

Large Language Models Under the Hood

Part 1: Training Deep Neural Networks

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NLDL 2025





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- 1 Tutorial technicalities
- 2 Basics of supervised machine learning
- 3 Advantages and limitations of linear models
- 4 Going deeply non-linear: multi-layered perceptrons
- 5 Non-linearities
- 6 Enters Transformer

Tutorial technicalities



Welcome to the NLDL 2025 tutorial on large language models!



Andrey Kutuzov, Egil Rønningstad, David Samuel
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Tutorial technicalities



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Schedule

- ▶ 13:00 - Part 1, Training Deep Neural Networks
- ▶ 14:00 - Part 2, Modern Generative Language Models
- ▶ 15:00 - Part 3, Hands-on with open-weights LLMs for Norwegian

Supported by NAIC



NAIC assists you to find the infrastructure fits your needs

- Self service, reduce administrative workload
- VMs with software modules and read-only access to data
- VMs as submit hosts to HPC
- user-support-user model, forums
- Consolidation of infrastructure, support and training
- Resource monitoring and feedback

NAIC Orchestrator

NAICvm	Creating
vCPUs	2
memory	8 GB
storage	20 GB
OS	GOLD CentOS Stream 9

Materials for the hands-on session

- ▶ Slides, code, instructions:

https://github.com/ltgoslo/nndl_llm_tutorial

- ▶ You are allocated Linux virtual machines for the hands-on session
- ▶ Connect via ssh and run Python; can also use Jupyter Lab.
- ▶ Find the ip address, username and key file for your virtual machine (VM) in the spreadsheet below:



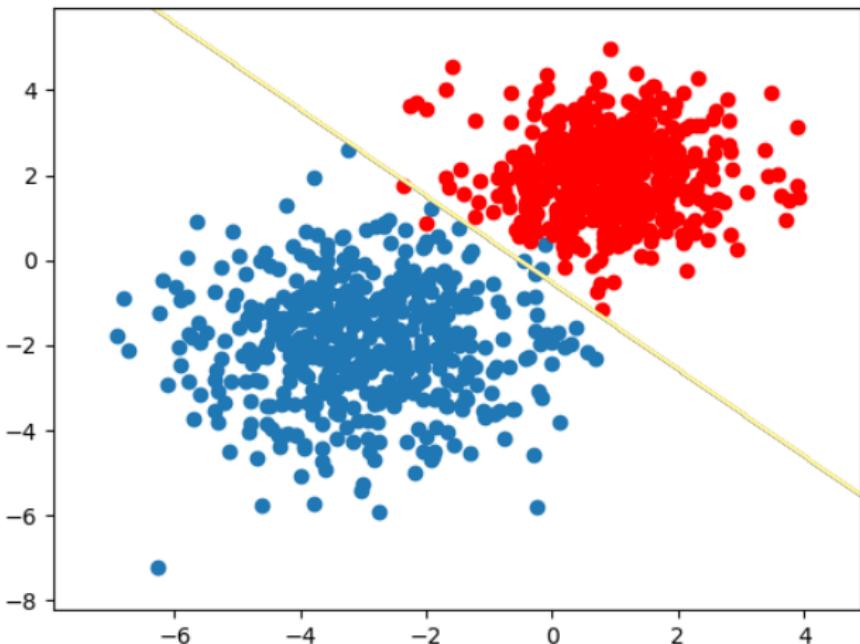
<https://docs.google.com/spreadsheets/d/13A-iyiBb2tmhm9RLMJp1wndfgm0Bt6HackY2miT5yeY>

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Basics of supervised machine learning



Basics of supervised machine learning



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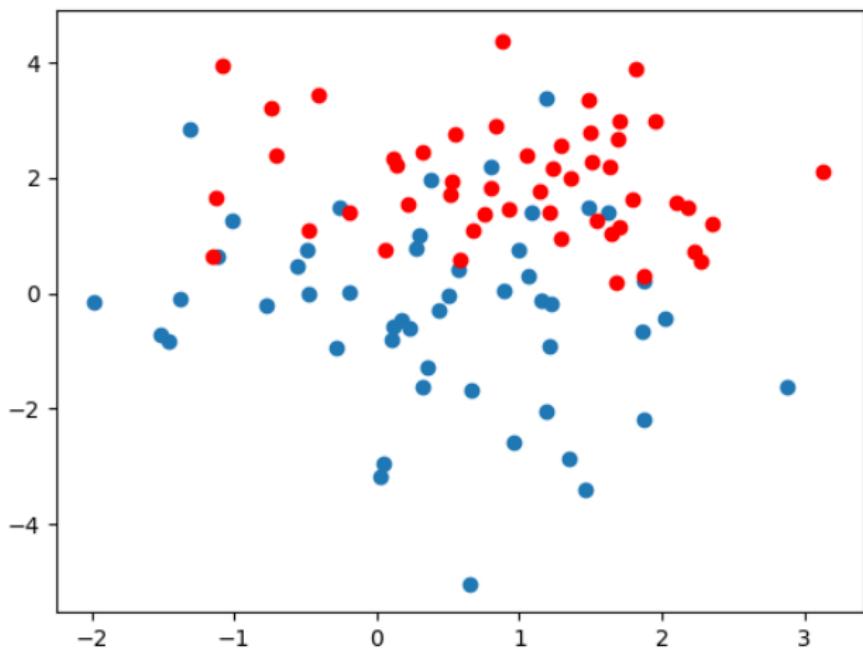


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 - ▶ for example, whether the message is spam (1) or not (0).
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- ▶ Generally: some program for mapping instances to labels.

Basics of supervised machine learning



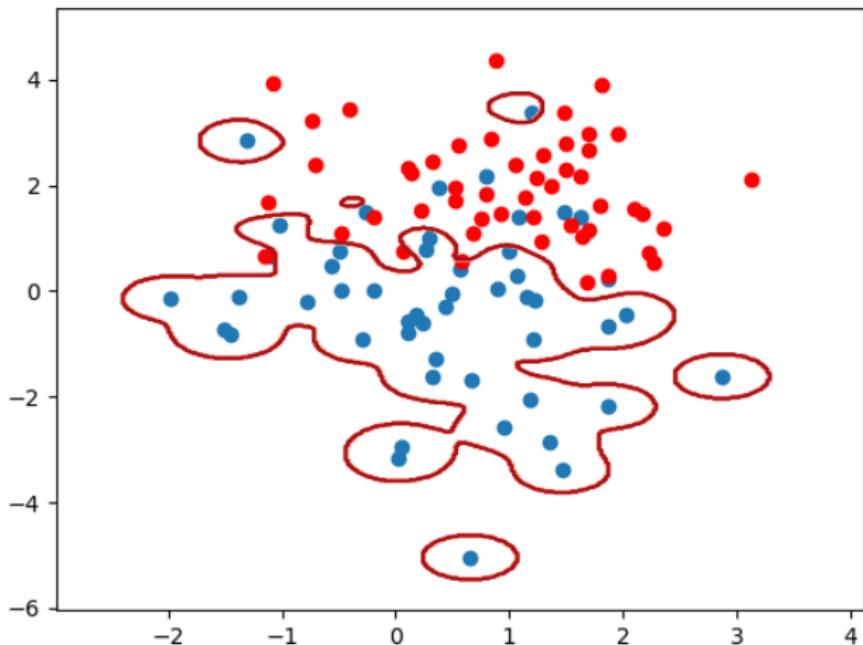
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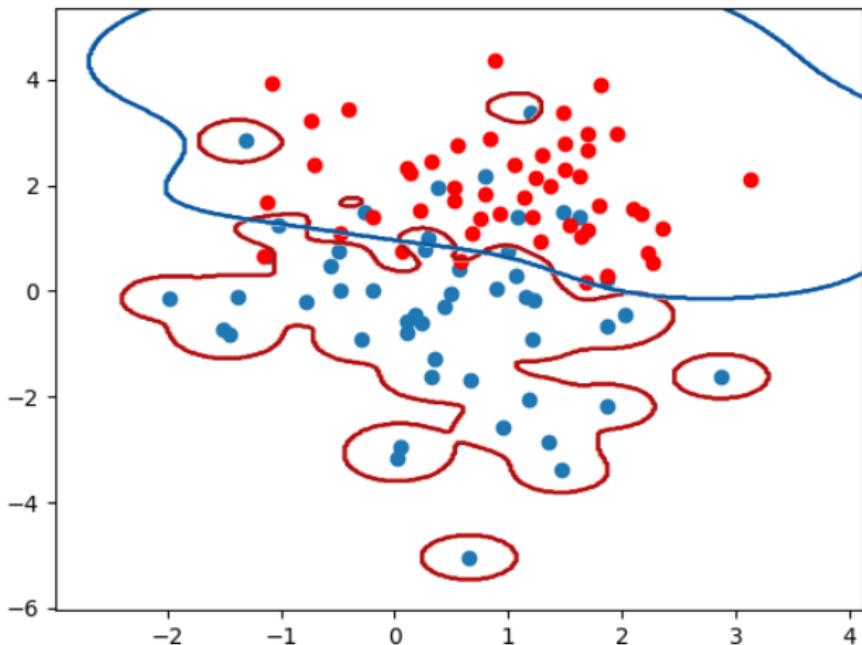
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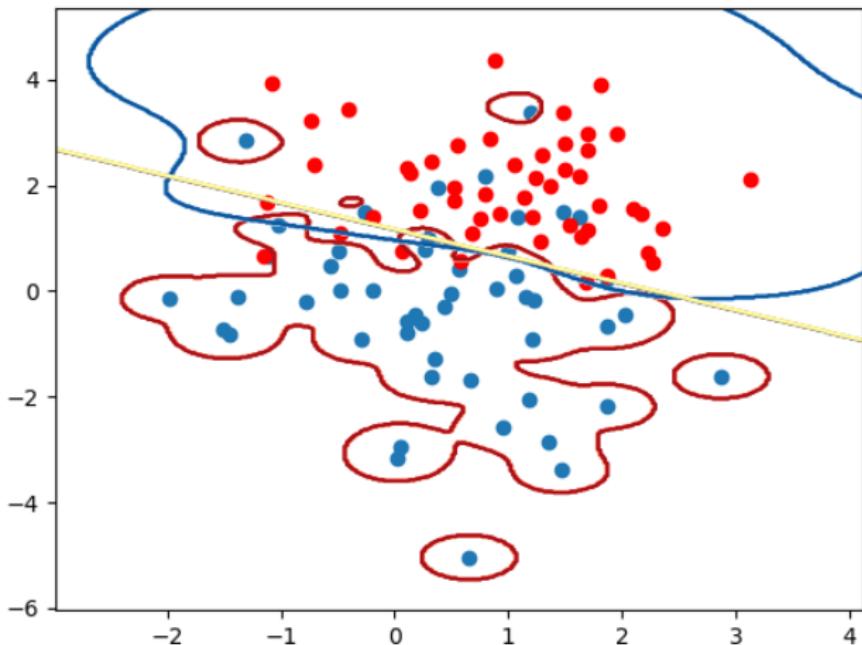
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Linear functions: a popular hypothesis class

Simple linear function

$$f(\mathbf{x}; \mathbf{W}, \mathbf{b}) = \mathbf{x} \cdot \mathbf{W} + \mathbf{b} \quad (1)$$

$$\theta = \mathbf{W}, \mathbf{b} \quad (2)$$

► Function input:

- feature vector $\mathbf{x} \in \mathbb{R}^{d_{in}}$;
- each training instance is represented with d_{in} **features**;
- for example, some properties of the documents.

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 - ▶ feature vector $\mathbf{x} \in \mathbb{R}^{d_{in}}$;
 - ▶ each training instance is represented with d_{in} **features**;
 - ▶ for example, some properties of the documents.
- ▶ Function parameters θ :
 - ▶ **matrix** $\mathbf{W} \in \mathbb{R}^{d_{in} \times d_{out}}$
 - ▶ d_{out} is the dimensionality of the desired prediction (number of classes)
 - ▶ **bias vector** $\mathbf{b} \in \mathbb{R}^{d_{out}}$
 - ▶ bias 'shifts' the function output to some direction.

Training of a linear classifier

$$f(\mathbf{x}; \mathbf{W}, b) = \mathbf{x} \cdot \mathbf{W} + b$$

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- ▶ Training is finding the optimal θ .
- ▶ 'Optimal' means '*producing predictions \hat{y} closest to the gold labels y on our n training instances*'.
- ▶ Ideally, $\hat{y} = y$

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Output of **binary** classification

Binary decision ($d_{out} = 1$):

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- ▶ \boldsymbol{W} is a vector, b is a scalar.

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Binary decision ($d_{out} = 1$):

- ▶ ‘*Is this message spam or not?*’
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- ▶ ‘Is this message spam or not?’
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- ▶ The prediction \hat{y} is also a scalar: either 1 ('yes') or -1 ('no').
- ▶ NB: the model can output any number, but we convert all negatives to -1 and all positives to 1 (*sign* function).

$$\theta = (\mathbf{W} \in \mathbb{R}^{d_{in}}, b \in \mathbb{R}^1)$$

Basics of supervised machine learning



$$f(x; W, b) = x \cdot W + b$$

$$\begin{matrix} x \\ \begin{array}{|c|} \hline 0 \\ \hline 1 \\ \hline 0 \\ \hline 1 \\ \hline 0 \\ \hline \end{array} \\ \cdot \quad \begin{array}{|c|c|c|c|c|} \hline 0 & 0 & 1 & 1 & 1 \\ \hline \end{array} \quad + \quad 0.5 \quad = \text{sign}(1.5) = 1 \end{matrix}$$

Basics of supervised machine learning



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Multi-class decision ($d_{out} = k$)

- ▶ ‘*Which languages appear in this document?*’
- ▶ \mathbf{W} is a matrix, \mathbf{b} is a vector of k components.

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- ▶ ‘Which languages appear in this document?’
- ▶ \mathbf{W} is a matrix, \mathbf{b} is a vector of k components.
- ▶ The prediction $\hat{\mathbf{y}}$ is also a one-hot vector of k components.
- ▶ The component corresponding to the correct language has the value of 1, others are zeros, for example:
 $\hat{\mathbf{y}} = [0, 0, 1, 0]$ (for $k = 4$)

$$\theta = (\mathbf{W} \in \mathbb{R}^{d_{in} \times d_{out}}, \mathbf{b} \in \mathbb{R}^{d_{out}})$$

Basics of supervised machine learning



$$f(x; W, b) = x \cdot W + b$$

$$\begin{matrix} x \\ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \end{matrix} \cdot \begin{matrix} W \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \end{bmatrix} \end{matrix} + \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} = \text{argmax}(\begin{bmatrix} 2 & 2 & 4 \end{bmatrix}) \\ = 3$$



Log-linear classification

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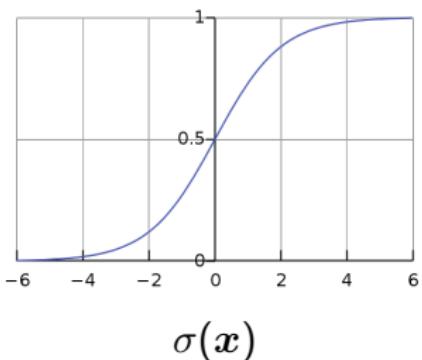
Log-linear classification

If we care about how confident is the classifier about each decision:

- ▶ Map the predictions to the range of $[0, 1] \dots$
- ▶ ...by a **squashing function**, for example, **sigmoid**:

$$\hat{y} = \sigma(f(x)) = \frac{1}{1 + e^{-(f(x))}} \quad (3)$$

- ▶ The result is the **probability** of the prediction!



Basics of supervised machine learning



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- ▶ But often it is more convenient to transform scores into a **probability distribution**, using the **softmax** function:

$$\hat{y} = \text{softmax}(xW + b) \quad (5)$$

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- ▶ $\hat{y} = \text{softmax}([0.4, 0.1, 0.9, 0.5]) = [0.22, 0.16, 0.37, 0.25]$
 - ▶ (all scores sum to 1)

Basics of supervised machine learning



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(expect this to be used for **language modeling**: it is secretly classification!)

Basics of supervised machine learning



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- ▶ It is averaged over all training instances and gives us estimation of the model ‘fitness’.
- ▶ $\hat{\theta}$ is the best set of parameters:

$$\hat{\theta} = \arg \min_{\theta} \mathcal{L}(\theta) \quad (7)$$



Now we can measure model performance. How can we change our parameters θ to improve?

Optimizing with gradient

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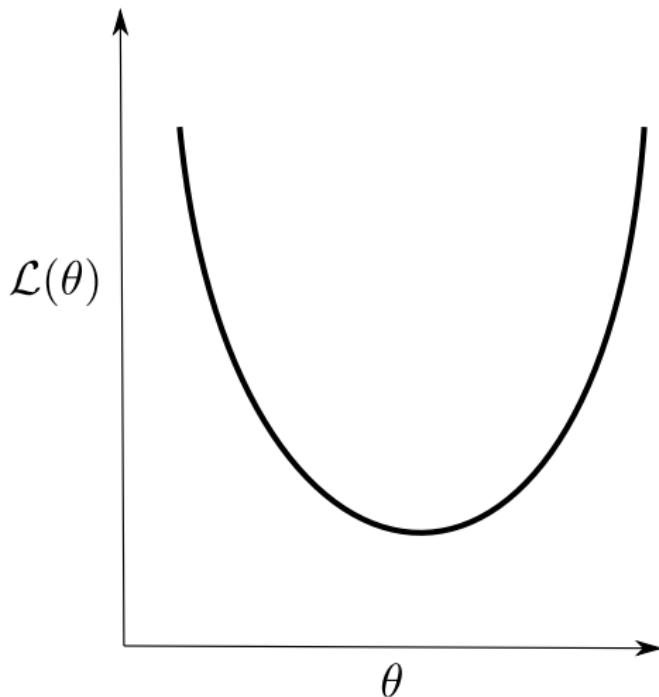
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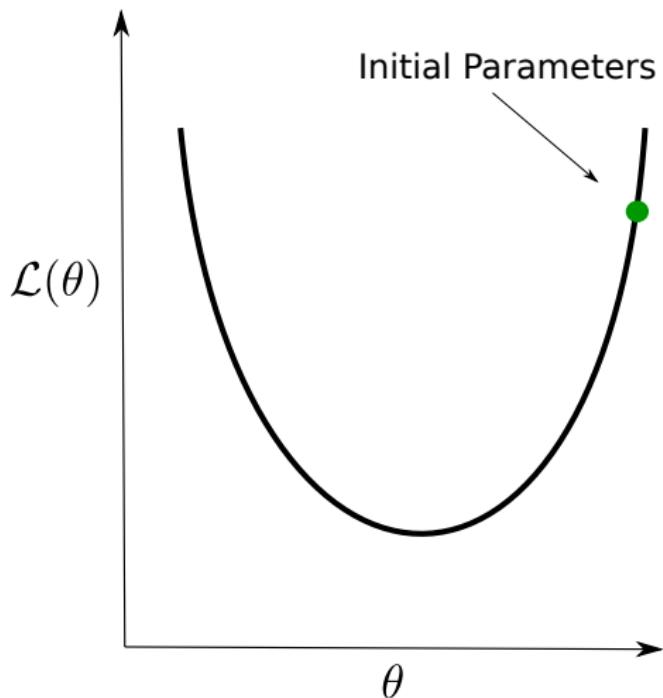
Convexity

- ▶ **Convex functions:** a single optimum point.
- ▶ **Non-convex functions:** multiple optimum points.

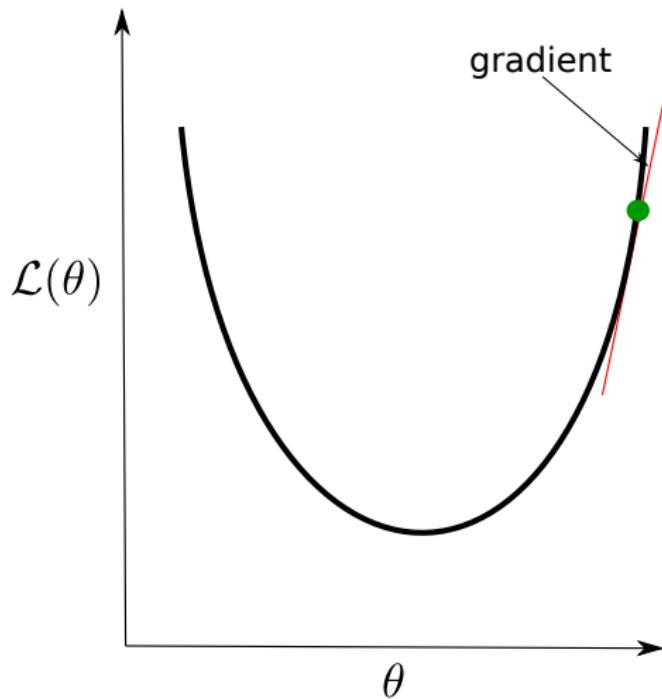
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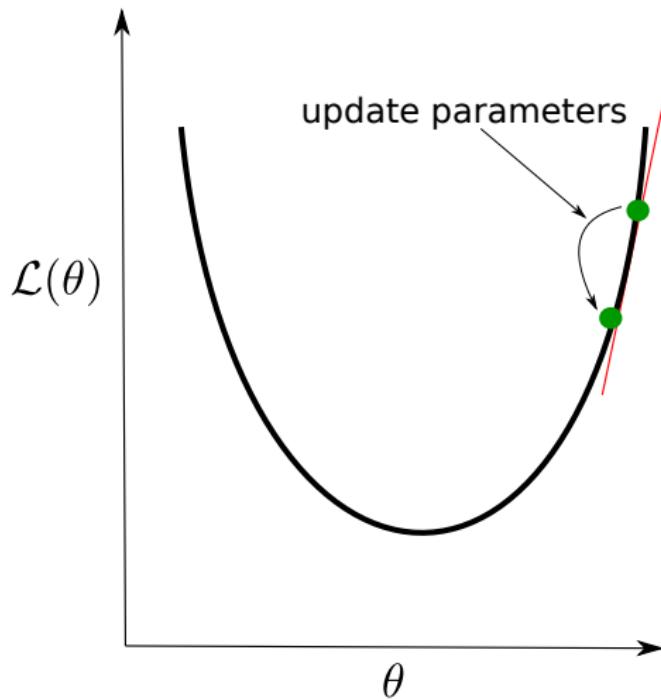
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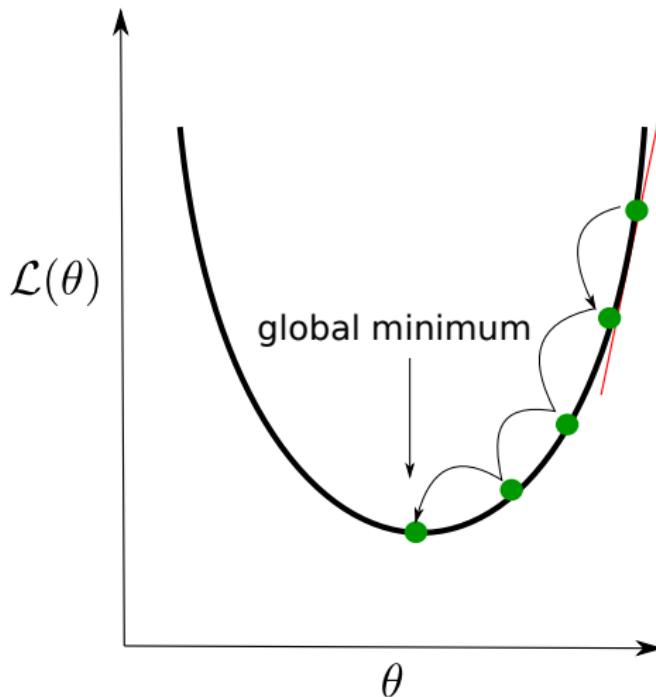
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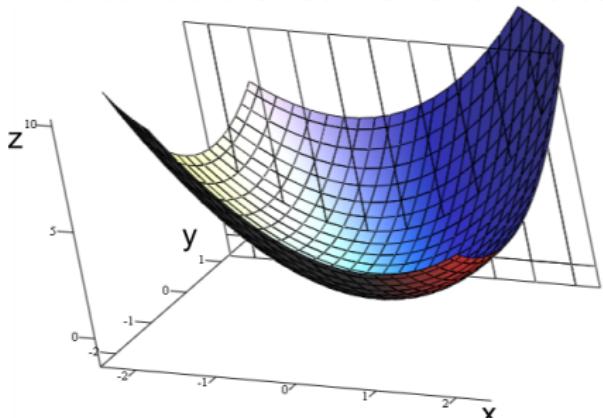
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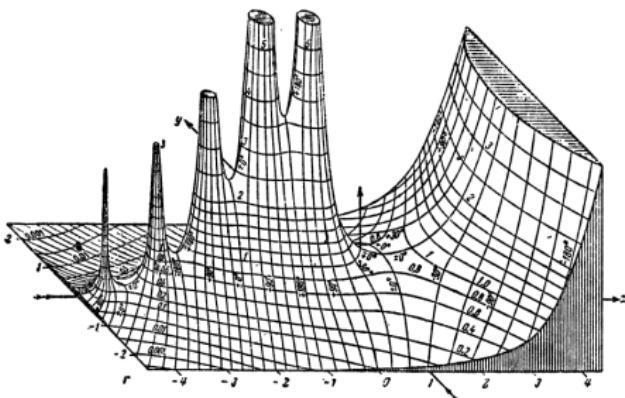
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Error surfaces of convex and not-convex functions:

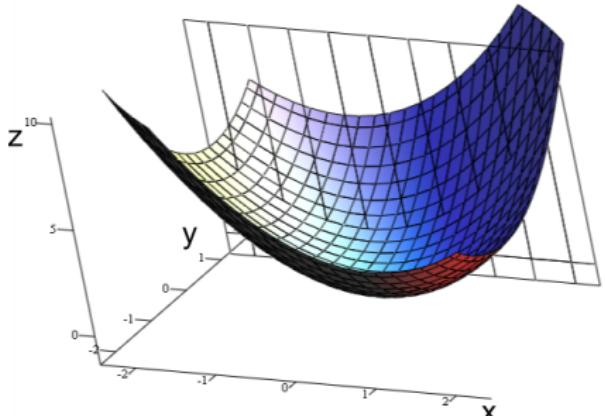


Convex function



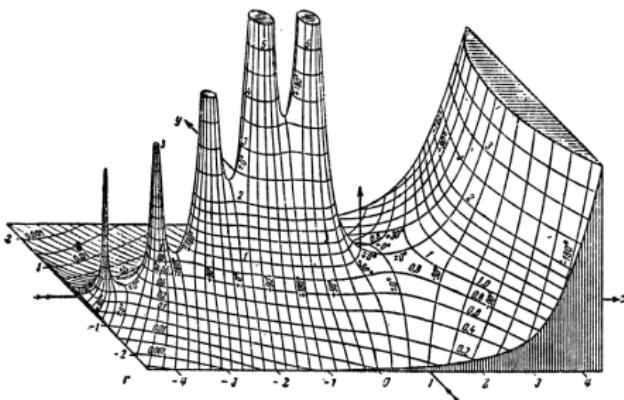
Non-convex function

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- Convex functions can be easily minimized with gradient methods, reaching the global optimum.
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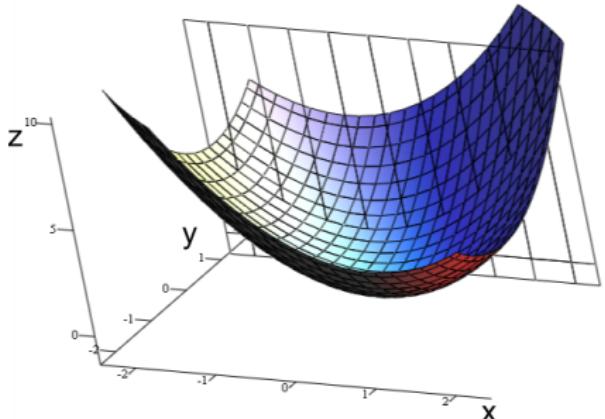


Non-convex function

Basics of supervised machine learning

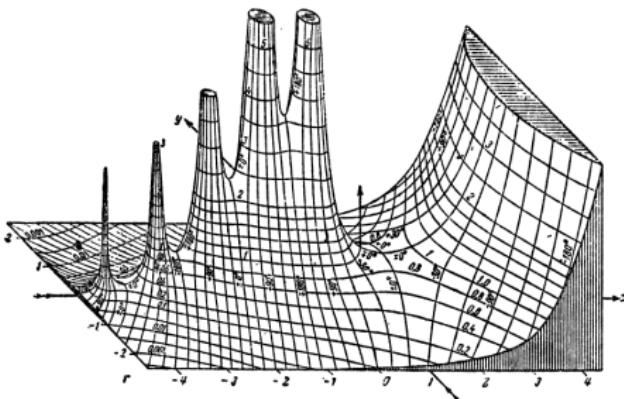


Error surfaces of convex and not-convex functions:



Convex function

- Convex functions can be easily minimized with gradient methods, reaching the global optimum.
- With non-convex functions, optimization can end up in a local optimum.
- Linear and log-linear models as a rule have convex error functions.
- ...unlike non-linear models (e.g., used in 'deep learning').



Non-convex function



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Advantages and limitations of linear models



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Unfortunately, linear models can represent only **linear relations** in the data

Advantages and limitations of linear models



- Are there **non-linear functions** that linear models can't deal with?

Advantages and limitations of linear models

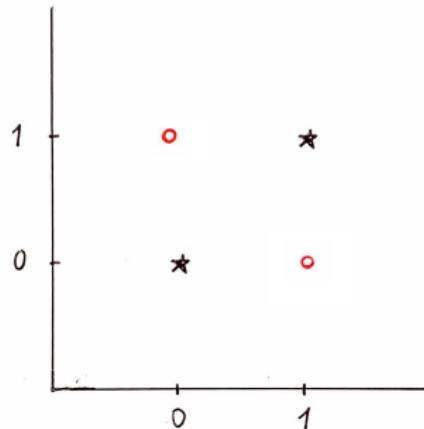


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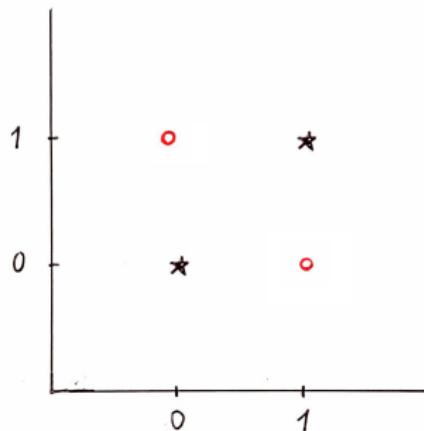
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Advantages and limitations of linear models



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It is clearly **not** linearly separable.



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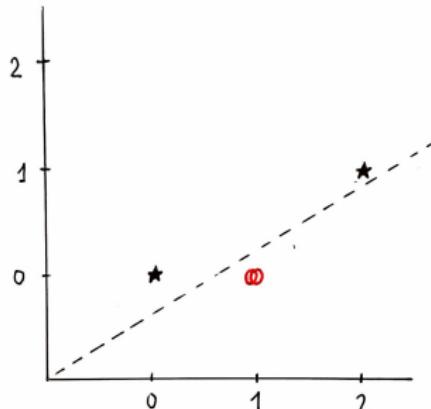
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For example, $\phi(x_1, x_2) = [x_1 + x_2, x_1 \times x_2]$ maps the instances to another representation and makes the XOR problem linearly separable:



Training mapping functions

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$$\phi(\mathbf{x}) = g(\mathbf{x}\mathbf{W}' + \mathbf{b}') \quad (8)$$

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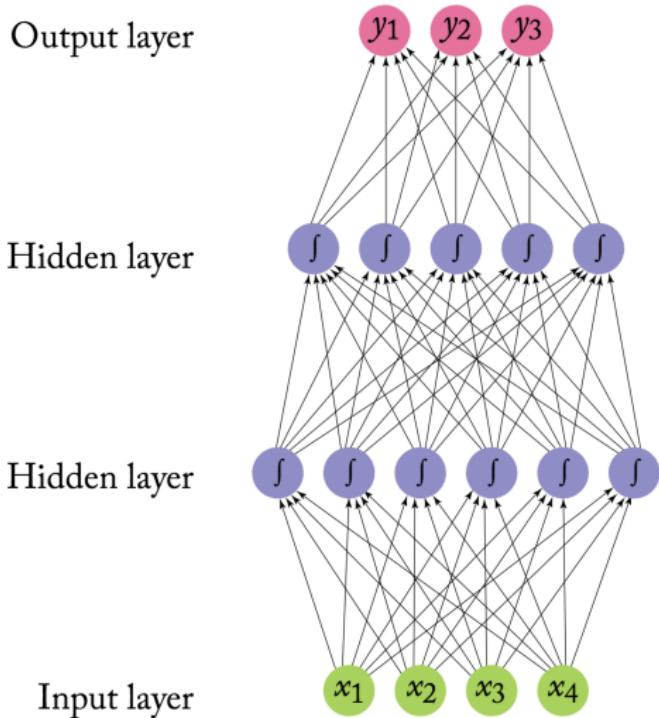
- ▶ ...where g is a non-linear **activation function**, and \mathbf{W}', \mathbf{b}' are its trainable parameters.
- ▶ The equation above defines a simple **multi-layer perceptron (MLP)**: a **neural model**.



Contents

- 1 Tutorial technicalities
- 2 Basics of supervised machine learning
- 3 Advantages and limitations of linear models
- 4 Going deeply non-linear: multi-layered perceptrons
- 5 Non-linearities
- 6 Enters Transformer

Going deeply non-linear: multi-layered perceptrons



A simple **feed-forward neural network** (perceptron) with 2 hidden layers.



The nature of perceptrons

- ▶ Input data **goes through successive transformations** at each layer.



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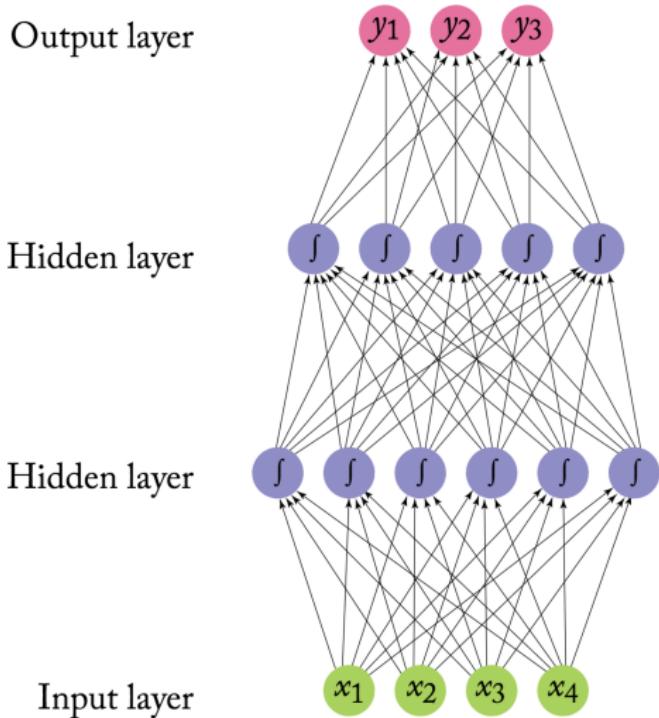


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Important: **neural networks with hidden layers can theoretically approximate any function** [Leshno et al., 1993].

Going deeply non-linear: multi-layered perceptrons



A simple **feed-forward neural network** (perceptron) with 2 hidden layers.

'Machines of this character can
behave in a very complicated
manner when the number of units
is large.'

Alan Turing, 'Intelligent Machinery'
[Turing, 1948]



Going deeply non-linear: multi-layered perceptrons



Brain metaphor

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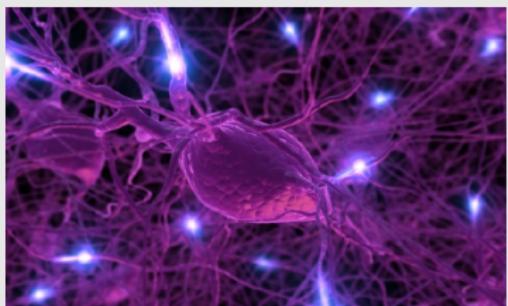


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...But brain metaphor is not optimal



- ▶ Although the first neural networks were based loosely on knowledge of neural activation at the time, we have long abandoned any real connection to neuroscience.

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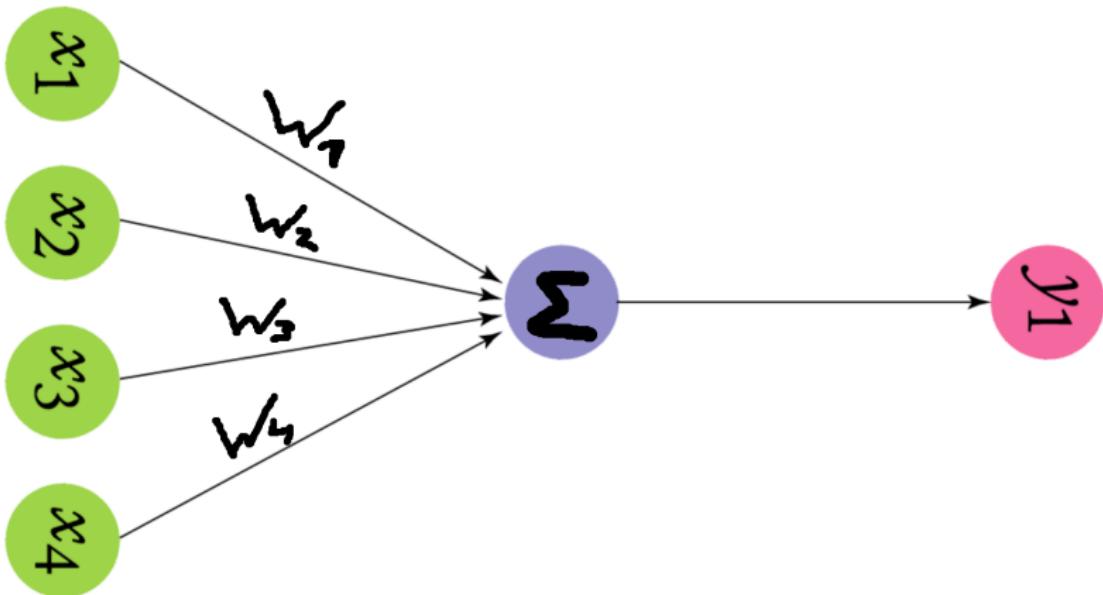
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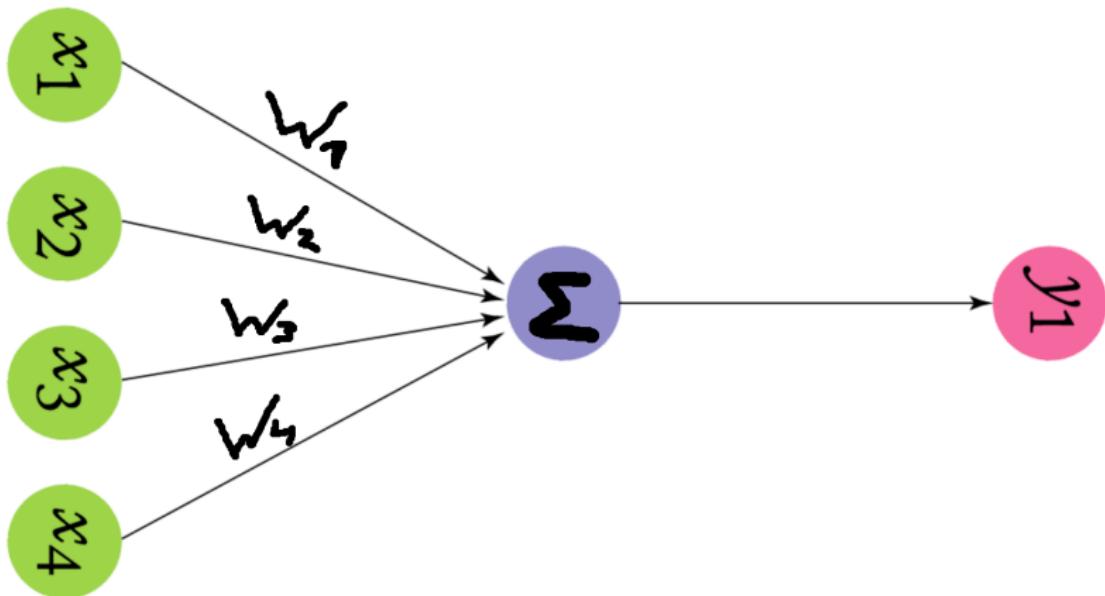


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 - ▶ vectors,
 - ▶ matrices,
 - ▶ sequential algebraic operations on them.

Going deeply non-linear: multi-layered perceptrons

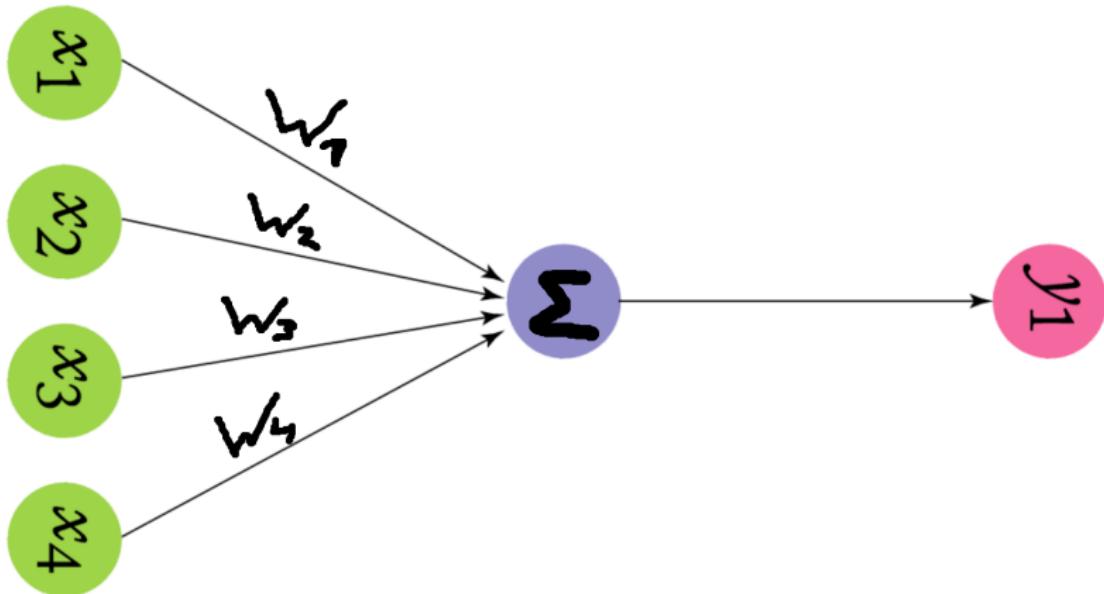


Going deeply non-linear: multi-layered perceptrons



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Going deeply non-linear: multi-layered perceptrons

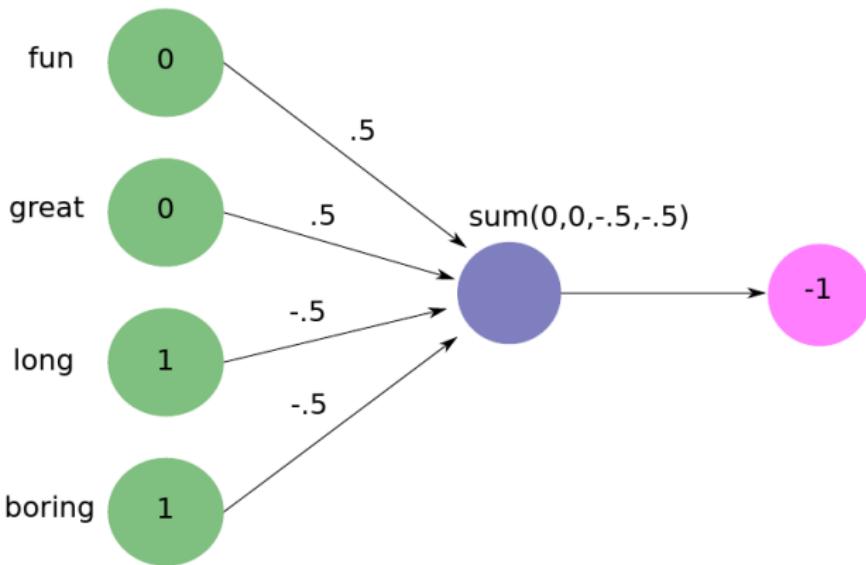


- ▶ In fact this is already a **simple neural network**...
- ▶ with **one neuron Σ** as a **computational unit**.
- ▶ Σ takes **4 input values** and returns their weighted sum as **output value**.

Going deeply non-linear: multi-layered perceptrons

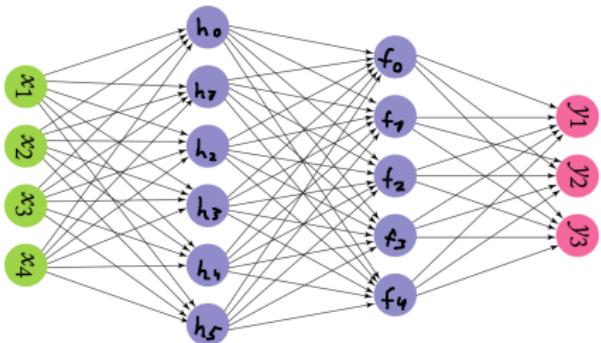


"It was long and a bit boring"

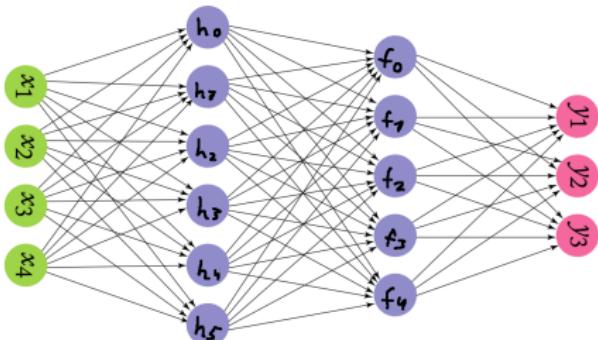


Sentiment analysis with only one neuron and binary bag-of-words.

Going deeply non-linear: multi-layered perceptrons



Going deeply non-linear: multi-layered perceptrons



Looks much like a 'deep neural network'

Stacked linear classifier with multiple computational units at each layer:

$$h = xW^0 \quad (10)$$

$$f = hW^1 \quad (11)$$

$$\hat{y} = fW^2 \quad (12)$$

Going deeply non-linear: multi-layered perceptrons



- ▶ But any stack of linear classifiers is still a linear classifier :-)

Going deeply non-linear: multi-layered perceptrons



- ▶ But any stack of linear classifiers is still a linear classifier :-(
- ▶ Still can't handle XOR and other non-linear problems.



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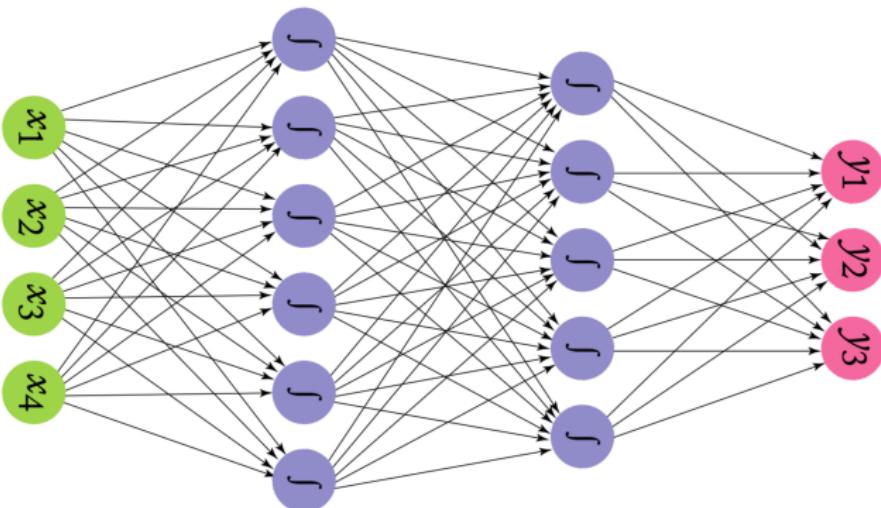
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- ▶ **Hyperbolic tangent**:

$$\tanh(x) = \frac{e^{2x} - 1}{e^{2x} + 1} \rightarrow [-1, 1]$$

Non-linearities



The only difference between this **deep neural network** and a stack of linear classifiers: **non-linearities** (\int) between linear transformations.



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- ▶ The whole system is trained end-to-end.

$$\phi(\mathbf{x}) = g(\mathbf{x}\mathbf{W}') \quad (13)$$

$$\hat{y} = \phi(\mathbf{x})\mathbf{W} \quad (14)$$



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1. The first layer contains input units,
 2. the last layer contains output units,
 3. all layers in between (hidden layers) contain hidden units, which form hidden representations of the input data.



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 - ▶ ...and reflect this in its hidden representations.

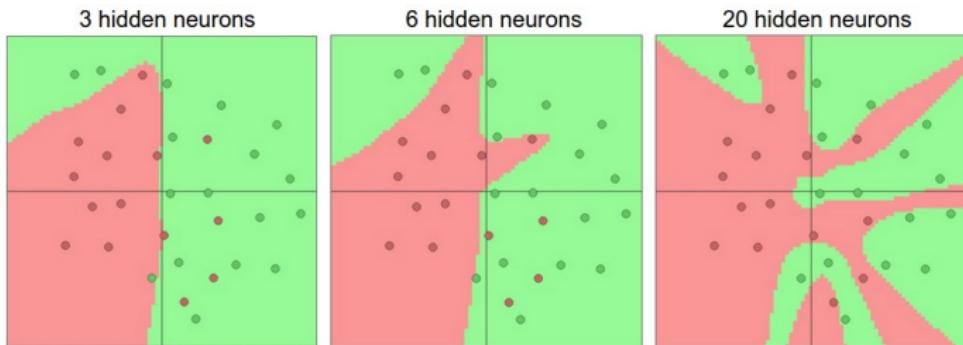
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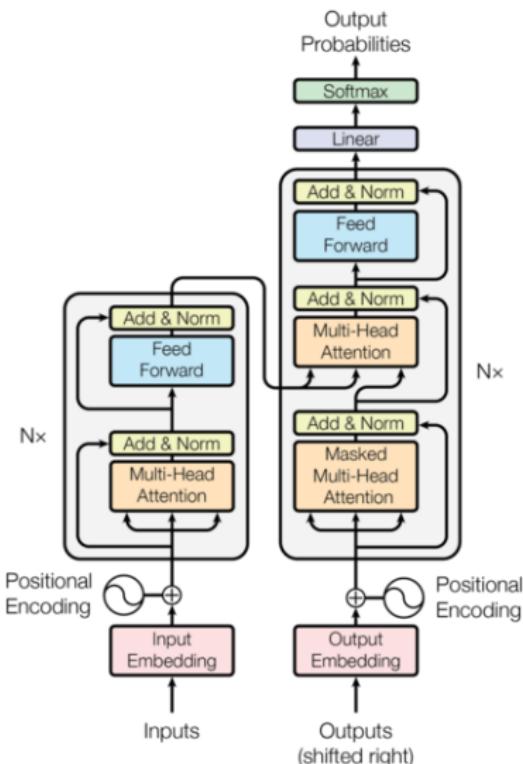
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The One

Almost all modern large language models are invariably based in a variation of feed-forward neural network: **Transformer** architecture.

- ▶ BERT, RoBERTa, BART
- ▶ GPT variants
- ▶ Text-to-Text Transfer Transformer (T5)
- ▶ PaLM, BARD, LaMDA, LLaMA bla bla bla
- ▶ Used for text-to-image in DALL-E, Stable Diffusion etc..



Attention is all you need

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Transformer revolution

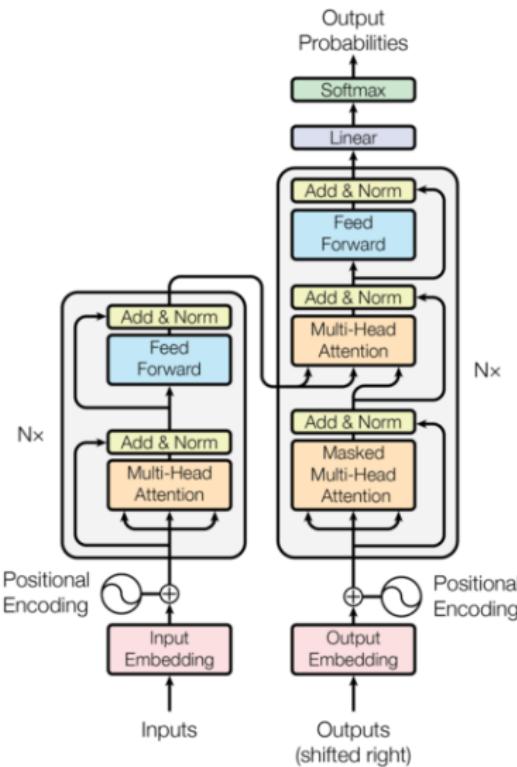
"The dominant sequence transduction models are based on complex recurrent or convolutional neural networks in an encoder and decoder configuration. The best performing models also connect the encoder and decoder through an **attention mechanism**. We propose a novel, simple network architecture, the Transformer, based solely on attention mechanisms, dispensing with recurrence and convolutions entirely."

– [Vaswani et al., 2017]

– Per 28.02.2023: cited 66,7k times

This simplification made Transformer **a very parallelizable architecture**. It makes very good use of modern computational hardware (GPUs and TPUs).

The Transformer, at a glance



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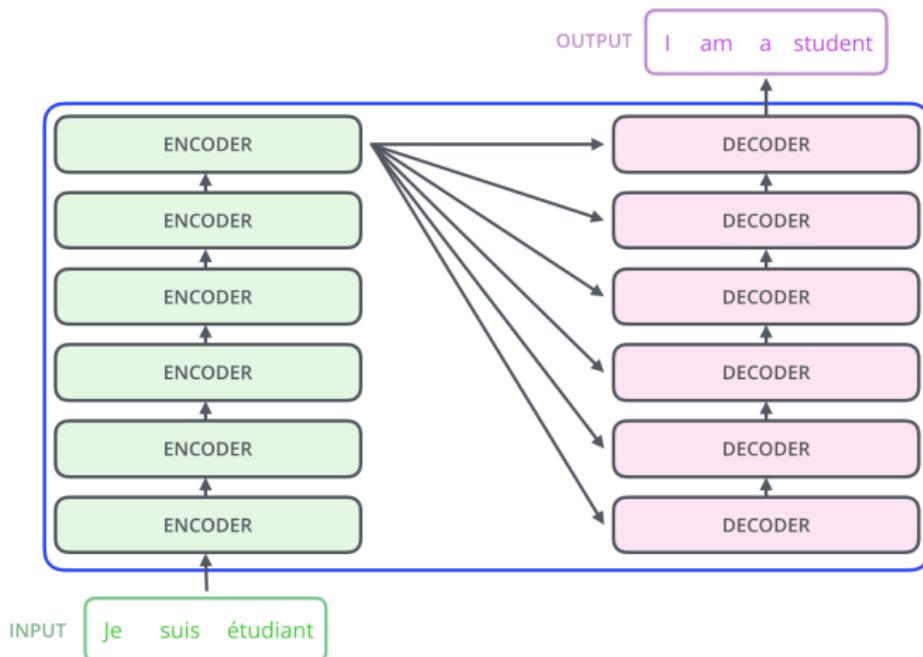


Figure from Jay Alammar's, *The Illustrated Transformer*¹

¹<https://jalammar.github.io/illustrated-transformer/>

Applications of Attention: high level explanation

Three different applications of attention in the Transformer:

1. In the **Encoder-Decoder** way: we use the ‘hidden state’ from the encoders and do some operation with the state in the decoder
2. **Encoder** Self-Attention: each position in the encoder can attend to all the positions in the input
3. **Decoder** Self-Attention: each position can attend to all previous positions up to that point (autoregressive via masking)

Those are also the **three types of modern large language models (LLMs)**.
Today we are mostly talking about **decoder-only** models.

So how good is it?

Transformer turned out to be extremely good in machine translation:

Table 2: The Transformer achieves better BLEU scores than previous state-of-the-art models on the English-to-German and English-to-French newstest2014 tests at a fraction of the training cost.

Model	BLEU		Training Cost (FLOPs)	
	EN-DE	EN-FR	EN-DE	EN-FR
ByteNet [18]	23.75			
Deep-Att + PosUnk [39]		39.2		$1.0 \cdot 10^{20}$
GNMT + RL [38]	24.6	39.92	$2.3 \cdot 10^{19}$	$1.4 \cdot 10^{20}$
ConvS2S [9]	25.16	40.46	$9.6 \cdot 10^{18}$	$1.5 \cdot 10^{20}$
MoE [32]	26.03	40.56	$2.0 \cdot 10^{19}$	$1.2 \cdot 10^{20}$
Deep-Att + PosUnk Ensemble [39]		40.4		$8.0 \cdot 10^{20}$
GNMT + RL Ensemble [38]	26.30	41.16	$1.8 \cdot 10^{20}$	$1.1 \cdot 10^{21}$
ConvS2S Ensemble [9]	26.36	41.29	$7.7 \cdot 10^{19}$	$1.2 \cdot 10^{21}$
Transformer (base model)	27.3	38.1	$3.3 \cdot 10^{18}$	
Transformer (big)	28.4	41.8		$2.3 \cdot 10^{19}$

'A small fraction of the training cost'

[Vaswani et al., 2017]

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Deep-Att + PosUnk [39]		39.2		$1.0 \cdot 10^{20}$
GNMT + RL [38]	24.6	39.92	$2.3 \cdot 10^{19}$	$1.4 \cdot 10^{20}$
ConvS2S [9]	25.16	40.46	$9.6 \cdot 10^{18}$	$1.5 \cdot 10^{20}$
MoE [32]	26.03	40.56	$2.0 \cdot 10^{19}$	$1.2 \cdot 10^{20}$
Deep-Att + PosUnk Ensemble [39]		40.4		$8.0 \cdot 10^{20}$
GNMT + RL Ensemble [38]	26.30	41.16	$1.8 \cdot 10^{20}$	$1.1 \cdot 10^{21}$
ConvS2S Ensemble [9]	26.36	41.29	$7.7 \cdot 10^{19}$	$1.2 \cdot 10^{21}$
Transformer (base model)	27.3	38.1	$3.3 \cdot 10^{18}$	
Transformer (big)	28.4	41.8		$2.3 \cdot 10^{19}$

'A small fraction of the training cost'

[Vaswani et al., 2017]

...not long after Transformer became the dominant architecture for language models, NLP models in general, and 'artificial intelligence' overall.

Technicalities again



Materials for the hands-on session

- ▶ Slides, code, instructions:
https://github.com/ltgoslo/nndl_llm_tutorial
- ▶ You are allocated Linux virtual machines for the hands-on session
- ▶ Connect via ssh and run Python; can also use Jupyter Lab.
- ▶ Find the ip address, username and key file for your virtual machine (VM) in the spreadsheet below:



<https://docs.google.com/spreadsheets/d/13A-iyiBb2tmhm9RLMJp1wndfgm0Bt6HackY2miT5yeY>

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