g*, *shift* and the size of the vectors in *c*. This provides perfect reconstruction in the painless case, see the references for more information.

References: [66]

5.3 Filter generators

5.3.1 CQTFILTERS - CQT-spaced filters

Usage

```
[g,a,fc]=cqtfilters(fs,fmin,fmax,bins,Ls,varargin);
```

Input parameters

fs	Sampling rate (in Hz).
fmin	Minimum frequency (in Hz)
fmax	Maximum frequency (in Hz)
bins	Vector consisting of the number of bins per octave.
Ls	Signal length.

Output parameters

g	Cell array of filters.
a	Downsampling rate for each channel.
fc	Center frequency of each channel.
L	Next admissible length suitable for the generated filters.

Description

[*g*, *a*, *fc*] = cqtfilters(*fs*, *fmin*, *fmax*, *bins*, *Ls*) constructs a set of band-limited filters *g* which cover the required frequency range *fmin*-*fmax* with *bins* filters per octave starting at *fmin*. All filters have (approximately) equal $Q = f_c/f_b$, hence constant-Q. The remaining frequency intervals not covered by these filters are captured by two additional filters (low-pass, high-pass). The signal length *Ls* is mandatory, since we need to avoid too narrow frequency windows.

By default, a Hann window on the frequency side is chosen, but the window can be changed by passing any of the window types from firwin as an optional parameter. Run `getfield(getfield(arg_firwin,'flags'),'window')` to get a cell array of window types available.

Because the downsampling rates of the channels must all divide the signal length, filterbank will only work for multiples of the least common multiple of the downsampling rates. See the help of `filterbanklength`.

[*g*, *a*] = cqtfilters(..., 'resampling') constructs a non-uniform filter bank. The downsampling rates are constant in the octaves but can differ among octaves. This approach was chosen in order to minimize the least common multiple of *a*, which determines a granularity of admissible input signal lengths.

[*g*, *a*] = cqtfilters(..., 'uniform') constructs a uniform filter bank where the downsampling rate is the same for all the channels. This results in most redundant representation, which produces nice plots.

[*g*, *a*] = cqtfilters(..., 'fractional') constructs a filter bank with fractional downsampling rates *a*. The rates are constructed such that the filter bank can handle signal lengths that are multiples of *L*, so the benefit of the fractional downsampling is that you get to choose the value returned by `filterbanklength`. This results in the least redundant system.

[*g*, *a*] = cqtfilters(..., 'fractionaluniform') constructs a filter bank with fractional downsampling rates *a*, which are uniform for all filters except the "filling" low-pass and high-pass filters can have different fractional downsampling rates. This is useful when uniform subsampling and low redundancy at the same time are desirable.

The filters are intended to work with signals with a sampling rate of *fs*.

cqtfilters accepts the following optional parameters:

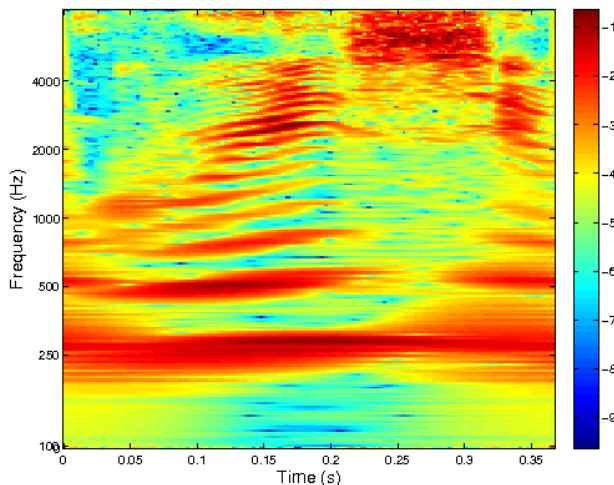
'Qvar' , 'Qvar'	Bandwidth variation factor. Multiplies the calculated bandwidth. Default value is 1. If the value is less than one, the system may no longer be painless.
'subprec'	Allow subsample window positions and bandwidths to better approximate the constant-Q property.
'complex'	Construct a filter bank that covers the entire frequency range. When missing, only positive frequencies are covered.

- 'min_win',min_win** Minimum admissible window length (in samples). Default is 4. This restrict the windows not to become too narrow when L is low. This however brakes the constant-Q property for such windows and creates rippling in the overall frequency response.
- 'redmul',redmul** Redundancy multiplier. Increasing the value of this will make the system more redundant by lowering the channel downsampling rates. Default value is 1. If the value is less than one, the system may no longer be painless.

Examples:

In the first example, we construct a highly redundant uniform filter bank and visualize the result:

```
[f,fs]=greasy; % Get the test signal
[g,a,fc]=cqtfilters(fs,100,fs,32,length(f),'uniform');
c=filterbank(f,g,a);
plotfilterbank(c,a,fc,fs,90,'audtck');
```



In the second example, we construct a non-uniform filter bank with fractional sampling that works for this particular signal length, and test the reconstruction. The plot displays the response of the filter bank to verify that the filters are well-behaved both on a normal and an log scale. The second plot shows frequency responses of filters used for analysis (top) and synthesis (bottom).

```
[f,fs]=greasy; % Get the test signal
L=length(f);
[g,a,fc]=cqtfilters(fs,100,fs,8,L,'fractional');
c=filterbank(f,['realdual',g],a);
r=2*real(ifilterbank(c,g,a));
norm(f-r)

% Plot the response
figure(1);
subplot(2,1,1);
R=filterbankresponse(g,a,L,fs,'real','plot');

subplot(2,1,2);
semiaudplot(linspace(0,fs/2,L/2+1),R(1:L/2+1));
ylabel('Magnitude');

% Plot frequency responses of individual filters
```

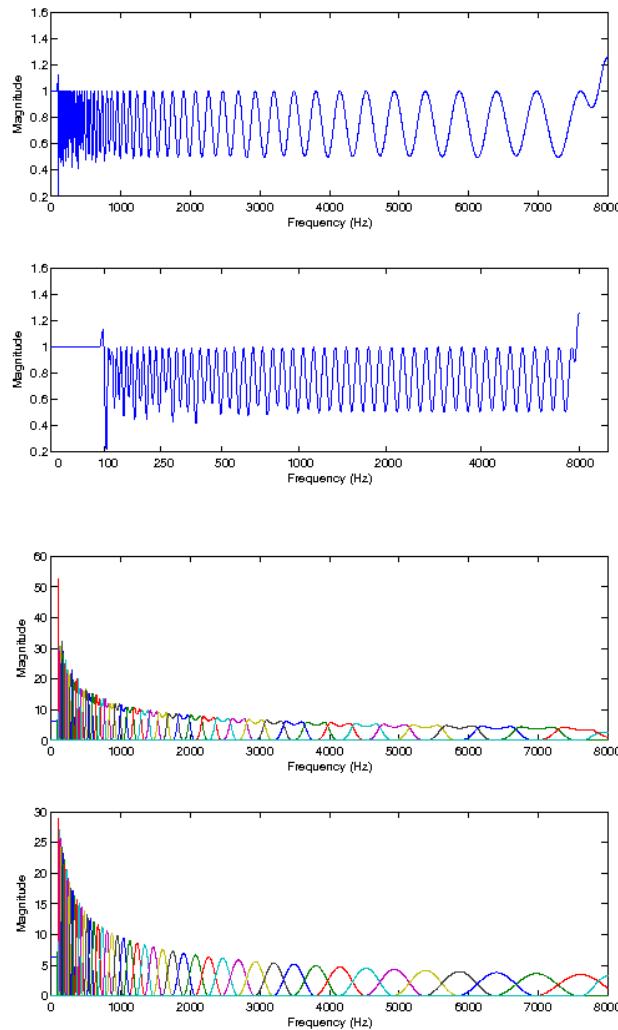
```
gd=filterbankrealdual(g,a,L);
figure(2);
subplot(2,1,1);
filterbankfreqz(gd,a,L,fs,'plot','linabs','posfreq');

subplot(2,1,2);
filterbankfreqz(g,a,L,fs,'plot','linabs','posfreq');
```

This code produces the following output:

ans =

8.299315044804507e-15



References: [93], [43], [81]

5.3.2 ERBFILTERS - ERB-spaced filters

Usage

```
[g,a,fc]=erbfilters(fs,Ls);
[g,a,fc]=erbfilters(fs,Ls,...);
```

Input parameters

fs	Sampling rate (in Hz).
Ls	Signal length.

Output parameters

g	Cell array of filters.
a	Downsampling rate for each channel.
fc	Center frequency of each channel.
L	Next admissible length suitable for the generated filters.

Description

`[g, a, fc]=erbfilters(fs, Ls)` constructs a set of filters g that are equidistantly spaced on the ERB-scale (see freqtoerb) with bandwidths that are proportional to the width of the auditory filters audfiltbw. The filters are intended to work with signals with a sampling rate of fs . The signal length Ls is mandatory, since we need to avoid too narrow frequency windows.

By default, a Hann window on the frequency side is chosen, but the window can be changed by passing any of the window types from firwin as an optional parameter. Run `getfield(getfield(arg_firwin, 'flags'), 'window')` to get a cell array of window types available.

The integer downsampling rates of the channels must all divide the signal length, filterbank will only work for input signal lengths being multiples of the least common multiple of the downsampling rates. See the help of filterbanklength. The fractional downsampling rates restrict the filterbank to a single length $L=Ls$.

`[g, a]=erbfilters(..., 'resampling')` constructs a non-uniform filterbank with integer subsampling factors.

`[g, a]=erbfilters(..., 'uniform')` constructs a uniform filterbank where the integer down-sampling rate is the same for all the channels. This results in most redundant representation which produces nice plots.

`[g, a]=erbfilters(..., 'fractional')` constructs a filterbank with fractional downsampling rates a . This results in the least redundant system.

`[g, a]=erbfilters(..., 'fractionaluniform')` constructs a filterbank with fractional down-sampling rates a , which are uniform for all filters except the "filling" low-pass and high-pass filters can have different fractional downsampling rates. This is useful when uniform subsampling and low redundancy at the same time are desirable.

`erbfilters` accepts the following optional parameters:

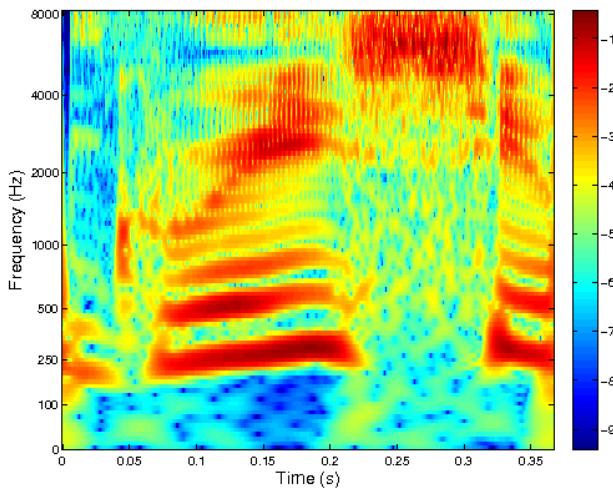
'spacing',b	Specify the spacing in ERBS between the filters. Default value is $b=1$.
'M',M	Specify the number of filters, M . If this parameter is specified, it overwrites the ' <code>spacing</code> ' parameter.
'redmul',redmul	Redundancy multiplier. Increasing the value of this will make the system more redundant by lowering the channel downsampling rates. It is only used if the filterbank is a non-uniform filterbank. Default value is 1 . If the value is less than one, the system may no longer be painless.
'symmetric'	Create filters that are symmetric around their centre frequency. This is the default. <code>'sqrtsquare'</code> , <code>'sqrtrect'</code>
'warped'	Create asymmetric filters that are symmetric on the Erb-scale.
'complex'	Construct a filterbank that covers the entire frequency range.
'bwmul',bwmul	Bandwidth of the filters relative to the bandwidth returned by <code>audfiltbw</code> . Default is $bwmul = 1$.

'min_win',min_win Minimum admissible window length (in samples). Default is 4. This restrict the windows not to become too narrow when L is low.

Examples:

In the first example, we construct a highly redundant uniform filterbank and visualize the result:

```
[f,fs]=greasy; % Get the test signal
[g,a,fc]=erbfilters(fs,length(f),'uniform','M',100);
c=filterbank(f,g,a);
plotfilterbank(c,a,fc,fs,90,'audtict');
```



In the second example, we construct a non-uniform filterbank with fractional sampling that works for this particular signal length, and test the reconstruction. The plot displays the response of the filterbank to verify that the filters are well-behaved both on a normal and an ERB-scale. The second plot shows frequency responses of filters used for analysis (top) and synthesis (bottom).

```
[f,fs]=greasy; % Get the test signal
L=length(f);
[g,a,fc]=erbfilters(fs,L,'fractional');
c=filterbank(f,'realdual',g),a;
r=2*real(ifilterbank(c,g,a));
norm(f-r)

% Plot the response
figure(1);
subplot(2,1,1);
R=filterbankresponse(g,a,L,fs,'real','plot');

subplot(2,1,2);
semiaudplot(linspace(0,fs/2,L/2+1),R(1:L/2+1));
ylabel('Magnitude');

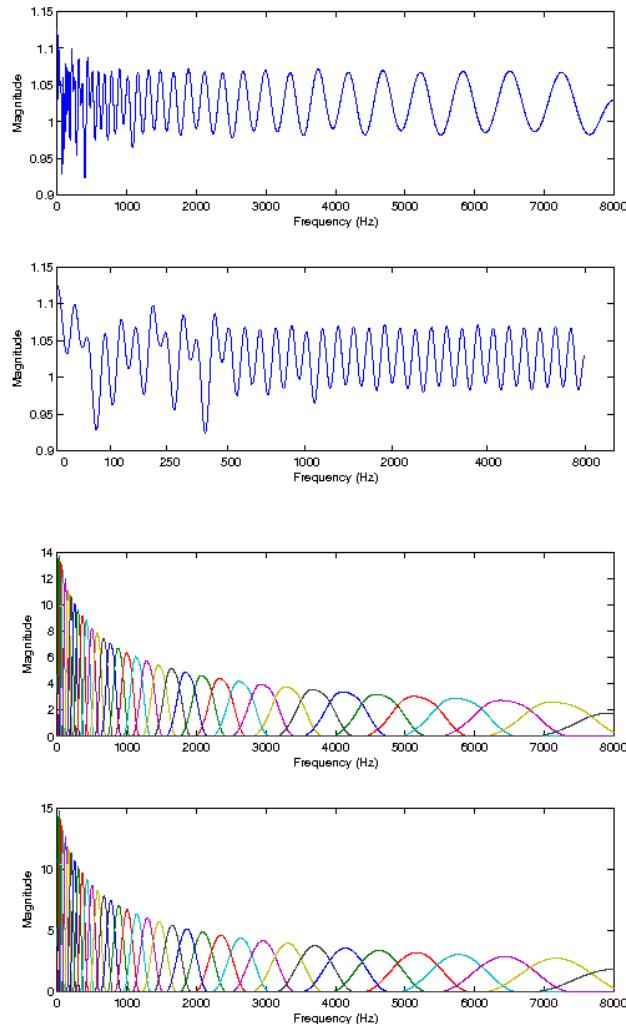
% Plot frequency responses of individual filters
gd=filterbankrealdual(g,a,L);
figure(2);
subplot(2,1,1);
filterbankfreqz(gd,a,L,fs,'plot','linabs','posfreq');

subplot(2,1,2);
filterbankfreqz(g,a,L,fs,'plot','linabs','posfreq');
```

This code produces the following output:

```
ans =
```

```
8.470876921240907e-15
```



References: [66]

5.3.3 WARPEDFILTERS - Frequency-warped band-limited filters

Usage

```
[g, a, fc] = warpedfilters(freqtoscale, scaletofreq, fs, fmin, fmax, bins, Ls);
```

Input parameters

freqtoscale Function converting frequency (Hz) to scale units

scaletofreq Function converting scale units to frequency (Hz)

fs Sampling rate (in Hz).

fmin Minimum frequency (in Hz)

fmax Maximum frequency (in Hz)

bins	Vector consisting of the number of bins per octave.
Ls	Signal length.

Output parameters

g	Cell array of filters.
a	Downsampling rate for each channel.
fc	Center frequency of each channel (in Hz).
L	Next admissible length suitable for the generated filters.

Description

`[g, a, fc] = warpedfilters(freqtoscale, scaletofreq, fs, fmin, fmax, bins, Ls)` constructs a set of band-limited filters g which cover the required frequency range f_{\min} - f_{\max} with bins filters per scale unit. The filters are always centered at full (fractional k/bins) scale units, where the first filter is selected such that its center is lower than f_{\min} .

By default, a Hann window on the frequency side is chosen, but the window can be changed by passing any of the window types from firwin as an optional parameter. Run `getfield(getfield(arg_firwin, 'flags'), 'window')` to get a cell array of window types available.

With respect to the selected scale, all filters have equal bandwidth and are uniformly spaced on the scale axis, e.g. if `freqtoscale` is $\log(x)$, then we obtain constant-Q filters with geometric spacing. The remaining frequency intervals not covered by these filters are captured one or two additional filters (high-pass always, low-pass if necessary). The signal length `Ls` is required in order to obtain the optimal normalization factors.

Attention: When using this function, the user needs to be aware of a number of things:

- a) Although the `freqtoscale` and `scaletofreq` can be chosen freely, it is assumed that `freqtoscale` is an invertible, increasing function from \mathbb{R} or \mathbb{R}^+ onto \mathbb{R} and that `freqtoscale` is the inverse function.
- b) If `freqtoscale` is from \mathbb{R}^+ onto \mathbb{R} , then necessarily $\text{freqtoscale}(0) = -\infty$.
- c) If the slope of `freqtoscale` is (locally) too steep, then there is the chance that some filters are effectively 0 or have extremely low bandwidth (1-3 samples), and consequently very poor localization in time. If `freqtoscale` is from \mathbb{R}^+ onto \mathbb{R} then this usually occurs close to the DC component and can be alleviated by increasing f_{\min} .
- d) Since the input parameter `bins` is supposed to be integer, `freqtoscale` and `scaletofreq` have to be scaled appropriately. Note that $\text{freqtoscale}(fs)$ is in some sense proportional to the resulting number of frequency bands and inversely proportional to the filter bandwidths. For example, the ERB scale defined by $21.4 \log_{10}(1 + f/228.8)$ works nicely out of the box, while the similar mel scale $2595 \log_{10}(1 + f/700)$ most likely has to be rescaled in order not to provide a filter bank with 1000s of channels.

If any of these guidelines are broken, this function is likely to break or give undesirable results.

By default, a Hann window is chosen as the transfer function prototype, but the window can be changed by passing any of the window types from firwin as an optional parameter.

The integer downsampling rates of the channels must all divide the signal length, filterbank will only work for input signal lengths being multiples of the least common multiple of the downsampling rates. See the help of `filterbanklength`. The fractional downsampling rates restrict the filterbank to a single length $L=L_s$.

`[g, a] = warpedfilters(..., 'regsampling')` constructs a non-uniform filterbank with integer subsampling factors.

`[g, a] =warpedfilters(..., 'uniform')` constructs a uniform filterbank where the the down-sampling rate is the same for all the channels. This results in most redundant representation, which produces nice plots.

`[g, a] =warpedfilters(..., 'fractional')` constructs a filterbank with fractional down-sampling rates a . This results in the least redundant system.

`[g, a] =warpedfilters(..., 'fractionaluniform')` constructs a filterbank with fractional downsampling rates a , which are uniform for all filters except the "filling" low-pass and high-pass filters can have different fractional downsampling rates. This is usefull when uniform subsampling and low redundancy at the same time are desirable.

The filters are intended to work with signals with a sampling rate of fs .

`warpedfilters` accepts the following optional parameters:

'bwmul',**'bwmul'** Bandwidth variation factor. Multiplies the calculated bandwidth. Default

value is 1. If the value is less than one, the system may no longer be painless.

'complex' Construct a filterbank that covers the entire frequency range. When missing, only positive frequencies are covered.

'redmul',**'redmul'** Redundancy multiplier. Increasing the value of this will make the system more redundant by lowering the channel downsampling rates. Default value is 1. If the value is less than one, the system may no longer be painless.

Examples:

In the first example, we use the ERB scale functions `freqtoerb` and `erbtotfreq` to construct a filter bank and visualize the result:

```
[s, fs] = gspi; % Get a test signal
Ls = numel(gspi);

% Fix some parameters
fmax = fs/2;
bins = 1;

% Compute filters, using fractional downsampling
[g, a, fc]=warpedfilters(@freqtoerb,@erbtotfreq,fs,0,fmax,bins, ...
    Ls,'bwmul',1.5,'real','fractional');

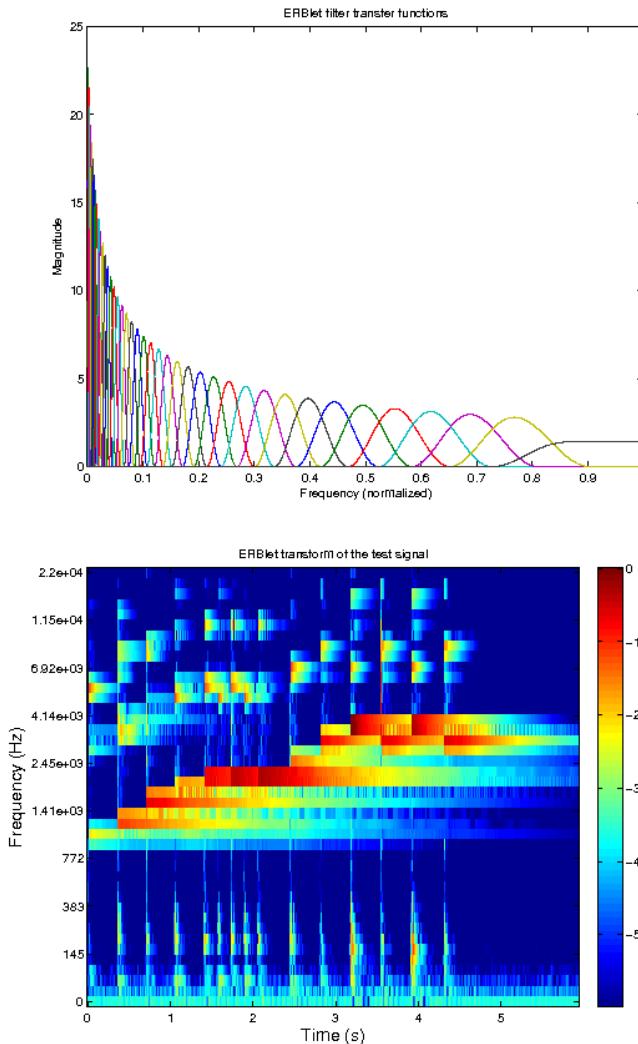
% Plot the filter transfer functions
figure(1);
filterbankfreqz(g,a,Ls,'plot','linabs','posfreq');
title('ERBlet filter transfer functions');

% Compute the frame bounds
gf=filterbankresponse(g,a,Ls,'real'); framebound_ratio = max(gf)/min(gf);
disp(['Painless system frame bound ratio of ERBlets: ', ...
    num2str(framebound_ratio)]);

% Plot the filter bank coefficients of the test signal
figure(2);
c=filterbank(s,g,a);
plotfilterbank(c,a,fc,fs,60);
title('ERBlet transform of the test signal');
```

This code produces the following output:

```
Painless system frame bound ratio of ERBlets: 1
```



In the second example, we look at the same test signal using a constant-Q filter bank with 4 bins per scale unit and the standard (semi-regular) sampling scheme:

```
[s,fs] = gspi; % Get a test signal
Ls = numel(gspi);

% Fix some parameters
fmax = fs/2;
bins = 1;

% Define the frequency-to-scale and scale-to-frequency functions
warpfun_log = @(x) 10*log(x);
invfun_log = @(x) exp(x/10);

bins_hi = 4; % Select bins/unit parameter
fmin = 50; % The logarithm's derivative 1/x tends to Inf for x towards 0

% Compute filters, using fractional downsampling
[g,a,fc]=warpedfilters(warpfun_log,invfun_log,fs,fmin,fmax,bins_hi,Ls,'bwmul',1,'');

% Plot the filter transfer functions
figure(1);
filterbankfreqz(g,a,Ls,'plot','linabs','posfreq');
```

```

title('constant-Q filter transfer functions (4 bins)');

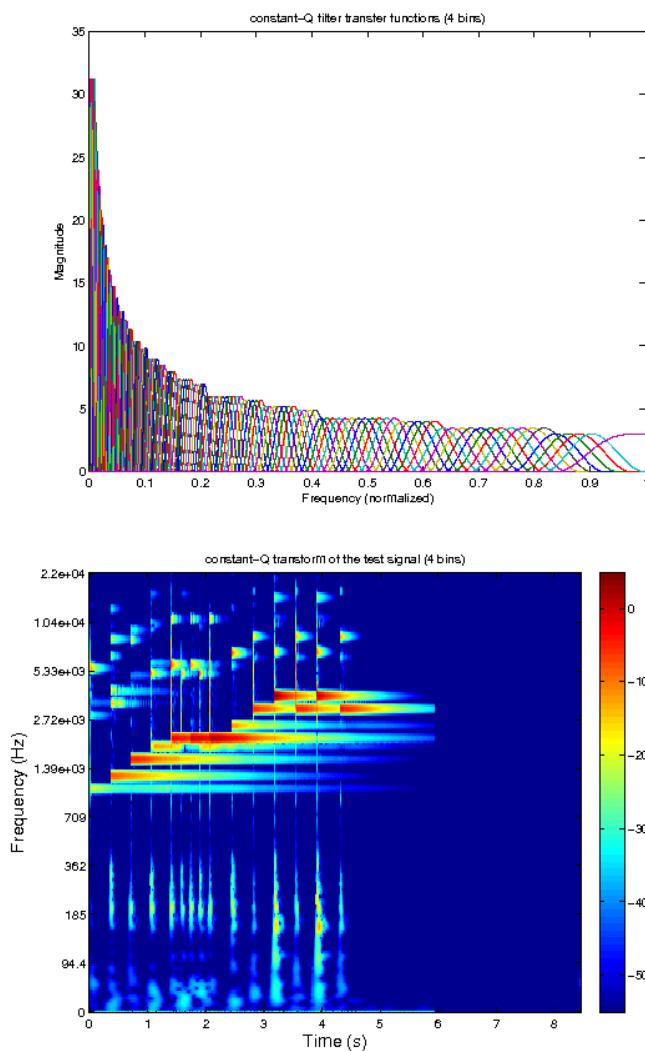
% Compute the frame bounds
gf=filterbankresponse(g,a,Ls,'real'); framebound_ratio = max(gf)/min(gf);
disp(['Painless system frame bound ratio (constant-Q - 4 bins): ', num2str(framebound_ratio)]);

% Plot the filter bank coefficients of the test signal
figure(2);
c=filterbank(s,g,a);
plotfilterbank(c,a,fc,fs,60);
title('constant-Q transform of the test signal (4 bins)');

```

This code produces the following output:

```
Painless system frame bound ratio (constant-Q - 4 bins): 1.0007
```



References: [45]

5.3.4 AUDFILTERS - generates AUD-spaced filters

Usage

```
[g,a,fc,L]=audfilters(fs,Ls);
[g,a,fc,L]=audfilters(fs,Ls,...);
```

Input parameters

fs	Sampling rate (in Hz).
Ls	Signal length.

Output parameters

g	Cell array of filters.
a	Downsampling rate for each channel.
fc	Center frequency of each channel.
L	Next admissible length suitable for the generated filters.

Description

`[g, a, fc, L]=audfilters(fs, Ls)` constructs a set of filters g that are equidistantly spaced on a perceptual frequency scale (see freqtoaud) between 0 and the Nyquist frequency and with bandwidths that are proportional to the critical bandwidth of the auditory filters audfiltbw. The filters are intended to work with signals with a sampling rate of fs . The signal length Ls is mandatory, since we need to avoid too narrow frequency windows.

By default the ERB scale is chosen but other frequency scales are possible. See 'freqtoaud' for all available options. The most scales are 'erb', 'bark', and 'mel'.

By default, a Hann window on the frequency side is chosen, but the window can be changed by passing any of the window types from firwin as an optional parameter. Run `getfield(getfield(arg_firwin, 'flags'), 'win')` to get a cell array of window types available.

The integer downsampling rates of the channels must all divide the signal length, filterbank will only work for input signal lengths being multiples of the least common multiple of the downsampling rates. See the help of filterbanklength. The fractional downsampling rates restrict the filterbank to a single length $L=Ls$.

`[g, a, fc, L]=audfilters(fs, Ls, flow, fhigh)` constructs a set of filters that are equidistantly spaced between $flow$ and $fhigh$. In that case two additional filters will be positioned at the 0 and Nyquist frequencies so as to cover the full spectrum. The values of $flow$ and $fhigh$ can be instead specified using a key/value pair as:

```
[g, a, fc, L]=audfilters(fs, Ls, ..., 'flow', flow, 'fhigh', fhigh)
```

`[g, a, fc, L]=audfilters(..., 'resampling')` constructs a non-uniform filterbank with integer subsampling factors.

`[g, a, fc, L]=audfilters(..., 'uniform')` constructs a uniform filterbank where the integer downsampling rate is the same for all the channels. This results in most redundant representation which produces nice plots.

`[g, a, fc, L]=audfilters(..., 'fractional')` constructs a filterbank with fractional downsampling rates a . This results in the least redundant system.

`[g, a, fc, L]=audfilters(..., 'fractionaluniform')` constructs a filterbank with fractional downsampling rates a , which are uniform for all filters except the "filling" low-pass and high-pass filters can have different fractional downsampling rates. This is useful when uniform subsampling and low redundancy at the same time are desirable.

`audfilters` accepts the following optional parameters:

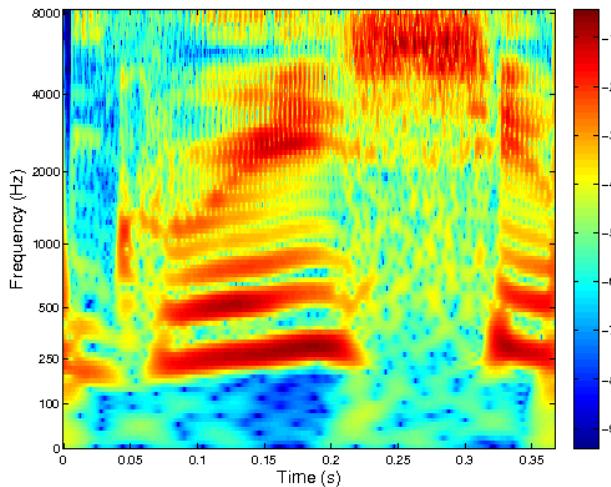
'spacing',b	Specify the spacing in Ecritical bandwidth (ERB or Bark depending on the scale) between the filters. Default value is $b=1$.
'M',M	Specify the total number of filters between 'flow' and 'fhigh', M . If this parameter is specified, it overwrites the 'spacing' parameter.

'redmul' , redmul	Redundancy multiplier. Increasing the value of this will make the system more redundant by lowering the channel downsampling rates. It is only used if the filterbank is a non-uniform filterbank. Default value is 1. If the value is less than one, the system may no longer be painless.
'symmetric'	Create filters that are symmetric around their centre frequency. This is the default.'sqrtsquare','sqrtrect'
'warped'	Create asymmetric filters that are asymmetric on the ERB scale. The warping does not work with other scales yet.
'complex'	Construct a filterbank that covers the entire frequency range.
'bwmul' , bwmul	Bandwidth of the filters relative to the bandwidth returned by audfltbw. Default is <i>bwmul</i> = 1.
'min_win' , min_win	Minimum admissible window length (in samples). Default is 4. This restrict the windows not to become too narrow when <i>L</i> is low.

Examples:

In the first example, we construct a highly redundant uniform filterbank on the ERB scale and visualize the result:

```
[f,fs]=greasy; % Get the test signal
[g,a,fc,L]=audfilters(fs,length(f),'uniform','M',100);
c=filterbank(f,g,a);
plotfilterbank(c,a,fc,fs,90,'audtick');
```



In the second example, we construct a non-uniform filterbank with fractional sampling that works for this particular signal length, and test the reconstruction. The plot displays the response of the filterbank to verify that the filters are well-behaved both on a normal and an ERB-scale. The second plot shows frequency responses of filters used for analysis (top) and synthesis (bottom).

```
[f,fs]=greasy; % Get the test signal
L=length(f);
[g,a,fc,L]=audfilters(fs,L,'fractional');
c=filterbank(f,['realdual',g],a);
r=2*real(ifilterbank(c,g,a));
norm(f-r)

% Plot the response
```

```

figure(1);
subplot(2,1,1);
R=filterbankresponse(g,a,L,fs,'real','plot');

subplot(2,1,2);
semiaudplot(linspace(0,fs/2,L/2+1),R(1:L/2+1));
ylabel('Magnitude');

% Plot frequency responses of individual filters
gd=filterbankrealdual(g,a,L);
figure(2);
subplot(2,1,1);
filterbankfreqz(gd,a,L,fs,'plot','linabs','posfreq');

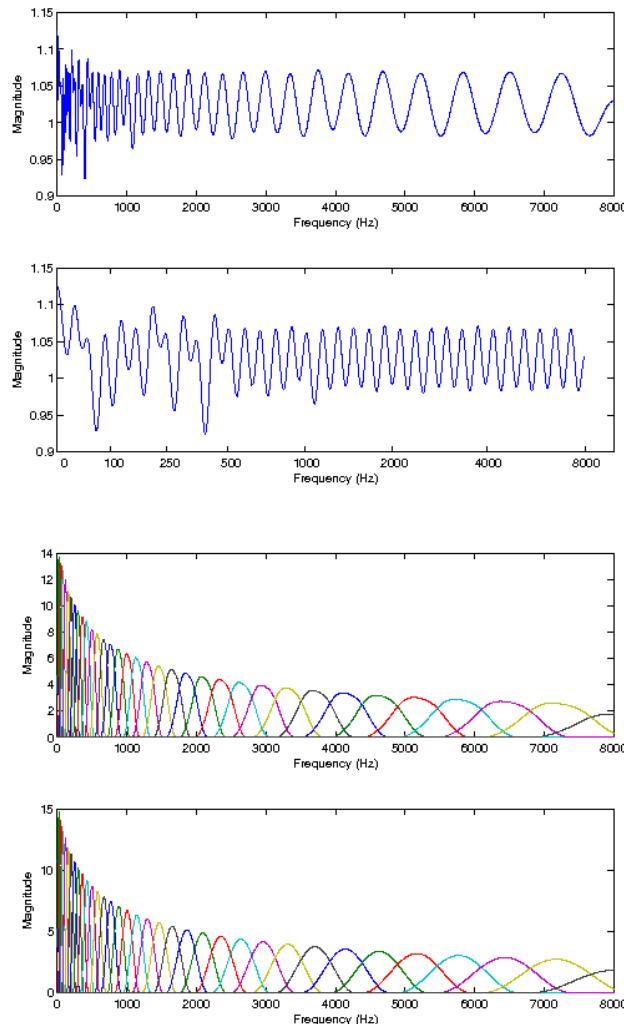
subplot(2,1,2);
filterbankfreqz(g,a,L,fs,'plot','linabs','posfreq');

```

This code produces the following output:

ans =

8.470876921240907e-15



References: [66]

5.4 Window construction and bounds

5.4.1 FILTERBANKDUAL - Dual filters

Usage

```
gd=filterbankdual(g,a,L);
gd=filterbankdual(g,a);
```

Description

`filterbankdual(g,a,L)` computes the canonical dual filters of g for a channel subsampling rate of a (hop-size) and system length L . L must be compatible with subsampling rate a as $L==\text{filterbanklength}(L,a)$. This will create a dual frame valid for signals of length L .

`filterbankrealDual(g,a)` does the same, but the filters must be FIR filters, as the transform length is unspecified. L will be set to next suitable length equal or bigger than the longest impulse response such that $L=\text{filterbanklength}(g_{\text{L_longest}},a)$.

The input and output format of the filters g are described in the help of `filterbank`.

In addition, the function recognizes a 'forcepainless' flag which forces treating the filterbank g and a as a painless case filterbank.

To actually invert the output of a filterbank, use the dual filters together with the `ifilterbank` function.

REMARK: In general, perfect reconstruction can be obtained for signals of length L . In some cases, using dual system calculated for shorter L might work but check the reconstruction error.

5.4.2 FILTERBANKTIGHT - Tight filterbank

Usage

```
gt=filterbanktight(g,a,L);
gt=filterbanktight(g,a);
```

Description

`filterbanktight(g,a,L)` computes the canonical tight filters of g for a channel subsampling rate of a (hop-size) and a system length L . L must be compatible with subsampling rate a as $L==\text{filterbanklength}(L,a)$.

`filterbanktight(g,a,L)` does the same, but the filters must be FIR filters, as the transform length is unspecified. L will be set to next suitable length equal or bigger than the longest impulse response.

The input and output format of the filters g are described in the help of `filterbank`.

REMARK: The resulting system is tight for length L . In some cases, using tight system calculated for shorter L might work but check the reconstruction error.

5.4.3 FILTERBANKREALDUAL - Dual filters of filterbank for real signals only

Usage

```
gd=filterbankrealDual(g,a,L);
gd=filterbankrealDual(g,a);
```

Description

`filterbankrealDual(g,a,L)` computes the canonical dual filters of g for a channel subsampling rate of a (hop-size) and a system length L . L must be compatible with subsampling rate a as $L==\text{filterbanklength}(L,a)$. The dual filters work only for real-valued signals. Use this function on the common construction where the filters in g only covers the positive frequencies.

`filterbankrealDual(g,a)` does the same, but the filters must be FIR filters, as the transform length is unspecified. L will be set to next suitable length equal or bigger than the longest impulse response.

The format of the filters g are described in the help of `filterbank`.

In addition, the function recognizes a 'forcepainless' flag which forces treating the filterbank g and a as a painless case filterbank.

To actually invert the output of a filterbank, use the dual filters together with $2 * \text{real}(\text{ifilterbank}(\dots))$.

REMARK: Perfect reconstruction can be obtained for signals of length L . In some cases, using dual system calculated for shorter L might work but check the reconstruction error.

5.4.4 FILTERBANKREALTIGHT - Tight filters of filterbank for real signals only

Usage

```
gt=filterbankrealtight(g,a,L);
gt=filterbankrealtight(g,a);
```

Description

`filterabankrealtight(g,a,L)` computes the canonical tight filters of g for a channel subsampling rate of a (hop-size) and a system length L . L must be compatible with subsampling rate a as $L==\text{filterbanklength}(L,a)$. The tight filters work only for real-valued signals. Use this function on the common construction where the filters in g only covers the positive frequencies.

`filterabankrealtight(g,a)` does the same, but the filters must be FIR filters, as the transform length is unspecified. L will be set to next suitable length equal or bigger than the longest impulse response.

The format of the filters g are described in the help of `filterbank`.

REMARK: The resulting system is tight for length L . In some cases, using tight system calculated for shorter L might work but check the reconstruction error.

5.4.5 FILTERBANKBOUNDS - Frame bounds of a filterbank

Usage

```
fcond=filterbankbounds(g,a,L);
[A,B]=filterbankbounds(g,a,L);
[...]=filterbankbounds(g,a);
```

Description

`filterbankbounds(g,a,L)` calculates the ratio B/A of the frame bounds of the filterbank specified by g and a for a system of length L . The ratio is a measure of the stability of the system.

`filterbankbounds(g,a)` does the same, but the filters must be FIR filters, as the transform length is unspecified. L will be set to next suitable length equal or bigger than the longest impulse response such that $L=\text{filterbanklength}(gl_longest,a)$.

`[A,B]=filterbankbounds(...)` returns the lower and upper frame bounds explicitly.

5.4.6 FILTERBANKREALBOUNDS - Frame bounds of filter bank for real signals only

Usage

```
fcond=filterbankrealbounds(g,a,L);
[A,B]=filterbankrealbounds(g,a,L);
[...]=filterbankrealbounds(g,a);
```

Description

`filterbankrealbounds(g,a,L)` calculates the ratio B/A of the frame bounds of the filterbank specified by g and a for a system of length L . The ratio is a measure of the stability of the system. Use this function on the common construction where the filters in g only covers the positive frequencies.

`filterbankrealbounds(g, a)` does the same, but the filters must be FIR filters, as the transform length is unspecified. L will be set to next suitable length equal or bigger than the longest impulse response such that $L=filterbanklength(gl_longest, a)$.

`[A, B]=filterbankrealbounds(g, a)` returns the lower and upper frame bounds explicitly.

5.4.7 FILTERBANKRESPONSE - Response of filterbank as function of frequency

Usage

```
gf=filterbankresponse(g, a, L);
```

Description

`gf=filterbankresponse(g, a, L)` computes the total response in frequency of a filterbank specified by g and a for a signal length of L . This corresponds to summing up all channels. The output is a useful tool to investigate the behaviour of the windows, as peaks indicate that a frequency is overrepresented in the filterbank, while a dip indicates that it is not well represented.

CAUTION: This function computes a sum of squares of modulus of the frequency responses, which is also the diagonal of the Fourier transform of the frame operator. Use `filterbankfreqz` for evaluation or plotting of frequency responses of filters.

`filterbankresponse(g, a, L, 'real')` does the same for a filterbank intended for positive-only filterbank.

`filterbankresponse(g, a, L, fs)` specifies the sampling rate fs . This is only used for plotting purposes.

`gf=filterbankresponse(g, a, L, 'individual')` returns responses in frequency of individual filters as columns of a matrix. The total response can be obtained by `gf = sum(gf, 2)`.

`filterbankresponse` takes the following optional parameters:

'fs', fs Sampling rate, used only for plotting.

'complex' Assume that the filters cover the entire frequency range. This is the default.

'real' Assume that the filters only cover the positive frequencies (and is intended to work with real-valued signals only).

'noplot' Don't plot the response, just return it.

'plot' Plot the response using `plotfftreal` or `plotfft`.

5.5 Auxilary

5.5.1 FILTERBANKFREQZ - Filterbank frequency responses

Usage

```
gf = filterbankfreqz(g, a, L)
```

Description

`gf = filterbankfreqz(g, a, L)` calculates length L frequency responses of filters in g and returns them as columns of gf .

If an optional parameters 'plot' is passed to `filterbankfreqz`, the frequency responses will be plotted using `plotfft`. Any optional parameter understood by `plotfft` can be passed in addition to 'plot'.

5.5.2 NONU2UFILTERBANK - Non-uniform to uniform filterbank transform

Usage

```
[gu, au]=nonu2ufilterbank(g, a)
```

Input parameters

g	Filters as a cell array of structs.
a	Subsampling factors.

Output parameters

gu	Filters as a cell array of structs.
au	Uniform subsampling factor.
pk	Numbers of copies of each filter.

Description

[gu, au]=nonu2ufilterbank (g, a) calculates uniform filterbank gu, au=lcm(a) which is identical to the (possibly non-uniform) filterbank g, a in terms of the equal output coefficients. Each filter $g\{k\}$ is replaced by $p(k) = au/a(k)$ advanced versions of itself such that $z^{ma(k)}G_k(z)$ for $m = 0, \dots, p - 1$.

This allows using the factorisation algorithm when determining filterbank frame bounds in filterbankbounds and filterbankrealbounds and in the computation of the dual filterbank in filterbankdual and filterbankreal-dual which do not work with non-uniform filterbanks.

One can change between the coefficient formats of gu, au and g, a using nonu2ucfmt and u2nonucfmt in the reverse direction.

References: [6]

5.5.3 U2NONUCFMT - Uniform to non-uniform filterbank coefficient format**Usage**

```
c=u2nonucfmt (cu, pk)
```

Input parameters

cu	Uniform filterbank coefficients.
-----------	----------------------------------

Output parameters

c	Non-uniform filterbank coefficients.
p	Numbers of copies of each filter.

Description

c = u2nonucfmt (cu, pk) changes the coefficient format from uniform filterbank coefficients cu ($M=\text{sum}(p)$ channels) to non-uniform coefficients c ($\text{numel}(p)$ channels) such that each channel of c consists of p (m) interleaved channels of cu.

The output c is a cell-array in any case.

References: [6]

5.5.4 NONU2UCFMT - Non-uniform to uniform filterbank coefficient format**Usage**

```
cu=nonu2ucfmt (c, pk)
```

Input parameters

c	Non-uniform filterbank coefficients.
----------	--------------------------------------

Output parameters

cu	Uniform filterbank coefficients.
p	Numbers of copies of each filter.

Description

`cu = nonu2ucfmt (c, p)` changes the coefficient format from non-uniform filterbank coefficients c ($M=\text{numel}(p)$ channels) to uniform coefficients cu ($\sum(p)$ channels) such that each channel of cu consists of de-interleaved samples of channels of c .

The output cu is a cell-array in any case.

References: [6]

5.6 Plots

5.6.1 PLOTFILTERBANK - Plot filterbank and ufilterbank coefficients

Usage

```
plotfilterbank (coef, a);
plotfilterbank (coef, a, fc);
plotfilterbank (coef, a, fc, fs);
plotfilterbank (coef, a, fc, fs, dynrange);
```

Description

`plotfilterbank (coef, a)` plots filterbank coefficients $coef$ obtained from either the filterbank or ufilterbank functions. The coefficients must have been produced with a time-shift of a . For more details on the format of the variables $coef$ and a , see the help of the filterbank or ufilterbank functions.

`plotfilterbank (coef, a, fc)` makes it possible to specify the center frequency for each channel in the vector fc .

`plotfilterbank (coef, a, fc, fs)` does the same assuming a sampling rate of fs Hz of the original signal.

`plotfilterbank (coef, a, fc, fs, dynrange)` makes it possible to specify the dynamic range of the coefficients.

`C=plotfilterbank (...)` returns the processed image data used in the plotting. Inputting this data directly to `imagesc` or similar functions will create the plot. This is useful for custom post-processing of the image data.

`plotfilterbank` supports all the optional parameters of `tfplot`. Please see the help of `tfplot` for an exhaustive list.

In addition to the flags and key/values in `tfplot`, `plotfilterbank` supports the following optional arguments:

'fc',fc	Centre frequencies of the channels. fc must be a vector with the length equal to the number of channels. The default value of [] means to plot the channel no. instead of its frequency.
'n tickpos',n	Number of tick positions along the y-axis. The position of the ticks are determined automatically. Default value is 10.
' tick',t	Array of tick positions on the y-axis. Use this option to specify the tick position manually.
'aud tick'	Use ticks suitable for visualizing an auditory filterbank. Same as ' <code>tick'</code> , [0,100,250,500,1000].

5.7 Reassignment and phase gradient

5.7.1 FILTERBANKPHASEGRAD - Phase gradient of a filterbank representation

Usage

```
[tgrad,fgrad,s,c] = filterbankphasegrad(f,g,a,L,minlvl);
[tgrad,fgrad,s,c] = filterbankphasegrad(f,g,a,L);
[tgrad,fgrad,s,c] = filterbankphasegrad(f,g,a,minlvl);
[tgrad,fgrad,s,c] = filterbankphasegrad(f,g,a);
[tgrad,fgrad,s] = filterbankphasegrad(...);
[tgrad,fgrad] = filterbankphasegrad(...)
```

Input parameters

f	Signal to be analyzed.
g	Cell array of filters
a	Vector of time steps.
L	Signal length (optional).
minlvl	Regularization parameter (optional, required < 1).

Output parameters

tgrad	Instantaneous frequency relative to original position.
fgrad	Group delay relative to original position.
cs	Filterbank spectrogram.
c	Filterbank coefficients.

Description

`[tgrad,fgrad,s,c] = filterbankphasegrad(f,g,a,L)` computes the group delay *fgrad* and instantaneous frequency *tgrad* of the filterbank spectrogram *s* obtained from the signal *f* and filterbank parameters *g* and *a*. Both quantities are specified relative to the original coefficient position. *tgrad* is given in samples, while *fgrad* is given as values on the unit circle, easily converted into relative frequencies by $\log(tgrad)/(pi*i)$. This routine uses the equivalence of the filterbank coefficients in each channel with coefficients obtained from an STFT obtained with a certain window (possibly different for every channel). As a consequence of this equivalence, the formulas derived in the reference apply.

References: [10], [44]

5.7.2 FILTERBANKREASSIGN - Reassign filterbank spectrogram

Usage

```
sr = filterbankreassign(s,tgrad,fgrad,a,cfreq);
sr = filterbankreassign(s,tgrad,fgrad,a,g);
[sr,repos,Lc] = filterbankreassign(...);
```

Input parameters

s	Spectrogram to be reassigned.
tgrad	Instantaneous frequency relative to original position.
fgrad	Group delay relative to original position.

a	Vector of time steps.
cfreq	Vector of relative center frequencies in]-1,1].
g	Set of filters.

Output parameters

sr	Reassigned filterbank spectrogram.
repos	Reassigned positions.
Lc	Subband lengths.

Description

`filterbankreassign(s,tgrad,fgrad,a,cfreq)` will reassign the values of the filterbank spectrogram *s* using the group delay *fgrad* and instantaneous frequency *tgrad*. The time-frequency sampling pattern is determined from the time steps *a* and the center frequencies *cfreq*.

`filterbankreassign(s,tgrad,fgrad,a,g)` will do the same thing except the center frequencies are estimated from a set of filters *g*.

`[sr, repos, Lc]=filterbankreassign(...)` does the same thing, but in addition returns a vector of subband lengths *Lc* (*Lc* = `cellfun(@numel,s)`) and cell array *repos* with `sum(Lc)` elements. Each element corresponds to a single coefficient obtained by `cell2mat(sr)` and it is a vector of indices identifying coefficients from `cell2mat(s)` assigned to the particular time-frequency position.

The arguments *s*, *tgrad* and *fgrad* must be cell-arrays of vectors of the same lengths. Arguments *a* and *cfreq* or *g* must have the same number of elements as the cell arrays with coefficients.

Examples:

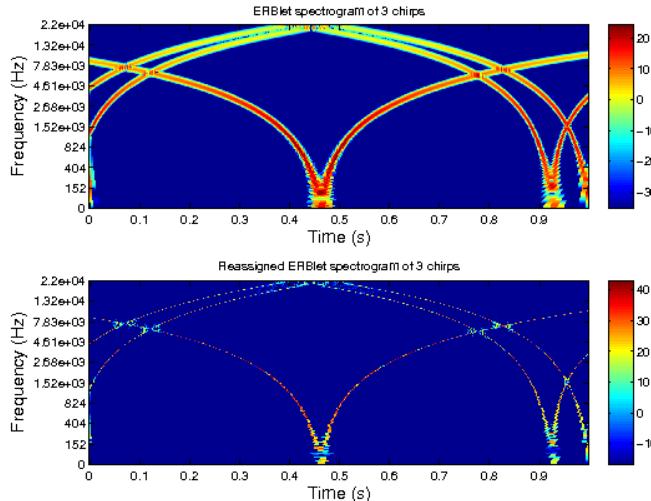
This example shows how to reassign a ERB filterbank spectrogram:

```
% Generate 3 chirps 1 second long
L = 44100; fs = 44100; l = 0:L-1;

f = sin(2*pi*(l/35+(l/300).^2)) + ...
    sin(2*pi*(l/10+(l/300).^2)) + ...
    sin(2*pi*(l/5-(l/450).^2));
f = 0.7*f';

% Create ERB filterbank
[g,a,fc]=erbfilters(fs,L,'fractional','spacing',1/12,'warped');

% Compute phase gradient
[tgrad,fgrad,cs,c]=filterbankphasegrad(f,g,a);
% Do the reassignment
sr=filterbankreassign(cs,tgrad,fgrad,a,cent_freqs(fs,fc));
figure(1); subplot(211);
plotfilterbank(cs,a,fc,fs,60);
title('ERBlet spectrogram of 3 chirps');
subplot(212);
plotfilterbank(sr,a,fc,fs,60);
title('Reassigned ERBlet spectrogram of 3 chirps');
```



References: [44]

5.7.3 FILTERBANKSYNCHROSQUEEZE - Synchrosqueeze filterbank spectrogram

Usage

```
cr = filterbanksynchrosqueeze(c,tgrad,cfreq);
cr = filterbanksynchrosqueeze(c,tgrad,g);
[cr,repos,Lc] = filterbanksynchrosqueeze(...);
```

Input parameters

- c** Coefficients to be synchrosqueezed.
- tgrad** Instantaneous frequency relative to original position.
- cfreq** Vector of relative center frequencies in]-1,1].
- g** Set of filters.

Output parameters

- cr** Synchrosqueezed filterbank coefficients.
- repos** Reassigned positions.
- Lc** Subband lengths.

Description

`filterbanksynchrosqueeze(c,tgrad,cfreq)` will reassign the values of the filterbank coefficients c according to instantaneous frequency $tgrad$. The frequency center frequencies of filters are given by $cfreq$. The filterbank coefficients c are assumed to be obtained from a non-subsampled filterbank ($a=1$).

`filterbanksynchrosqueeze(s,tgrad,g)` will do the same thing except the center frequencies are estimated from a set of filters g .

`[sr,repos,Lc]=filterbanksynchrosqueeze(...)` does the same thing, but in addition returns a vector of subband lengths Lc ($Lc = \text{cellfun}(@\text{numel}, s)$) and cell array $repos$ with $\sum(Lc)$ elements. Each element corresponds to a single coefficient obtained by `cell2mat(sr)` and it is a vector of indices identifying coefficients from `cell2mat(s)` assigned to the particular time-frequency position.

The arguments s , $tgrad$ must be cell-arrays of vectors of the same lengths. Arguments $cfreq$ or g must have the same number of elements as the cell arrays with coefficients.

Examples:

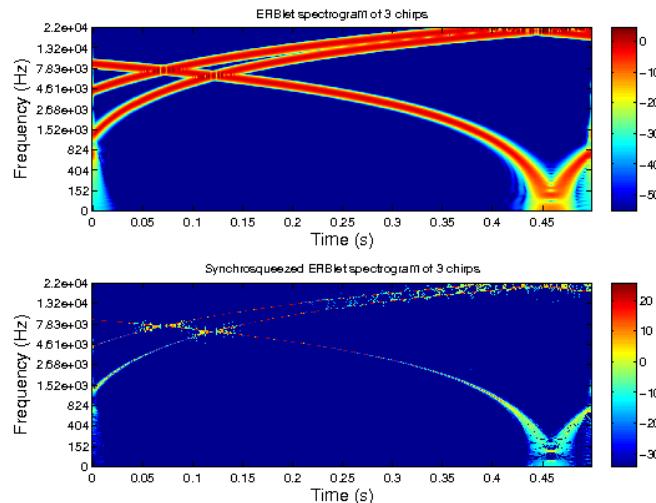
This example shows how to synchrosqueeze a ERB filterbank spectrogram:

```
% Generate 3 chirps half a second long
L = 22050; fs = 44100; l = 0:L-1;

f = sin(2*pi*(l/35+(l/300).^2)) + ...
    sin(2*pi*(l/10+(l/300).^2)) + ...
    sin(2*pi*(l/5-(l/450).^2));
f = 0.7*f';

% Create ERB filterbank
[g,~,fc]=erbfilters(fs,L,'uniform','spacing',1/12,'warped');

% Compute phase gradient
[tgrad,~,~,c]=filterbankphasegrad(f,g,1);
% Do the reassignment
sr=filterbanksynchrosqueeze(c,tgrad,cent_freqs(fs,fc));
figure(1); subplot(211);
plotfilterbank(c,1,fc,fs,60);
title('ERBlet spectrogram of 3 chirps');
subplot(212);
plotfilterbank(sr,1,fc,fs,60);
title('Synchrosqueezed ERBlet spectrogram of 3 chirps');
```



References: [44]

Chapter 6

LTFAT - Non-stationary Gabor systems

6.1 Transforms

6.1.1 NSDGT - Non-stationary Discrete Gabor transform

Usage

```
c=nsdgt(f,g,a,M);  
[c,Ls]=nsdgt(f,g,a,M);
```

Input parameters

f	Input signal.
g	Cell array of window functions.
a	Vector of time shifts.
M	Vector of numbers of frequency channels.

Output parameters

c	Cell array of coefficients.
Ls	Length of input signal.

Description

`nsdgt(f,g,a,M)` computes the non-stationary Gabor coefficients of the input signal f . The signal f can be a multichannel signal, given in the form of a 2D matrix of size $L_s \times W$, with L_s the signal length and W the number of signal channels.

The non-stationary Gabor theory extends standard Gabor theory by enabling the evolution of the window over time. It is therefore necessary to specify a set of windows instead of a single window. This is done by using a cell array for g . In this cell array, the n 'th element $g\{n\}$ is a row vector specifying the n 'th window.

The resulting coefficients also require a storage in a cell array, as the number of frequency channels is not constant over time. More precisely, the n 'th cell of c , $c\{n\}$, is a 2D matrix of size $M(n) \times W$ and containing the complex local spectra of the signal channels windowed by the n 'th window $g\{n\}$ shifted in time at position $a(n)$. $c\{n\}(m,w)$ is thus the value of the coefficient for time index n , frequency index m and signal channel w .

The variable a contains the distance in samples between two consecutive blocks of coefficients. The variable M contains the number of channels for each block of coefficients. Both a and M are vectors of integers.

The variables g , a and M must have the same length, and the result c will also have the same length.

The time positions of the coefficients blocks can be obtained by the following code. A value of 0 correspond to the first sample of the signal:

```
timepos = cumsum(a)-a(1);
```

`[c, Ls]=nsdgt(f, g, a, M)` additionally returns the length L_s of the input signal f . This is handy for reconstruction:

```
[c, Ls]=nsdgt(f, g, a, M);
fr=insdgt(c, gd, a, Ls);
```

will reconstruct the signal f no matter what the length of f is, provided that gd are dual windows of g .

Notes:

`nsdgt` uses circular border conditions, that is to say that the signal is considered as periodic for windows overlapping the beginning or the end of the signal.

The phaselocking convention used in `nsdgt` is different from the convention used in the `dgt` function. `nsdgt` results are phaselocked (a phase reference moving with the window is used), whereas `dgt` results are not phaselocked (a fixed phase reference corresponding to time 0 of the signal is used). See the help on `phaselock` for more details on phaselocking conventions.

References: [11]

6.1.2 UNSDGT - Uniform Non-stationary Discrete Gabor transform

Usage

```
c=unsdgt(f, g, a, M);
[c, Ls]=unsdgt(f, g, a, M);
```

Input parameters

f	Input signal.
g	Cell array of window functions.
a	Vector of time positions of windows.
M	Numbers of frequency channels.

Output parameters

c	Cell array of coefficients.
Ls	Length of input signal.

Description

`unsdgt(f, g, a, M)` computes the uniform non-stationary Gabor coefficients of the input signal f . The signal f can be a multichannel signal, given in the form of a 2D matrix of size $L_s \times W$, with L_s being the signal length and W the number of signal channels.

The non-stationary Gabor theory extends standard Gabor theory by enabling the evolution of the window over time. It is therefore necessary to specify a set of windows instead of a single window. This is done by using a cell array for g . In this cell array, the n 'th element $g\{n\}$ is a row vector specifying the n 'th window. However, the uniformity means that the number of channels is fixed.

The resulting coefficients is stored as a $M \times N \times W$ array. $c(m, n, w)$ is thus the value of the coefficient for time index n , frequency index m and signal channel w .

The variable a contains the distance in samples between two consecutive blocks of coefficients. a is a vectors of integers. The variables g and a must have the same length.

The time positions of the coefficients blocks can be obtained by the following code. A value of 0 correspond to the first sample of the signal:

```
timepos = cumsum(a)-a(1);
```

`[c, Ls] = nsdgt(f, g, a, M)` additionally returns the length L_s of the input signal f . This is handy for reconstruction:

```
[c, Ls] = unsdgt(f, g, a, M);
fr = iunsdgt(c, gd, a, Ls);
```

will reconstruct the signal f no matter what the length of f is, provided that gd are dual windows of g .

Notes:

`unsdgt` uses circular border conditions, that is to say that the signal is considered as periodic for windows overlapping the beginning or the end of the signal.

The phaselocking convention used in `unsdgt` is different from the convention used in the `dgt` function. `unsdgt` results are phaselocked (a phase reference moving with the window is used), whereas `dgt` results are not phaselocked (a fixed phase reference corresponding to time 0 of the signal is used). See the help on `phaselock` for more details on phaselocking conventions.

References: [11]

6.1.3 INSDGT - Inverse non-stationary discrete Gabor transform

Usage

```
f = insdgt(c, g, a, Ls);
```

Input parameters

c	Cell array of coefficients.
g	Cell array of window functions.
a	Vector of time positions of windows.
Ls	Length of input signal.

Output parameters

f	Signal.
----------	---------

Description

`insdgt(c, g, a, Ls)` computes the inverse non-stationary Gabor transform of the input coefficients c .

`insdgt` is used to invert the functions `nsdgt` and `unsdgt`. Please read the help of these functions for details of variables format and usage.

For perfect reconstruction, the windows used must be dual windows of the ones used to generate the coefficients. The windows can be generated using `nsgabdual` or `nsgabtight`.

References: [11]

6.1.4 NSDGTRREAL - Non-stationary Discrete Gabor transform for real valued signals

Usage

```
c = nsdgtrreal(f, g, a, M);
[c, Ls] = nsdgtrreal(f, g, a, M);
```

Input parameters

f	Input signal.
g	Cell array of window functions.
a	Vector of time positions of windows.
M	Vector of numbers of frequency channels.

Output parameters

c	Cell array of coefficients.
Ls	Length of input signal.

Description

`nsdgtreal (f, g, a, M)` computes the non-stationary Gabor coefficients of the input signal f . The signal f can be a multichannel signal, given in the form of a 2D matrix of size $L_s \times W$, with L_s the signal length and W the number of signal channels.

As opposed to `nsdgtr` only the coefficients of the positive frequencies of the output are returned. `nsdgtreal` will refuse to work for complex valued input signals.

The non-stationary Gabor theory extends standard Gabor theory by enabling the evolution of the window over time. It is therefore necessary to specify a set of windows instead of a single window. This is done by using a cell array for g . In this cell array, the n 'th element $g\{n\}$ is a row vector specifying the n 'th window.

The resulting coefficients also require a storage in a cell array, as the number of frequency channels is not constant over time. More precisely, the n 'th cell of c , $c\{n\}$, is a 2D matrix of size $M(n)/2 + 1 \times W$ and containing the complex local spectra of the signal channels windowed by the n 'th window $g\{n\}$ shifted in time at position $a(n)$. $c\{n\}(m, 1)$ is thus the value of the coefficient for time index n , frequency index m and signal channel l .

The variable a contains the distance in samples between two consecutive blocks of coefficients. The variable M contains the number of channels for each block of coefficients. Both a and M are vectors of integers.

The variables g , a and M must have the same length, and the result c will also have the same length.

The time positions of the coefficients blocks can be obtained by the following code. A value of 0 correspond to the first sample of the signal:

```
timepos = cumsum(a)-a(1);
```

`[c, Ls]=nsdgtreal (f, g, a, M)` additionally returns the length L_s of the input signal f . This is handy for reconstruction:

```
[c, Ls]=nsdgtreal (f, g, a, M);
fr=insdgtreal(c, gd, a, Ls);
```

will reconstruct the signal f no matter what the length of f is, provided that gd are dual windows of g .

Notes:

`nsdgtreal` uses circular border conditions, that is to say that the signal is considered as periodic for windows overlapping the beginning or the end of the signal.

The phaselocking convention used in `nsdgtreal` is different from the convention used in the `dgt` function. `nsdgtreal` results are phaselocked (a phase reference moving with the window is used), whereas `dgt` results are not phaselocked (a fixed phase reference corresponding to time 0 of the signal is used). See the help on `phaselock` for more details on phaselocking conventions.

References: [11]

6.1.5 UNSDGTREAL - Uniform non-stationary Discrete Gabor transform

Usage

```
c=unsdgtreal(f,g,a,M);
[c,Ls]=unsdgtreal(f,g,a,M);
```

Input parameters

f	Input signal.
g	Cell array of window functions.
a	Vector of time positions of windows.
M	Vector of numbers of frequency channels.

Output parameters

c	Cell array of coefficients.
Ls	Length of input signal.

Description

`unsdgtreal(f,g,a,M)` computes the non-stationary Gabor coefficients of the input signal f . The signal f can be a multichannel signal, given in the form of a 2D matrix of size $L_s \times W$, with L_s the signal length and W the number of signal channels.

As opposed to `nsdgt` only the coefficients of the positive frequencies of the output are returned. `unsdgtreal` will refuse to work for complex valued input signals.

The non-stationary Gabor theory extends standard Gabor theory by enabling the evolution of the window over time. It is therefore necessary to specify a set of windows instead of a single window. This is done by using a cell array for g . In this cell array, the n 'th element $g\{n\}$ is a row vector specifying the n 'th window. The uniformity means that the number of channels is not allowed to vary over time.

The resulting coefficients is stored as a $M/2 + 1 \times N \times W$ array. $c(m,n,l)$ is thus the value of the coefficient for time index n , frequency index m and signal channel l .

The variable a contains the distance in samples between two consecutive blocks of coefficients. The variable M contains the number of channels for each block of coefficients. Both a and M are vectors of integers.

The variables g , a and M must have the same length, and the result c will also have the same length.

The time positions of the coefficients blocks can be obtained by the following code. A value of 0 correspond to the first sample of the signal:

```
timepos = cumsum(a)-a(1);
```

`[c,Ls]=unsdgtreal(f,g,a,M)` additionally returns the length L_s of the input signal f . This is handy for reconstruction:

```
[c,Ls]=unsdgtreal(f,g,a,M);
fr=insdgtreal(c,gd,a,Ls);
```

will reconstruct the signal f no matter what the length of f is, provided that gd are dual windows of g .

Notes:

`unsdgtreal` uses circular border conditions, that is to say that the signal is considered as periodic for windows overlapping the beginning or the end of the signal.

The phaselocking convention used in `unsdgtreal` is different from the convention used in the `dgt` function. `unsdgtreal` results are phaselocked (a phase reference moving with the window is used), whereas `dgt` results are not phaselocked (a fixed phase reference corresponding to time 0 of the signal is used). See the help on `phaselock` for more details on phaselocking conventions.

References: [11]

6.1.6 INSDGTREAL - Inverse NSDGT for real-valued signals

Usage

```
f=insdgt(c,g,a,M,Ls);
```

Input parameters

c	Cell array of coefficients.
g	Cell array of window functions.
a	Vector of time positions of windows.
M	Vector of numbers of frequency channels.
Ls	Length of input signal.

Output parameters

f	Signal.
----------	---------

Description

`insdgt(c,g,a,Ls)` computes the inverse non-stationary Gabor transform of the input coefficients c .
`insdgt` is used to invert the functions `nsdgt` and `unsdgt`. Please read the help of these functions for details of variables format and usage.

For perfect reconstruction, the windows used must be dual windows of the ones used to generate the coefficients. The windows can be generated using `nsgabdual` or `nsgabtight`.

References: [11]

6.2 Window construction and bounds

6.2.1 NSGABDUAL - Canonical dual window for non-stationary Gabor frames

Usage

```
gd=nsgabdual(g,a,M);
gd=nsgabdual(g,a,M,L);
```

Input parameters

g	Cell array of windows.
a	Vector of time shift.
M	Vector of numbers of channels.
L	Transform length.

Output parameters

gd	Cell array of canonical dual windows
-----------	--------------------------------------

Description

`nsgabdual(g, a, M, L)` computes the canonical dual windows of the non-stationary discrete Gabor frame defined by windows given in g and time-shifts given by a .

`nsgabdual` is designed to be used with the functions `nsdgt` and `insdgt`. See the help on `nsdgt` for more details about the variables structure.

The computed dual windows are only valid for the 'painless case', that is to say that they ensure perfect reconstruction only if for each window the number of frequency channels used for computation of `nsdgt` is greater than or equal to the window length. This correspond to cases for which the frame operator is diagonal.

References: [11]

6.2.2 NSGABTIGHT - Canonical tight window for non-stationary Gabor frames**Usage**

```
gt=nsgabtight(g, a, M);
gt=nsgabtight(g, a, M, L);
```

Input parameters

g	Cell array of windows
a	Vector of time shifts of windows.
M	Vector of numbers of channels.
L	Transform length.

Output parameters

gt	Cell array of canonical tight windows
-----------	---------------------------------------

Description

`nsgabtight(g, a, M)` computes the canonical tight windows of the non-stationary discrete Gabor frame defined by windows given in g and time-shifts given by a .

`nsgabtight` is designed to be used with functions `nsdgt` and `insdgt`. Read the help on `nsdgt` for more details about the variables structure.

The computed tight windows are only valid for the 'painless case', that is to say that they ensure perfect reconstruction only if for each window the number of frequency channels used for computation of `nsdgt` is greater than or equal to the window length. This correspond to cases for which the frame operator is diagonal.

References: [11]

6.2.3 NSGABFRAMEBOUNDS - Frame bounds of non-stationary Gabor frame**Usage**

```
fcond=nsgabframebounds(g, a, M);
[A, B]=nsgabframebounds(g, a, M);
```

Input parameters

g	Cell array of windows
a	Vector of time positions of windows.
M	Vector of numbers of frequency channels.

Output parameters

fcond	Frame condition number (B/A)
A, B	Frame bounds.

Description

`nsgabframebounds(g, a, Ls)` calculates the ratio B/A of the frame bounds of the non-stationary discrete Gabor frame defined by windows given in g at positions given by a . Please see the help on `nsdgt` for a more thorough description of g and a .

`[A, B] = nsgabframebounds(g, a, Ls)` returns the actual frame bounds A and B instead of just the their ratio.

The computed frame bounds are only valid for the 'painless case' when the number of frequency channels used for computation of `nsdgt` is greater than or equal to the window length. This correspond to cases for which the frame operator is diagonal.

References: [11]

6.2.4 NSGABFRAMEDIAG - Diagonal of Gabor frame operator**Usage**

```
d=nsgabframediag(g, a, M);
```

Input parameters

g	Window function.
a	Length of time shift.
M	Number of channels.

Output parameters

d	Diagonal stored as a column vector
----------	------------------------------------

Description

`nsgabframediag(g, a, M)` computes the diagonal of the non-stationary Gabor frame operator with respect to the window g and parameters a and M . The diagonal is stored as a column vector of length $L=\text{sum}(a)$.

The diagonal of the frame operator can for instance be used as a preconditioner.

6.3 Plots**6.3.1 PLOTNSDGT - Plot non-stationary Gabor coefficients****Usage**

```
plotnsdgt(c, a, fs, dynrange);
```

Input parameters

coef	Cell array of coefficients.
a	Vector of time positions of windows.
fs	signal sample rate in Hz (optional)
dynrange	Color scale dynamic range in dB (optional).

Description

`plotnsdgt (coef, a)` plots coefficients computed using nsdgt or unsdgt. For more details on the format of the variables *coef* and *a*, please read the function help for these functions.

`plotnsdgt (coef, a, fs)` does the same assuming a sampling rate of *fs* Hz of the original signal.

`plotnsdgt (coef, a, fs, dynrange)` additionally limits the dynamic range.

`C=plotnsdgt (...)` returns the processed image data used in the plotting. Inputting this data directly to `imagesc` or similar functions will create the plot. This is useful for custom post-processing of the image data.

`plotnsdgt` supports all the optional parameters of `tfplot`. Please see the help of `tfplot` for an exhaustive list. In addition, the following parameters may be specified:

'xres',xres Approximate number of pixels along x-axis / time. The default value is 800

'yres',yres Approximate number of pixels along y-axis / frequency The default value is 600

6.3.2 PLOTNSDGTREAL - Plot NSDGTRREAL coefficients

Usage

```
plotnsdgtrreal(c,a,fs,dynrange);
```

Input parameters

coef Cell array of coefficients.

a Vector of time positions of windows.

fs signal sample rate in Hz (optional).

dynrange Colorscale dynamic range in dB (optional).

Description

`plotnsdgtrreal (coef, a)` plots coefficients computed using nsdgtrreal or unsdgtrreal. For more details on the format of the variables *coef* and *a*, please read the function help for these functions.

`plotnsdgtrreal (coef, a, fs)` does the same assuming a sampling rate of *fs* Hz of the original signal.

`plotnsdgtrreal (coef, a, fs, dynrange)` additionally limits the dynamic range.

`C=plotnsdgtrreal (...)` returns the processed image data used in the plotting. Inputting this data directly to `imagesc` or similar functions will create the plot. This is useful for custom post-processing of the image data.

`plotnsdgtrreal` supports all the optional parameters of `tfplot`. Please see the help of `tfplot` for an exhaustive list. In addition, the following parameters may be specified:

'xres',xres Approximate number of pixels along x-axis /time. Default value is 800

'yres',yres Approximate number of pixels along y-axis / frequency Default value is 600

Chapter 7

LTFAT - Quadratic time-frequency distributions

7.1 Quadratic distributions

7.1.1 AMBIGUITYFUNCTION - Ambiguity function

Usage

```
A = ambiguityfunction(f);  
A = ambiguityfunction(f, g);
```

Input parameters

f, g Input vector(s).

Output parameters

A ambiguity function

Description

`ambiguityfunction(f)` computes the (symmetric) ambiguity function of `f`. The ambiguity function is computed as the two-dimensional Fourier transform of the Wigner-Ville distribution `wignerville`.

WARNING: The quadratic time-frequency distributions are highly redundant. For an input vector of length L , the quadratic time-frequency distribution will be a $L \times L$ matrix.

7.1.2 WIGNERVILLEDIST - Wigner-Ville distribution

Usage

```
W = wignerville(f);  
W = wignerville(f, g);
```

Input parameters

f, g Input vector(s)

Output parameters

w Wigner-Ville distribution

Description

`wignervilleedist (f)` computes the Wigner-Ville distribution of the vector f . The Wigner-Ville distribution is computed by

$$W(n+1, k+1) = \sum_{m=0}^{L-1} R(n+1, m+1) e^{-i2\pi mk/L},$$

where $R(n, m)$ is the instantaneous correlation matrix given by

$$R(n, m) = z(n+m) \overline{z(n-m)},$$

where $m \in -L/2, \dots, L/2 - 1$, and where z is the analytical representation of f , when f is real-valued.

`wignervilleedist (f, g)` computes the cross-Wigner-Ville distribution of f and g .

WARNING: The quadratic time-frequency distributions are highly redundant. For an input vector of length L , the quadratic time-frequency distribution will be a $L \times L$ matrix.

7.1.3 DRIHACZEKDIST - discrete Rihaczek distribution**Usage**

```
r = drihaczekdist (f);
```

Description

`drihaczekdist (f)` computes a discrete Rihaczek distribution of vector f . The discrete Rihaczek distribution is computed by

$$r(k+1, l+1) = f(l+1) \overline{c(k+1)} e^{-2\pi i kl/L}$$

where $k, l = 0, \dots, L-1$ and c is the Fourier transform of f .

WARNING: The quadratic time-frequency distributions are highly redundant. For an input vector of length L , the quadratic time-frequency distribution will be a $L \times L$ matrix. If f is multichannel ($L \times W$ matrix), the resulting distributions are stacked along the third dimension such that the result is $L \times L \times W$ cube.

7.1.4 QUADTFDIST - Quadratic time-frequency distribution**Usage**

```
p = quadtdist (f, q);
```

Input parameters

f	Input vector:w
q	Kernel

Output parameters

p	Quadratic time-frequency distribution
----------	---------------------------------------

Description

For an input vector of length L , the kernel should be a $L \times L$ matrix. `quadtdist (f, q);` computes a discrete quadratic time-frequency distribution.

7.2 Plots**7.2.1 PLOTQUADTFDIST - Plot quadratic time-frequency distribution****Usage**

```
plotquadtdist (p);
```

Description

'plotquadtfdist(p)' plots the quadratic time-frequency distribution on the time-frequency plane. The quadratic time-frequency distribution should be a square matrix.

PLOTQUADTFDIST takes the following additional arguments:

'dynrange',r	Limit the dynamical range to r by using a colormap in the interval [chigh-r,chigh], where chigh is the highest value in the plot. The default value of [] means to not limit the dynamical range.
'db'	Apply $20 \log_{10}$ to the coefficients. This makes it possible to see very weak phenomena, but it might show too much noise. A logarithmic scale is more adapted to perception of sound. This is the default.
'dbsq'	Apply $10 \log_{10}$ to the coefficients. Same as the 'db' option, but assume that the input is already squared.
'lin'	Show the coefficients on a linear scale. This will display the raw input without any modifications. Only works for real-valued input.
'linsq'	Show the square of the coefficients on a linear scale.
'linabs'	Show the absolute value of the coefficients on a linear scale.
'tc'	Time centring. Move the beginning of the signal to the middle of the plot.
'clim',clim	Use a colormap ranging from clim(1) to clim(2). These values are passed to imagesc. See the help on imagesc.
'image'	Use imagesc to display the plot. This is the default.
'contour'	Do a contour plot.
'surf'	Do a surf plot.
'colorbar'	Display the colorbar. This is the default.
'nocolorbar'	Do not display the colorbar.
'display'	Display the figure. This is the default.

Chapter 8

LTFAT - Frames

8.1 Creation of a frame object

8.1.1 FRAME - Construct a new frame

Usage

```
F=frame (ftype, ...);
```

Description

`F=frame (ftype, ...)` constructs a new frame object F of type $ftype$. Arguments following $ftype$ are specific to the type of frame chosen.

Time-frequency frames

`frame ('dgt', g, a, M)` constructs a Gabor frame with window g , time-shift a and M channels. See the help on `dgt` for more information.

`frame ('dgtrreal', g, a, M)` constructs a Gabor frame for real-valued signals with window g , time-shift a and M channels. See the help on `dgtrreal` for more information.

`frame ('dwilt', g, M)` constructs a Wilson basis with window g and M channels. See the help on `dwilt` for more information.

`frame ('wmdct', g, M)` constructs a windowed MDCT basis with window g and M channels. See the help on `wmdct` for more information.

`frame ('filterbank', g, a, M)` constructs a filterbank with filters g , time-shifts of a and M channels. For the ease of implementation, it is necessary to specify M , even though it strictly speaking could be deduced from the size of the windows. See the help on `filterbank` for more information on the parameters. Similarly, you can construct a uniform filterbank by selecting '`ufilterbank`', a positive-frequency filterbank by selecting '`filterbankreal`' or a uniform positive-frequency filterbank by selecting '`ufilterbankreal`'.

`frame ('nsdgt', g, a, M)` constructs a non-stationary Gabor frame with filters g , time-shifts of a and M channels. See the help on `nsdgt` for more information on the parameters. Similarly, you can construct a uniform NSDGT by selecting '`unsdgt`', an NSDGT for real-valued signals only by selecting '`nsdgtreal`' or a uniform NSDGT for real-valued signals by selecting '`unsdgtreal`'.

Wavelet frames

`frame ('fwt', w, J)` constructs a wavelet frame with wavelet definition w and J number of filterbank iterations. Similarly, a redundant time invariant wavelet representation can be constructed by selecting '`ufwt`'. See the help on `fwt` and `ufwt` for more information.

`frame ('wfbt', wt)` constructs a wavelet filterbank tree defined by the wavelet filterbank tree definition wt . Similarly, an undecimated wavelet filterbank tree can be constructed by selecting '`uwfbt`'. See the help on `wfbt` and `uwfbt` for more information.

`frame('wpfbt', wt)` constructs a wavelet packet filterbank tree defined by the wavelet filterbank tree definition `wt`. Similarly, an undecimated wavelet packet filterbank tree can be constructed by selecting '`uwpfbt`'. See the help on `wpfbt` and `uwpfbt` for more information.

Pure frequency frames

`frame('dft')` constructs a basis where the analysis operator is the dft, and the synthesis operator is its inverse, `idft`. Completely similar to this, you can enter the name of any of the cosine or sine transforms `dcti`, `dctii`, `dctiii`, `dctiv`, `dsti`, `dstii`, `dstiii` or `dstiv`.

`frame('dftreal')` constructs a normalized `fftreal` basis for real-valued signals of even length only. The basis is normalized to ensure that is it orthonormal.

Special / general frames

`frame('gen', g)` constructs an general frame with analysis matrix `g`. The frame atoms must be stored as column vectors in the matrices.

`frame('identity')` constructs the canonical orthonormal basis, meaning that all operators return their input as output, so it is the dummy operation.

Container frames

`frame('fusion', w, F1, F2, ...)` constructs a fusion frame, which is the collection of the frames specified by `F1, F2, ...`. The vector `w` contains a weight for each frame. If `w` is a scalar, this weight will be applied to all the sub-frames.

`frame('tensor', F1, F2, ...)` constructs a tensor product frame, where the frames `F1, *F2, ...` are applied along the 1st, 2nd etc. dimensions. If you don't want any action along a specific dimension, use the `identity` frame along that dimension. Any remaining dimensions in the input signal are left alone.

Wrapper frames

Frames types in this section are "virtual". They serve as a wrapper for a different type of frame.

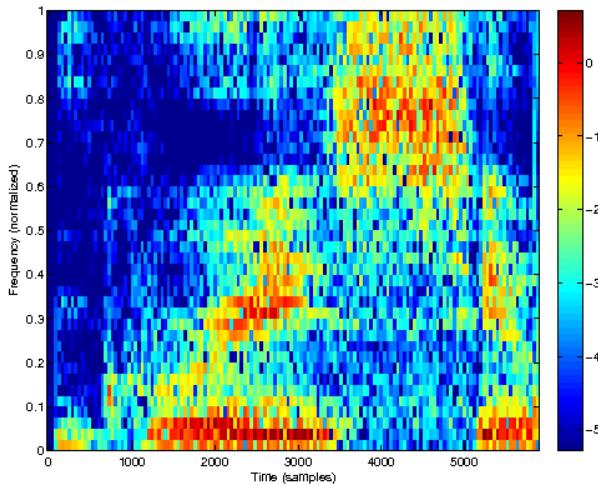
`frame('erbletfb', fs, Ls, ...)` constructs an Erb-let filterbank frame for a given samp. frequency `fs` working with signals of length `Ls`. See `erbfilters` for a description of additional parameters as all parameters other than the frame type string '`erbletfb`' are passed to it. NOTE: The resulting frame is defined only for a single signal length `Ls`. Shorter signals will be zero-padded, signals longer than `Ls` cannot be processed. The actual frame type is '`filterbank`' or '`filterbankreal`'.

`frame('cqtfb', fs, fmin, fmax, bins, Ls, ...)` constructs a CQT filterbank frame for a given samp. frequency `fs` working with signals of length `Ls`. See `cqtfilters` for a description of other parameters. NOTE: The resulting frame is defined only for a single signal length `Ls`. Shorter signals will be zero-padded, signals longer than `Ls` cannot be processed. The actual frame type is '`filterbank`' or '`filterbankreal`'.

Examples

The following example creates a Modified Discrete Cosine Transform frame, analyses an input signal and plots the frame coefficients:

```
F=frame('wmdct','gauss',40);
c=frana(F,greasy);
plotframe(F,c,'dynrange',60);
```



8.1.2 FRAMEPAIR - Construct a new frame

Usage

```
[F1,F2]=framepair(ftype,g1,g2,...);
```

Description

`[F1,F2]=framepair(ftype,g1,g2,...)` constructs two new frame objects *F1* and *F2* of the same type *ftype* using the windows *g1* and *g2*. The windows are specific to chosen frame type. See the help on *frame* for the windows and arguments.

This function makes it easy to create a pair of canonical dual frames: simply specify '*dual*' as window if one frame should be the dual of the other.

This is most easily explained through some examples. The following example creates a Gabor frame for real-valued signals with a Gaussian analysis window and its canonical dual frame as the synthesis frame:

```
f=greasy;
[Fa,Fs]=framepair('dgtreal','gauss','dual',20,294);
c=frana(Fa,f);
r=frsyn(Fs,c);
norm(f-r)
```

This code produces the following output:

```
ans =
5.644413339694357e-15
```

The following example creates a Wilson basis with a Gaussian synthesis window, and its canonical dual frame as the analysis frame:

```
[Fa,Fs]=framepair('dwilt','dual','gauss',20);
```

8.1.3 FRAMEDUAL - Construct the canonical dual frame

Usage

```
Fd=framedual(F);
```

Description

`Fd=framedual(F)` returns the canonical dual frame of F .

The canonical dual frame can be used to get perfect reconstruction as in the following example:

```
% Create a frame and its canonical dual
F=frame('dgt','hamming',32,64);
Fd=framedual(F);

% Compute the frame coefficients and test for perfect
% reconstruction
f=gspi;
c=frana(F,f);
r=frsyn(Fd,c);
norm(r(1:length(f))-f)
```

This code produces the following output:

```
ans =
9.757245647085724e-15
```

8.1.4 FRAMETIGHT - Construct the canonical tight frame

Usage

```
Ft=frametight(F);
```

Description

`Ft=frametight(F)` returns the canonical tight frame of F .

The canonical tight frame can be used to get perfect reconstruction if it is used for both analysis and synthesis. This is demonstrated in the following example:

```
% Create a frame and its canonical tight
F=frame('dgt','hamming',32,64);
Ft=frametight(F);

% Compute the frame coefficients and test for perfect
% reconstruction
f=gspi;
c=frana(Ft,f);
r=frsyn(Ft,c);
norm(r(1:length(f))-f)
```

This code produces the following output:

```
ans =
1.067814285565954e-14
```

8.1.5 FRAMEACCEL - Precompute structures

Usage

```
F=frameaccel(F,Ls);
```

Description

`F=frameaccel(F,Ls)` precomputes certain structures that makes the basic frame operations `frana` and `frsyn` faster (like instantiating the window from a textual description). If you only need to call the routines once, calling `frameaccel` first will not provide any total gain, but if you are repeatedly calling these routines, for instance in an iterative algorithm, it will be a benefit.

Notice that you need to input the signal length L_s , so this routines will only be a benefit if L_s stays fixed.

If `frameaccel` is called twice for the same transform length, no additional computations will be done.

8.2 Linear operators

8.2.1 FRANA - Frame analysis operator

Usage

```
c=frana(F,f);
```

Description

`c=frana(F,f)` computes the frame coefficients c of the input signal f using the frame F . The frame object F must have been created using `frame` or `framepair`.

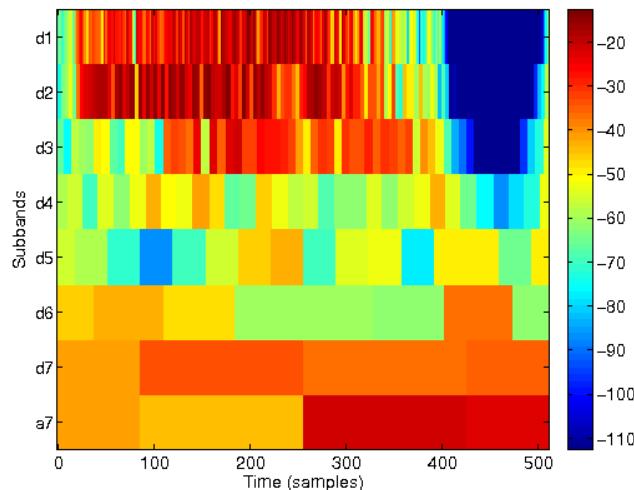
If f is a matrix, the transform will be applied along the columns of f . If f is an N-D array, the transform will be applied along the first non-singleton dimension.

The output coefficients are stored as columns. This is usually **not** the same format as the 'native' format of the frame. As an examples, the output from `frana` for a gabor frame cannot be passed to `idgt` without a reshape.

Examples:

In the following example the signal `bat` is analyzed through a wavelet frame. The result are the frame coefficients associated with the input signal `bat` and the analysis frame '`fwt`':

```
f = bat;
w = 'sym8';
J = 7;
F = frame('fwt', w, J);
c = frana(F, f);
% A plot of the frame coefficients
plotframe(F, c, 'dynrange', 100);
```



8.2.2 FRSYN - Frame synthesis operator

Usage

```
f=frsyn(F, c);
```

Description

`f=frsyn(F, c)` constructs a signal f from the frame coefficients c using the frame F . The frame object F must have been created using `frame`.

Examples:

In the following example a signal f is constructed through the frame synthesis operator using a Gabor frame. The coefficients associated with this Gabor expansion are contained in an identity matrix. The identity matrix corresponds to a diagonal in the time-frequency plane, that is, one atom at each time position with increasing frequency.:

```
a = 10;
M = 40;

F = frame('dgt', 'gauss', a, M);

c = frmenative2coef(F, eye(40));

f = frsyn(F, c);
```

8.2.3 FRSYNMATRIX - Frame synthesis operator matrix

Usage

```
G=frsynmatrix(F, L);
```

Description

`G=frsynmatrix(F, L)` returns the matrix representation G of the frame synthesis operator for a frame F of length L . The frame object F must have been created using `frame`.

The frame synthesis operator matrix contains all the frame atoms as column vectors. It has dimensions $L \times Ncoef$, where $Ncoef$ is the number of coefficients. The number of coefficients can be found as `Ncoef=frameclength(L)`. This means that the frame matrix is usually **very** large, and this routine should only be used for small values of L .

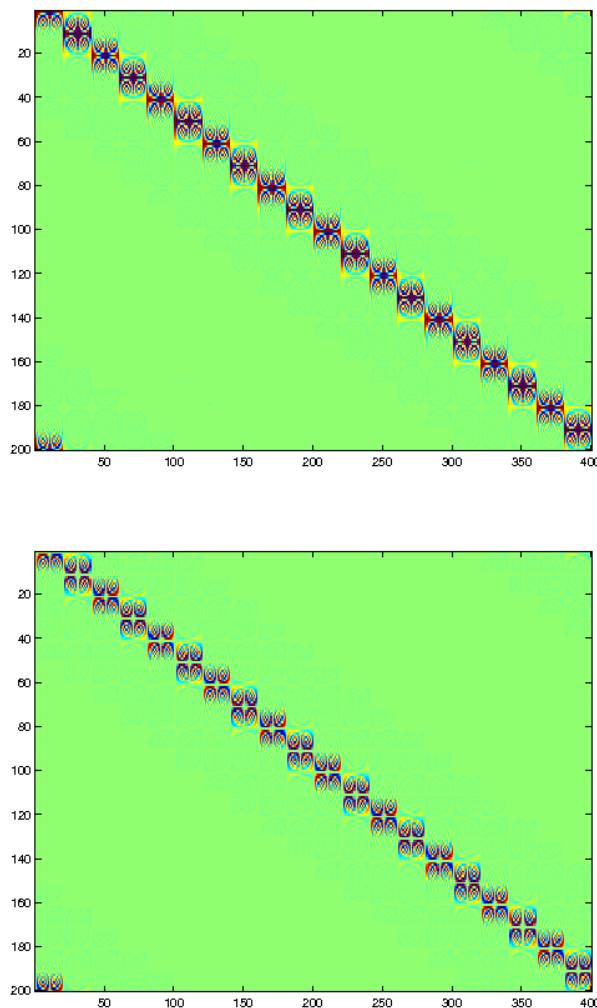
The action of the frame analysis operator `frana` is equal to multiplication with the Hermitean transpose of the frame matrix. Consider the following simple example:

```
L=200;
F=frame('dgt','gauss',10,20);
G=frsynmatrix(F,L);
testsig = randn(L,1);
res = frana(F,testsig)-G'*testsig;
norm(res)
% Show the matrix (real and imaginary parts)
figure(1); imagesc(real(G));
figure(2); imagesc(imag(G));
```

This code produces the following output:

```
ans =
```

```
7.106774012374892e-15
```



8.2.4 FRGRAMIAN - Frame Gramian operator

Usage

```
o=frgramian(c, F);
o=frgramian(c, Fa, Fs);
```

Input parameters

c	Input coefficients
Fa	Analysis frame
Fs	Synthesis frame

Output parameters

o	Output coefficients
----------	---------------------

Description

`o=frgramian(c, F)` applies the Gramian operator or Gram matrix of the frame F . The entries of the Gram matrix are the inner products of the frame elements of F . The frame must have been created using

frame. If the frame F is a Parseval frame, the Gramian operator is a projection onto the range of the frame analysis operator.

`o=frgramian(c, Fa, Fs)` applies the (cross) Gramian operator with the frames Fa and Fs . Here Fs is the frame associated with the frame synthesis operator and Fa the frame that is associated with the frame analysis operator. The entries of the matrix that is constructed through the Gramian operator are the inner products of the frame elements of Fa and Fs . If Fa and Fs are canonical dual frames, the Gramian operator is a projection onto the range of the frame analysis operator.

8.2.5 FRAMEOPERATOR - Frame Operator

Usage

```
o=frameoperator(F, f);
```

Input parameters

F	frame
f	input vector

Description

Output parameter: `h` : output vector

`h=frameoperator(F, f)` applies the frame operator associated with the frame F to the input f .

If the frame F is a tight frame, then h equals f up to the constant $\frac{1}{A}$ where A is the lower frame bound of F . If the frame F is an orthonormal basis, or more general a Parseval frame, then h equals f .

8.2.6 FRAMEDIAG - Compute the diagonal of the frame operator

Usage

```
d=framediag(F, L);
```

Description

`framediag(F, L)` computes the diagonal of the frame operator for a frame of type F of length L .

The diagonal of the frame operator can for instance be used as a preconditioner.

8.2.7 FRANAITER - Iterative analysis

Usage

```
c=franaiter(F, f);
[c, relres, iter]=franaiter(F, f, ...);
```

Input parameters

F	Frame.
f	Signal.
Ls	Length of signal.

Output parameters

c	Array of coefficients.
relres	Vector of residuals.
iter	Number of iterations done.

Description

`c=franaiter(F, f)` computes the frame coefficients c of the signal f using an iterative method such that perfect reconstruction can be obtained using `frsyn`. `franaiter` always works, even when `frana` cannot generate perfect reconstruction coefficients.

`[c, relres, iter]=franaiter(...)` additionally returns the relative residuals in a vector `relres` and the number of iteration steps `iter`.

Note: If it is possible to explicitly calculate the canonical dual frame then this is usually a much faster method than invoking `franaiter`.

`franaiter` takes the following parameters at the end of the line of input arguments:

'tol',t	Stop if relative residual error is less than the specified tolerance. Default is 1e-9 (1e-5 for single precision)
'maxit',n	Do at most n iterations.
'pg'	Solve the problem using the Conjugate Gradient algorithm. This is the default.
'pcg'	Solve the problem using the Preconditioned Conjugate Gradient algorithm.
'print'	Display the progress.
'quiet'	Don't print anything, this is the default.

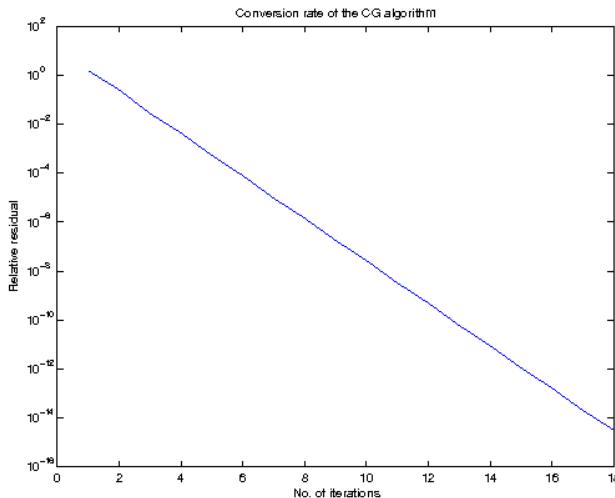
Examples

The following example shows how to reconstruct a signal without ever using the dual frame:

```
f=greasy;
F=frame('dgtreal','gauss',40,60);
[c,relres,iter]=franaiter(F,f,'tol',1e-14);
r=frsyn(F,c);
norm(f-r)/norm(f)
semilogy(relres);
title('Conversion rate of the CG algorithm');
xlabel('No. of iterations');
ylabel('Relative residual');
```

This code produces the following output:

```
ans =
2.023584411446212e-15
```



8.2.8 FRSYNITER - Iterative synthesis

Usage

```
f=frsyniter(F,c);
f=frsyniter(F,c,Ls);
[f,relres,iter]=frsyniter(F,c,...);
```

Input parameters

F	Frame
c	Array of coefficients.
Ls	length of signal.

Output parameters

f	Signal.
relres	Vector of residuals.
iter	Number of iterations done.

Description

`f=frsyniter(F,c)` iteratively inverts the analysis operator of F , so `frsyniter` always performs the inverse operation of `frana`, even when a perfect reconstruction is not possible by using `frsyn`.

`[f,relres,iter]=frsyniter(...)` additionally returns the relative residuals in a vector `relres` and the number of iteration steps `iter`.

Note: If it is possible to explicitly calculate the canonical dual frame then this is usually a much faster method than invoking `frsyniter`.

`frsyniter` takes the following parameters at the end of the line of input arguments:

'tol',t	Stop if relative residual error is less than the specified tolerance. Default is 1e-9 (1e-5 for single precision)
'maxit',n	Do at most n iterations.
'cg'	Solve the problem using the Conjugate Gradient algorithm. This is the default.

'pcg'	Solve the problem using the Preconditioned Conjugate Gradient algorithm.
'print'	Display the progress.
'quiet'	Don't print anything, this is the default.

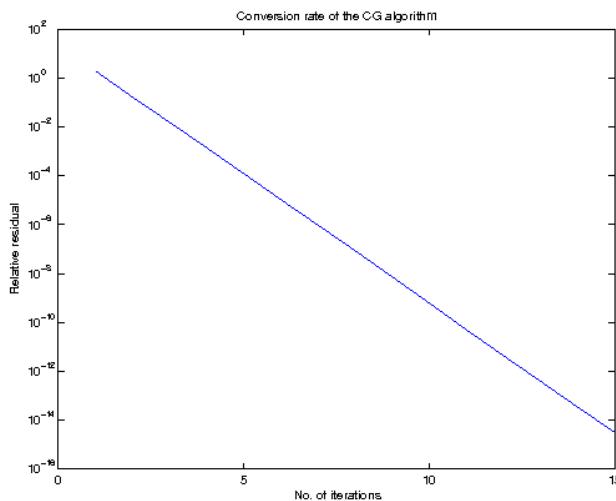
Examples

The following example shows how to reconstruct a signal without ever using the dual frame:

```
F=frame('dgtreal','gauss',10,20);
c=frana(F,bat);
[r,relres]=frsyniter(F,c,'tol',1e-14);
norm(bat-r)/norm(bat)
semilogy(relres);
title('Conversion rate of the CG algorithm');
xlabel('No. of iterations');
ylabel('Relative residual');
```

This code produces the following output:

```
ans =
1.591570805698530e-15
```



8.3 Visualization

8.3.1 PLOTFRAME - Plot frame coefficients

Usage

```
plotframe(F,c,...);
C = plotframe(...);
```

Description

`plotframe(F, c)` plots the frame coefficients c using the plot command associated to the frame F .
`C=plotframe(...)` for frames with time-frequency plots returns the processed image data used in the plotting. The function produces an error for frames which does not have a time-frequency plot.

`plotframe(F, c, ...)` passes any additional parameters to the native plot routine. Please see the help on the specific plot routine for a complete description.

The following common set of parameters are supported by all plotting routines:

'dynrange',r	Limit the dynamical range to r . The default value of [] means to not limit the dynamical range.
'db'	Apply $20 \cdot \log_{10}$ to the coefficients. This makes it possible to see very weak phenomena, but it might show too much noise. A logarithmic scale is more adapted to perception of sound. This is the default.
'dbsq'	Apply $10 \cdot \log_{10}$ to the coefficients. Same as the ' <code>db</code> ' option, but assume that the input is already squared.
'lin'	Show the coefficients on a linear scale. This will display the raw input without any modifications. Only works for real-valued input.
'linsq'	Show the square of the coefficients on a linear scale.
'linabs'	Show the absolute value of the coefficients on a linear scale.
'clim',clim	Only show values in between $clim(1)$ and $clim(2)$. This is usually done by adjusting the colormap. See the help on <code>imagesc</code> .

8.3.2 FRAMEGRAM - Easy visualization of energy in transform domain

Usage

```
framegram(F, x, ...);
```

Description

`framegram(F, x)` plots the energy of the frame coefficients computed from the input signal x using the frame F for analysis. This is just a shorthand for:

```
plotframe(F, abs(frana(F, x)) .^2);
```

Any additional arguments given to `framegram` are passed onto `plotframe`.

8.4 Information about a frame

8.4.1 FRAMEBOUNDS - Frame bounds

Usage

```
fcond=framebounds(F);
[A, B]=framebounds(F);
[...]=framebounds(F, Ls);
```

Description

`framebounds(F)` calculates the ratio B/A of the frame bounds of the frame given by F . The length of the system the frame bounds are calculated for is given by `L=framelength(F, 1)`.

`framebounds(F, Ls)` additionally specifies a signal length for which the frame should work. The actual length used is `L=framelength(F, Ls)`.

`[A, B]=framebounds(F)` returns the frame bounds A and B instead of just their ratio.

'`framebounds`' accepts the following optional parameters:

'fac'	Use a factorization algorithm. The function will throw an error if no algorithm is available.
--------------	---

'iter'	Call <code>eigs</code> to use an iterative algorithm.
'full'	Call <code>eig</code> to solve the full problem.
'auto'	Choose the <code>fac</code> method if possible, otherwise use the <code>full</code> method for small problems and the <code>iter</code> method for larger problems. This is the default.
'crossover',c	Set the problem size for which the ' <code>auto</code> ' method switches between <code>full</code> and <code>iter</code> . Default is 200.

The following parameters specifically related to the `iter` method:

'tol',t	Stop if relative residual error of <code>eigs</code> is less than the specified tolerance. Default is <code>1e-9</code>
'maxit',n	Do at most n iterations in <code>eigs</code> . Default is 100.
'pcgtol',t	Stop if relative residual error of <code>pcg</code> is less than the specified tolerance. Default is <code>1e-6</code>
'pcgmaxit',n	Do at most n iterations in <code>pcg</code> . Default is 150.
'p',p	The number of Lanczos basis vectors to use. More vectors will result in faster convergence, but a larger amount of memory. The optimal value of p is problem dependent and should be less than L. The default value chosen automatically by <code>eigs</code> .
'print'	Display the progress.
'quiet'	Don't print anything, this is the default.

8.4.2 FRAMERED - Redundancy of a frame

Usage

```
red=framered(F);
```

Description

`framered(F)` computes the redundancy of a given frame F . If the redundancy is larger than 1 (one), the frame transform will produce more coefficients than it consumes. If the redundancy is exactly 1 (one), the frame is a basis.

Examples:

The following simple example shows how to obtain the redundancy of a Gabor frame:

```
F=frame('dgt','gauss',30,40);
framered(F)
```

This code produces the following output:

```
ans =
1.3333333333333333
```

The redundancy of a basis is always one:

```
F=frame('wmdct','gauss',40);
framered(F)
```

This code produces the following output:

```
ans =
```

8.4.3 FRAMELENGTH - Frame length from signal

Usage

```
L=framelength(F,Ls);
```

Description

`framelength(F,Ls)` returns the length of the frame F , such that F is long enough to expand a signal of length L_s .

If the frame length is longer than the signal length, the signal will be zero-padded by `frana`.

If instead a set of coefficients are given, call `framelengthcoef`.

8.4.4 FRAMELENGTHCOEF - Frame length from coefficients

Usage

```
L=framelengthcoef(F,Ncoef);
```

Description

`framelengthcoef(F,Ncoef)` returns the length of the frame F , such that F is long enough to expand the coefficients of length N_{coef} .

If instead a signal is given, call `framelength`.

8.4.5 FRAMECLLENGTH - Number of coefficients from length of signal

Usage

```
Ncoef=framecllength(F,Ls);
[Ncoef,L]=framecllength(...);
```

Description

`Ncoef=framecllength(F,Ls)` returns the total number of coefficients obtained by applying the analysis operator of frame F to a signal of length L_s i.e. `size(frana(F,f),1)` for $L_s=\text{length}(f)$.

`[Ncoef,L]=framecllength(F,Ls)` additionally returns L , which is the same as returned by `framelength`.

If the frame length L is longer than the signal length L_s , the signal will be zero-padded to L by `frana`.

8.5 Coefficients conversions

8.5.1 FRAMECOEF2NATIVE - Convert coefficients to native format

Usage

```
coef=framecoef2native(F,coef);
```

Description

`framecoef2native(F,coef)` converts the frame coefficients $coef$ into the native coefficient format of the frame. The frame object F must have been created using `frame`.

8.5.2 FRAMENATIVE2COEF - Convert coefficient from native format

Usage

```
coef=framenative2coef(F,coef);
```

Description

`framenative2coef(F, coef)` converts the frame coefficients from the native format of the transform into the common column format.

8.5.3 FRAMECOEF2TF - Convert coefficients to time-frequency plane**Usage**

```
cout=framecoef2tf(F, cin);
```

Description

`framecoef2tf(F, cin)` converts the frame coefficients *cin* into the time-frequency plane layout. The frame object *F* must have been created using `frame`.

The time-frequency plane layout is a matrix, where the first dimension indexes frequency and the second dimension time. This is similar to the output format from `dgt` and `wmdct`.

Not all types of frames support this coefficient conversion. The supported types of frames are: '`dgt`', '`dgtreal`', '`dwilt`', '`wmdct`', '`ufilterbank`', '`ufwt`', '`uwfbt`' and '`uwpfbt`'.

8.5.4 FRAMETF2COEF - Convert coefficients from TF-plane format**Usage**

```
cout=frametf2coef(F, cin);
```

Description

`frametf2coef(F, cin)` converts the frame coefficients from the time-frequency plane layout into the common column format.

Not all types of frames support this coefficient conversion. The supported types of frames are: '`dgt`', '`dgtreal`', '`dwilt`', '`wmdct`', '`ufilterbank`', '`ufwt`', '`uwfbt`' and '`uwpfbt`'.

8.5.5 FRAMECOEF2TFPLOT - Convert coefficients to time-frequency plane matrix**Usage**

```
cout=framecoef2tfplot(F, cin);
```

Description

`framecoef2tfplot(F, coef)` converts the frame coefficients *coef* into the time-frequency plane layout matrix. The frame object *F* must have been created using `frame`. The function acts exactly as `framecoef2tf` for frames which admit regular (rectangular) sampling of a time-frequency plane and converts irregularly sampled coefficients to a rectangular matrix. This is usefull for custom plotting.

8.6 Non-linear analysis and synthesis**8.6.1 FRANABP - Frame Analysis Basis Pursuit****Usage**

```
c = franabp(F, f)
c = franabp(F, f, lambda, C, tol, maxit)
[c, relres, iter, freq, cd] = franabp(...)
```

Input parameters

F	Frame definition
f	Input signal
lambda	Regularisation parameter.
C	Step size of the algorithm.
tol	Reative error tolerance.
maxit	Maximum number of iterations.

Output parameters

c	Sparse coefficients.
relres	Last relative error.
iter	Number of iterations done.
frec	Reconstructed signal such that $frec = frsyn(F,c)$
cd	The min $\ c\ _2$ solution using the canonical dual frame.

Description

`c = franabp (F, f)` solves the basis pursuit problem

$$\operatorname{argmin}_c \|c\|_1$$

$$\text{subject to } Fc = f$$

for a general frame F using SALSA (Split Augmented Lagrangian Shrinkage algorithm) which is an application of ADMM (Alternating Direction Method of Multipliers) to the basis pursuit problem.

The algorithm given F and f and parameters $C > 0$, $\lambda > 0$ (see below) acts as follows:

```

Initialize c,d
repeat
    v <- soft (c+d, lambda/C) - d
    d <- F*(FF*)^(-1) (f - Fv)
    c <- d + v
end

```

When compared to other algorithms, $Fc = f$ holds exactly (up to a num. prec) in each iteration.

For a quick execution, the function requires analysis operator of the canonical dual frame $F^*(FF^*)^{(-1)}$. By default, the function attempts to call `framedual` to create the canonical dual frame explicitly. If it is not available, the conjugate gradient method algorithm is used for inverting the frame operator in each iteration of the algorithm. Optionally, the canonical dual frame object or an anonymous function acting as the analysis operator of the canonical dual frame can be passed as a key-value pair '`Fd`', `Fd` see below.

Optional positional parameters (`lambda,C,tol,maxit`)

lambda A parameter for weighting coefficients in the objective function. For $\lambda \approx 1$ the basis pursuit problem changes to

$$\arg \min_c \|\lambda c\|_1$$

subject to $Fc = f$

`lambda` can either be a scalar or a vector of the same length as `c` (in such case the product is carried out elementwise). One can obtain length of `c` from length of `f` by `frameclength`. `framecoef2native` and `framenative2coef` will help with defining weights specific to some regions of coefficients (e.g. channel-specific weighting can be achieved this way). The default value of `lambda` is 1.

C A step parameter of the SALSA algorithm. The default value of `C` is the upper frame bound of `F`. Depending on the structure of the frame, this can be an expensive operation.

tol Defines tolerance of `relres` which is a norm or a relative difference of coefficients obtained in two consecutive iterations of the algorithm. The default value 1e-2.

maxit Maximum number of iterations to do. The default value is 100.

Other optional parameters

Key-value pairs:

'Fd', `Fd` A canonical dual frame object or an anonymous function acting as the analysis operator of the canonical dual frame.

'printstep', `printstep` Print current status every `printstep` iteration.

Flag groups (first one listed is the default):

'print', **'quiet'** Enables/disables printing of notifications.

'zeros', **'frana'** Starting point of the algorithm. With `'zeros'` enabled, the algorithm starts from coefficients set to zero, with `'frana'` the algorithm starts from `c=frana(F, f)`.

Returned arguments:

`[c, relres, iter] = franabp(...)` returns the residuals `relres` in a vector and the number of iteration steps done `iter`.

`[c, relres, iter, freq, cd] = franabp(...)` returns the reconstructed signal from the coefficients, `freq` (this requires additional computations) and a coefficients `cd` minimising the $\|c\|_2$ norm (this is a byproduct of the algorithm).

The relationship between the output coefficients `freq` and `c` is given by

```
freq = frsyn(F, c);
```

And `cd` and `f` by

```
cd = frana(framedual(F), f);
```

Examples:

The following example shows how `franabp` produces a sparse representation of a test signal `greasy` still maintaining a perfect reconstruction:

```
f = greasy;
% Gabor frame with redundancy 8
F = frame('dgtreal','gauss',64,512);
% Solve the basis pursuit problem
[c,~,~,freq,cd] = franabp(F,f);
% Plot sparse coefficients
```

```

figure(1);
plotframe(F,c,'dynrange',50);

% Plot coefficients obtained by applying an analysis operator of a
% dual Gabor system to *f*
figure(2);
plotframe(F,cd,'dynrange',50);

% Check the reconstruction error (should be close to zero).
% f_rec is obtained by applying the synthesis operator of frame *F*
% to sparse coefficients *c*.
norm(f-f_rec)

% Compare decay of coefficients sorted by absolute values
% (compressibility of coefficients)
figure(3);
semilogx([sort(abs(c),'descend')/max(abs(c)),...
sort(abs(cd),'descend')/max(abs(cd))]);
legend({'sparsified coefficients','dual system coefficients'});

```

This code produces the following output:

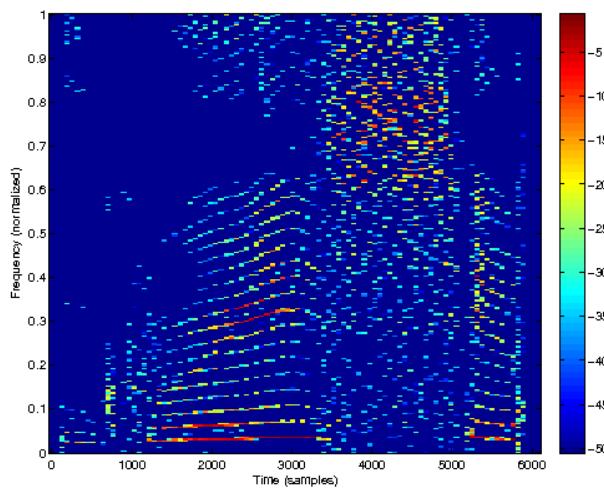
```

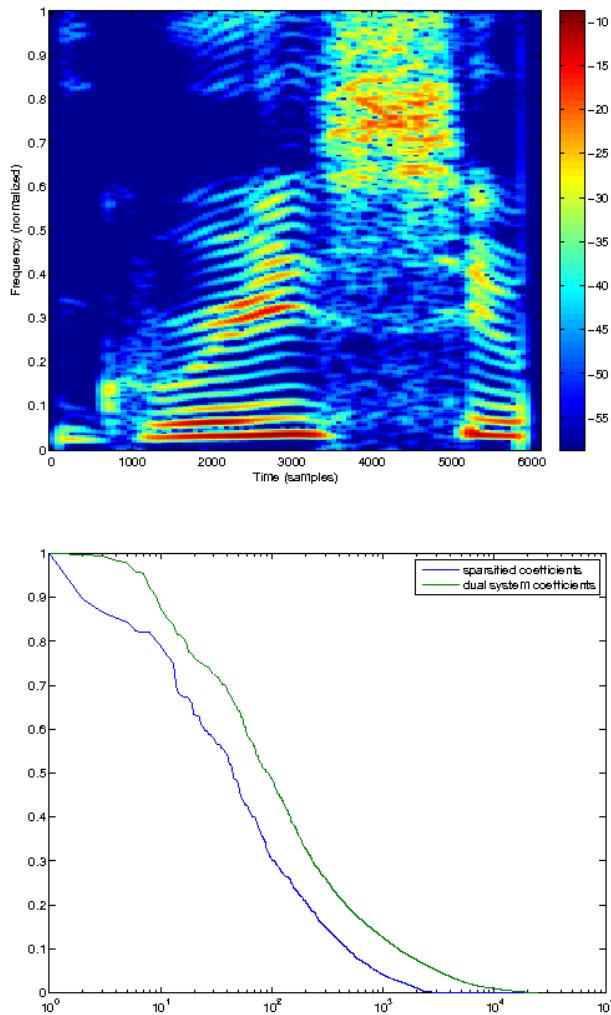
Iteration 10: relative error = 0.068384
Iteration 20: relative error = 0.034516
Iteration 30: relative error = 0.022865
Iteration 40: relative error = 0.016747
Iteration 50: relative error = 0.013124
Iteration 60: relative error = 0.010508

```

```
ans =
```

```
2.334642657924850e-14
```





References: [84], [18]

8.6.2 FRANALASSO - Frame LASSO regression

Usage

```
tc = franalasso(F,f,lambda)
tc = franalasso(F,f,lambda,C,tol,maxit)
[tc,relres,iter,frec,cd] = franalasso(....)
```

Input parameters

F	Frame definition
f	Input signal
lambda	Regularisation parameter, controls sparsity of the solution
C	Step size of the algorithm.
tol	Reative error tolerance.
maxit	Maximum number of iterations.

Output parameters

tc	Thresholded coefficients
relres	Vector of residuals.
iter	Number of iterations done.
frec	Reconstructed signal
cd	The min $\ c\ _2$ solution using the canonical dual frame.

Description

`franalasso(F, f, lambda)` solves the LASSO (or basis pursuit denoising) regression problem for a general frame: minimize a functional of the synthesis coefficients defined as the sum of half the l^2 norm of the approximation error and the l^1 norm of the coefficient sequence, with a penalization coefficient `lambda` such that

$$\operatorname{argmin}_c \lambda \|c\|_1 + \frac{1}{2} \|Fc - f\|_2^2$$

The solution is obtained via an iterative procedure, called Landweber iteration, involving iterative soft thresholdings.

The following flags determining an algorithm to be used are recognized at the end of the argument list:

'ista' The basic (Iterative Soft Thresholding) algorithm given `F` and `f` and parameters $C > 0$, `lambda > 0` acts as follows:

```
Initialize c
repeat until stopping criterion is met
    c <- soft(c + F*(f - Fc)/C, lambda/C)
end
```

'fista' The fast version of the previous. This is the default option.

```
Initialize c(0), z, tau(0)=1, n=1
repeat until stopping criterion is met
    c(n) <- soft(z + F*(f - Fz)/C, lambda/C)
    tau(n) <- (1+sqrt(1+4*tau(n-1)^2))/2
    z     <- c(n) + (c(n)-c(n-1))(tau(n-1)-1)/tau(n)
    n     <- n + 1
end
```

`[tc, relres, iter] = franalasso(...)` returns the residuals `relres` in a vector and the number of iteration steps done `iter`.

`[tc, relres, iter, frec, cd] = franalasso(...)` returns the reconstructed signal from the coefficients, `frec` and coefficients `cd` obtained by analysing using the canonical dual system. Note that this requires additional computations.

The relationship between the output coefficients and `frec` is given by

```
frec = frsyn(F, tc);
```

The function takes the following optional parameters at the end of the line of input arguments:

'C',eval Landweber iteration parameter: must be larger than square of upper frame bound. Default value is the upper frame bound.

'tol',tol Stopping criterion: minimum relative difference between norms in two consecutive iterations. Default value is 1e-2.

'maxit',maxit Stopping criterion: maximal number of iterations to do. Default value is 100.

'print' Display the progress.

'quiet' Don't print anything, this is the default.

'printstep',*p* If 'print' is specified, then print every *p*'th iteration. Default value is 10;

The parameters *C*, *itermax* and *tol* may also be specified on the command line in that order: `franalasso(F, x, lambda, C, itermax, tol)`

Note: If you do not specify *C*, it will be obtained as the upper framebound. Depending on the structure of the frame, this can be an expensive operation.

Examples:

The following example shows how `franalasso` produces a sparse representation of a test signal *greasy*:

```
f = greasy;
% Gabor frame with redundancy 8
F = frame('dgtreal','gauss',64,512);
% Choosing lambda (weight of the sparse regularization param.)
lambda = 0.1;
% Solve the basis pursuit problem
[c,~,~,frec,cd] = franalasso(F,f,lambda);
% Plot sparse coefficients
figure(1);
plotframe(F,c,'dynrange',50);

% Plot coefficients obtained by applying an analysis operator of a
% dual Gabor system to f
figure(2);
plotframe(F,cd,'dynrange',50);

% Check the (NON-ZERO) reconstruction error .
% frec is obtained by applying the synthesis operator of frame F
% to sparse coefficients c.
norm(f-frec)

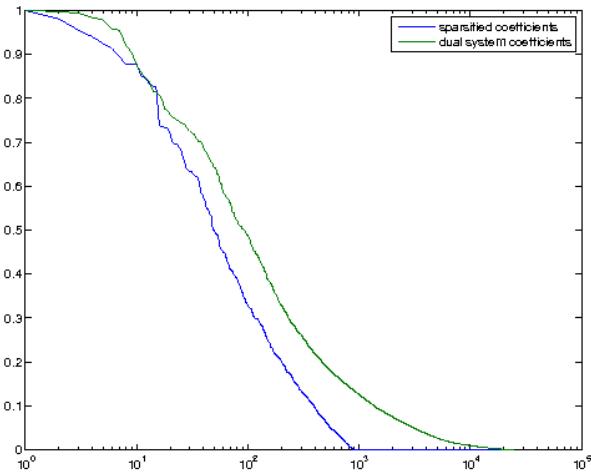
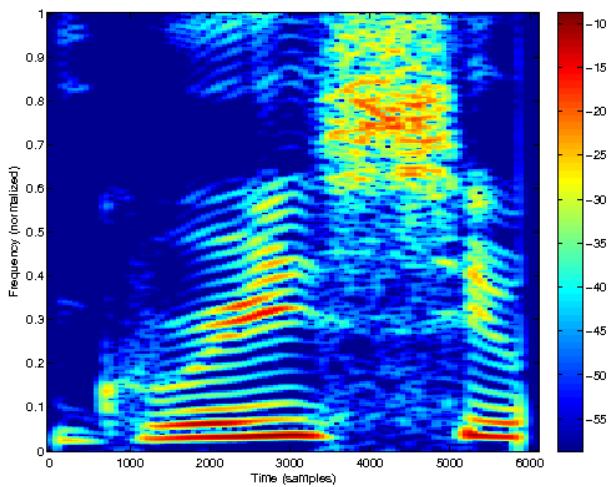
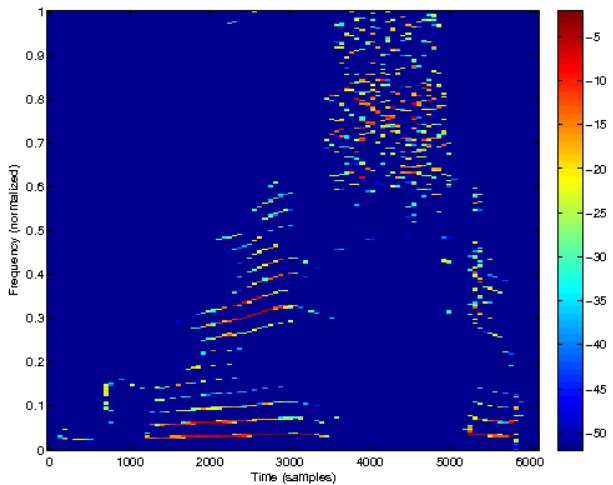
% Compare decay of coefficients sorted by absolute values
% (compressibility of coefficients)
figure(3);
semilogx([sort(abs(c),'descend')/max(abs(c)),...
sort(abs(cd),'descend')/max(abs(cd))]);
legend({'sparsified coefficients','dual system coefficients'});
```

This code produces the following output:

```
Iteration 10: relative error = 0.049584
Iteration 20: relative error = 0.032723
Iteration 30: relative error = 0.023291
Iteration 40: relative error = 0.015511
Iteration 50: relative error = 0.010411
```

```
ans =
```

```
3.163620955029133
```



References: [25], [13]

8.6.3 FRANAGROUPASSO - Group LASSO regression in the TF-domain

Usage

```
tc = franagroupasso(F,f,lambda)
tc = franagroupasso(F,f,lambda,C,tol,maxit)
[tc,relres,iter,frec] = franagroupasso(...)
```

Input parameters

F	Frame definition
f	Input signal
lambda	Regularisation parameter, controls sparsity of the solution
C	Step size of the algorithm.
tol	Reative error tolerance.
maxit	Maximum number of iterations.

Output parameters

tc	Thresholded coefficients
relres	Vector of residuals.
iter	Number of iterations done.
frec	Reconstructed signal

Description

`franagroupasso(F, f, lambda)` solves the group LASSO regression problem in the time-frequency domain: minimize a functional of the synthesis coefficients defined as the sum of half the l^2 norm of the approximation error and the mixed l^1 / l^2 norm of the coefficient sequence, with a penalization coefficient `lambda`.

The matrix of time-frequency coefficients is labelled in terms of groups and members. By default, the obtained expansion is sparse in terms of groups, no sparsity being imposed to the members of a given group. This is achieved by a regularization term composed of l^2 norm within a group, and l^1 norm with respect to groups. See the help on `groupthresh` for more information.

Note the involved frame `F` must support regular time-frequency layout of coefficients.

`[tc, relres, iter] = franagroupasso(...)` returns the residuals `relres` in a vector and the number of iteration steps done, `maxit`.

`[tc, relres, iter, frec] = franagroupasso(...)` returns the reconstructed signal from the coefficients, `frec`. Note that this requires additional computations.

The function takes the following optional parameters at the end of the line of input arguments:

'freq'	Group in frequency (search for tonal components). This is the default.
'time'	Group in time (search for transient components).
'C',eval	Landweber iteration parameter: must be larger than square of upper frame bound. Default value is the upper frame bound.
'maxit',maxit	Stopping criterion: maximal number of iterations. Default value is 100.
'tol',tol	Stopping criterion: minimum relative difference between norms in two consecutive iterations. Default value is 1e-2.
'print'	Display the progress.

'quiet'	Don't print anything, this is the default.
'printstep',p	If 'print' is specified, then print every p'th iteration. Default value is 10;

In addition to these parameters, this function accepts all flags from the groupthresh and thresh functions. This makes it possible to switch the grouping mechanism or inner thresholding type.

The parameters C , $maxit$ and tol may also be specified on the command line in that order: `franagrouplasso(F, x, lambda, C, maxit, tol)`

The solution is obtained via an iterative procedure, called Landweber iteration, involving iterative group thresholdings.

The relationship between the output coefficients is given by

```
frec = frsyn(F,tc);
```

References: [54], [53]

8.6.4 FRSYNABS - Reconstruction from magnitude of coefficients

Usage

```
f=frsynabs(F,s);
f=frsynabs(F,s,Ls);
[f,relres,iter,c]=frsynabs(...);
```

Input parameters

F	Frame
s	Array of coefficients.
Ls	length of signal.

Output parameters

f	Signal.
relres	Vector of residuals.
iter	Number of iterations done.
c	Coefficients with the reconstructed phase

Description

`frsynabs(F, s)` attempts to find a signal which has s as the absolute value of its frame coefficients

```
s = abs(frina(F,f));
```

using an iterative method.

`frsynabs(F, s, Ls)` does as above but cuts or extends f to length Ls .

If the phase of the coefficients s is known, it is much better to use `frsyn`.

`[f,relres,iter]=frsynabs(...)` additionally returns the residuals in a vector $relres$ and the number of iteration steps $iter$. The residuals are computed as:

$$relres = \frac{\| |c_n| - s \|_{fro}}{\| s \|_{fro}},$$

where c_n is the Gabor coefficients of the signal in iteration n .

`[f,relres,iter,c]=frsynabs(...,'griflim'|'fgriflim')` additionally returns coefficients c with the reconstructed phase prior to the final reconstruction. This is useful for determining the consistency (energy lost in the nullspace of F) of the reconstructed spectrogram. c will only be equal

to `frana(F, f)` if the spectrogram is already consistent (i.e. already in the range space of F^*). This is possible only for '`griflim`' and '`fgriflim`' methods.

Generally, if the absolute value of the frame coefficients has not been modified, the iterative algorithm will converge slowly to the correct result. If the coefficients have been modified, the algorithm is not guaranteed to converge at all.

`frsynabs` takes the following parameters at the end of the line of input arguments.

Initial phase guess:

'input'	Choose the starting phase as the phase of the input s . This is the default
'zero'	Choose a starting phase of zero.
'rand'	Choose a random starting phase.

The Griffin-Lim algorithm related parameters:

'griflim'	Use the Griffin-Lim iterative method. This is the default.
'fgriflim'	Use the Fast Griffin-Lim iterative method.
'Fd',F_d	A canonical dual frame object or an anonymous function acting as the synthesis operator of the canonical dual frame. If not provided, the function attempts to create one using $F_d = \text{framedual}(F)$.
'alpha',a	Parameter of the Fast Griffin-Lim algorithm. It is ignored if not used together with ' <code>fgriflim</code> ' flag.

The BFGS method related parameters:

'bfgs'	Use the limited-memory Broyden Fletcher Goldfarb Shanno (BFGS) method.
'p',p	Parameter for the compressed version of the <code>obj.</code> function in the l-BFGS method. It is ignored if not used together with ' <code>bfgs</code> ' flag.

Other:

'tol',t	Stop if relative residual error is less than the specified tolerance.
'maxit',n	Do at most n iterations.
'print'	Display the progress.
'quiet'	Don't print anything, this is the default.
'printstep',p	If ' <code>print</code> ' is specified, then print every p 'th iteration. Default value is $p=10$;

The BFGS method makes use of the `minFunc` software. To use the BFGS method, please install the `minFunc` software from: <http://www.cs.ubc.ca/~schmidtm/Software/minFunc.html>.

References: [39], [71], [28]

Chapter 9

LTFAT - Signal processing tools

9.1 General

9.1.1 RMS - RMS value of signal

Usage

```
y = rms(f);  
y = rms(f, ...);
```

Description

`RMS(f)` computes the RMS (Root Mean Square) value of a finite sampled signal sampled at a uniform sampling rate. This is a vector norm equal to the l^2 averaged by the length of the signal.

If the input is a matrix or ND-array, the RMS is computed along the first (non-singleton) dimension, and a vector of values is returned.

The RMS value of a signal x of length N is computed by

$$rms(f) = \frac{1}{\sqrt{N}} \left(\sum_{n=1}^N |f(n)|^2 \right)^{\frac{1}{2}}$$

RMS takes the following flags at the end of the line of input parameters:

<code>'ac'</code>	Consider only the AC component of the signal (i.e. the mean is removed).
<code>'dim',d</code>	Work along specified dimension. The default value of <code>[]</code> means to work along the first non-singleton one.

9.1.2 NORMALIZE - Normalize input signal by specified norm

Usage

```
h=normalize(f, ...);
```

Description

`normalize(f, ...)` will normalize the signal f by the specified norm.

`[f, fnorm]=normalize(f, ...)` does the same thing, but in addition returns norm $fnorm$ of a signal f .

The norm is specified as a string and may be one of:

<code>'1'</code>	Normalize the l^1 norm to be 1.
------------------	-----------------------------------

'area'	Normalize the area of the signal to be 1 . This is exactly the same as ' 1 '.
'2'	Normalize the l^2 norm to be 1 .
'energy'	Normalize the energy of the signal to be 1 . This is exactly the same as ' 2 '.
'inf'	Normalize the l^∞ norm to be 1 .
'peak'	Normalize the peak value of the signal to be 1 . This is exactly the same as ' inf '.
'rms'	Normalize the Root Mean Square (RMS) norm of the signal to be 1 .
's0'	Normalize the S0-norm to be 1 .
'wav'	Normalize to the l^∞ norm to be 0.99 to avoid possible clipping introduced by the quantization procedure when saving as a wav file. This only works with floating point data types.
'null'	Do NOT normalize, output is identical to input.

It is possible to specify the dimension:

'dim',d Work along specified dimension. The default value of [] means to work along the first non-singleton one.

9.1.3 GAINDB - Increase/decrease level of signal

Usage

```
outsig = gaindb(insig,gn);
```

Description

`gaindb(insig,gn)` increases the energy level of the signal by gn dB.

If gn is a scalar, the whole input signal is scaled.

If gn is a vector, each column is scaled by the entries in gn . The length of gn must match the number of columns.

`gaindb(insig,gn,dim)` scales the signal along dimension dim .

9.1.4 CRESTFACTOR - Crest factor of input signal in dB

Usage

```
c=crestfactor(insig);
```

Description

`crestfactor(insig)` computes the crest factor of the input signal $insig$. The output is measured in dB.

9.1.5 UQUANT - Simulate uniform quantization

Usage

```
x=uquant(x);
x=uquant(x,nbits,xmax,...);
```

Description

`uquant (x, nbits, xmax)` simulates the effect of uniform quantization of x using $nbits$ bits. The output is simply x rounded to 2^{nbits} different values. The $xmax$ parameters specify the maximal value that should be quantifiable.

`uquant (x, nbits)` assumes a maximal quantifiable value of 1.

`uquant (x)` additionally assumes 8 bit quantization.

`uquant` takes the following flags at the end of the input arguments:

'nbits'	Number of bits to use in the quantization. Default is 8.
'xmax'	Maximal quantifiable value. Default is 1.
's'	Use signed quantization. This assumes that the signal has a both positive and negative part. Useful for sound signals. This is the default.
'u'	Use unsigned quantization. Assumes the signal is positive. Negative values are silently rounded to zero. Useful for images.

If this function is applied to a complex signal, it will be applied to the real and imaginary part separately.

9.2 Window functions

9.2.1 FIRWIN - FIR window

Usage

```
g=firwin(name,M);
g=firwin(name,M,...);
g=firwin(name,x);
```

Description

`firwin(name, M)` will return an FIR window of length M of type *name*.

All windows are symmetric and generate zero delay and zero phase filters. They can be used for the Wilson and WMDCT transform, except when noted otherwise.

`firwin(name, x)` where x is a vector will sample the window definition as the specified points. The normal sampling interval for the windows is $-.5 < x < .5$.

In the following PSL means "Peak Sidelobe level", and the main lobe width is measured in normalized frequencies.

If a window g forms a "partition of unity" (PU) it means specifically that:

```
g+fftshift(g)==ones(L,1);
```

A PU can only be formed if the window length is even, but some windows may work for odd lengths anyway.

If a window is the square root of a window that forms a PU, the window will generate a tight Gabor frame / orthonormal Wilson/WMDCT basis if the number of channels is less than M .

The windows available are:

'hann'	von Hann window. Forms a PU. The Hann window has a mainlobe width of $8/M$, a PSL of -31.5 dB and decay rate of 18 dB/Octave.
'sine'	Sine window. This is the square root of the Hanning window. The sine window has a mainlobe width of $8/M$, a PSL of -22.3 dB and decay rate of 12 dB/Octave. Aliases: ' <code>'cosine'</code> ', ' <code>'sqrthann'</code> '
'rect'	(Almost) rectangular window. The rectangular window has a mainlobe width of $4/M$, a PSL of -13.3 dB and decay rate of 6 dB/Octave. Forms a PU if the order is odd. Alias: ' <code>'square'</code> '

'tria'	(Almost) triangular window. Forms a PU. Alias: ' <code>bartlett</code> '
'sqrttria'	Square root of the triangular window.
'hamming'	Hamming window. Forms a PU that sums to 1.08 instead of 1.0 as usual. The Hamming window has a mainlobe width of $8/M$, a PSL of -42.7 dB and decay rate of 6 dB/Octave. This window should not be used for a Wilson basis, as a reconstruction window cannot be found by <code>wildual</code> .
'blackman'	Blackman window. The Blackman window has a mainlobe width of $12/M$, a PSL of -58.1 dB and decay rate of 18 dB/Octave.
'blackman2'	Alternate Blackman window. This window has a mainlobe width of $12/M$, a PSL of -68.24 dB and decay rate of 6 dB/Octave.
'itersine'	Iterated sine window. Generates an orthonormal Wilson/WMDCT basis. This window is described in Wesfreid and Wickerhauser (1993) and is used in the ogg sound codec. Alias: ' <code>ogg</code> '
'nuttall'	Nuttall window. The Nuttall window has a mainlobe width of $16/M$, a PSL of -93.32 dB and decay rate of 18 dB/Octave.
'nuttall10'	2-term Nuttall window with 1 continuous derivative. Alias: ' <code>hann</code> ', ' <code>hanning</code> '.
'nuttall01'	2-term Nuttall window with 0 continuous derivatives. This is a slightly improved Hamming window. It has a mainlobe width of $8/M$, a PSL of -43.19 dB and decay rate of 6 dB/Octave.
'nuttall20'	3-term Nuttall window with 3 continuous derivatives. The window has a mainlobe width of $12/M$, a PSL of -46.74 dB and decay rate of 30 dB/Octave.
'nuttall11'	3-term Nuttall window with 1 continuous derivative. The window has a mainlobe width of $12/M$, a PSL of -64.19 dB and decay rate of 18 dB/Octave.
'nuttall02'	3-term Nuttall window with 0 continuous derivatives. The window has a mainlobe width of $12/M$, a PSL of -71.48 dB and decay rate of 6 dB/Octave.
'nuttall30'	4-term Nuttall window with 5 continuous derivatives. The window has a mainlobe width of $16/M$, a PSL of -60.95 dB and decay rate of 42 dB/Octave.
'nuttall21'	4-term Nuttall window with 3 continuous derivatives. The window has a mainlobe width of $16/M$, a PSL of -82.60 dB and decay rate of 30 dB/Octave.
'nuttall12'	4-term Nuttall window with 1 continuous derivatives. Alias: ' <code>nuttall</code> '.
'nuttall03'	4-term Nuttall window with 0 continuous derivatives. The window has a mainlobe width of $16/M$, a PSL of -98.17 dB and decay rate of 6 dB/Octave.

`firwin` understands the following flags at the end of the list of input parameters:

'shift',s	Shift the window by s samples. The value can be a fractional number.
'wp'	Output is whole point even. This is the default. It corresponds to a shift of $s = 0$.

'hp'	Output is half point even, as most Matlab filter routines. This corresponds to a shift of $s = -0.5$
'taper',t	Extend the window by a flat section in the middle. The argument t is the ratio of the rising and falling parts as compared to the total length of the window. The default value of 1 means no tapering. Accepted values lie in the range from 0 to 1.

Additionally, `firwin` accepts flags to normalize the output. Please see the help of `normalize`. Default is to use '`peak`' normalization, which is useful for using the output from `firwin` for windowing in the time-domain. For filtering in the time-domain, a normalization of '`1`' or '`area`' is preferable.

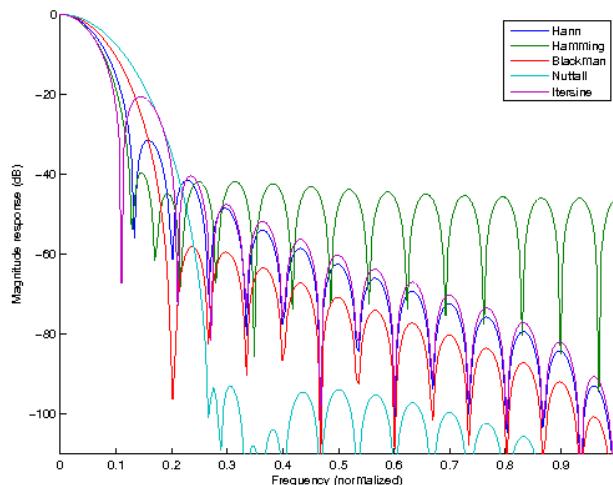
Examples:

The following plot shows the magnitude response for some common windows:

```
hold all;
L=30;
dr=110;

magresp(firwin('hanning',L,'1'),'fir','dynrange',dr);
magresp(firwin('hamming',L,'1'),'fir','dynrange',dr);
magresp(firwin('blackman',L,'1'),'fir','dynrange',dr);
magresp(firwin('nuttall',L,'1'),'fir','dynrange',dr);
magresp(firwin('itersine',L,'1'),'fir','dynrange',dr);

legend('Hann','Hamming','Blackman','Nuttall','Itersine');
```



References: [69], [42], [68], [95]

9.2.2 FIRKAISER - Kaiser-Bessel window

Usage

```
g=firkaiser(L,beta);
g=firkaiser(L,beta,...);
```

Description

`firkaiser(L,beta)` computes the Kaiser-Bessel window of length L with parameter β . The smallest element of the window is set to zero when the window has an even length. This gives the window perfect whole-point even symmetry, and makes it possible to use the window for a Wilson basis.

`firkaiser` takes the following flags at the end of the input arguments:

'normal'	Normal Kaiser-Bessel window. This is the default.
'derived'	Derived Kaiser-Bessel window.
'wp'	Generate a whole point even window. This is the default.
'hp'	Generate half point even window.

Additionally, `firkaiser` accepts flags to normalize the output. Please see the help of `normalize`. Default is to use '`peak`' normalization.

References: [69]

9.2.3 FIR2LONG - Extend FIR window to LONG

Usage

```
g=fir2long(g,Llong);
```

Description

`fir2long(g,Llong)` will extend the FIR window `g` to a length `Llong` window by inserting zeros. Note that this is a slightly different behaviour than `middlepad`.

`fir2long` can also be used to extend a FIR window to a longer FIR window, for instance in order to satisfy the usual requirement that the window length should be divisible by the number of channels.

If the input to `fir2long` is a cell, `fir2long` will recurse into the cell array.

9.2.4 LONG2FIR - Cut LONG window to FIR

Usage

```
g=long2fir(g,L);
```

Description

`long2fir(g,L)` will cut the LONG window `g` to a length `L` FIR window by cutting out the middle part. Note that this is a slightly different behaviour than `middlepad`.

`long2fir(g,L,'wp')` or `long2fir(g,L,'hp')` does the same assuming the input window is a whole-point even or half-point even window, respectively.

9.3 Filtering

9.3.1 FIRFILTER - Construct an FIR filter

Usage

```
g=firfilter(name,M);
g=firfilter(name,M,...);
```

Description

`firfilter(name,M)` creates an FIR filter of length `M`. This is exactly the same as calling `firwin`. The name must be one of the accepted window types of `firwin`.

`firfilter(name,M,fc)` constructs a filter with a centre frequency of `fc` measured in normalized frequencies.

If one of the inputs is a vector, the output will be a cell array with one entry in the cell array for each element in the vector. If more input are vectors, they must have the same size and shape and the filters will be generated by stepping through the vectors. This is a quick way to create filters for filterbank and ufilterbank.

`firfilter` accepts the following optional parameters:

'fs',fs	If the sampling frequency fs is specified then the length M is specified in seconds and the centre frequency fc in Hz.
'complex'	Make the filter complex valued if the centre frequency is non-zero. This is the default.
'real'	Make the filter real-valued if the centre frequency is non-zero.
'delay',d	Set the delay of the filter. Default value is zero.
'causal'	Create a causal filter starting at the first sample. If specified, this flag overwrites the delay setting.

It is possible to normalize the impulse response of the filter by passing any of the flags from the normalize function. The default normalization is '`energy`'.

The filter can be used in the `pfilt` routine to filter a signal, or in can be placed in a cell-array for use with filterbank or `ufilterbank`.

9.3.2 BLFILTER - Construct a band-limited filter

Usage

```
g=blfilter(winname, fsupp, fc);
g=blfilter(winname, fsupp, fc, ...);
```

Input parameters

winname	Name of prototype
fsupp	Support length of the prototype

Description

`blfilter(winname, fsupp)` constructs a band-limited filter. The parameter `winname` specifies the shape of the frequency response. The name must be one of the shapes accepted by `firwin`. The support of the frequency response measured in normalized frequencies is specified by `fsupp`.

`blfilter(winname, fsupp, fc)` constructs a filter with a centre frequency of `fc` measured in normalized frequencies.

If one of the inputs is a vector, the output will be a cell array with one entry in the cell array for each element in the vector. If more input are vectors, they must have the same size and shape and the filters will be generated by stepping through the vectors. This is a quick way to create filters for filterbank and `ufilterbank`.

`blfilter` accepts the following optional parameters:

'fs',fs	If the sampling frequency fs is specified then the support $fsupp$ and the centre frequency fc is specified in Hz.
'complex'	Make the filter complex valued if the centre frequency is non-zero.necessary. This is the default.
'real'	Make the filter real-valued if the centre frequency is non-zero.
'delay',d	Set the delay of the filter. Default value is zero.
'scal',s	Scale the filter by the constant s . This can be useful to equalize channels in a filter bank.
'pedantic'	Force window frequency offset (<code>g.foff</code>) to a subsample precision by a subsample shift of the <code>firwin</code> output.

It is possible to normalize the transfer function of the filter by passing any of the flags from the normalize function. The default normalization is '`energy`'.

The filter can be used in the `pfilt` routine to filter a signal, or in can be placed in a cell-array for use with filterbank or `ufilterbank`.

Output format:

The output g from `blfilter` is a structure. This type of structure can be used to describe any bandlimited filter defined in terms of its transfer function. The structure contains the following fields:

--Xg_DOT_H	This is an anonymous function taking the transform length L as input and producing the bandlimited transfer function in the form of a vector.
--Xg_DOT_foff	This is an anonymous function taking the transform length L as input and procing the frequency offset of H as an integer. The offset is the value of the lowest frequency of H measured in frequency samples. f_{off} is used to position the bandlimited tranfer function stored in H correctly when multiplying in the frequency domain.
--Xg_DOT_delay	This is the desired delay of the filter measured in samples.
--Xg_DOT_realonly	This is an integer with value 1 if the filter defined a real-valued filter. In this case, the bandlimited transfer function H will be mirrored from the positive frequencies to the negative frequencies. If the filter is a natural lowpass filter correctly centered around 0 , <code>realonly</code> does not need to be 1 .
--Xg_DOT_fs	The intended sampling frequency. This is an optional parameter that is only used for plotting and visualization.

9.3.3 WARPEDBLFILTER - Construct a warped band-limited filter**Usage**

```
g=warpedblfilter(winname,fsupp,fc,fs,freqtoscale,scaletofreq);
```

Input parameters

winname	Name of prototype.
fsupp	Support length of the prototype (in scale units).
fc	Centre frequency (in Hz).
fs	Sampling rate
freqtoscale	Function handle to convert Hz to scale units
scaletofreq	Function to convert scale units into Hz.

Output parameters

g	Filter definition, see <code>blfilter</code> .
----------	--

Description

`warpedblfilter(winname,fsupp,fc,fs,freqtoscale,scaletofreq)` constructs a band-limited filter that is warped on a given frequency scale. The parameter `winname` specifies the basic shape of the frequency response. The name must be one of the shapes accepted by `firwin`. The support of the frequency response measured on the selected frequency scale is specified by `fsupp`, the centre frequency by `fc` and the scale by the function handle `freqtoscale` of a function that converts Hz into the choosen scale and `scaletofreq` doing the inverse.

If one of the inputs is a vector, the output will be a cell array with one entry in the cell array for each element in the vector. If more input are vectors, they must have the same size and shape and the the filters will be generated by stepping through the vectors. This is a quick way to create filters for filterbank and ufilterbank.

`warpedblfilter` accepts the following optional parameters:

'complex'	Make the filter complex valued if the centre frequency is non-zero. This is the default.
'real'	Make the filter real-valued if the centre frequency is non-zero.
'symmetric'	The filters with $fc < 0$ (or $fc > fs/2$) will be created on the positive frequencies and mirrored. This allows using <i>freqtoscale</i> defined only for the positive numbers.
'delay',d	Set the delay of the filter. Default value is zero.
'scal',s	Scale the filter by the constant <i>s</i> . This can be useful to equalize channels in a filterbank.

It is possible to normalize the transfer function of the filter by passing any of the flags from the normalize function. The default normalization is '*energy*'.

The filter can be used in the pfilt routine to filter a signal, or it can be placed in a cell-array for use with filterbank or ufilterbank.

The output format is the same as that of blfilter.

9.3.4 PFILT - Apply filter with periodic boundary conditions

Usage

```
h=pfilt(f,g);
h=pfilt(f,g,a,dim);
```

Description

pfilt(*f*,*g*) applies the filter *g* to the input *f*. If *f* is a matrix, the filter is applied along each column.

pfilt(*f*,*g*,*a*) does the same, but downsamples the output keeping only every *a*'th sample (starting with the first one).

pfilt(*f*,*g*,*a*,*dim*) filters along dimension *dim*. The default value of [] means to filter along the first non-singleton dimension.

The filter *g* can be a vector, in which case the vector is treated as a zero-delay FIR filter.

The filter *g* can be a cell array. The following options are possible:

- If the first element of the cell array is the name of one of the windows from firwin, the whole cell array is passed onto firfilter.
- If the first element of the cell array is '*bl*', the rest of the cell array is passed onto blfilter.
- If the first element of the cell array is '*pgauss*', '*psech*', the rest of the parameters is passed onto the respective function. Note that you do not need to specify the length *L*.

The coefficients obtained from filtering a signal *f* by a filter *g* are defined by

$$c(n+1) = \sum_{l=0}^{L-1} f(l+1) g(an - l + 1)$$

where *an* - *l* is computed modulo *L*.

9.3.5 MAGRESP - Magnitude response plot of window

Usage

```
magresp(g,...);
magresp(g,fs,...);
magresp(g,fs,dynrange,...);
```

Description

`magresp(g)` will display the magnitude response of the window on a log scale (dB);

`magresp(g, fs)` does the same for windows that are intended to be used with signals with sampling rate fs . The x-axis will display Hz.

`magresp(g, fs, dynrange)` will limit the dynamic range (see below).

`magresp` takes the following parameters at the end of the line of input arguments.

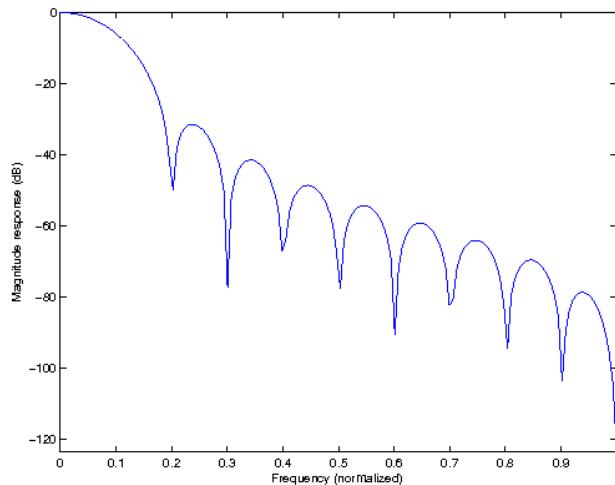
'dynrange',r	Limit the dynamic range of the plot to r dB.
'fir'	Indicate that the input is an FIR window. <code>magresp</code> will zero-extend the window to display a smooth magnitude response.
'L',L	Zero-extend the window to length L.
'posfreq'	Show only positive frequencies.
'nf'	Show also negative frequencies
'autoposfreq'	Show positive frequencies for real-valued signals, otherwise show also the negative frequencies. This is the default.
'opts',op	Pass options onto the plot command. The extra options <i>op</i> are specified as a cell array

In addition to these flags, it is possible to specify any of the normalization flags from `normalize` to normalize the input before calculation of the magnitude response. Specifying '`1`' or '`area`' will display a magnitude response which peaks at 0 dB.

Examples:

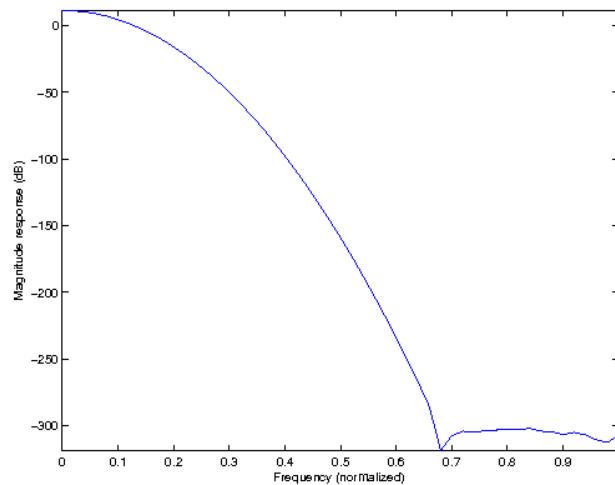
The following will display the magnitude response of a Hann window of length 20 normalized to a peak of 0 dB:

```
magresp({'hann', 20}, '1');
```



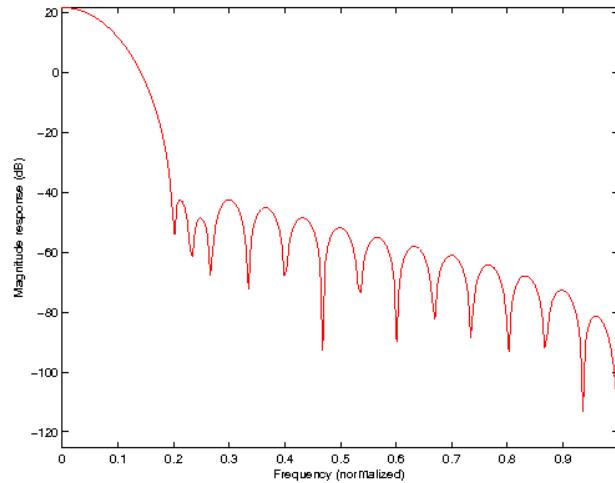
The following will display the magnitude response of a Gaussian window of length 100:

```
magresp('gauss', 'L', 100)
```



The following passes additional options to the plot command to draw in red:

```
magresp({'nuttall11',30}, 'opts', {'r'});
```



9.3.6 TRANSFERFUNCTION - The transferfunction of a filter

Usage

```
H=transferfunction(g,L);
```

Description

`transferfunction(g,L)` computes the transferfunction of length L of the filter defined by g .

9.3.7 PGRPDELAY - Group delay of a filter with periodic boundaries

Usage

```
ggd = pgrpdelay(g,L);
```

Description

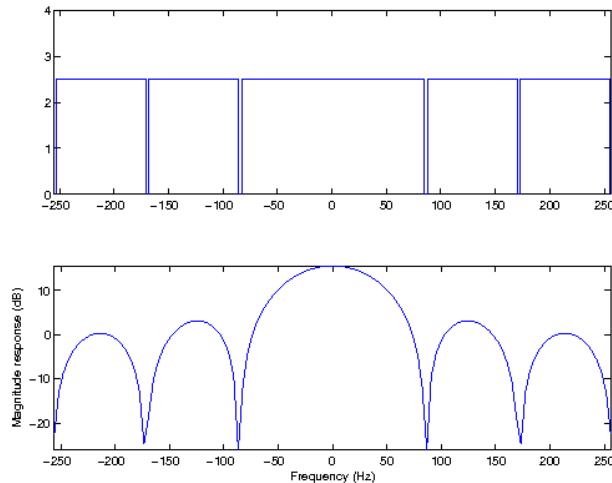
`pgrpdelay(g, L)` computes group delay of filter g as a negative derivative of the phase frequency response of filter g assuming periodic (cyclic) boundaries i.e. the delay may be a negative number. The derivative is calculated using the second order centered difference approximation. The resulting group delay is in samples.

Example:

The following example shows a group delay of causal, moving average 6tap FIR filter and it's magnitude frequency response for comparison. The dips in the group delay correspond to places where modulus of the frequency response falls to zero.:

```
g = struct(struct('h',ones(6,1),'offset',0));
L = 512;
figure(1);
subplot(2,1,1);
plot(-L/2+1:L/2,fftshift(pgrpdelay(g,512)));
axis tight; ylim([0,4]);

subplot(2,1,2);
magresp(g,L,'nf');
```



9.4 Ramping

9.4.1 RAMPUP - Rising ramp function

Usage

```
outsig=rampup(L);
```

Description

`rampup(L)` will return a rising ramp function of length L . The ramp is a sinusoid starting from zero and ending at one. The ramp is centered such that the first element is always 0 and the last element is not quite 1, such that the ramp fits with following ones.

`rampup(L,wintype)` will use another window for ramping. This may be any of the window types from firwin. Please see the help on firwin for more information. The default is to use a piece of the Hann window.

9.4.2 RAMPDOWN - Falling ramp function

Usage

```
outsig=rampdown(siglen);
```

Description

`rampdown(siglen)` will return a falling ramp function of length *siglen*. The ramp is a sinusoid starting from one and ending at zero. The ramp is centered such that the first element is always one and the last element is not quite zero, such that the ramp fits with following zeros.

`rampdown(L,wintype)` will use another window for ramping. This may be any of the window types from firwin. Please see the help on firwin for more information. The default is to use a piece of the Hann window.

9.4.3 RAMPSIGNAL - Ramp signal

Usage

```
outsig=rampsignal(insig,L);
```

Description

`rampsignal(insig,L)` applies a ramp function of length *L* to the beginning and the end of the input signal. The default ramp is a sinusoide starting from zero and ending at one (also known as a cosine squared ramp).

If *L* is scalar, the starting and ending ramps will be of the same length. If *L* is a vector of length 2, the first entry will be used for the rising ramp, and the second for the falling.

If the input is a matrix or an N-D array, the ramp will be applied along the first non-singleton dimension.

`rampsignal(insig)` will use a ramp length of half the signal.

`rampsignal(insig,L,wintype)` will use another window for ramping. This may be any of the window types from firwin. Please see the help on firwin for more information. The default is to use a piece of the Hann window.

`rampsignal` accepts the following optional parameters:

'dim',d	Apply the ramp along dimension d. The default value of [] means to use the first non-singleton dimension.
----------------	---

9.5 Thresholding methods

9.5.1 THRESH - Coefficient thresholding

Usage

```
x=thresh(x,lambda,...);
[x,N]=thresh(x,lambda,...);
```

Description

`thresh(x,lambda)` will perform hard thresholding on *x*, i.e. all elements with absolute value less than scalar *lambda* will be set to zero.

`thresh(x,lambda,'soft')` will perform soft thresholding on *x*, i.e. *lambda* will be subtracted from the absolute value of every element of *x*.

The *lambda* parameter can also be a vector with number of elements equal to `numel(xi)` or it can be a numeric array of the same shape as *xi*. *lambda* is then applied element-wise and in a column major order if *lambda* is a vector.

`[x,N]=thresh(x,lambda)` additionally returns a number *N* specifying how many numbers were kept.

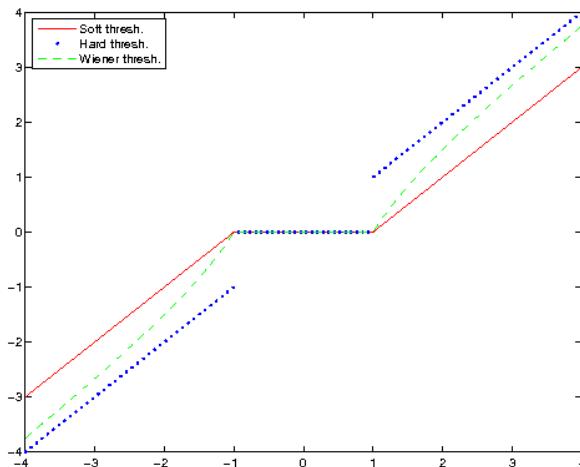
`thresh` takes the following flags at the end of the line of input arguments:

'hard'	Perform hard thresholding. This is the default.
'wiener'	Perform empirical Wiener shrinkage. This is in between soft and hard thresholding.
'soft'	Perform soft thresholding.
'full'	Returns the output as a full matrix. This is the default.
'sparse'	Returns the output as a sparse matrix.

The function `wthresh` in the Matlab Wavelet toolbox implements some of the same functionality.

The following code produces a plot to demonstrate the difference between hard and soft thresholding for a simple linear input:

```
t=linspace(-4, 4, 100);
plot(t,thresh(t,1,'soft'),'r',...
      t,thresh(t,1,'hard'),'b',...
      t,thresh(t,1,'wiener'),'--g');
legend('Soft thresh.', 'Hard thresh.', 'Wiener thresh.', 'Location','NorthWest');
```



References: [56], [36]

9.5.2 LARGESTR - Keep fixed ratio of largest coefficients

Usage

```
xo=largestr(x,p);
xo=largestr(x,p,mtype);
[xo,N]=largestr(...);
```

Description

`largestr(x,p)` returns an array of the same size as x keeping the fraction p of the coefficients. The coefficients with the largest magnitude are kept.

`[xo, n]=largestr(xi, p)` additionally returns the number of coefficients kept.

Note: If the function is used on coefficients coming from a redundant transform or from a transform where the input signal was padded, the coefficient array will be larger than the original input signal. Therefore, the number of coefficients kept might be higher than expected.

`largestr` takes the following flags at the end of the line of input arguments:

'hard'	Perform hard thresholding. This is the default.
---------------	---

'wiener'	Perform empirical Wiener shrinkage. This is in between soft and hard thresholding.
'soft'	Perform soft thresholding.
'full'	Returns the output as a full matrix. This is the default.
'sparse'	Returns the output as a sparse matrix.

Note: If soft- or Wiener thresholding is selected, one less coefficient will actually be returned. This is caused by that coefficient being set to zero.

References: [60]

9.5.3 LARGESTN - Keep N largest coefficients

Usage

```
xo=largestn(x,N);
xo=largestn(x,N,mtype);
```

Description

largestn(x,N) returns an array of the same size as *x* keeping the *N* largest coefficients.

largestn takes the following flags at the end of the line of input arguments:

'hard'	Perform hard thresholding. This is the default.
'wiener'	Perform empirical Wiener shrinkage. This is in between soft and hard thresholding.
'soft'	Perform soft thresholding.
'full'	Returns the output as a full matrix. This is the default.
'sparse'	Returns the output as a sparse matrix.

If the coefficients represents a signal expanded in an orthonormal basis then this will be the best *N*-term approximation.

Note: If soft- or Wiener thresholding is selected, only *N* – 1 coefficients will actually be returned. This is caused by the *N*'th coefficient being set to zero.

References: [60]

9.5.4 DYNLIMIT - Limit the dynamical range of the input

Usage

```
xo=dynlimit(xi,dynrange);
```

Description

dynlimit(xi,dynrange) will threshold the input such that the difference between the maximum and minimum value of *xi* is exactly *dynrange*.

9.5.5 GROUPTHRESH - Group thresholding

Usage

```
xo=groupthresh(xi,lambda);
```

Description

`groupthresh(x, lambda)` performs group thresholding on x , with threshold $lambda$. x must be a two-dimensional array, the first dimension labelling groups, and the second one labelling members. This means that the groups are the row vectors of the input (the vectors along the 2nd dimension).

Several types of grouping behaviour are available:

- `groupthresh(x, lambda, 'group')` shrinks all coefficients within a given group according to the value of the l^2 norm of the group in comparison to the threshold $lambda$. This is the default.
- `groupthresh(x, lambda, 'elite')` shrinks all coefficients within a given group according to the value of the l^1 norm of the group in comparison to the threshold value $lambda$.

`groupthresh(x, lambda, dim)` chooses groups along dimension dim . The default value is $dim = 2$.

`groupthresh` accepts all the flags of `thresh` to choose the thresholding type within each group and the output type (full / sparse matrix). Please see the help of `thresh` for the available options. Default is to use soft thresholding and full matrix output.

References: [54], [53], [98]

9.6 Image processing

9.6.1 RGB2JPEG - Converts from RGB format to the YCbCr format used by JPEG

Usage

```
YCbCr = rgb2jpeg(RGB);
```

Input parameters

RGB	3d data-cube, containing RGB information of the image
------------	---

Output parameters

YCbCr	3d data-cube, containing the YCbCr information of the image
--------------	---

Description

'`rgb2jpeg(RGB)`' performs a transformation of the 3d data-cube RGB with dimensions $N \times M \times 3$, which contains information of the colours "red", "green" and "blue". The output variable $YCbCr$ is a 3d data-cube of the same size containing information about "luminance", "chrominance blue" and "chrominance red". The output will be of the `uint8` type.

See <http://en.wikipedia.org/wiki/YCbCr> and <http://de.wikipedia.org/wiki/JPEG>.

Examples:

In the following example, the Lichtenstein test image is split into its three components. The very first subplot is the original image:

```
f=lichtenstein;
f_jpeg=rgb2jpeg(f);
subplot(2,2,1);
image(f);axis('image');

Ymono=zeros(512,512,3,'uint8');
Ymono(:,:,1)=f_jpeg(:,:,1);
```

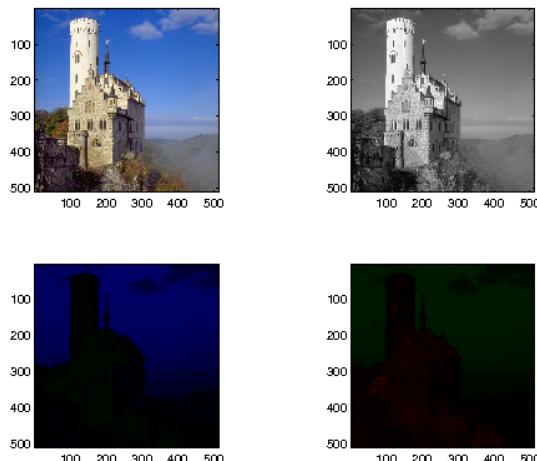
```

Ymono(:,:,2:3)=128;
fmono=jpeg2rgb(Ymono);
subplot(2,2,2);
image(fmono);axis('image');

Cbmono=zeros(512,512,3,'uint8');
Cbmono(:,:,2)=f_jpeg(:,:,2);
Cbmono(:,:,3)=128;
fmono=jpeg2rgb(Cbmono);
subplot(2,2,3);
image(fmono);axis('image');

Crmono=zeros(512,512,3,'uint8');
Crmono(:,:,3)=f_jpeg(:,:,3);
Crmono(:,:,2)=128;
fmono=jpeg2rgb(Crmono);
subplot(2,2,4);
image(fmono);axis('image');

```



9.6.2 JPEG2RGB - Converts from RGB format to YCbCr format

Usage

```
RGB = jpeg2rgb(YCbCr);
```

Input parameters

YCbCr 3d data-cube, containing the YCbCr information of the image

Output parameters

RGB 3d data-cube, containing RGB information of the image

Description

'jpeg2rgb(YCbCr)' performs a transformation of the 3d data-cube *YCbCr* with dimensions $N \times M \times 3$, which contains information of "luminance", "chrominance blue" and "chrominance red". The output variable *RGB* is a 3d data-cube of the same size containing information about the colours "red", "green" and "blue". The output will be of the `uint8` type.

For more information, see <http://en.wikipedia.org/wiki/YCbCr> and <http://de.wikipedia.org/wiki/JPEG>

9.7 Tools for OFDM

9.7.1 QAM4 - Quadrature amplitude modulation of order 4

Usage

```
xo=qam4(xi);
```

Description

`qam4(xi)` converts a vector of 0's and 1's into the complex roots of unity (QAM4 modulation). Every 2 input coefficients are mapped into 1 output coefficient.

9.7.2 IQAM4 - Inverse QAM of order 4

```
iqam4(xi)
```

`iqam4(xi)` demodulates a signal mapping the input coefficients to the closest complex root of unity, and returning the associated bit pattern. This is the inverse operation of `qam4`.

Chapter 10

LTFAT - Simple auditory processing

10.1 Plots

10.1.1 SEMIAUDPLOT - 2D plot on auditory scale

Usage

```
h=semiaudplot(x,y);
```

Description

semiaudplot (x, y) plots the data (x, y) on an auditory scale. By default the values of the x-axis will be shown on the Erb-scale.

semiaudplot takes the following parameters at the end of the line of input arguments:

'x'	Make the x-axis use the auditory scale. This is the default.
'y'	Make the y-axis use the auditory scale.
'opts',c	Pass options stored in a cell array onto the plot function.

In addition to these parameters, the auditory scale can be specified. All scales supported by freqtoaud are supported. The default is to use the erb-scale.

10.2 Auditory scales

10.2.1 AUDTOFREQ - Converts auditory units to frequency (Hz)

Usage

```
freq = audtofreq(aud);
```

Description

audtofreq ($aud, scale$) converts values on the selected auditory scale to frequencies measured in Hz.

See the help on freqtoaud to get a list of the supported values of the $scale$ parameter. If no scale is given, the erb-scale will be selected by default.

10.2.2 FREQTOAUD - Converts frequencies (Hz) to auditory scale units

Usage

```
aud = freqtoaud(freq,scale);
```

Description

`freqtoaud(freq, scale)` converts values on the frequency scale (measured in Hz) to values on the selected auditory scale. The value of the parameter *scale* determines the auditory scale:

'erb'	A distance of 1 erb is equal to the equivalent rectangular bandwidth of the auditory filters at that point on the frequency scale. The scale is normalized such that 0 erbs corresponds to 0 Hz. The width of the auditory filters were determined by a notched-noise experiment. The erb scale is defined in Glasberg and Moore (1990). This is the default.
'mel'	The mel scale is a perceptual scale of pitches judged by listeners to be equal in distance from one another. The reference point between this scale and normal frequency measurement is defined by equating a 1000 Hz tone, 40 dB above the listener's threshold, with a pitch of 1000 mels. The mel-scale is defined in Stevens et. al (1937).
'mel1000'	Alternative definition of the mel scale using a break frequency of 1000 Hz. This scale was reported in Fant (1968).
'bark'	The bark-scale is originally defined in Zwicker (1961). A distance of 1 on the bark scale is known as a critical band. The implementation provided in this function is described in Traunmuller (1990).
'erb83'	This is the original defintion of the erb scale given in Moore. et al. (1983).
'freq'	Return the frequency in Hz.

If no flag is given, the erb-scale will be selected.

References: [89], [100], [30], [37], [92], [65]

10.2.3 AUDSPACE - Equidistantly spaced points on auditory scale**Usage**

```
y=audspace(flow,fhigh,n,scale);
```

Description

`audspace(flow, fhigh, n, scale)` computes a vector of length *n* containing values equidistantly scaled on the selected auditory scale between the frequencies *flow* and *fhigh*. All frequencies are specified in Hz.

See the help on `freqtoaud` to get a list of the supported values of the *scale* parameter.

`[y, bw]=audspace(. . .)` does the same but outputs the bandwidth between each sample measured on the selected scale.

10.2.4 AUDSPACEBW - Auditory scale points specified by bandwidth**Usage**

```
y=audspacebw(flow,fhigh,bw,hitme);
y=audspacebw(flow,fhigh,bw);
y=audspacebw(flow,fhigh);
[y,n]=audspacebw( . . . );
```

Description

`audspacebw(flow, fhigh, bw, scale)` computes a vector containing values equistantly scaled between frequencies `flow` and `fhigh` on the selected auditory scale. All frequencies are specified in Hz. The distance between two consecutive values is `bw` on the selected scale, and the points will be centered on the scale between `flow` and `fhigh`.

See the help on `freqtoaud` to get a list of the supported values of the `scale` parameter.

`audspacebw(flow, fhigh, bw, hitme, scale)` will do as above, but one of the points is guaranteed to be the frequency `hitme`.

`[y, n] = audspacebw(...)` additionally returns the number of points `n` in the output vector `y`.

10.2.5 ERBTOFREQ - Converts erb units to frequency (Hz)**Usage**

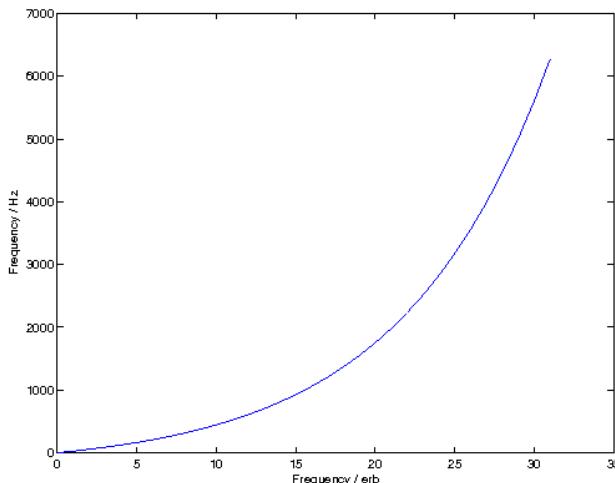
```
freq = erbtufreq(erb);
```

Description

This is a wrapper around `autofreq` that selects the erb-scale. Please see the help on `autofreq` for more information.

The following figure shows the corresponding frequencies for erb values up to 31:

```
erbs=0:31;
freqs=erbtufreq(erbs);
plot(erbs,freqs);
xlabel('Frequency / erb');
ylabel('Frequency / Hz');
```

**10.2.6 FREQTOERB - Converts frequencies (Hz) to erbs****Usage**

```
erb = freqtoerb(freq);
```

Description

This is a wrapper around `freqtoaud` that selects the erb-scale. Please see the help on `freqtoaud` for more information.

10.2.7 ERBSPACE - Equidistantly spaced points on erb-scale

Usage

```
y=erbspace(flow,fhigh,n);
```

Description

This is a wrapper around audspace that selects the erb-scale. Please see the help on audspace for more information.

10.2.8 ERBSPACEBW - Erbscale points specified by bandwidth

Usage

```
y=erbspacebw(flow,fhigh,bw,hitme);
y=erbspacebw(flow,fhigh,bw);
y=erbspacebw(flow,fhigh);
```

Description

This is a wrapper around audspacebw that selects the erb-scale. Please see the help on audspacebw for more information.

10.2.9 AUDFILTBW - Bandwidth of auditory filter

Usage

```
bw = audfiltbw(fc)
```

Description

`audfiltbw(fc)` returns the critical bandwidth of the auditory filter at center frequency fc defined in equivalent rectangular bandwidth. The function uses the relation:

$$bw = 24.7 + \frac{fc}{9.265}$$

as estimated in Glasberg and Moore (1990).

`audfiltbw(fc, 'bark')` returns the critical bandwidth at fc according to the Bark scale using the relation:

$$bw = 25 + 75(1 + 1.4 \times 10^{-6}fc^2)^{0.69}$$

as estimated by Zwicker and Terhardt (1980).

References: [37], [101]

10.3 Range compression

10.3.1 RANGECOMPRESS - Compress the dynamic range of a signal

Usage

```
[outsig, sigweight] = rangecompress(insig,mu);
```

Description

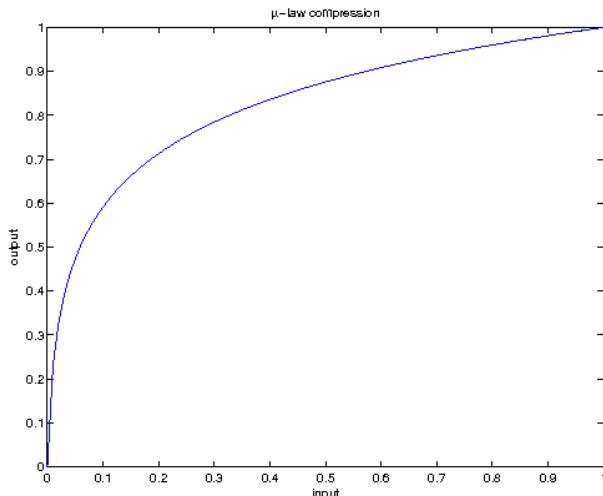
`[outsig, sigweight] = rangecompress(insig, mu)` range-compresses the input signal *insig* using μ -law range-compression with parameter *mu*.

`rangecompress` takes the following optional arguments:

'mulaw'	Do mu-law compression, this is the default.
'alaw'	Do A-law compression.
'mu',mu	μ -law parameter. Default value is 255.
'A',A	A-law parameter. Default value is 87.7.

The following plot shows how the output range is compressed for input values between 0 and 1:

```
x=linspace(0,1,100);
xc=rangecompress(x);
plot(x,xc);
xlabel('input');
ylabel('output');
title('\mu-law compression');
```



References: [49]

10.3.2 RANGEEXPAND - Expand the dynamic range of a signal**Usage**

```
sig = rangeexpand(insig,mu,sigweight);
```

Description

`rangeexpand(insig,mu,sigweight)` inverts a previously applied μ -law companding to the signal *insig*. The parameters *mu* and *sigweight* must match those from the call to `rangecompress`.

`rangeexpand` takes the following optional arguments:

'mulaw'	Do mu-law compression, this is the default.
'alaw'	Do A-law compression.
'mu',mu	μ -law parameter. Default value is 255.

References: [49]

10.4 Auditory filters

10.4.1 GAMMATONEFIR - Gammatone filter coefficients

Usage

```
b = gammatonefir(fc, fs, n, betamul);
b = gammatonefir(fc, fs, n);
b = gammatonefir(fc, fs);
```

Input parameters

fc	center frequency in Hz.
fs	sampling rate in Hz.
n	max. filter length.
beta	bandwidth of the filter.

Output parameters

b	FIR filters as an cell-array of structs.
----------	--

Description

`gammatonefir(fc, fs, n, betamul)` computes the filter coefficients of a digital FIR gammatone filter with length at most n , center frequency fc , 4th order rising slope, sampling rate fs and bandwidth determined by $betamul$. The bandwidth β of each filter is determined as $betamul$ times audfiltbw of the center frequency of corresponding filter. The actual length of the impulse response depends on fc (the filter is longer for low center frequencies), fs and $betamul$ but it is never bigger than n .

`gammatonefir(fc, fs, n)` will do the same but choose a filter bandwidth according to Glasberg and Moore (1990). $betamul$ is chosen to be 1.0183.

`gammatonefir(fc, fs)` will do as above and choose a sufficiently long filter to accurately represent the lowest subband channel.

If fc is a vector, each entry of fc is considered as one center frequency, and the corresponding coefficients are returned as column vectors in the output.

The impulse response of the gammatone filter is given by

$$g(t) = at^{4-1} \cos(2\pi \cdot fc \cdot t) e^{-2\pi\beta \cdot t}$$

The gammatone filters as implemented by this function generate complex valued output, because the filters are modulated by the exponential function. Using `real` on the output will give the coefficients of the corresponding cosine modulated filters.

To create the filter coefficients of a 1-erb spaced filter bank using gammatone filters use the following construction:

```
g = gammatonefir(erbspacebw(flow, fhigh), fs);
```

References: [5], [37]

Chapter 11

LTFAT - Signals

11.1 Signal generators

11.1.1 CTESTFUN - Complex 1-D test function

Usage

```
ftest=ctestfun(L);
```

Description

`ctestfun(L)` returns a test signal consisting of a superposition of a chirp and an indicator function.

11.1.2 NOISE - Stochastic noise generator

Usage

```
outsig = noise(siglen,nsigs,type);
```

Input parameters

siglen	Length of the noise (samples)
nsigs	Number of signals (default is 1)
type	type of noise. See below.

Output parameters

outsig	$\text{siglen} \times \text{nsigs}$ signal vector
---------------	---

Description

`noise(siglen,nsigs)` generates $nsigs$ channels containing white noise of the given type with the length of `siglen`. The signals are arranged as columns in the output. If only `siglen` is given, a column vector is returned.

`noise` takes the following optional parameters:

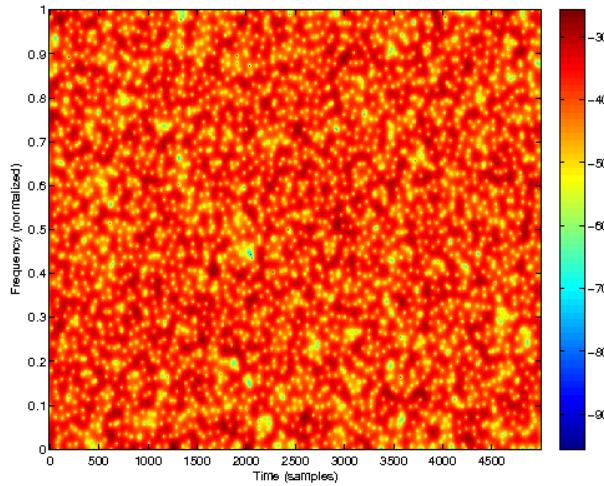
'white'	Generate white (gaussian) noise. This is the default.
'pink'	Generate pink noise.
'brown'	Generate brown noise.
'red'	This is the same as brown noise.

By default, the noise is normalized to have a unit energy, but this can be changed by passing a flag to normalize.

Examples:

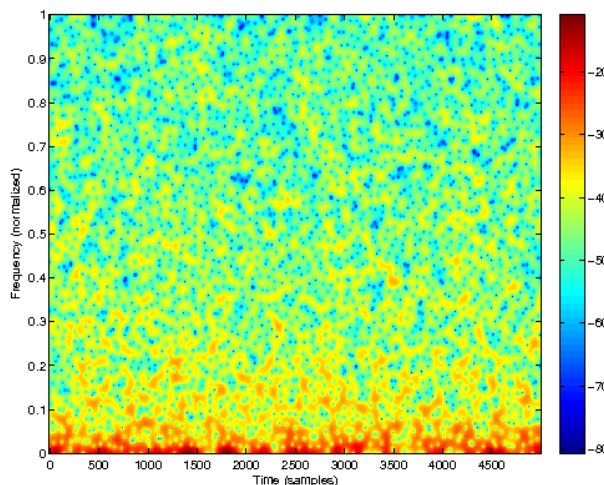
White noise in the time-frequency domain:

```
sgram(noise(5000,'white'),'dynrange',70);
```



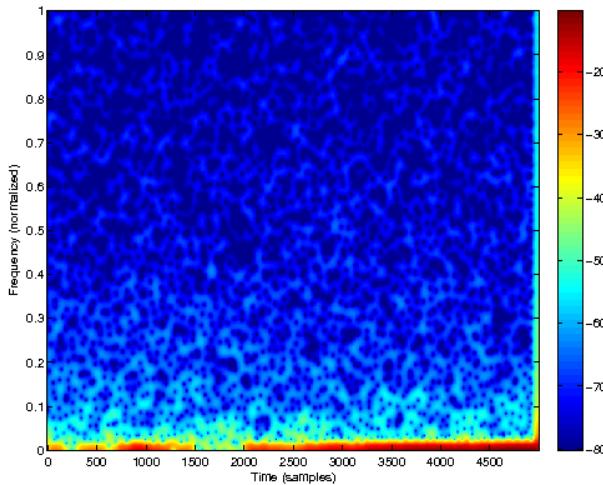
Pink noise in the time-frequency domain:

```
sgram(noise(5000,'pink'),'dynrange',70);
```



Brown/red noise in the time-frequency domain:

```
sgram(noise(5000,'brown'),'dynrange',70);
```



11.1.3 PINKNOISE - Generates a pink noise signal

Usage

```
outsig = pinknoise(siglen,nsigs);
```

Input parameters

siglen	Length of the noise (samples)
nsigs	Number of signals (default is 1)

Output parameters

outsig	$siglen \times nsigs$ signal vector
---------------	-------------------------------------

Description

`pinknoise(siglen,nsigs)` generates $nsigs$ channels containing pink noise ($1/f$ spectrum) with the length of $siglen$. The signals are arranged as columns in the output.

```
pinknoise is just a wrapper around noise(...,'pink');
```

11.1.4 EXPCHIRP - Exponential chirp

Usage

```
outsig=expchirp(L,fstart,fend)
```

Description

`expchirp(L,fstart,fend)` computes an exponential chirp of length L starting at frequency $fstart$ and ending at frequency $fend$. The frequencies are assumed to be normalized to the Nyquist frequency.

`expchirp` takes the following parameters at the end of the line of input arguments:

'fs',fs	Use a sampling frequency of fs Hz. If this option is specified, $fstart$ and $fend$ will be measured in Hz.
'phi',phi	Starting phase of the chirp. Default value is 0.
'fc',fc	Shift the chirp by fc in frequency. Default values is 0.

11.2 Sound signals.

11.2.1 BAT - Load the 'bat' test signal

Usage

```
s=bat;
```

Description

`bat` loads the 'bat' signal. It is a 400 samples long recording of a bat chirp sampled with a sampling period of 7 microseconds. This gives a sampling rate of 143 kHz.

`[sig, fs]=bat` additionally returns the sampling frequency fs .

The signal can be obtained from <http://dsp.rice.edu/software/bat-echolocation-chirp>

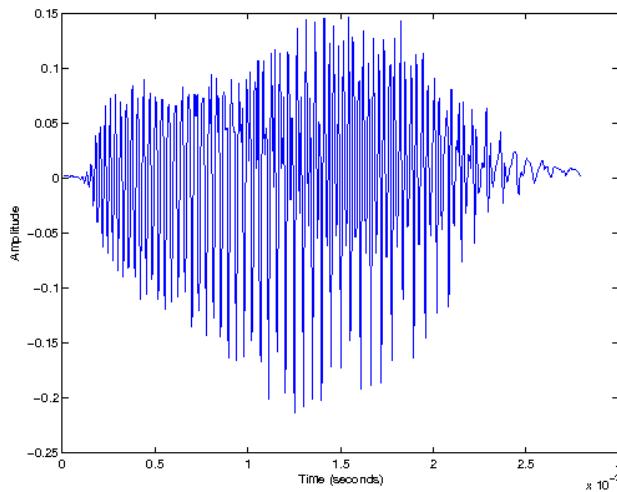
Please acknowledge use of this data in publications as follows:

The author wishes to thank Curtis Condon, Ken White, and Al Feng of the Beckman Institute of the University of Illinois for the bat data and for permission to use it in this paper.

Examples:

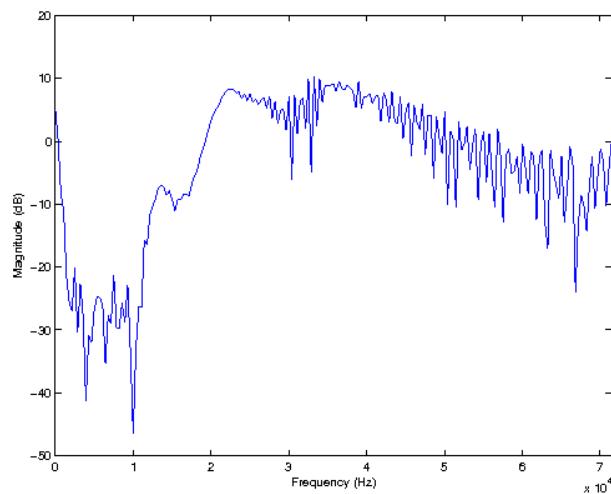
Plot of 'bat' in the time-domain:

```
plot((1:400)/143000,bat);
xlabel('Time (seconds)');
ylabel('Amplitude');
```



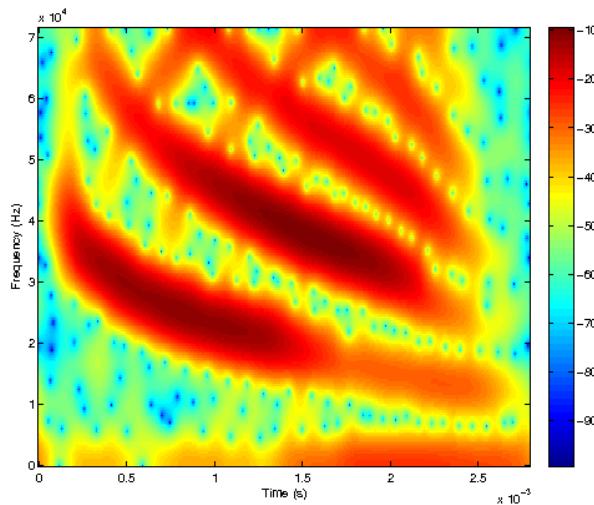
Plot of 'bat' in the frequency-domain:

```
plotfftreal(fftreal(bat),143000,90);
```



Plot of 'bat' in the time-frequency-domain:

```
sgram(bat, 143000, 90);
```



11.2.2 BATMASK - Load a Gabor multiplier symbol for the 'bat' test signal

Usage

```
c=batmask;
```

Description

batmask loads a Gabor multiplier with a 0/1 symbol that masks out the main contents of the 'bat' signal. The symbol fits a Gabor multiplier with lattice given by $a = 10$ and $M = 40$.

The mask was created manually using a image processing program. The mask is symmetric, such that the result will be real valued if the multiplier is applied to a real valued signal using a real valued window.

11.2.3 GREASY - Load the 'greasy' test signal

Usage

```
s=greasy;
```

Description

`greasy` loads the 'greasy' signal. It is a recording of a woman pronouncing the word "greasy".

The signal is 5880 samples long and recorded at 16 kHz with around 11 bits of effective quantization.

`[sig, fs]=greasy` additionally returns the sampling frequency fs .

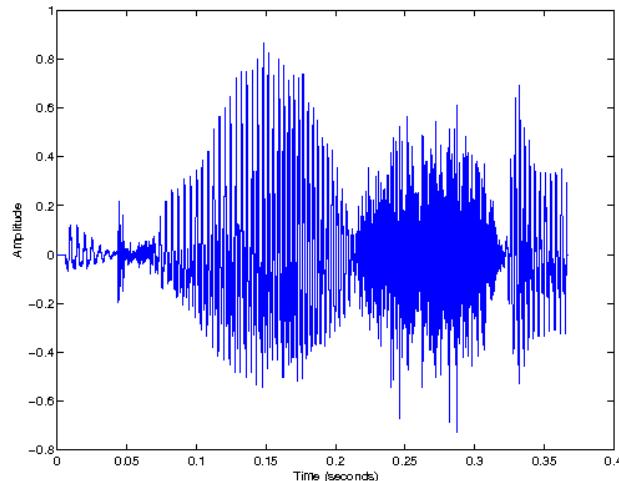
The signal has been scaled to not produce any clipping when played. To get integer values use `round(greasy*2048)`.

The signal was obtained from Wavelab: <http://www-stat.stanford.edu/~wavelab/>, it is a part of the first sentence of the TIMIT speech corpus "She had your dark suit in greasy wash water all year": <http://www.ldc.upenn.edu/Catalog/CatalogEntry.jsp?catalogId=LDC93S1>.

Examples:

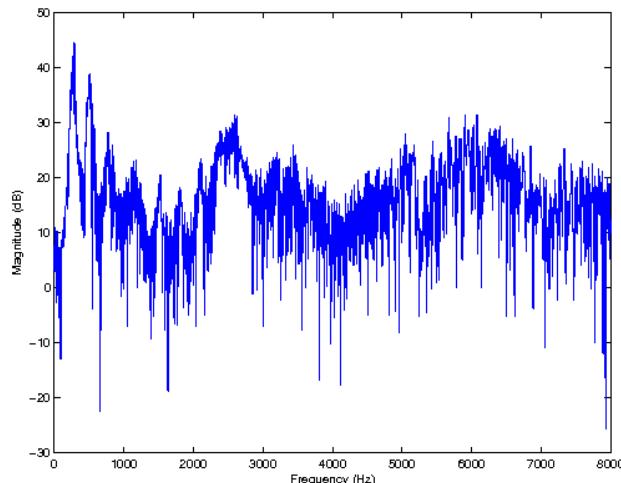
Plot of 'greasy' in the time-domain:

```
plot ((1:5880)/16000, greasy);
xlabel('Time (seconds)');
ylabel('Amplitude');
```



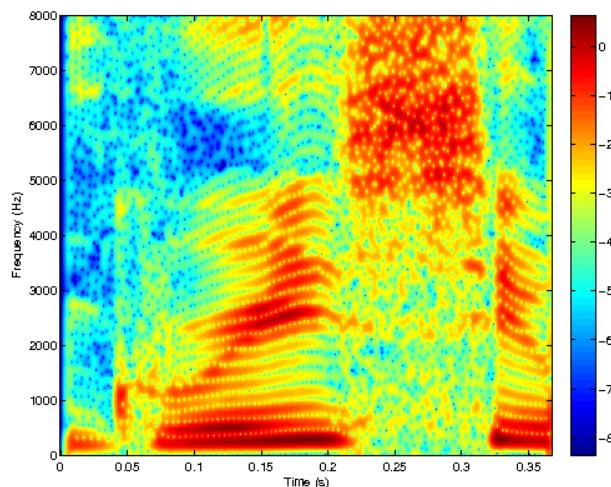
Plot of 'greasy' in the frequency-domain:

```
plotfftreal(fftreal(greasy), 16000, 90);
```



Plot of 'greasy' in the time-frequency-domain:

```
sgram(greasy, 16000, 90);
```



References: [62]

11.2.4 COCKTAILPARTY - Load the 'cocktailparty' test signal

Usage

```
s=cocktailparty;
```

Description

`cocktailparty` loads the 'cocktailparty' signal. It is a recording of a male native English speaker pronouncing the sentence "The cocktail party effect refers to the ability to focus on a single talker among a mixture of conversations in background noises".

`[sig, fs]=cocktailparty` additionally returns the sampling frequency fs .

The signal is 363200 samples long and recorded at 44.1 kHz in an anechoic environment.

11.2.5 GSPI - Load the 'glockenspiel' test signal

`gspi` loads the 'glockenspiel' signal. This is a recording of a simple tune played on a glockenspiel. It is 262144 samples long, mono, recorded at 44100 Hz using 16 bit quantization.

`[sig, fs]=gspi` additionally returns the sampling frequency fs .

This signal, and other similar audio tests signals, can be found on the EBU SQAM test signal CD <http://tech.ebu.ch/publications/sqamcd>.

11.2.6 LINUS - Load the 'linus' test signal

Usage

```
s=linus;
```

Description

`linus` loads the 'linus' signal. It is a recording of Linus Thorvalds pronouncing the words "Hello. My name is Linus Thorvalds, and I pronounce Linux as Linux".

The signal is 41461 samples long and is sampled at 8 kHz.

`[sig, fs]=linus` additionally returns the sampling frequency fs .

See <http://www.paul.sladen.org/pronunciation/>.

11.2.7 LTFATLOGO - Load the 'ltfatlogo' test signal

Usage

```
s=ltfatlogo;
```

Description

`ltfatlogo` loads the 'ltfatlogo' signal. This is a sound synthesised from an artificial spectrogram of the word 'LTFAT'. See the help of `lftattext`.

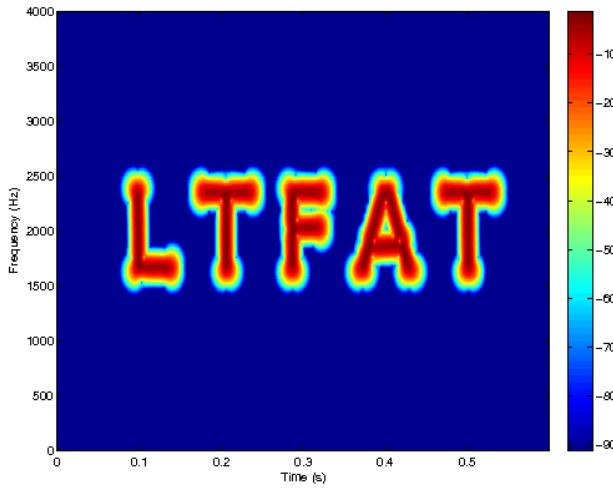
`[sig, fs]=ltfatlogo` additionally returns the sampling frequency fs .

The signal is 7200 samples long and recorded at 8 kHz. It has been scaled to not produce any clipping.

Examples:

To produce a spectrogram of the logo, use:

```
sgram(ltfatlogo, 8000, 90);
```



11.2.8 OTOCLICK - Load the 'otoclick' test signal

Usage

```
s=otoclick;
```

Description

`otoclick` loads the 'otoclick' signal. The signal is a click-evoked otoacoustic emission. It consists of two clear clicks followed by a ringing. The ringing is the actual otoacoustic emission.

`[sig, fs]=otoclick` additionally returns the sampling frequency fs .

It was measured by Sarah Verhulst at CAHR (Centre of Applied Hearing Research) at Department of Electrical Engineering, Technical University of Denmark

The signal is 2210 samples long and sampled at 44.1 kHz.

11.2.9 TRAINDOPPLER - Load the 'traindoppler' test signal

Usage

```
s=traindoppler;
```

Description

`traindoppler` loads the 'traindoppler' signal. It is a recording of a train passing close by with a clearly audible doppler shift of the train whistle sound.

`[sig, fs]=traindoppler` additionally returns the sampling frequency fs .

The signal is 157058 samples long and sampled at 8 kHz.

The signal was obtained from <http://www.fourmilab.ch/cship/doppler.html>

11.3 Images.

11.3.1 CAMERAMAN - Load the 'cameraman' test image

Usage

```
s=cameraman;
```

Description

`cameraman` loads a 256×256 greyscale image of a cameraman.

The returned matrix `s` consists of integers between 0 and 255, which have been converted to double precision.

To display the image, use `imagesc` with a gray colormap:

```
imagesc(cameraman); colormap(gray); axis('image');
```



See <ftp://nic.funet.fi/pub/graphics/misc/test-images/> or <http://sipi.usc.edu/database/database.cgi?volume=misc>.

11.3.2 LICHTENSTEIN - Load the 'lichtenstein' test image

Usage

```
s=lichtenstein;
```

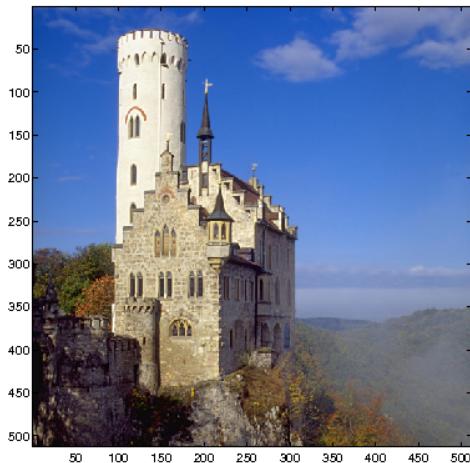
Description

`lichtenstein` loads a 512×512 color image of a castle Lichtenstein, http://en.wikipedia.org/wiki/Lichtenstein_Castle.

The returned matrix `s` consists of integers between 0 and 255.

To display the image, simply use `image`:

```
image(lichtenstein); axis('image');
```



See http://commons.wikimedia.org/wiki/File:Lichtenstein_img_processing_test.png.

11.3.3 LTFATTEXT - Load the 'ltfattext' test image

Usage

```
s=ltfattext;
```

Description

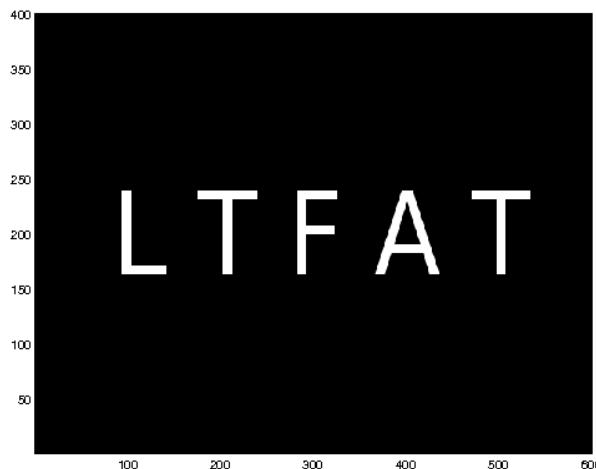
`ltfattext` loads a 401×600 black and white image of the word 'LTFAT'.

The image is assumed to be used as a spectrogram with 800 channels as produced by `dgtreal`.

The returned matrix `s` consists of the integers 0 and 1, which have been converted to double precision.

To display the image, use `imagesc` with a gray colormap:

```
imagesc(ltfattext);
colormap(gray);
axis('xy');
```



Chapter 12

LTFAT - Demos

12.1 Basic demos

12.1.1 DEMO_DGT - Basic introduction to DGT analysis/synthesis

This demo shows how to compute Gabor coefficients of a signal.

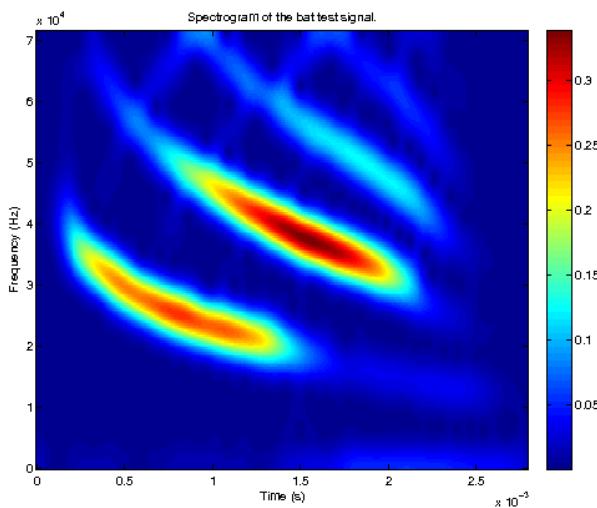


Figure 12.1: Spectrogram of the 'bat' signal.

The figure shows a spectrogram of the 'bat' signal. The coefficients are shown on a linear scale.

Output

Type "help demo_dgt" to see a description of how this demo works.

----- Spectrogram analysis -----

The spectrogram is highly redundant.

No. of coefficients in the signal: 400

No. of coefficients in the spectrogram: 80400

Redundancy of the spectrogram: 201.000000

----- Simple Gabor analysis using a standard Gaussian window. -----

Setup parameters for a Discrete Gabor Transform.

Time shift:

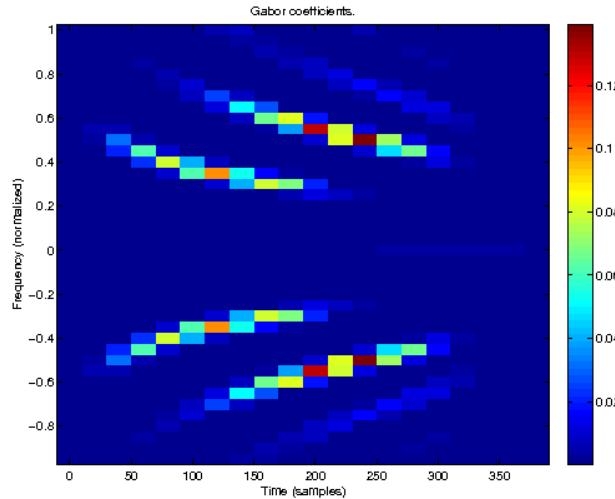


Figure 12.2: Gabor coefficients of the 'bat' signal.

The figure show a set of Gabor coefficients for the 'bat' signal, computed using a DGT with a Gaussian window. The coefficients contains all the information to reconstruct the signal, even though there are far fewer coefficients than the spectrogram contains.

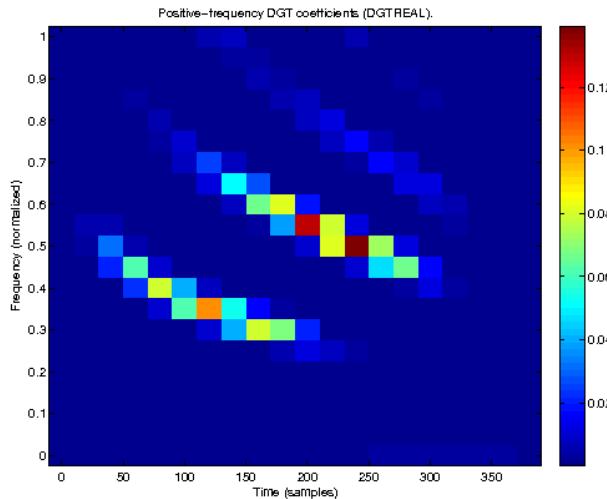


Figure 12.3: Real-valued Gabor analysis

This figure shows only the coefficients for the positive frequencies. As the signal is real-value, these coefficients contain all the necessary information. Compare to the shape of the spectrogram shown on Figure 1.

$a =$

20

Number of frequency channels.

$M =$

40

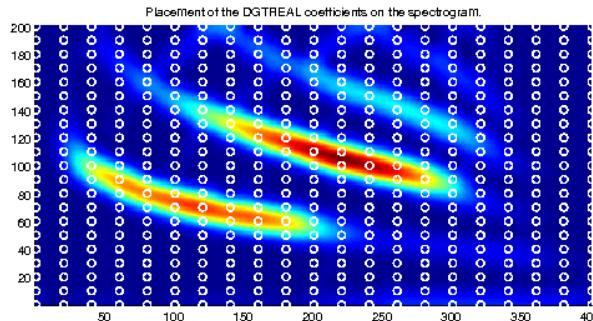


Figure 12.4: DGT coefficients on a spectrogram

This figure shows how the coefficients from DGTREAL can be picked from the coefficients computed by a full Short-time Fourier transform, as visualized by a spectrogram.

Note that it must hold that $L = M \cdot b = N \cdot a$ for some integers b , N and L , and that $a < M$. L is the transform length, and the DGT will choose the smallest possible value of L that is larger or equal to the length of the signal. Choosing $a < M$ makes the transform redundant, otherwise the transform will be lossy, and reconstruction will not be possible.

Number of time shifts in transform:

Length of transform:

```
L =
```

```
400
```

The redundancy of the Gabor transform can be reduced without loosing information.

No. of coefficients in the signal: 400

No. of output coefficients from the DGT: 800

Redundacy of the DGT (in this case) 2.000000

----- Real valued Gabor analysis. -----

----- Perfect reconstruction. -----

Reconstruction error using IDGT: 5.384302e-16

Reconstruction error using IDGTREAL: 5.664802e-16

12.1.2 DEMO_GABFIR - Working with FIR windows

This demo demonstrates how to work with FIR windows in Gabor systems.

FIR windows are the windows traditionally used in signal processing. They are short, much shorter than the signal, and this is used to make effecient algorithms. They are also the only choice for applications involving streaming data.

It is very easy to compute a spectrogram or Gabor coefficients using a FIR window. The hard part is

reconstruction, because both the window and the dual window used for reconstruction must be FIR, and this is hard to obtain, if the window is longer than the number of channels.

This demo demonstrates two methods:

- 1) Using a Gabor frame with a simple structure, for which dual/tight FIR windows are easy to construct. This is a very common technique in traditional signal processing, but it limits the choice of windows and lattice parameters.
- 2) Cutting a canonical dual/tight window. We compute the canonical dual window of the analysis window, and cut away the parts that are close to zero. This will work for any analysis window and any lattice constant, but the reconstruction obtained is not perfect.

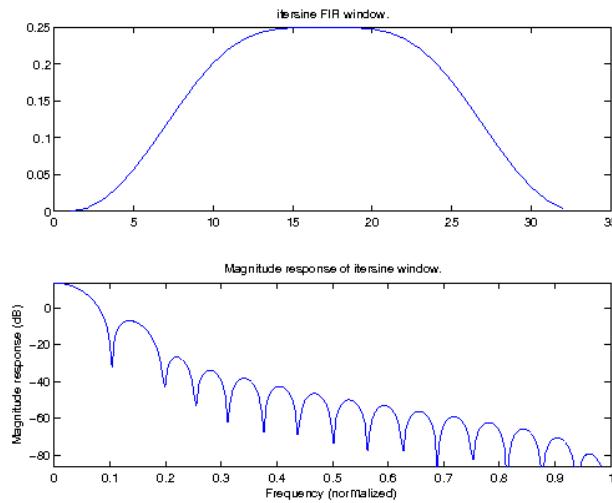


Figure 12.5: Hanning FIR window

This figure shows the a Hanning window in the time domain and its magnitude response.

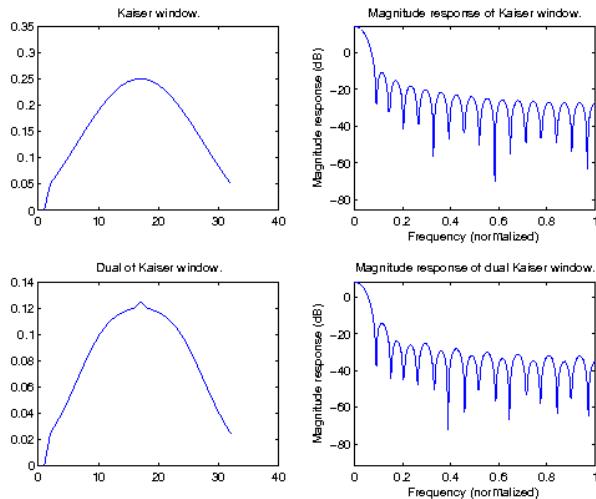


Figure 12.6: Kaiser-Bessel FIR window

This figure shows a Kaiser Bessel window and its magnitude response, and the same two plots for the canonical dual of the window.

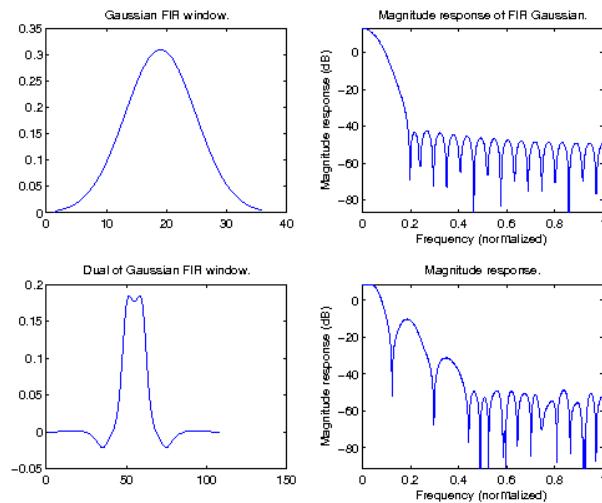


Figure 12.7: Gaussian FIR window for low redundancy

This figure shows a truncated Gaussian window and its magnitude response. The same two plots are shown for the truncated canonical dual window.

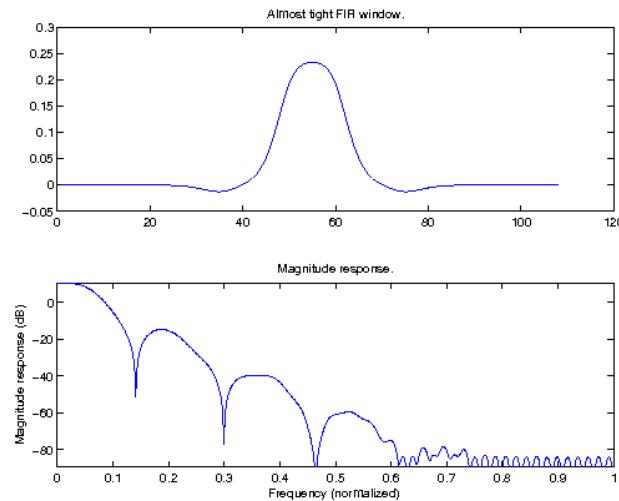


Figure 12.8: Almost tight Gaussian FIR window

This figure shows a tight Gaussian window that has been truncated and its magnitude response.

Output

Reconstruction error using itersine window, should be close to zero:

```
ans =
```

```
0.5000000000000000
```

Reconstruction error canonical dual of Kaiser window, should be close to zero:

```
ans =
```

```
6.401998278383549e-17
```

Reconstruction error using cutted dual window.

```
recerr =
0.001762485642153
or expressed in dB:
ans =
-27.538744122424180
```

Reconstruction error using cutted tight window.

```
recerr =
9.858293436835774e-04
or expressed in dB:
ans =
-30.061982590052025
```

12.1.3 DEMO_WAVELETS - Wavelet filter banks

This demo exemplifies the use of the wavelet filter bank trees. All representations use "least asymmetric" Daubechies wavelet orthonormal filters 'sym8' (8-regular, length 16).

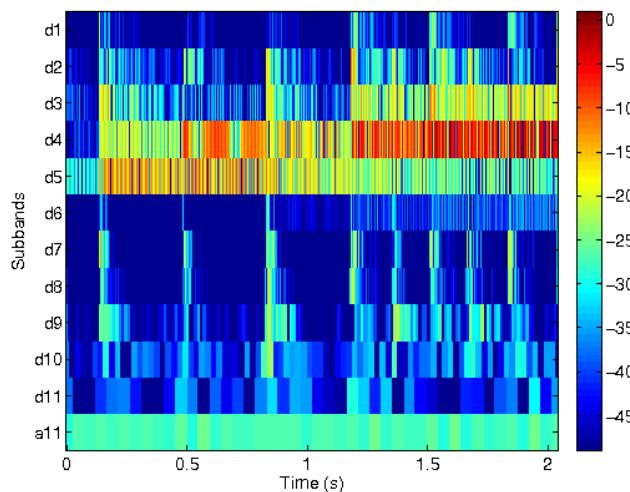


Figure 12.9: DWT representation

The filter bank tree consists of 11 levels of iterated 2-band basic wavelet filter bank, where only the low-pass output is further decomposed. This results in 12 bands with octave resolution.

Output

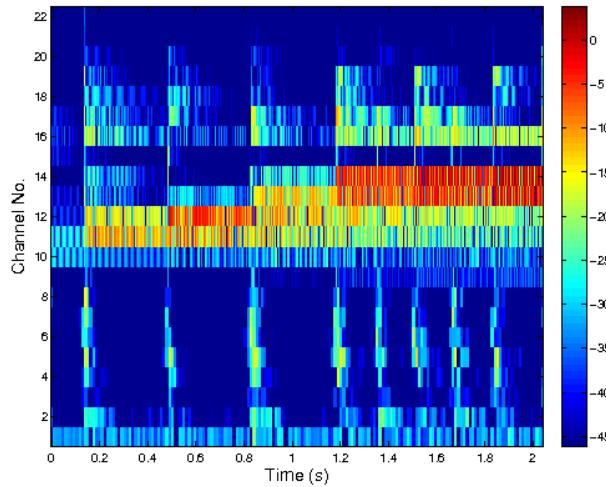


Figure 12.10: 8-band DWT representation

The filter bank tree (effectively) consists of 3 levels of iterated 8-band basic wavelet filter bank resulting in 22 bands. Only the low-pass output is decomposed at each level.

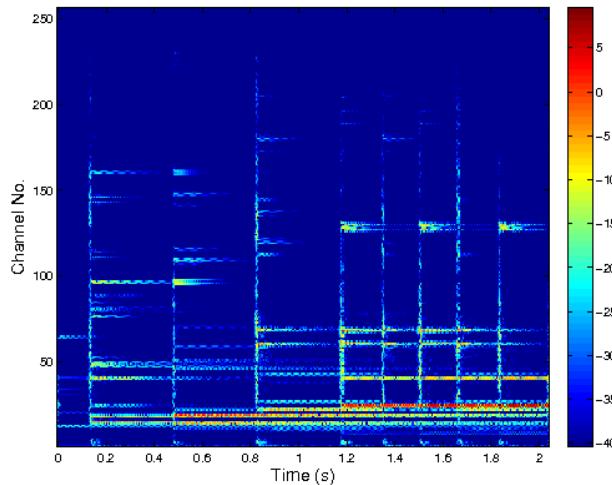


Figure 12.11: Full Wavelet filter bank tree representation

The filter bank tree depth is 8 and it is fully decomposed meaning both outputs (low-pass and high-pass) of the basic filter bank is plot further. This results in 256 bands linearly covering the frequency axis.

12.2 Compression

12.2.1 DEMO_IMAGECOMPRESSION - Image compression using N-term approximation

This demo shows how to perform a simple imagecompression using either a Wilson basis or a Wavelet. The compression step is done by retaining only 5% of the coefficients.

Output

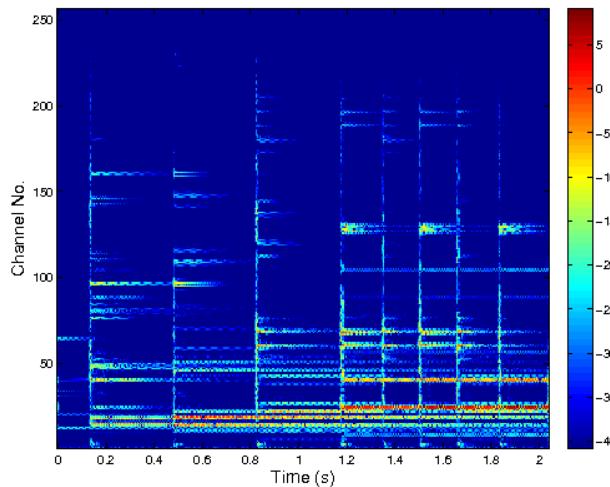


Figure 12.12: Full Wavelet filter bank tree representation
The same case as before, but symmetric nearly orthogonal basic filter bank is used.

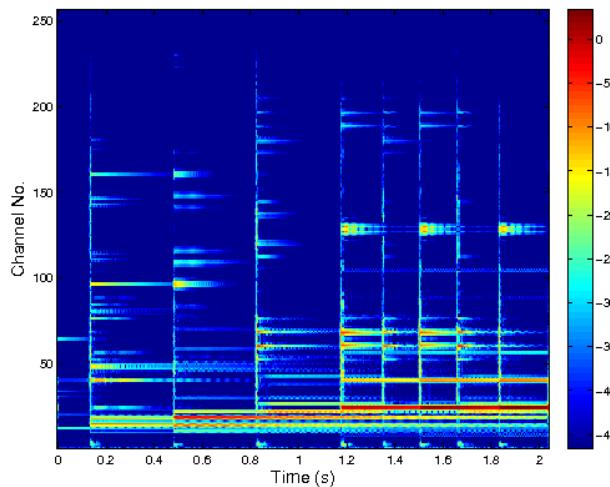


Figure 12.13: Full Dual-tree Wavelet filter bank representation
This is a 2 times redundant representation using Q-shift dual-tree wavelet filters.

12.2.2 DEMO_AUDIOCOMPRESSON - Audio compression using N-term approx

This demos shows how to do audio compression using best N-term approximation of an wmdct transform.

The signal is transformed using an orthonormal wmdct transform. Then approximations with a fixed number N of coefficients are obtained by:

- Linear approximation: The N coefficients with lowest frequency index are kept.
- Non-linear approximation: The N largest coefficients (in magnitude) are kept.

The corresponding approximated signal can be computed using iwmdct.

Note: The inverse WMDCT is not needed for computing computing SNRs. Instead Parseval theorem states that the norm of a signal equals the norm of the sequence of its wmdct coefficients.

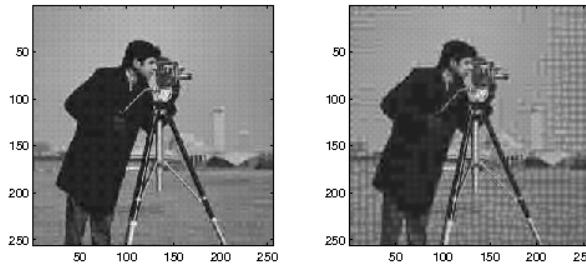


Figure 12.14: Wilson and WMDCT basis

This right figure shows the image compressed using a dwilt basis with 8 channels. This corresponds quite closely to JPEG compression, except that the borders between neighbouring blocs are smoother, since the dwilt uses a windowing function.

The left figure shows the same, now using a MDCT basis. The MDCT produces more visible artifacts, as the slowest changing frequency in each block has a half-wave modulation. This is visible on otherwise smooth backgrounds.

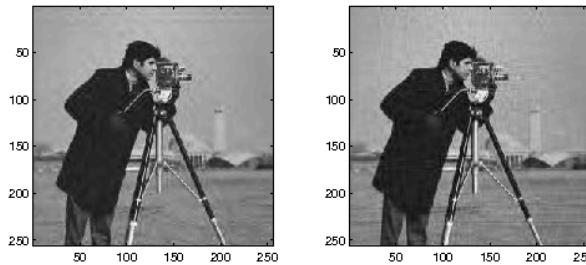


Figure 12.15: Wavelet

The Wavelet used is the DB6 with $J = 5$ levels. On the right figure the standard layout has been used, on the left the tensor layout was used.

Output

12.3 Denoising

12.3.1 DEMO_AUDIO_DENOISE - Audio denoising using thresholding

This demo shows how to do audio denoising using thresholding of WMDCT transform.

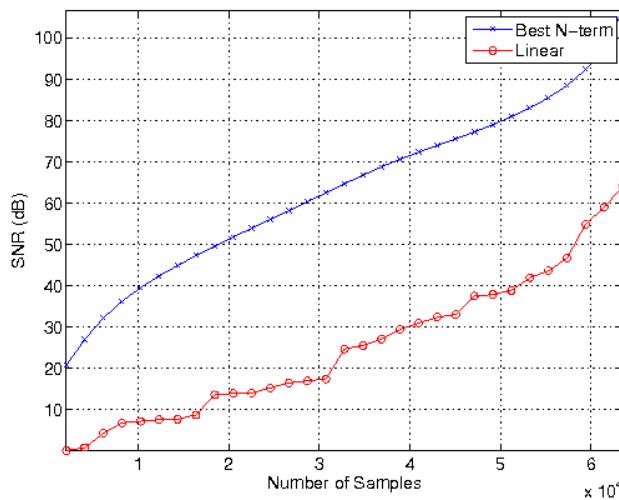


Figure 12.16: Rate-distortion plot

The figure shows the output Signal to Noise Ratio (SNR) as a function of the number of retained coefficients.

The signal is transformed using an orthonormal WMDCT transform followed by a thresholding. Then the signal is reconstructed and compared with the original.

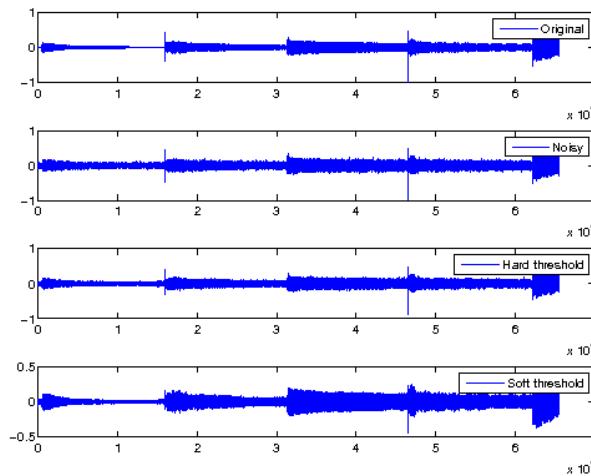


Figure 12.17: Denoising

The figure shows the original signal, the noisy signal and denoised signals using hard and soft thresholding applied to the WMDCT of the noise signal.

Output

RESULTS:

Input SNR: 6.044694 dB.
 Output SNR (hard): 9.091411 dB.
 Output SNR (soft): 14.283694 dB.
 Signals are stored in variables `sig`, `nsig`, `hrec`, `srec`

12.4 Applications

12.4.1 DEMO_OFDM - Demo of Gabor systems used for OFDM

This demo shows how to use a Gabor Riesz basis for OFDM.

We want to transmit a signal consisting of 0's and 1's through a noisy communication channel. This is accomplished in the following steps in the demo:

- 1) Convert this digital signal into complex valued coefficients by QAM modulation.
- 2) Construct the signal to be transmitted by an inverse Gabor transform of the complex coefficients
- 3) "Transmit" the signal by applying a spreading operator to the signal and adding white noise
- 4) Convert the received signal into noisy coefficients by a Gabor transform
- 5) Convert the noisy coefficients into bits by inverse QAM.

Some simplifications used to make this demo simple:

- We assume that the whole spectrum is available for transmission.
- The window and its dual have full length support. This is not practical, because all data would have to be processed at once. Instead, an FIR should be used, with both the window and its dual having a short length.
- The window is periodic. The data at the very end interferes with the data at the very beginning. A simple way to solve this is to transmit zeros at the beginning and at the end, to flush the system properly.

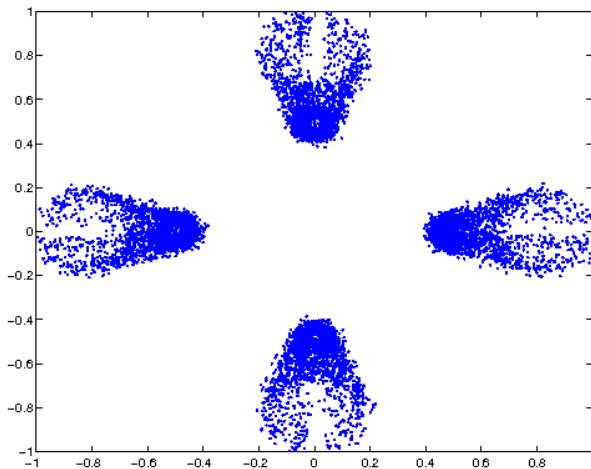


Figure 12.18: Received coefficients.

This figure shows the distribution in the complex plane of the received coefficients. If the channel was perfect, all the points should appear at the complex roots of unity ($1, i, -1$ and $-i$). This demo is random, so everytime it is run it produces a new plot, and the error rate may vary.

Output

Type "help demo_ofdm" to see a description of how this demo works.

Number of faulty bits:

```
faulty =
```

```
0
```

```
Error rate:
```

```
ans =
```

```
0
```

12.4.2 DEMO_AUDIO SHRINK - Decomposition into tonal and transient parts

This demos shows how to do audio coding and "tonal + transient" decomposition using group lasso shrinkage of two wmdct transforms with different time-frequency resolutions.

The signal is transformed using two orthonormal wmdct bases. Then group lasso shrinkage is applied to the two transforms in order to:

- select fixed frequency lines of large wmdct coefficients on the wide window wmdct transform
- select fixed time lines of large wmdct coefficients on the narrow window wmdct transform

The corresponding approximated signals are computed with the corresponding inverse, iwmddct.

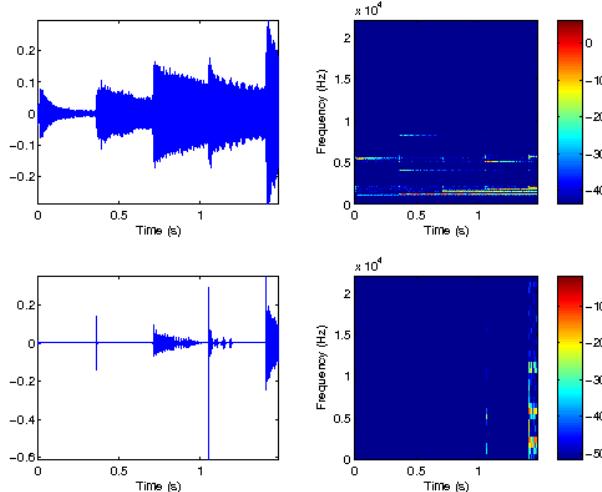


Figure 12.19: Plots and time-frequency images

The upper plots in the figure show the tonal parts of the signal, the lower plots show the transients. The TF-plots on the left are the corresponding wmdct coefficients found by appropriate group lasso shrinkage

Corresponding reconstructed tonal and transient sounds may be listened from arrays rec1 and rec2 (sampling rate: 44.1 kHz)

Output

Percentage of retained coefficients: $10.156250 + 12.988281 = 23.144531$

To play the original, type "soundsc(sig,fs)"

To play the tonal part, type "soundsc(rec1,fs)"

To play the transient part, type "soundsc(rec2,fs)"

12.4.3 DEMO_GABMULAPPR - Approximate a slowly time variant system by a Gabor multiplier

This script construct a slowly time variant system and performs the best approximation by a Gabor multiplier with specified parameters (a and L see below). Then it shows the action of the slowly time variant system (A) as well as of the best approximation of (A) by a Gabor multiplier (B) on a sinusoids and an exponential sweep.

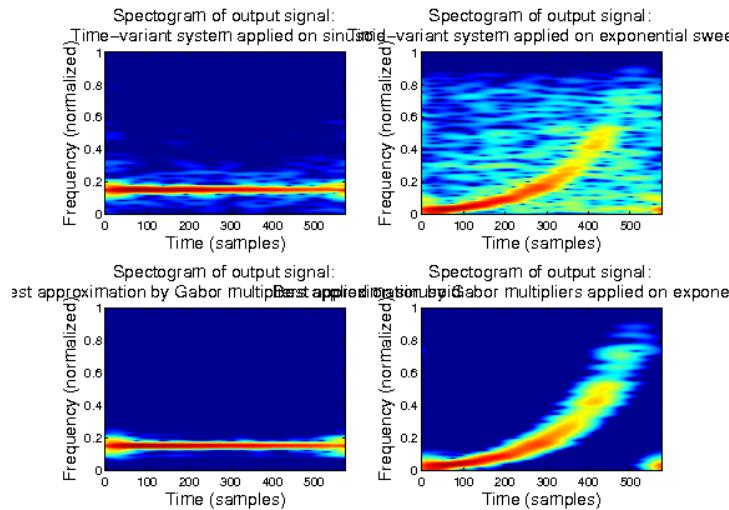


Figure 12.20: Spectrogram of signals

The figure shows the spectrogram of the output of the two systems applied on a sinusoid (left) and an exponential sweep.

Output

Type "help demo_gabmulappr" to see a description of how this demo works.

12.4.4 DEMO_BPFRAMEMUL - Frame multiplier acting as a time-varying band-pass filter

This demo demonstrates creation and effect of a Gabor multiplier. The multiplier performs a time-varying bandpass filtering. The band-pass filter with center frequency changing over time is explicitly created but it is treated as a "black box" system. The symbol is identified by "probing" the system with a white noise and dividing DGT of the output by DGT of the input element-wise. The symbol is smoothed out by a 5x5 median filter.

Output

The original signal can be played by typing: `sound(forig, 44100);`

The signal obtained by a direct filtering can be played by (this is what is approximated):
`sound(fcen, 44100);`

The signal obtained by applying the Gabor multiplier: `sound(fmul, 44100);`

12.4.5 DEMO_FRSYNABS - Construction of a signal with a given spectrogram

This demo demonstrates iterative reconstruction of a spectrogram.

The BFGS method makes use of the minFunc software. To use the BFGS method, please install the minFunc software from: <http://www.cs.ubc.ca/~schmidtm/Software/minFunc.html>.

Output

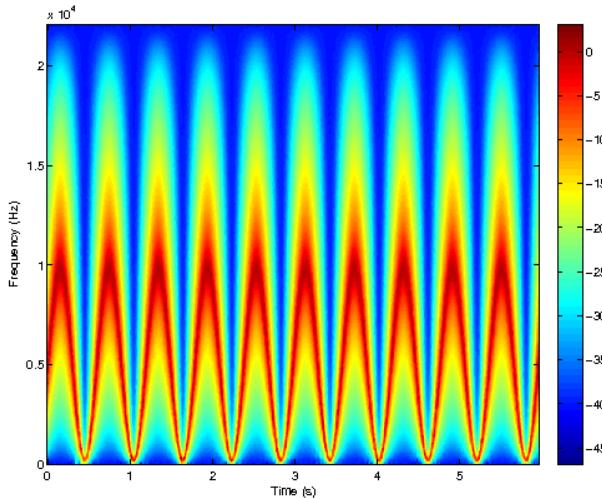


Figure 12.21: The symbol of the multiplier.
This figure shows a symbol used in the Gabor multiplier.

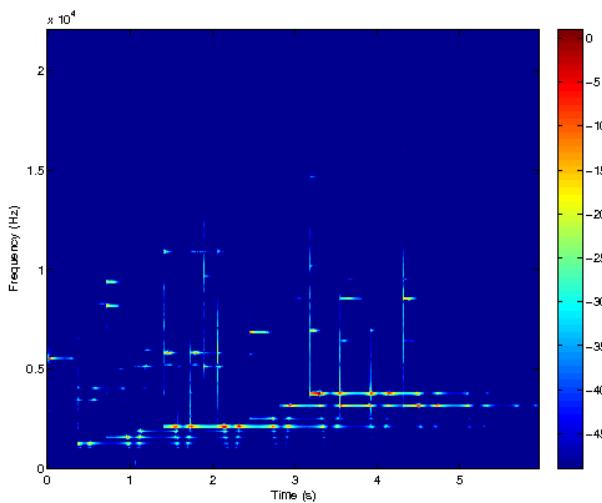


Figure 12.22: Spectrogram obtained by re-analysis of the test signal after applying the multiplier.
This figure shows a spectrogram of the test signal after applying the estimated Gabor multiplier.

12.4.6 DEMO_FILTERBANKSYNCHROSQUEEZE - Filterbank synchrosqueezing and inversion

The demo shows that the synchrosqueezed filterbank representation can be directly used to reconstruct the original signal. Since we do not work with a filterbank which forms a tight frame (its filterbank response is not constant) the direct reconstruction (mere summing all the channels) does not work well. We can fix that by filtering (equalizing) the result by the inverse of the overall analysis filterbank frequency response.

References: [44]

Output

```
Direct reconstruction MSE:  
From the coefficients: 2.176541e-03  
From the synchrosqueezed coefficients: 2.176541e-03
```

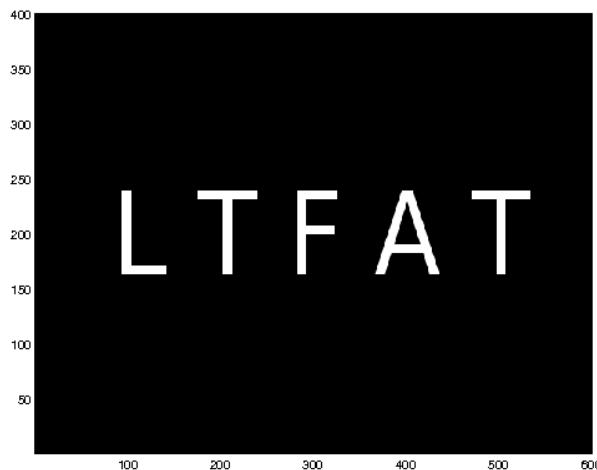


Figure 12.23: Original spectrogram

This figure shows the target spectrogram

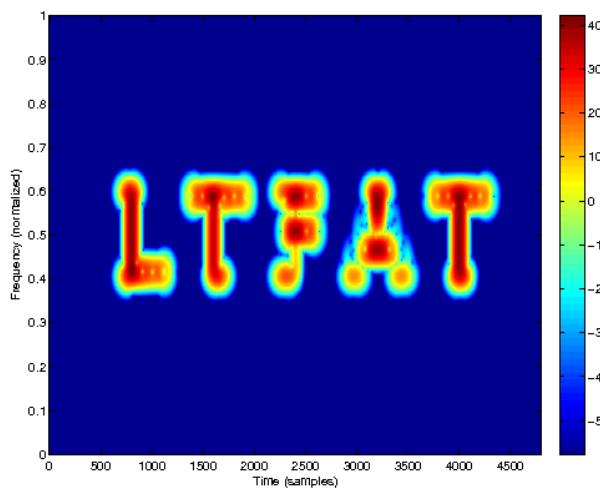


Figure 12.24: Linear reconstruction

This figure shows a spectrogram of a linear reconstruction of the target spectrogram.

Equalized reconstruction MSE:

From the coefficients: $8.587864\text{e-}16$

From the synchrosqueezed coefficients: $7.161705\text{e-}16$

12.5 Aspects of particular functions

12.5.1 DEMO_NSDGT - Non-stationary Gabor transform demo

This script sets up a non-stationary Gabor frame with the specified parameters, computes windows and corresponding canonical dual windows and a test signal, and plots the windows and the energy of the coefficients.

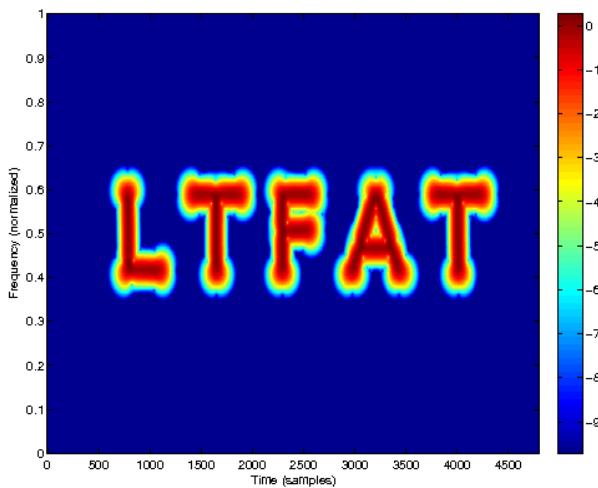


Figure 12.25: Iterative reconstruction using the Griffin-Lim method.

This figure shows a spectrogram of an iterative reconstruction of the target spectrogram using the Griffin-Lim projection method.

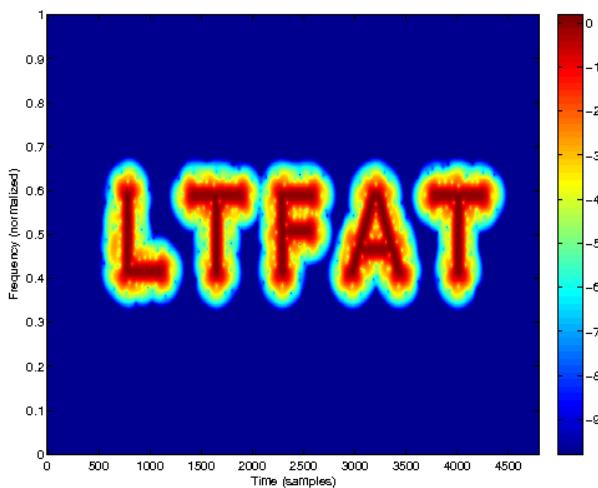


Figure 12.26: Iterative reconstruction using the BFGS method.

This figure shows a spectrogram of an iterative reconstruction of the target spectrogram using the BFGS method.

Output

```
Type "help demo_nsdgt" to see a description of how this example works.  
Relative error of reconstruction (should be close to zero.): 3.227678e-16
```

12.5.2 DEMO_PGAUSS - How to use PGAUSS

This script illustrates various properties of the Gaussian function.

Output

```
Type "help demo_pgauss" to see a description of how this demo works.  
Test of DFT invariance: Should be close to zero.
```

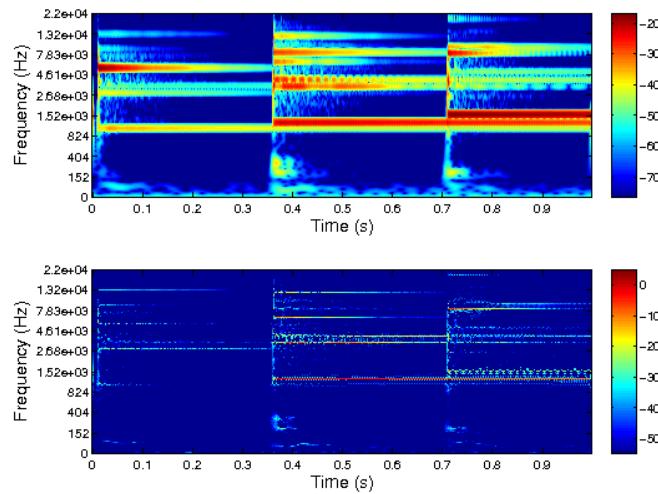


Figure 12.27: ERBlet spectrogram (top) and synchrosqueezed ERBlet spectrogram (bottom)
The signal used is the first second from gspi. Only the energy of the coefficients is show. Both representations are in fact complex and invertible.

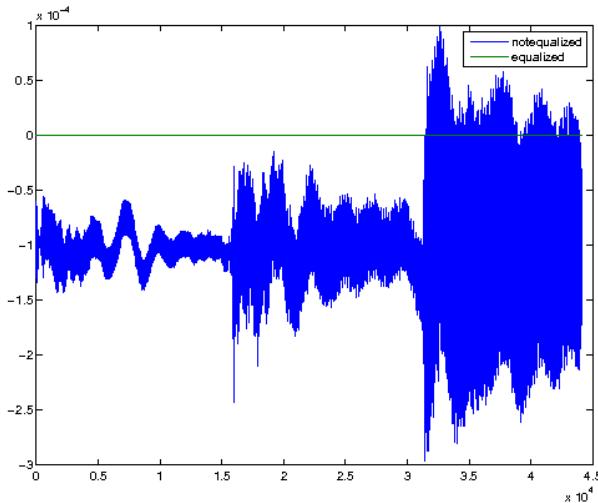


Figure 12.28: Errors of the direct and the equalized reconstructions
There is still a small DC offset of the signal obtained by the direct summation.

```
ans =
```

```
2.481524897647706e-15
```

The function is WP even. The following should be 1.

```
ans =
```

```
1
```

Therefore, its DFT is real.

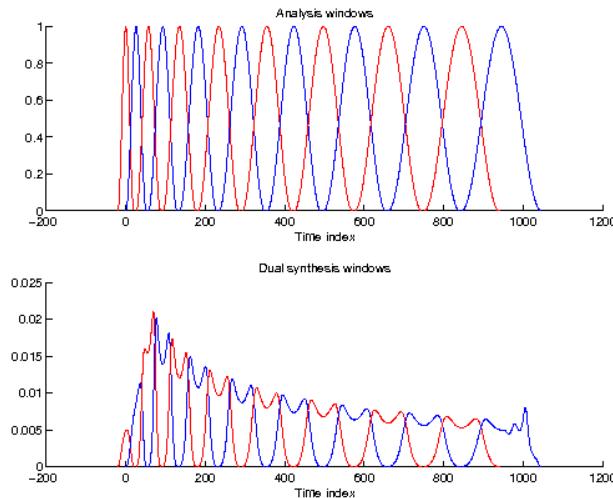


Figure 12.29: Windows + dual windows

This figure shows the window functions used and the corresponding canonical dual windows.

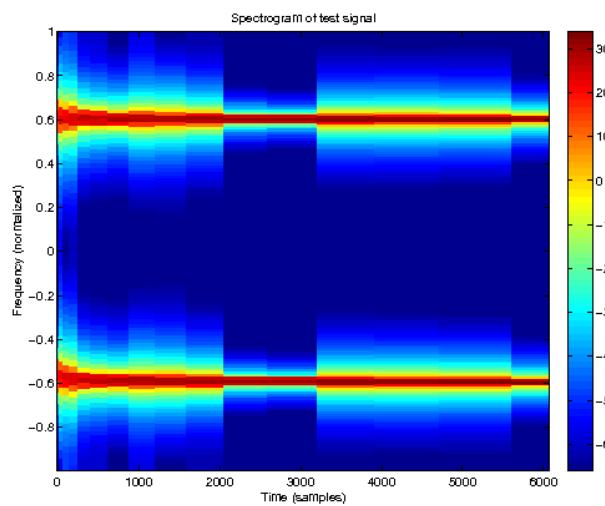


Figure 12.30: Spectrogram (absolute value of coefficients in dB)

This figure shows a (colour-coded) image of the nsdgt coefficient modulus.

The norm of the imaginary part should be close to zero.

```
ans =
```

```
1.501312721600895e-15
```

12.5.3 DEMO_PBSPLINE - How to use PBSPLINE

This script illustrates various properties of the pbspline function.

Output

Type "help demo_pbspline" to see a description of how this demo works.

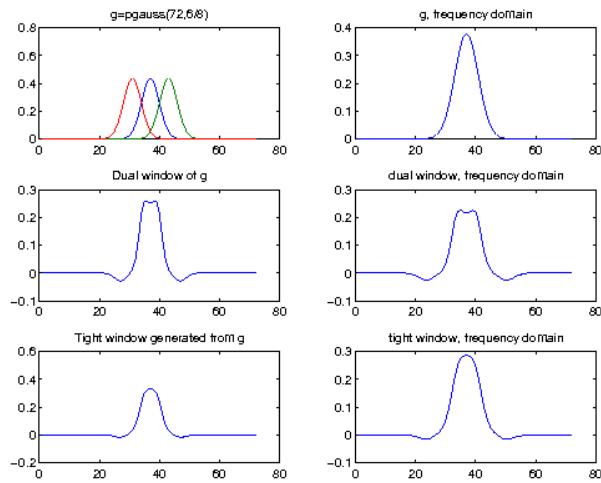


Figure 12.31: Window+Dual+Tight

This figure shows an optimally centered Gaussian for a given Gabor system, its canonical dual and tight windows and the DFTs of these windows.

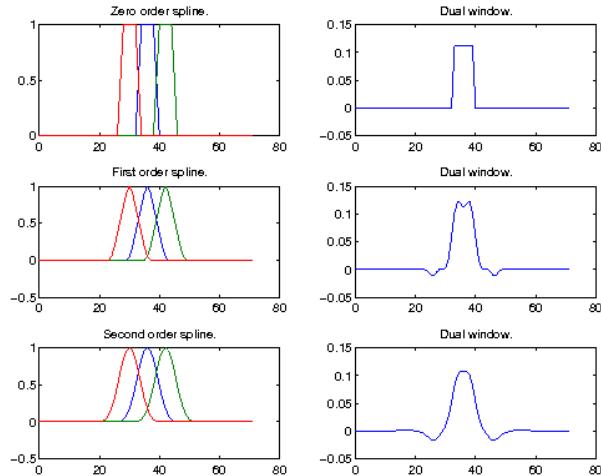


Figure 12.32: Three first splines

This figure shows the three first splines (order 0, 1 and 2) and their dual windows.

Note that they are calculated for an even number of the parameter a , meaning that they are not exactly splines, but a slightly smoother construction, that still form a partition of unity.

Length of the generated window:

```
ntaps =
```

```
19
```

Norm of imaginary part. Should be close to zero.

```
ans =
```

```
4.035410778421974e-16
```

Window is whole point even. Should be 1.

```
ans =
```

```
1
```

Length of g after cutting.

```
ans =
```

```
19
```

difference between original g, and gextend.
Should be close to zero.

```
ans =
```

```
6.204997459932044e-16
```

12.5.4 DEMO_GABMIXDUAL - How to use GABMIXDUAL

This script illustrates how one can produce dual windows using GABMIXDUAL

The demo constructs a dual window that is more concentrated in the time domain by mixing the original Gabor window by one that is extremely well concentrated. The result is somewhat in the middle of these two.

The lower framebound of the mixing Gabor system is horrible, but this does not carry over to the gabmixdual.

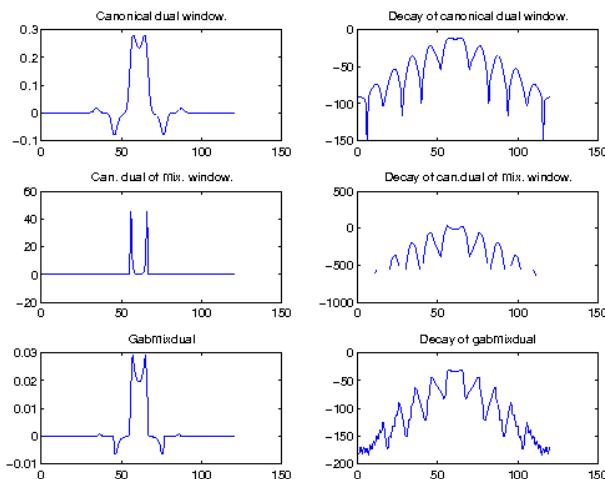


Figure 12.33: Gabmixdual of two Gaussians.

The first row of the figure shows the canonical dual window of the input window, which is a Gaussian function perfectly localized in the time and frequency domains.

The second row shows the canonical dual window of the window we will be mixing with: This is a Gaussian that is 10 times more concentrated in the time domain than in the frequency domain. The resulting canonical dual window has rapid decay in the time domain.

The last row shows the gabmixdual of these two. This is a non-canonical dual window of the first Gaussian, with decay resembling that of the second.

Output

Type "help demo_gabmixdual" to see a description of how this demo works.
 Framebounds of initial Gabor system:

```
A1 =
0.605813442913889
```

```
B1 =
1.690164970019093
```

Framebounds of mixing Gabor system:

```
A2 =
2.024110119704336e-05
```

```
B2 =
4.898979421758012
```

Framebounds of gabmixdual Gabor system:

```
Am =
0.004131003699532
```

```
Bm =
0.011463008168063
```

12.5.5 DEMO_FRAMEMUL - Time-frequency localization by a Gabor multiplier

This script creates several different time-frequency symbols and demonstrate their effect on a random, real input signal.

Output

Type "help demo_framemul" to see a description of how this demo works.

12.5.6 DEMO_PHASEPLOT - Give demos of nice phaseplots

This script creates a synthetic signal and then uses phaseplot on it, using several of the possible options.

For real-life signal only small parts should be analyzed. In the chosen demo the fundamental frequency of the speaker can be nicely seen.

References: [21], [20], [41]

Output

Type "help demo_phaseplot" to see a description of how this demo works.

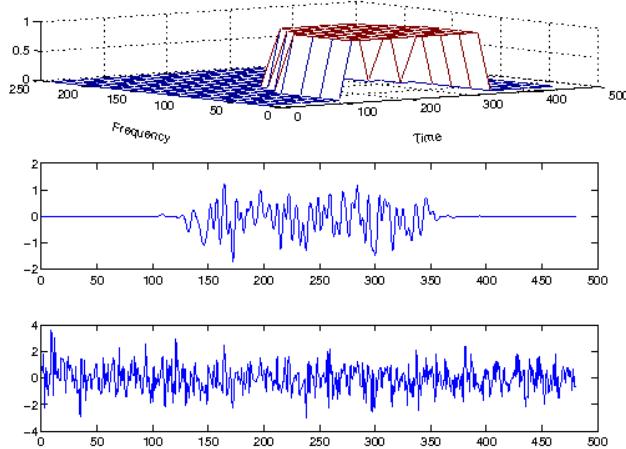


Figure 12.34: Cut a circle in the TF-plane

This figure shows the symbol (top plot, only the positive frequencies are displayed), the input random signal (bottom) and the output signal (middle).

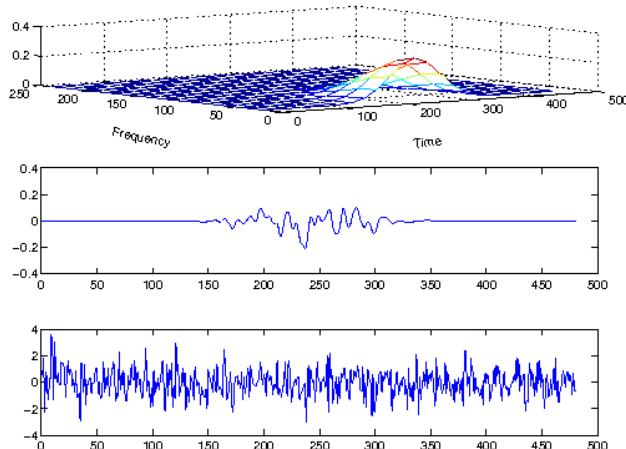


Figure 12.35: Keep low frequencies (low-pass)

This figure shows the symbol (top plot, only the positive frequencies are displayed), the input random signal (bottom) and the output signal (middle).

12.5.7 DEMO_PHASERET - Phase retrieval and phase difference

This demo demonstrates iterative reconstruction of a spectrogram and the phase difference.

Output

12.5.8 DEMO_NEXTPFASTFFT - Next fast FFT number

This demo shows the behaviour of the nextfastfft function.

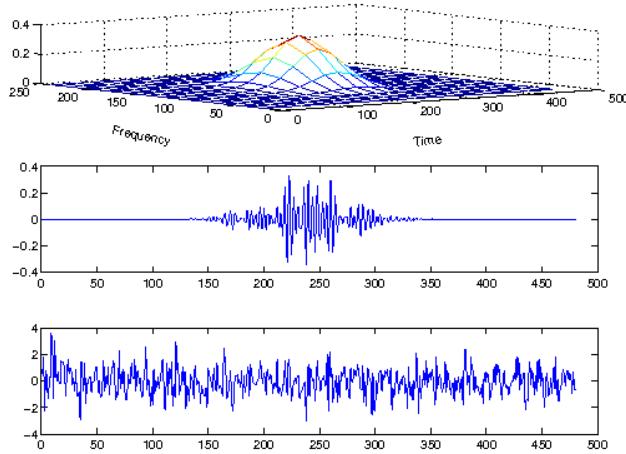


Figure 12.36: Keep middle frequencies (band-pass)

This figure shows the symbol (top plot, only the positive frequencies are displayed), the input random signal (bottom) and the output signal (middle).

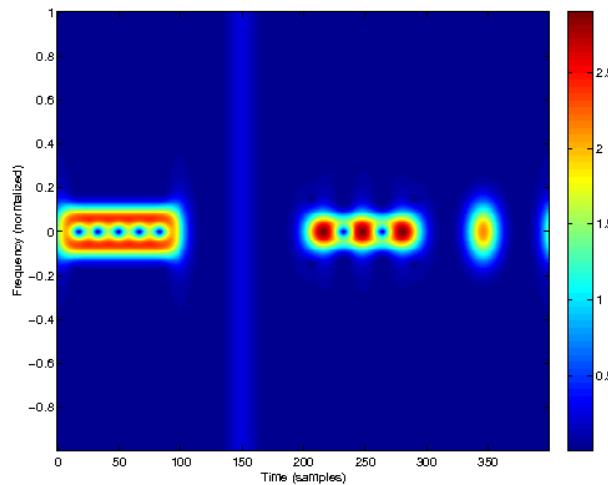


Figure 12.37: Synthetic signal

Compare this to the pictures in reference 2 and 3. In the first two figures a synthetic signal is analyzed. It consists of a sinusoid, a small Delta peak, a periodic triangular function and a Gaussian. In the time-invariant version in the first part the periodicity of the sinusoid can be nicely seen also in the phase coefficients. Also the points of discontinuities can be seen as asymptotic lines approached by parabolic shapes. In the third part both properties, periodicity and discontinuities can be nicely seen. A comparison to the spectrogram shows that the rectangular part in the middle of the signal can be seen by the phase plot, but not by the spectrogram.

In the frequency-invariant version, the fundamental frequency of the sinusoid can still be guessed as the position of an horizontal asymptotic line.

Output

```
ans =
```

```
1.007891912889226
```

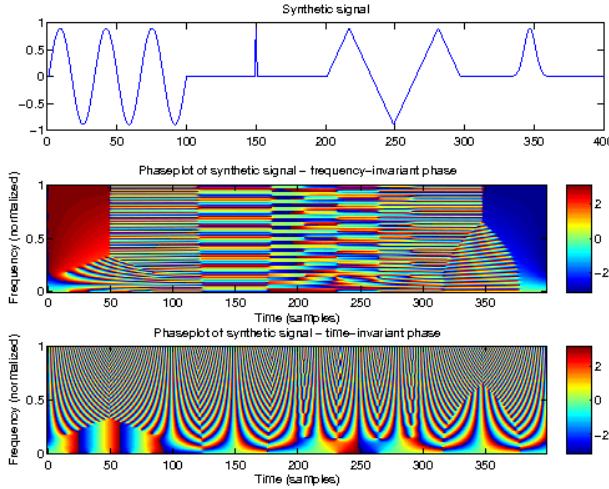


Figure 12.38: Synthetic signal, thresholded.

This figure shows the same as Figure 1, except that values with low magnitude has been removed.

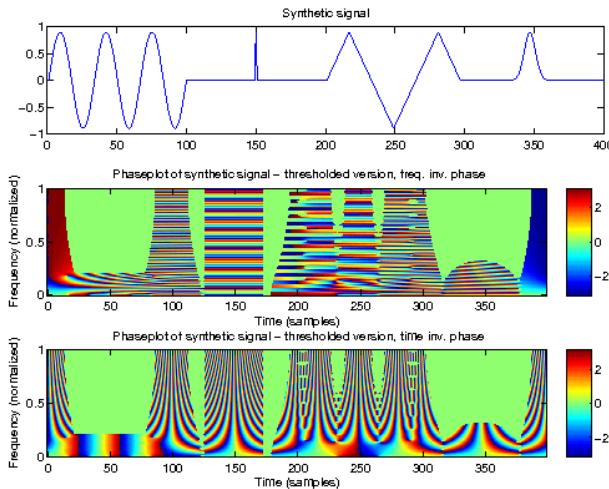


Figure 12.39: Speech signal.

The figure shows a part of the 'linus' signal. The fundamental frequency of the speaker can be nicely seen.

12.5.9 DEMO_FILTERBANKS - CQT, ERBLEET and AUDLET filterbanks

This demo shows CQT (Constant Quality Transform), ERBLEET (Equivalent Rectangular Bandwidth -let transform), and AUDLET (Auditory -let) representations acting as filterbanks with high and low redundancies. Note that ERBLEET and AUDLET are similar concepts. The main difference is that ERBlet uses only the perceptual ERB scale while AUDlet allows for various perceptual scales like Bark or Mel scales. In short, ERBFILTERS is a wrapper of AUDFILTERS for the ERB scale. Filterbanks are build such that the painless condition is always satisfied. Real input signal and filters covering only the positive frequency range are used. The redundancy is calculated as a ratio of the number of (complex) coefficients and the input length times two to account for the storage requirements of complex numbers.

- The high redundancy representation uses 'uniform' subsampling i.e. all channels are subsampled with the same subsampling factor which is the lowest from the filters according to the painless condition rounded towards zero.

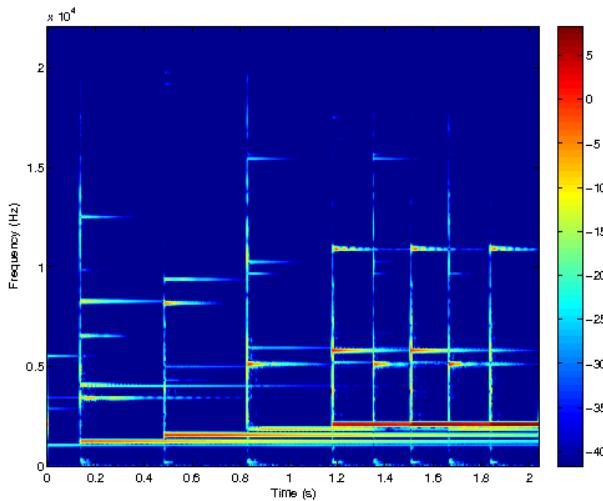


Figure 12.40: Original spectrogram

This figure shows the target spectrogram of an excerpt of the gspi signal

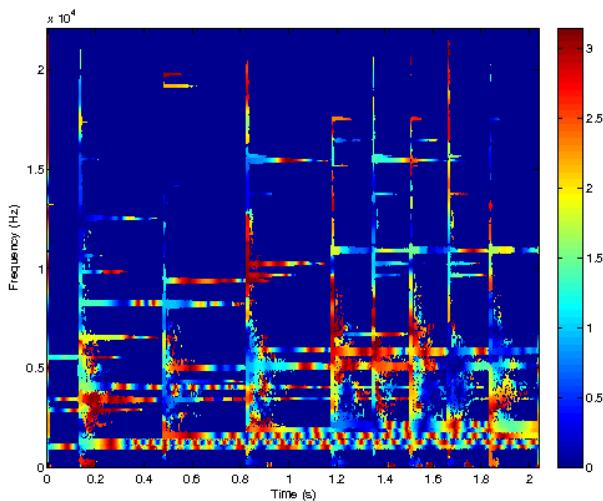


Figure 12.41: Phase difference

This figure shows a difference between the original phase and the reconstructed using 100 iterations of a Fast Griffin-Lim algorithm. Note: The figure in the LTFAT 2.0 paper differs slightly because it is generated using 1000 iterations.

- The low redundancy representation uses 'fractional' subsampling which results in the least redundant representation still satisfying the painless condition. Actual time positions of atoms can be non-integer, hence the word fractional.

Output

ERBLET high redundancy 133.33, low redundancy 2.85.

CQT high redundancy 50.91, low redundancy 2.02.

BARKLET redundancy 6.09

MELLET redundancy 7.27

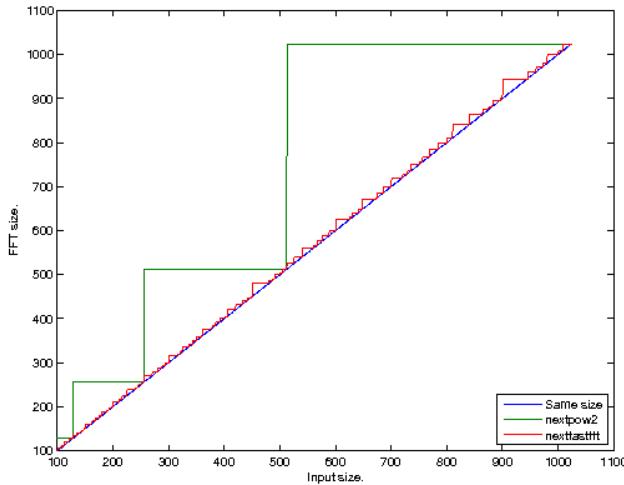


Figure 12.42: Benchmark of the FFT routine

The figure shows the sizes returned by the `nextfastfft` function compared to using `nextpow2`. As can be seen, the `nextfastfft` approach gives FFT sizes that are much closer to the input size.

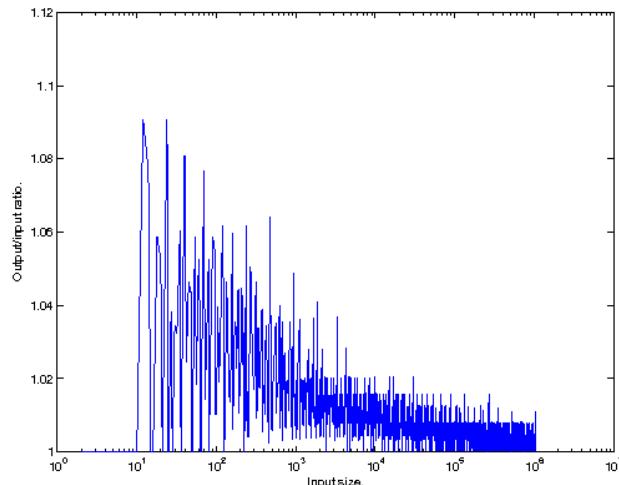


Figure 12.43: Efficiency of the table

The figure show the highest output/input ratio for varying input sizes. As can be seen, the efficiency is better for larger input values, where the output size is at most a few percent larger than the input size.

12.6 Auditory scales and filters

12.6.1 DEMO_AUDSCALES - Plot of the different auditory scales

This demos generates a simple figure that shows the behaviour of the different audiology scales in the frequency range from 0 to 8000 Hz.

Output

Type "help demo_audscales" to see a description of how this demo works.

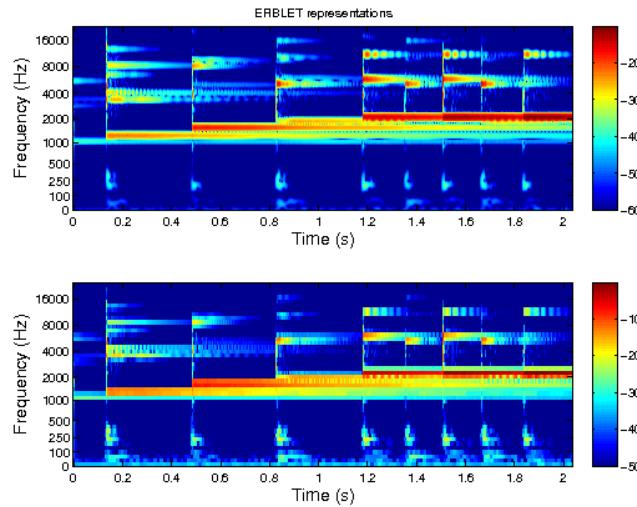


Figure 12.44: ERBLET representations

The high-redundancy plot (top) consists of 400 channels (~9 filters per ERB) and low-redundancy plot (bottom) consists of 44 channels (1 filter per ERB).

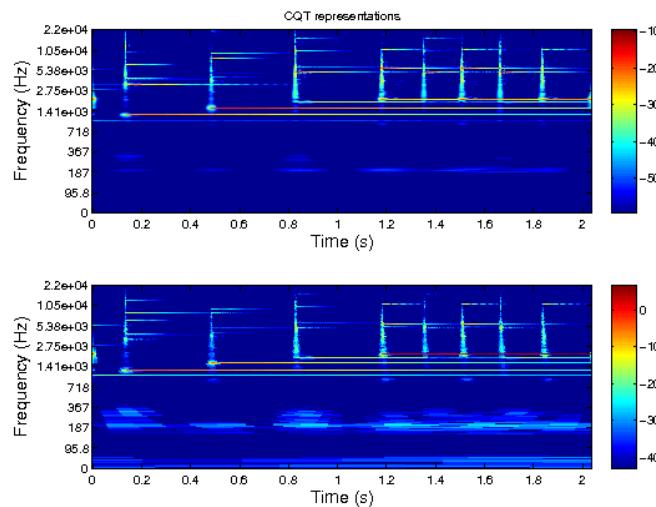


Figure 12.45: CQT representations

Both representations consist of 280 channels (32 channels per octave, frequency range 50Hz-20kHz). The high-redundancy representation is on the top and the low-redundancy repr. is on the bottom.

12.6.2 DEMO_AUDITORYFILTERBANK - Construct an auditory filterbank

In this file we construct a uniform filterbank using a the impulse response of a 4th order gammatone for each channel. The center frequencies are equidistantly spaced on an ERB-scale, and the width of the filter are choosen to match the auditory filter bandwidth as determined by Moore.

Each channel is subsampled by a factor of 8 ($a=8$), and to generate a nice plot, 4 channels per Erb has been used.

The filterbank covers only the positive frequencies, so we must use filterbankreal dual and filterbankreal bounds.

References: [37]

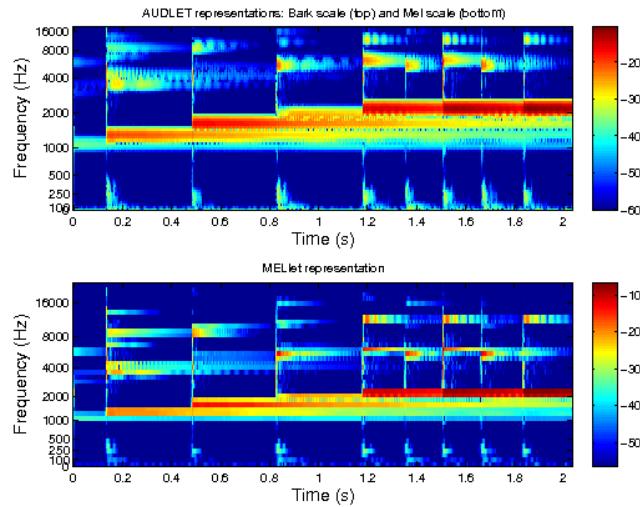


Figure 12.46: AUDLET representations

The first representation consists of 72 channels BARKlet FB (3 filters per Bark in the frequency range 100Hz-16kHz using fractional subsampling. The second representation consists of 40 channels MELlet FB using uniform subsampling and triangular windows.

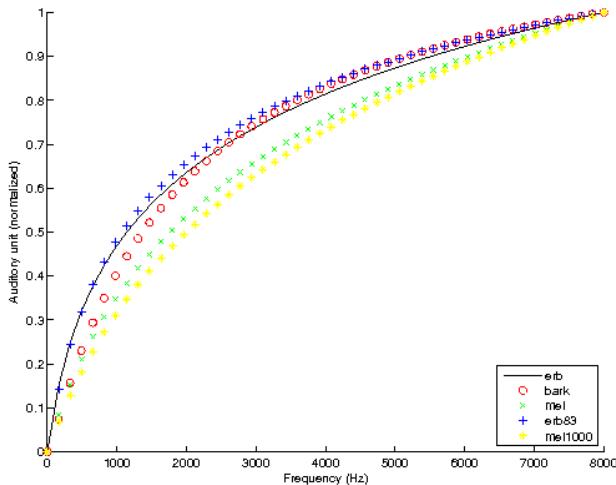


Figure 12.47: Auditory scales

The figure shows the behaviour of the auditory scales on a normalized frequency plot.

Output

Frame bound ratio for gammatone filterbank, should be close to 1 if the filters are ch

```
ans =
```

```
1.598850055103133
```

Relative error in reconstruction, should be close to zero.

```
ans =
```

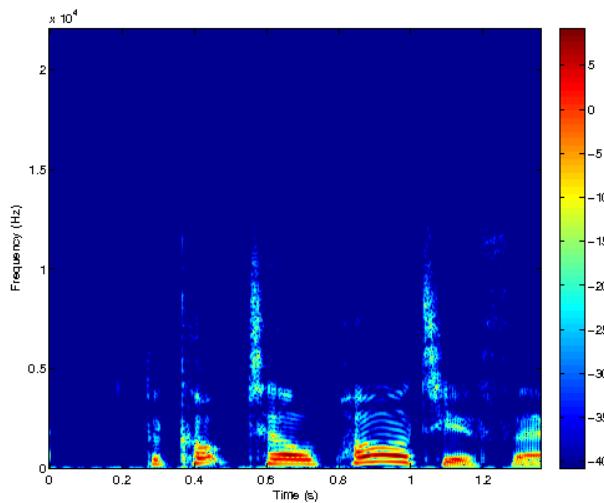


Figure 12.48: Classic spectrogram

A classic spectrogram of the spoken sentence. The dynamic range has been set to 50 dB, to highlight the most important features.

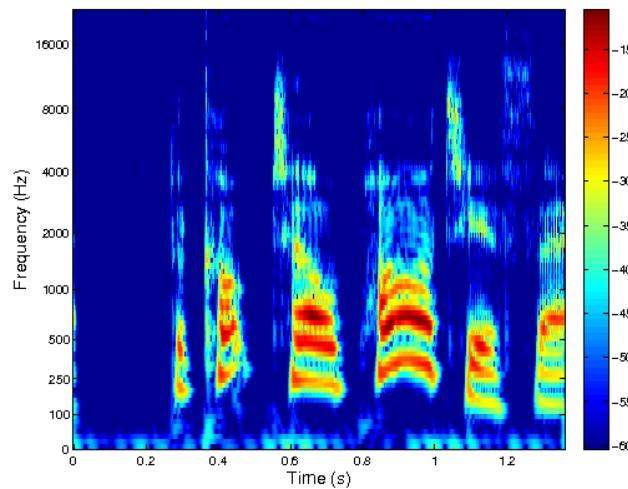


Figure 12.49: Auditory filterbank representation

Auditory filterbank representation of the spoken sentence using gammatone filters on an Erb scale. The dynamic range has been set to 50 dB, to highlight the most important features.

```
6.351688463338187e-05
```

Relative error in interative reconstruction, should be close to zero.

```
ans =
```

```
1.883372347722060e-10
```

12.6.3 DEMO_WFBT - Auditory filterbanks built using filterbank tree structures

This demo shows two specific constructions of wavelet filterbank trees using several M-band filterbanks with possibly different M (making the thee non-homogenous) in order to approximate auditory frequency bands and a musical scale respectively. Both wavelet trees produce perfectly reconstructable non-redundant representations.

The constructions are the following:

- 1) Auditory filterbank, approximately dividing the frequency band into intervals reminiscent of the bask scale.
- 2) Musical filterbank, approximately dividing the freq. band into intervals reminiscent of the well-tempered musical scale.

Shapes of the trees were taken from fig. 8 and fig. 9 from the refernece. Sampling frequency of the test signal is 48kHz as used in the article.

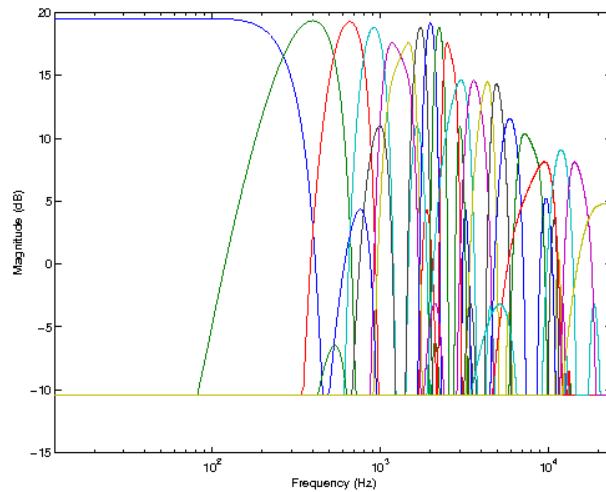


Figure 12.50: Frequency responses of the auditory filterbank.

Both axes are in logarithmic scale.

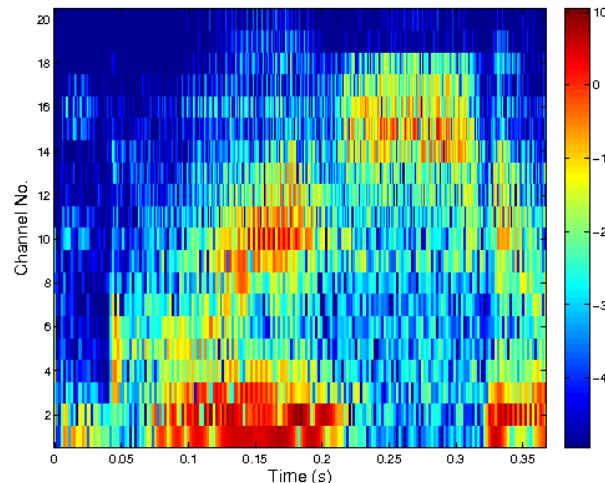


Figure 12.51: TF plot of the test signal using the auditory filterbank.

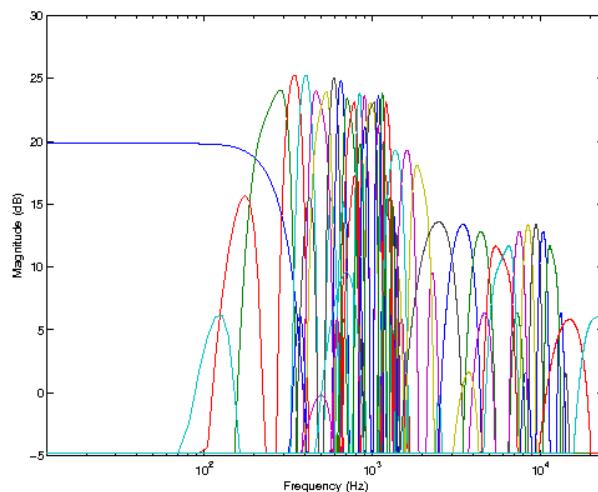


Figure 12.52: Frequency responses of the musical filterbank.
Both axes are in logarithmic scale.

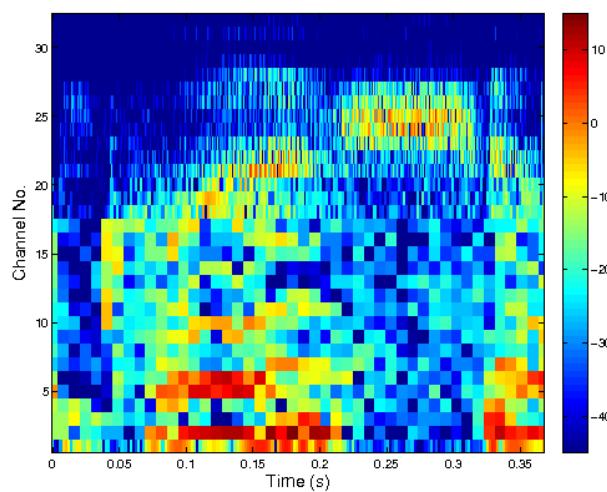


Figure 12.53: TF plot of the test signal using the musical filterbank.

References: [55]

Output

The reconstruction should be close to zero:

```
ans =
4.783697746283275e-14
```

The reconstruction should be close to zero:

```
ans =
8.871444670941919e-14
```

12.7 Block-processing demos

12.7.1 DEMO_BLOCKPROC_BASICLOOP - Basic real-time audio manipulation

Usage

```
demo_blockproc_basicloop('gspi.wav')
```

Description

For additional help call `demo_blockproc_basicloop` without arguments.

The demo runs simple playback loop allowing to set gain in dB.

Output

`DEMO_BLOCKPROC_BASICLOOP`:

To run the demo, use one of the following:

```
demo_blockproc_basicloop('gspi.wav') to play gspi.wav (any wav file will do).
demo_blockproc_basicloop('dialog') to choose the wav file via file chooser dialog GUI
demo_blockproc_basicloop(f,'fs',fs) to play from a column vector f using sampling freq fs
demo_blockproc_basicloop('playrec') to record from a mic and play simultaneously.
```

Available input and output devices can be listed by `|blockdevices|`.

Particular device can be chosen by passing additional key-value pair `'devid'`, `devid`.

Output channels of the device can be selected by additional key-value pair `'playch'`, `[ch1 ch2 ...]`.

Input channels of the device can be selected by additional key-value pair `'recch'`, `[ch1 ch2 ...]`.

12.7.2 DEMO_BLOCKPROC_PARAMEQUALIZER - Real-time equalizer demonstration

Usage

```
demo_blockproc_paramequalizer('gspi.wav')
```

Description

For additional help call `demo_blockproc_paramequalizer` without arguments.

This demonstration shows an example of a octave parametric equalizer. See chapter 5.2 in the book by Zolzer.

References: [99]

Output

`DEMO_BLOCKPROC_PARAMEQUALIZER`:

To run the demo, use one of the following:

```
demo_blockproc_paramequalizer('gspi.wav') to play gspi.wav (any wav file will do).
demo_blockproc_paramequalizer('dialog') to choose the wav file via file chooser dialog GUI
demo_blockproc_paramequalizer(f,'fs',fs) to play from a column vector f using sampling freq fs
demo_blockproc_paramequalizer('playrec') to record from a mic and play simultaneously.
```

Available input and output devices can be listed by `|blockdevices|`.

Particular device can be chosen by passing additional key-value pair `'devid'`, `devid`.

Output channels of the device can be selected by additional key-value pair 'playch', [ch]
 Input channels of the device can be selected by additional key-value pair 'recch', [ch]

12.7.3 DEMO_BLOCKPROC_DENOISING - Variable coefficients thresholding

Usage

```
demo_blockproc_denoising('gspi.wav')
```

Description

For additional help call demo_blockproc_denoising without arguments.

The present demo allows you to set the coefficient threshold during the playback using the control panel.

Output

DEMO_BLOCKPROC_DENOISING:

To run the demo, use one of the following:

demo_blockproc_denoising('gspi.wav') to play gspi.wav (any wav file will do).
 demo_blockproc_denoising('dialog') to choose the wav file via file chooser dialog GUI
 demo_blockproc_denoising(f,'fs',fs) to play from a column vector f using sampling frequency fs
 demo_blockproc_denoising('playrec') to record from a mic and play simultaneously.

Available input and output devices can be listed by |blockdevices|.

Particular device can be chosen by passing additional key-value pair 'devid', devid.

Output channels of the device can be selected by additional key-value pair 'playch', [ch]

Input channels of the device can be selected by additional key-value pair 'recch', [ch]

12.7.4 DEMO_BLOCKPROC_SLIDINGSGRAM - Basic real-time rolling spectrogram visualization

Usage

```
demo_blockproc_slidingsgram('gspi.wav')
```

Description

For additional help call demo_blockproc_slidingsgram without arguments.

This demo shows a simple rolling spectrogram of whatever is specified in source.

Output

DEMO_BLOCKPROC_SLIDINGSGRAM:

To run the demo, use one of the following:

demo_blockproc_slidingsgram('gspi.wav') to play gspi.wav (any wav file will do).
 demo_blockproc_slidingsgram('dialog') to choose the wav file via file chooser dialog GUI
 demo_blockproc_slidingsgram(f,'fs',fs) to play from a column vector f using sampling frequency fs
 demo_blockproc_slidingsgram('playrec') to record from a mic and play simultaneously.

Available input and output devices can be listed by |blockdevices|.

Particular device can be chosen by passing additional key-value pair 'devid', devid.

Output channels of the device can be selected by additional key-value pair 'playch', [ch]

Input channels of the device can be selected by additional key-value pair 'recch', [ch]

12.7.5 DEMO_BLOCKPROC_SLIDINGCQT - Basic real-time rolling CQT-spectrogram visualization

Usage

```
demo_blockproc_slidingcqt('gspi.wav')
```

Description

For additional help call `demo_blockproc_slidingcqt` without arguments.

This demo shows a simple rolling CQT-spectrogram of whatever is specified in source.

Output

`DEMO_BLOCKPROC_SLIDINGCQT`:

To run the demo, use one of the following:

```
demo_blockproc_slidingcqt('gspi.wav') to play gspi.wav (any wav file will do).
demo_blockproc_slidingcqt('dialog') to choose the wav file via file chooser dialog GUI.
demo_blockproc_slidingcqt(f,'fs',fs) to play from a column vector f using sampling frequency fs.
demo_blockproc_slidingcqt('playrec') to record from a mic and play simultaneously.
```

Available input and output devices can be listed by `|blockdevices|`.

Particular device can be chosen by passing additional key-value pair `'devid'`, `devid`.

Output channels of the device can be selected by additional key-value pair `'playch'`, `[ch]`.

Input channels of the device can be selected by additional key-value pair `'recch'`, `[ch]`.

12.7.6 DEMO_BLOCKPROC_SLIDINGERBLETS - Basic real-time rolling erblet-spectrogram visualization

Usage

```
demo_blockproc_slidingerblets('gspi.wav')
```

Description

For additional help call `demo_blockproc_slidingerblets` without arguments.

This demo shows a simple rolling erblet-spectrogram of whatever is specified in source.

Output

`DEMO_BLOCKPROC_SLIDINGERBLETS`:

To run the demo, use one of the following:

```
demo_blockproc_slidingerblets('gspi.wav') to play gspi.wav (any wav file will do).
demo_blockproc_slidingerblets('dialog') to choose the wav file via file chooser dialog GUI.
demo_blockproc_slidingerblets(f,'fs',fs) to play from a column vector f using sampling frequency fs.
demo_blockproc_slidingerblets('playrec') to record from a mic and play simultaneously.
```

Available input and output devices can be listed by `|blockdevices|`.

Particular device can be chosen by passing additional key-value pair `'devid'`, `devid`.

Output channels of the device can be selected by additional key-value pair `'playch'`, `[ch]`.

Input channels of the device can be selected by additional key-value pair `'recch'`, `[ch]`.

12.7.7 DEMO_BLOCKPROC_DGTEQUALIZER - Real-time audio manipulation in the transform domain

Usage

```
demo_blockproc_dgtequalizer('gspi.wav')
```

Description

For additional help call demo_blockproc_dgtequalizer without arguments.

This script demonstrates a real-time Gabor coefficient manipulation. Frequency bands of Gabor coefficients are multiplied (weighted) by values taken from sliders having a similar effect as a octave equalizer. The shown spectrogram is a result of a re-analysis of the synthetized block to show a frequency content of what is actually played.

Output

DEMO_BLOCKPROC_DGTEQUALIZER:

To run the demo, use one of the following:

```
demo_blockproc_dgtequalizer('gspi.wav') to play gspi.wav (any wav file will do).
demo_blockproc_dgtequalizer('dialog') to choose the wav file via file chooser dialog
demo_blockproc_dgtequalizer(f,'fs',fs) to play from a column vector f using sampling rate fs
demo_blockproc_dgtequalizer('playrec') to record from a mic and play simultaneously.
```

Available input and output devices can be listed by |blockdevices|.

Particular device can be chosen by passing additional key-value pair 'devid', devid. Output channels of the device can be selected by additional key-value pair 'playch', [ch]. Input channels of the device can be selected by additional key-value pair 'recch', [ch].

12.7.8 DEMO_BLOCKPROC_EFFECTS - Various vocoder effects using DGT

Usage

```
demo_blockproc_effects('gspi.wav')
```

Description

For additional help call demo_blockproc_effects without arguments. This demo works correctly only with the sampling rate equal to 44100 Hz.

This script demonstrates several real-time vocoder effects. Namely:

- 1) Robotization effect: Achieved by doing DGT reconstruction using absolute values of coefficients only.
- 2) Whisperization effect: Achieved by randomizing phase of DGT coefficients.
- 3) Pitch shifting effect: Achieved by stretching/compressing coefficients along frequency axis.
- 4) Audio morphing: Input is morphed with a background sound such that the phase of DGT coefficients is substituted by phase of DGT coefficients of the background signal. File beat.wav (at 44,1kHz) (any sound will do) is expected to be in the search path, otherwise the effect will be disabled.

This demo was created for the Lange Nacht der Forschung 4.4.2014 event.

Output

DEMO_BLOCKPROC_EFFECTS:

To run the demo, use one of the following:

```
demo_blockproc_effects('gspi.wav') to play gspi.wav (any wav file will do).  
demo_blockproc_effects('dialog') to choose the wav file via file chooser dialog GUI.  
demo_blockproc_effects(f,'fs',fs) to play from a column vector f using sampling frequency fs.  
demo_blockproc_effects('playrec') to record from a mic and play simultaneously.
```

Available input and output devices can be listed by |blockdevices|.

Particular device can be chosen by passing additional key-value pair 'devid', devid.

Output channels of the device can be selected by additional key-value pair 'playch', [ch1 ch2 ...].

Input channels of the device can be selected by additional key-value pair 'recch', [ch1 ch2 ...].

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