# Large Language Models Under the Hood
# Part 1: Training Deep Neural Networks

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University of Oslo

NLDL 2025

# Contents

# Tutorial technicalities

Welcome to the NLDL 2025 tutorial on large language models!



Andrey Kutuzov, Egil Rønningstad, David Samuel
Language Technology Group, UiO

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## Schedule

- 13:00 - Part 1, Training Deep Neural Networks
- 14:00 - Part 2, Modern Generative Language Models
- 15:00 - Part 3, Hands-on with open-weights LLMs for Norwegian

# Supported by NAIC

# Tutorial technicalities

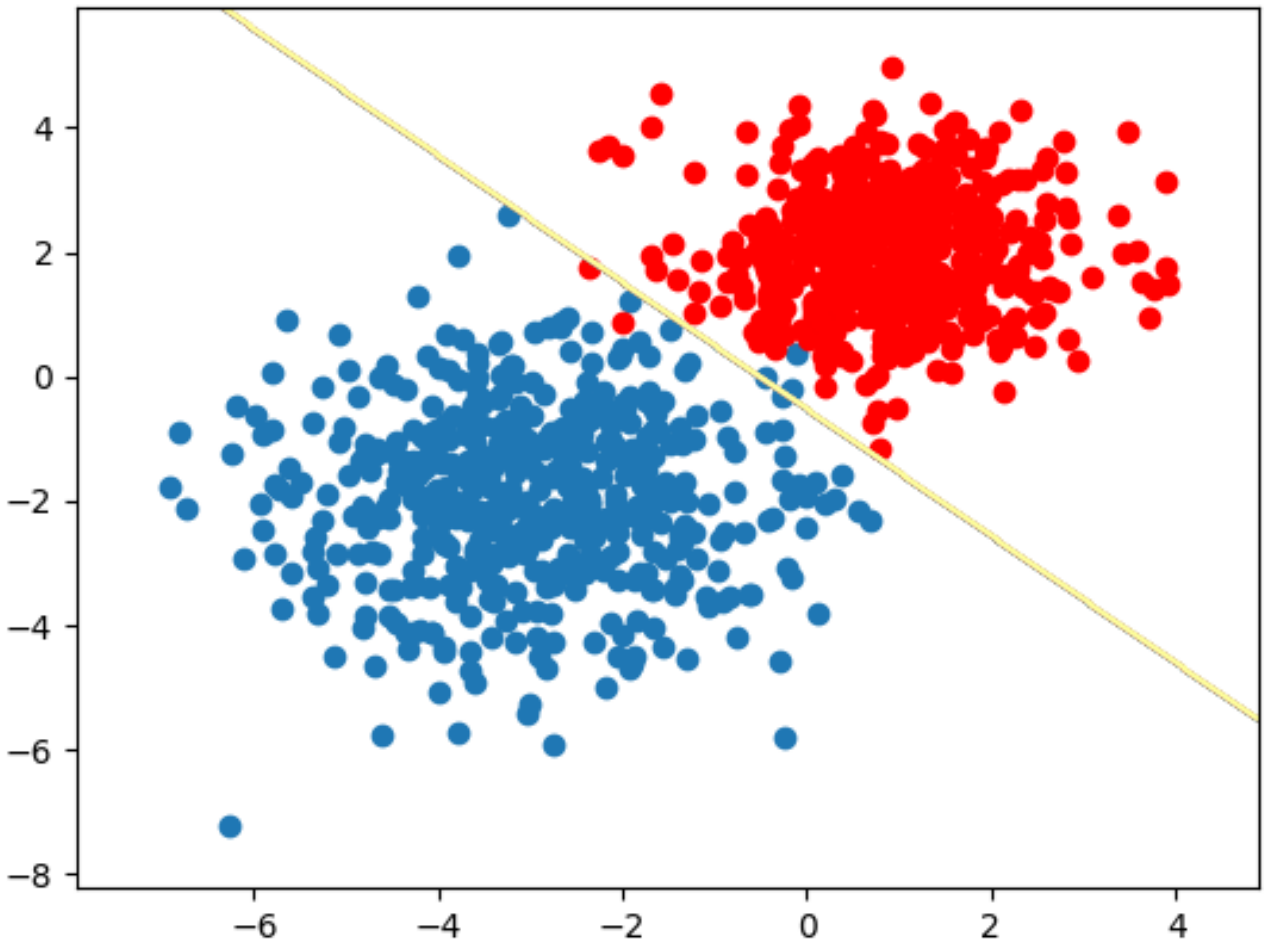
## Materials for the hands-on session

- Slides, code, instructions:
  `https://github.com/ltgoslo/nldl_llm_tutorial`
- You are allocated Linux virtual machines for the hands-on session
- Connect via `ssh` and run Python; can also use Jupyter Lab.
- Find the ip address, username and key file for your virtual machine (VM) in the spreadsheet below:

`https://docs.google.com/spreadsheets/d/13A-iyiBb2tmhm9RLMJp1wndfgm0Bt6HackY2miT5yeY`

# Contents

# Basics of supervised machine learning



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- Generally: some program for mapping instances to labels.

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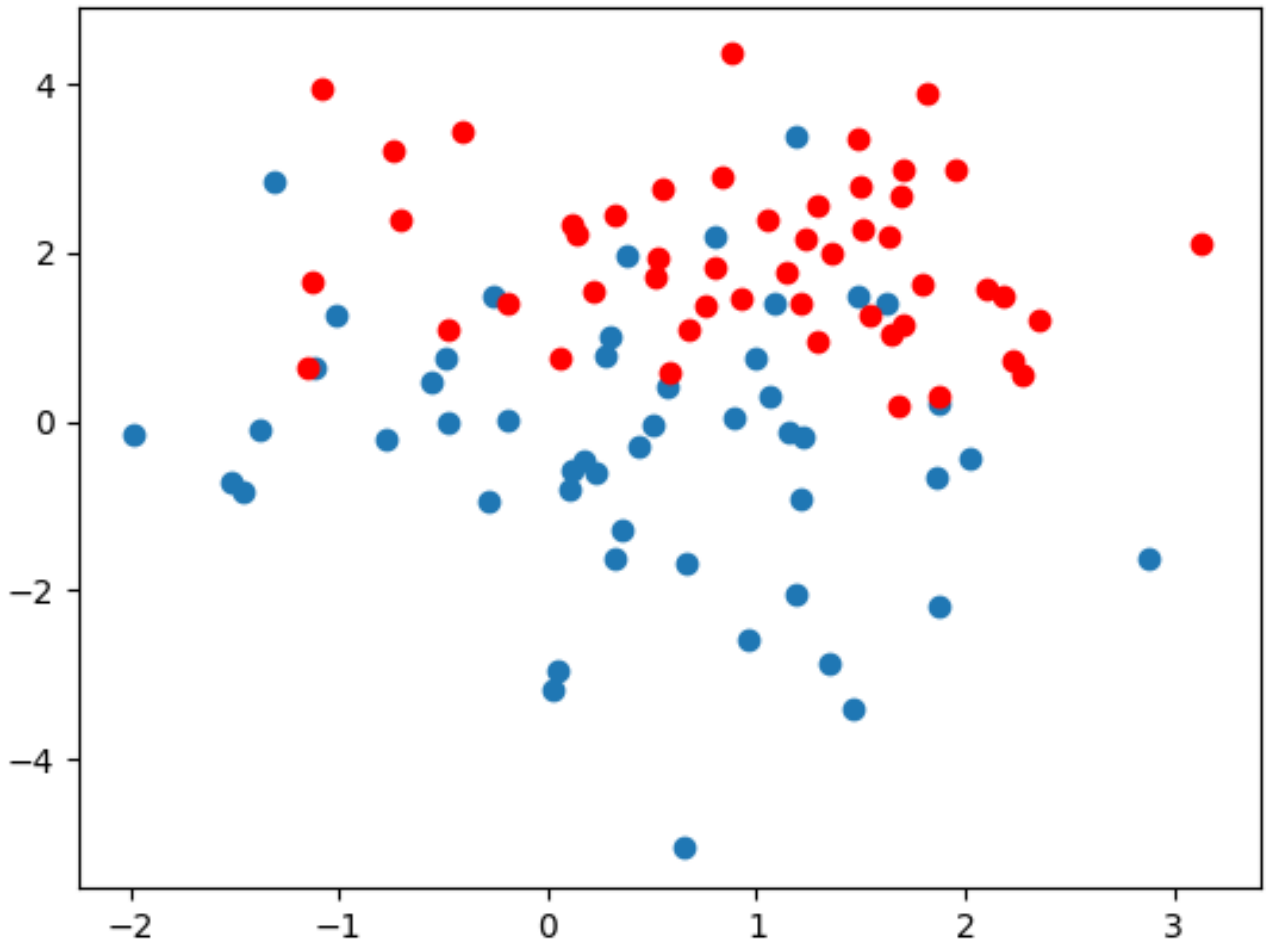

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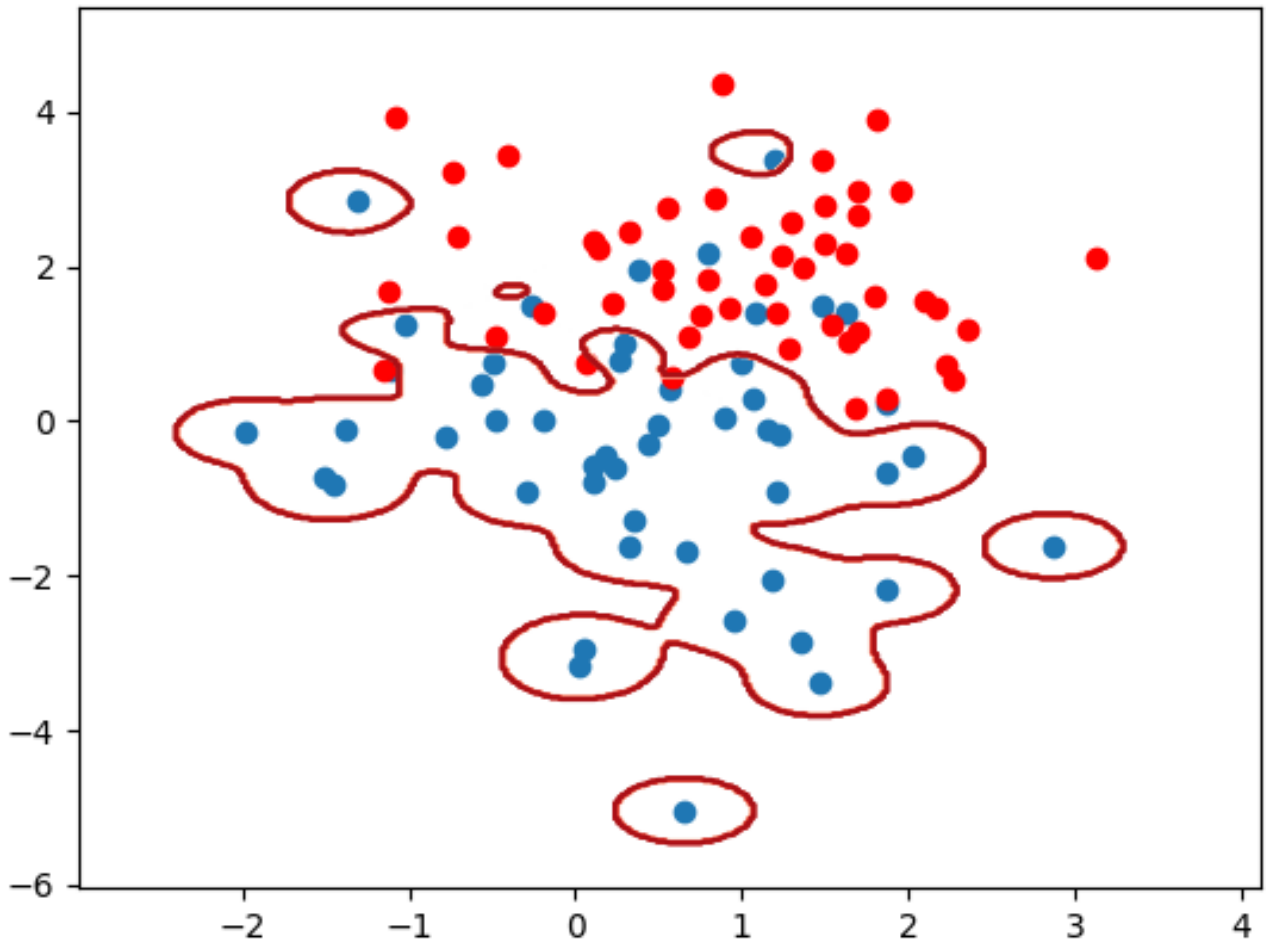

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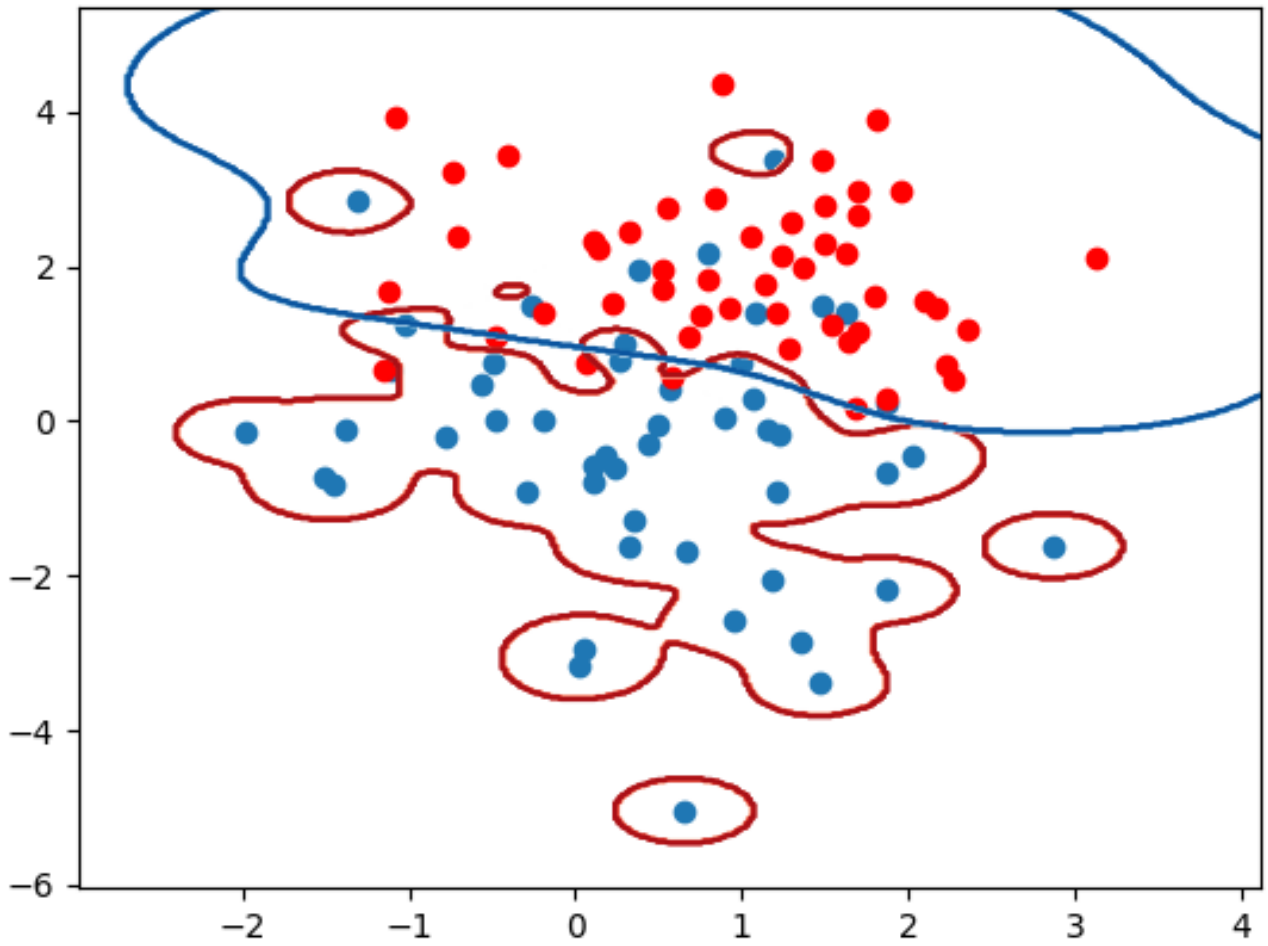

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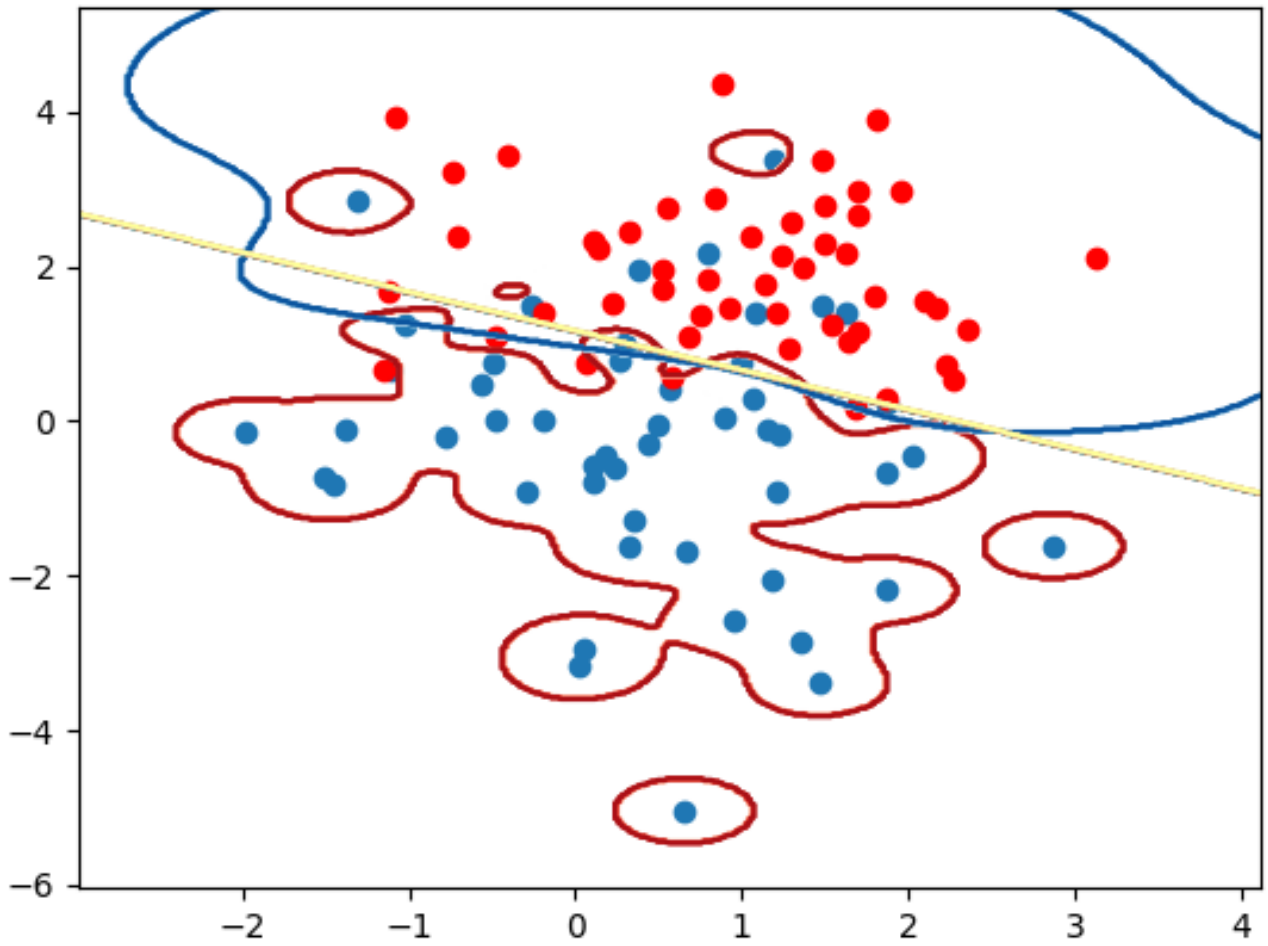

Linear functions: a popular hypothesis class

## Simple linear function

$$f(\boldsymbol{x}; \boldsymbol{W}, \boldsymbol{b}) = \boldsymbol{x} \cdot \boldsymbol{W} + \boldsymbol{b} \tag{1}$$

$$\theta = \boldsymbol{W}, \boldsymbol{b} \tag{2}$$

- Function input:
  - feature vector $\boldsymbol{x} \in \mathbb{R}^{d_{in}}$;
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- Function parameters $\theta$:
  - matrix $\boldsymbol{W} \in \mathbb{R}^{d_{in} \times d_{out}}$
    - $d_{out}$ is the dimensionality of the desired prediction (number of classes)
  - bias vector $\boldsymbol{b} \in \mathbb{R}^{d_{out}}$
    - bias 'shifts' the function output to some direction.

## Training of a linear classifier

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- Ideally, $\hat{\boldsymbol{y}} = \boldsymbol{y}$

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## Output of binary classification

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- NB: the model can output any number, but we convert all negatives to $-1$ and all positives to 1 (*sign* function).

$$\theta = (\boldsymbol{W} \in \mathbb{R}^{d_{in}}, \boldsymbol{b} \in \mathbb{R}^1)$$

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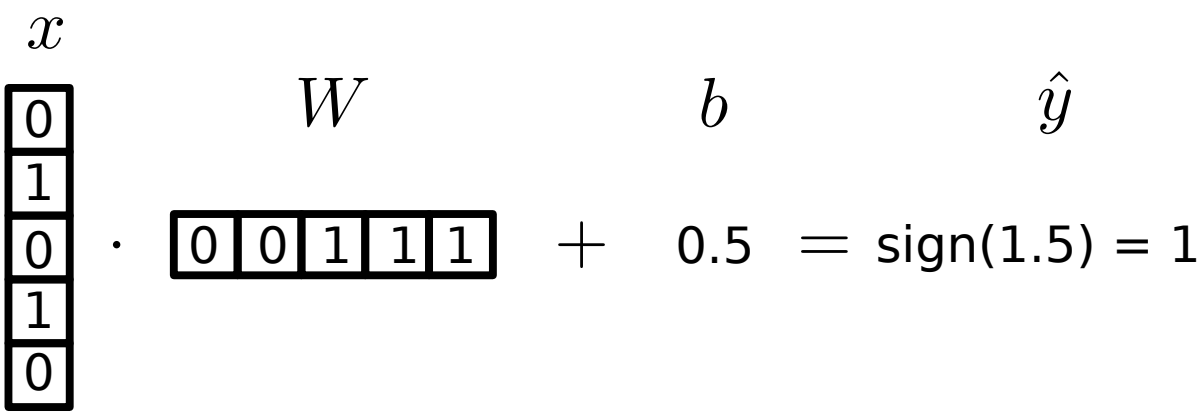

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- The prediction $\hat{\boldsymbol{y}}$ is also a one-hot vector of $k$ components.
- The component corresponding to the correct language has the value of 1, others are zeros, for example:
  $\hat{\boldsymbol{y}} = [0, 0, 1, 0]$ (for $k = 4$)

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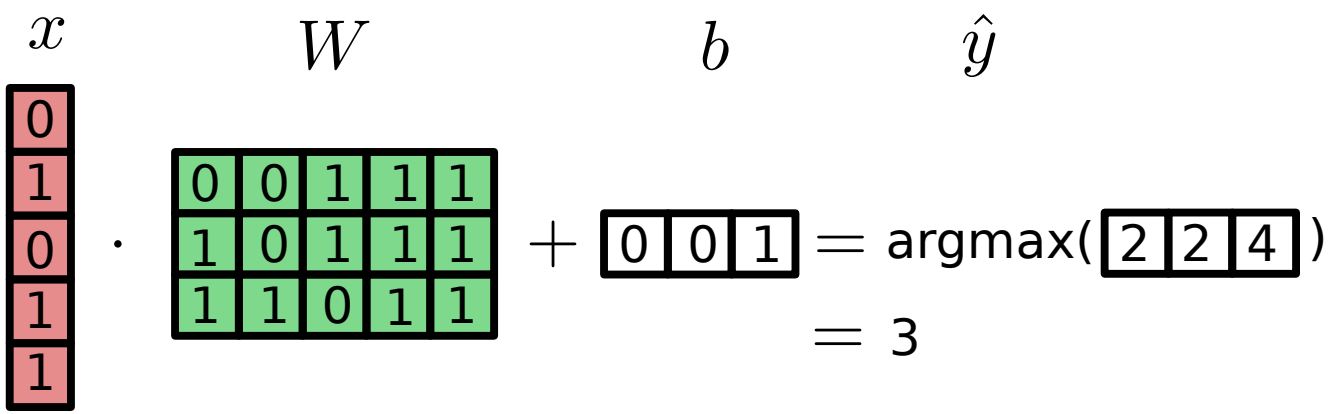

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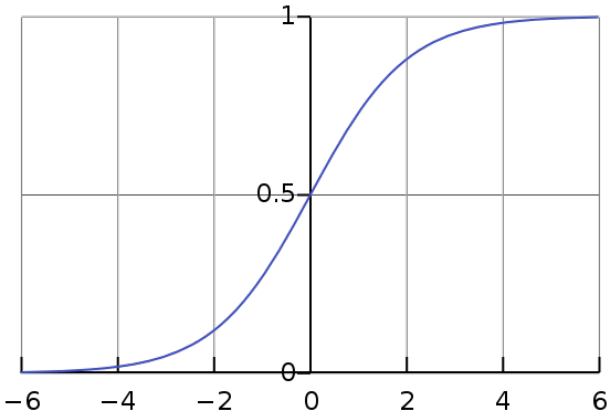
- ...by a squashing function, for example, sigmoid:

$$\hat{y} = \sigma(f(\boldsymbol{x})) = \frac{1}{1 + e^{-(f(\boldsymbol{x}))}} \tag{3}$$

- The result is the probability of the prediction!




$\sigma(\boldsymbol{x})$

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- But often it is more convenient to transform scores into a probability distribution, using the softmax function:

$$\hat{\boldsymbol{y}} = softmax(\boldsymbol{x}\boldsymbol{W} + \boldsymbol{b}) \tag{5}$$

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- $\hat{\boldsymbol{y}} = softmax([0.4, 0.1, 0.9, 0.5]) = [0.22, 0.16, 0.37, 0.25]$
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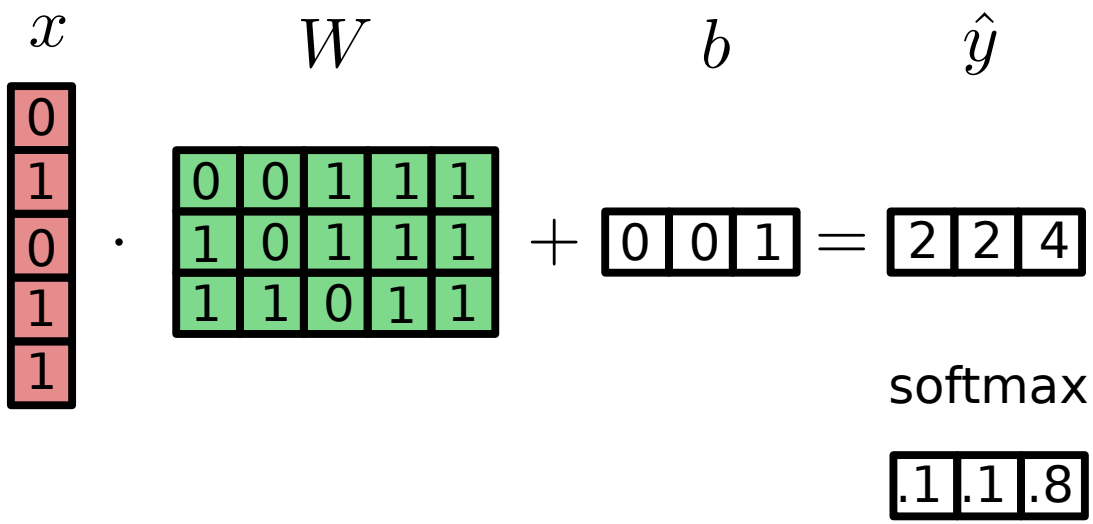


(expect this to be used for language modeling: it is secretly classification!)

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- It is averaged over all training instances and gives us estimation of the model 'fitness'.
- $\hat{\theta}$ is the best set of parameters:

$$\hat{\theta} = \arg\min_{\theta} \mathcal{L}(\theta) \tag{7}$$

Now we can measure model performance. How can we change our parameters $\theta$ to improve?

## Optimizing with gradient

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## Convexity

- Convex functions: a single optimum point.
- Non-convex functions: multiple optimum points.

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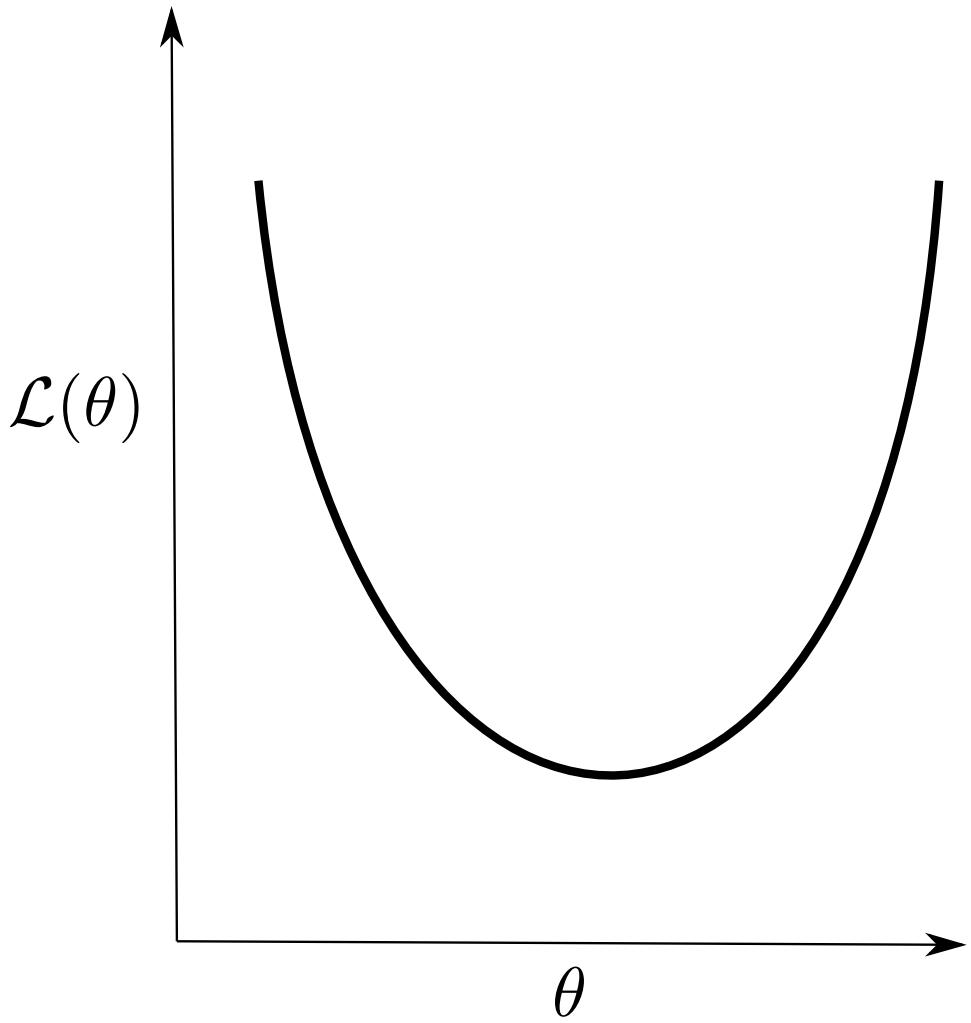

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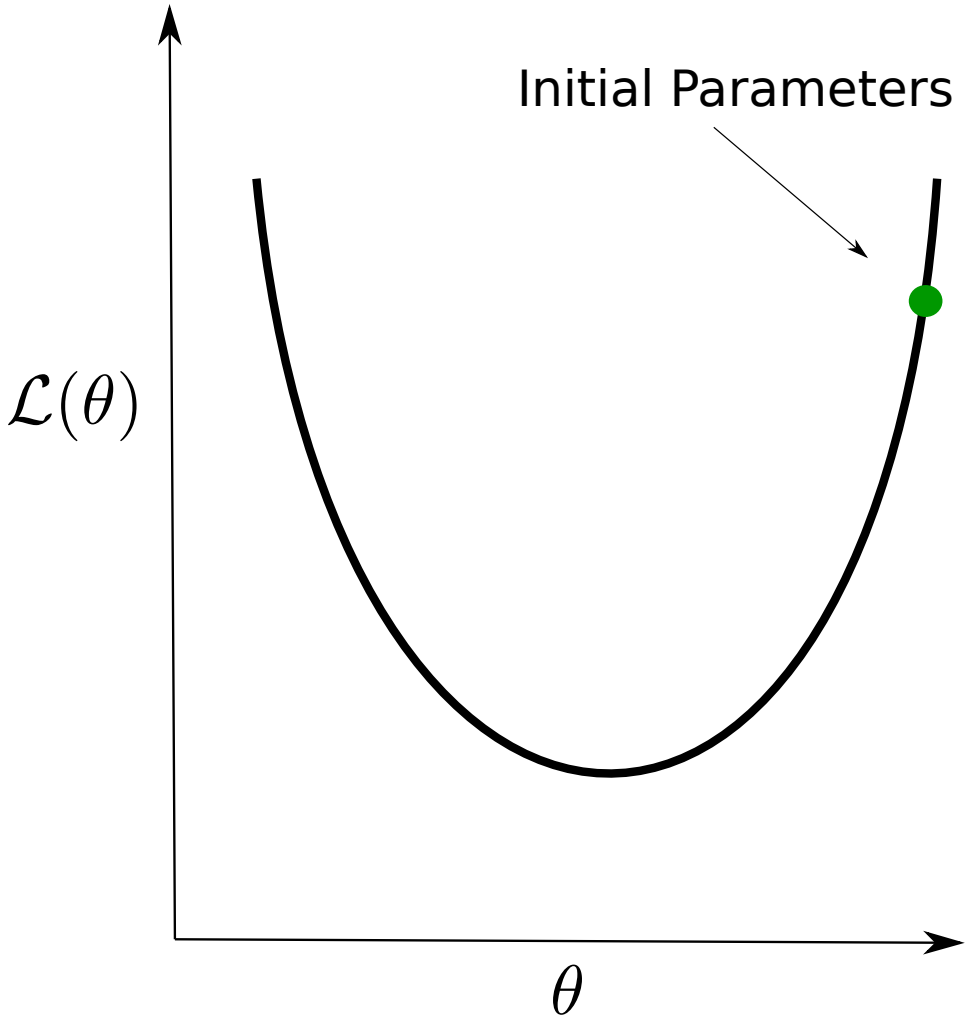

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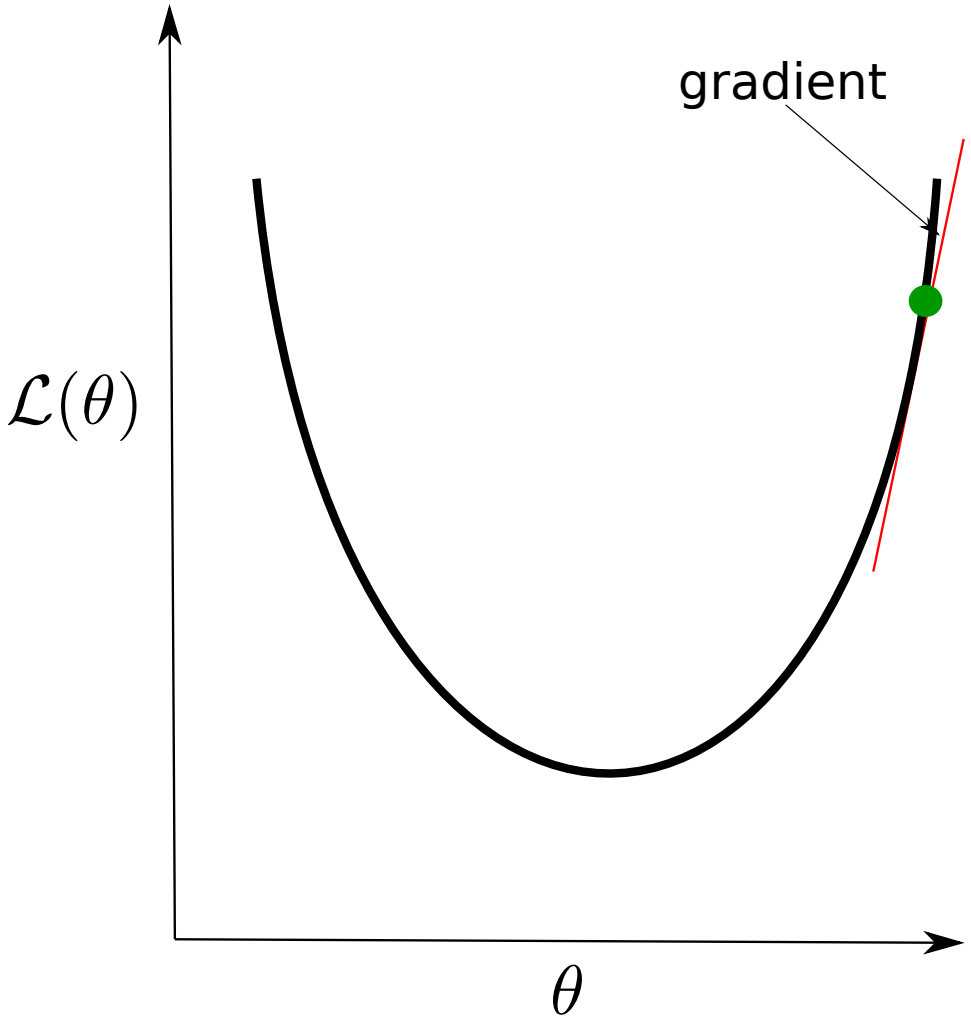

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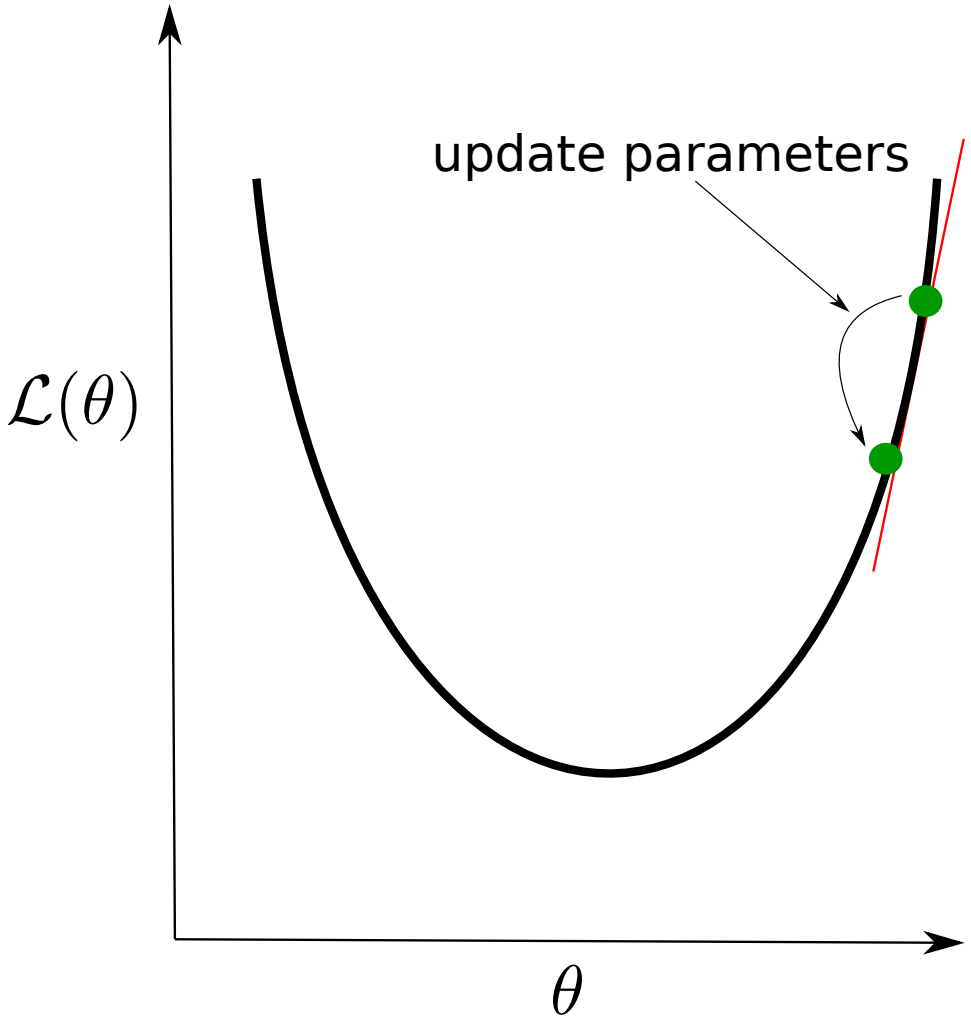

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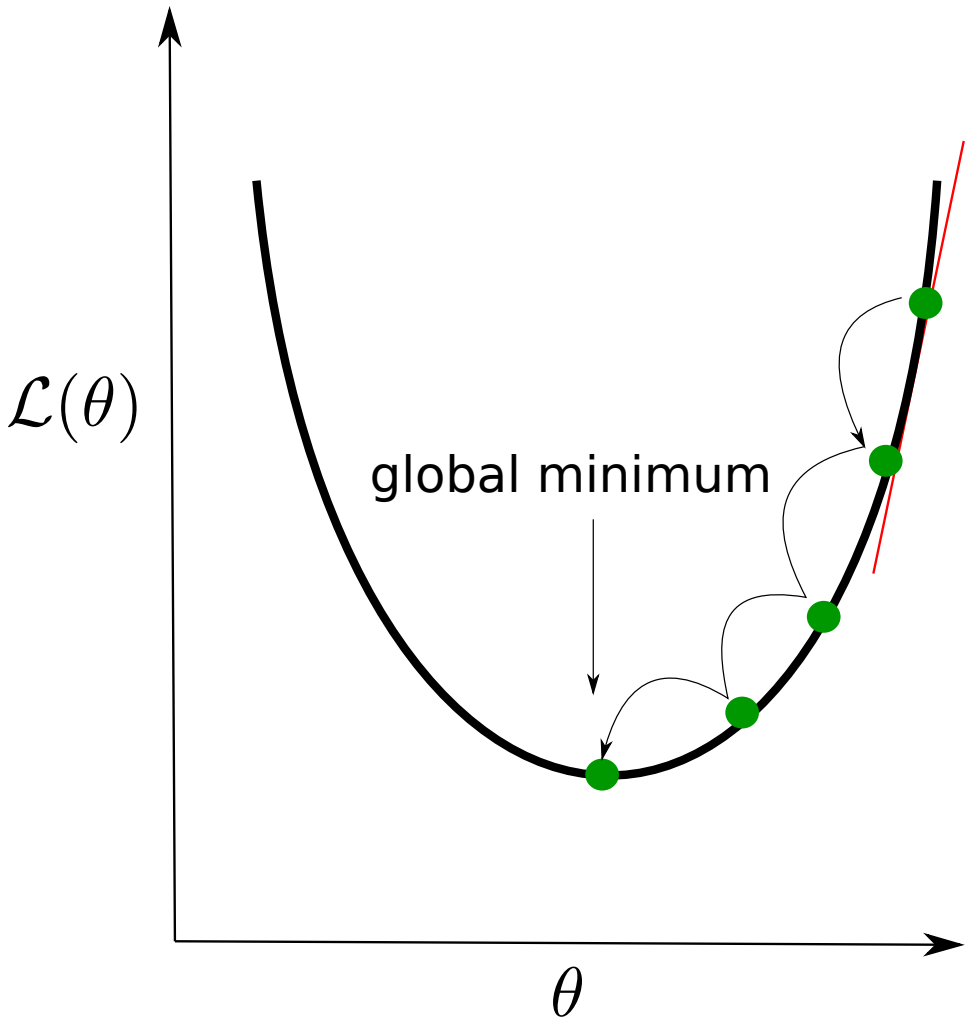

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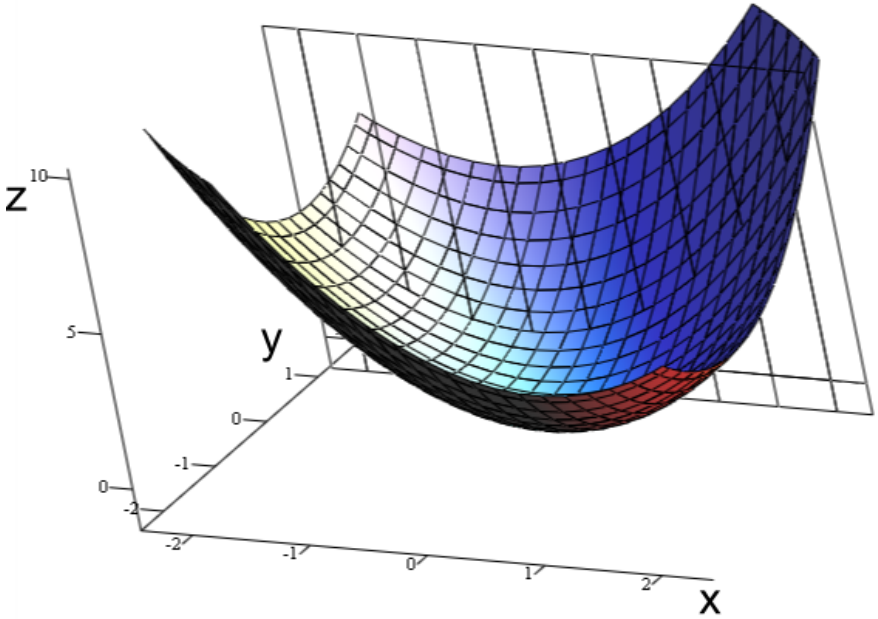
Error surfaces of convex and not-convex functions:

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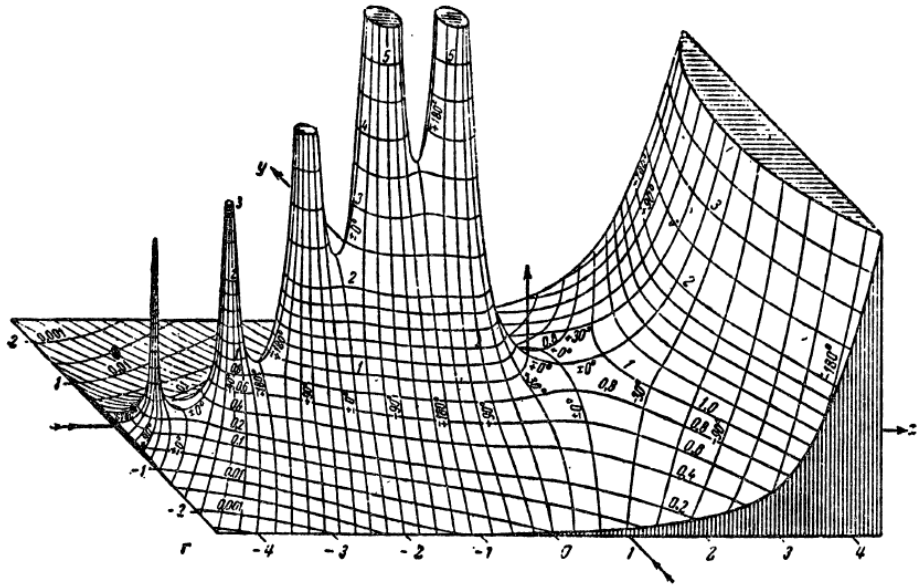

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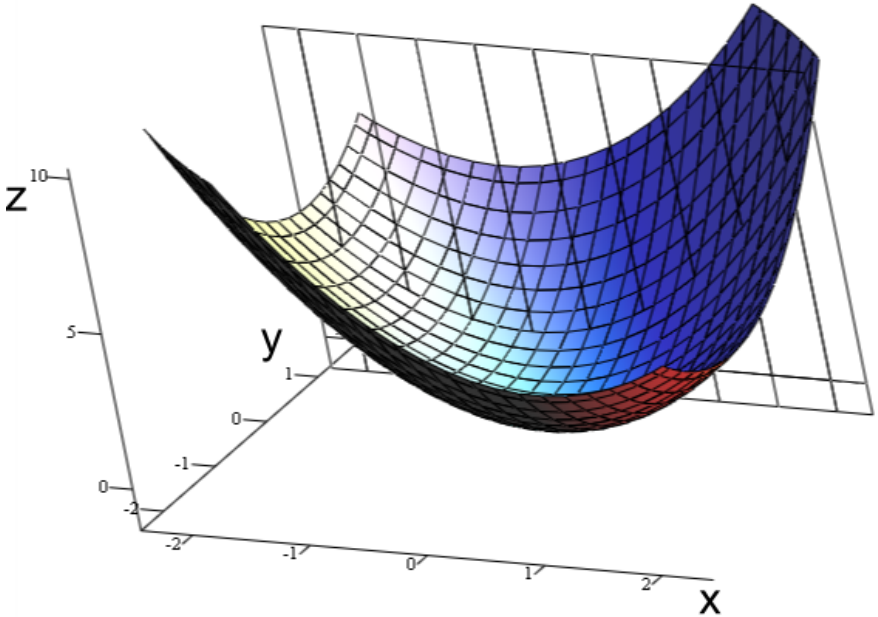


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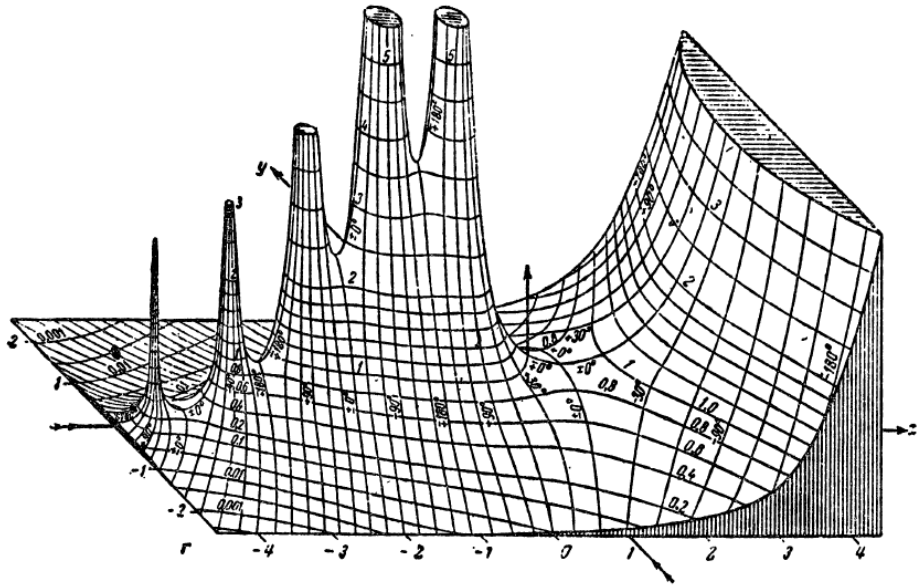
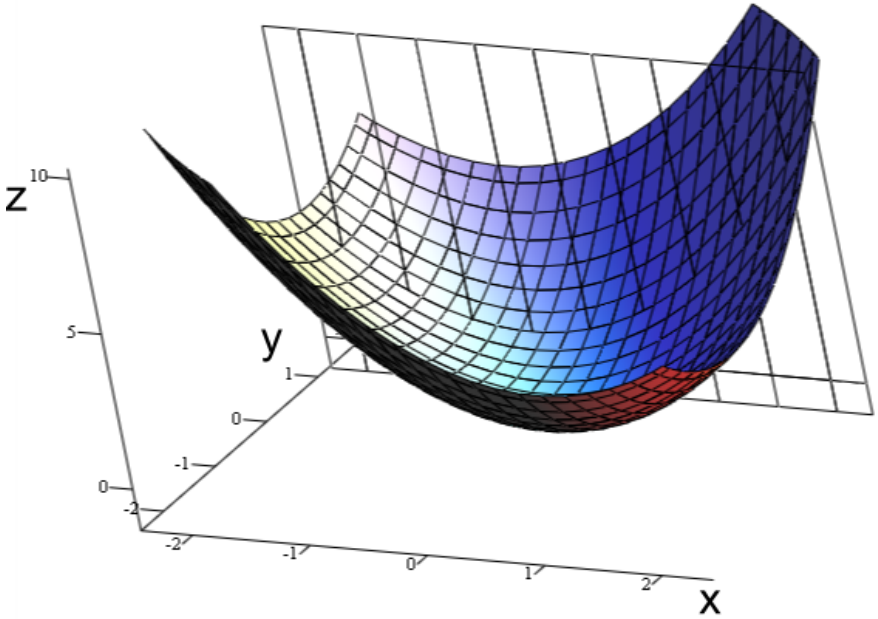


Non-convex function

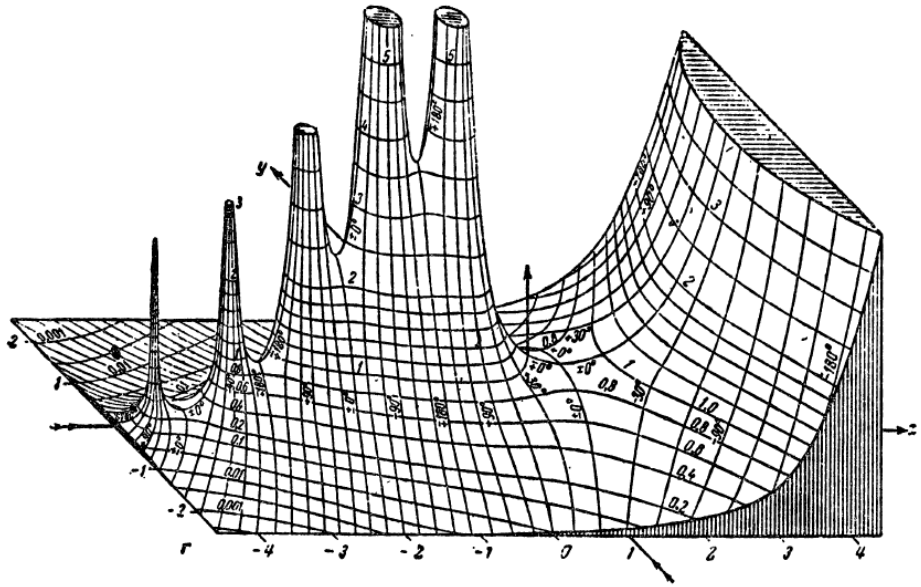
- Convex functions can be easily minimized with gradient methods, reaching the global optimum.
- With non-convex functions, optimization can end up in a local optimum.

# Basics of supervised machine learning

Error surfaces of convex and not-convex functions:




Convex function



Non-convex function

- Convex functions can be easily minimized with gradient methods, reaching the global optimum.
- With non-convex functions, optimization can end up in a local optimum.
- Linear and log-linear models as a rule have convex error functions.
- ...unlike non-linear models (e.g., used in 'deep learning').

# Contents

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Unfortunately, linear models can represent only linear relations in the data

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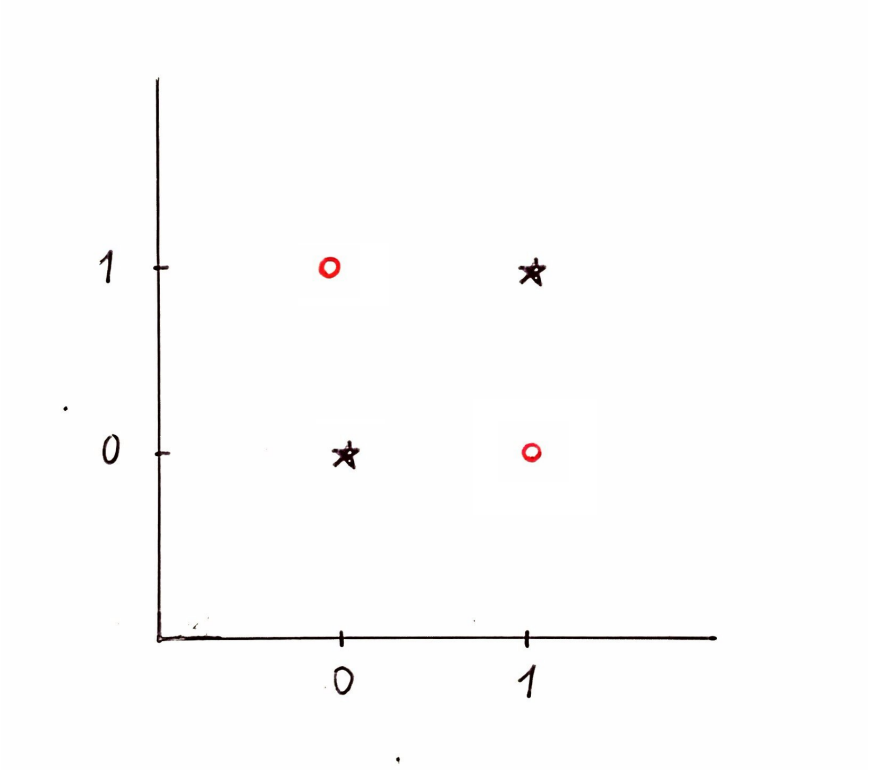

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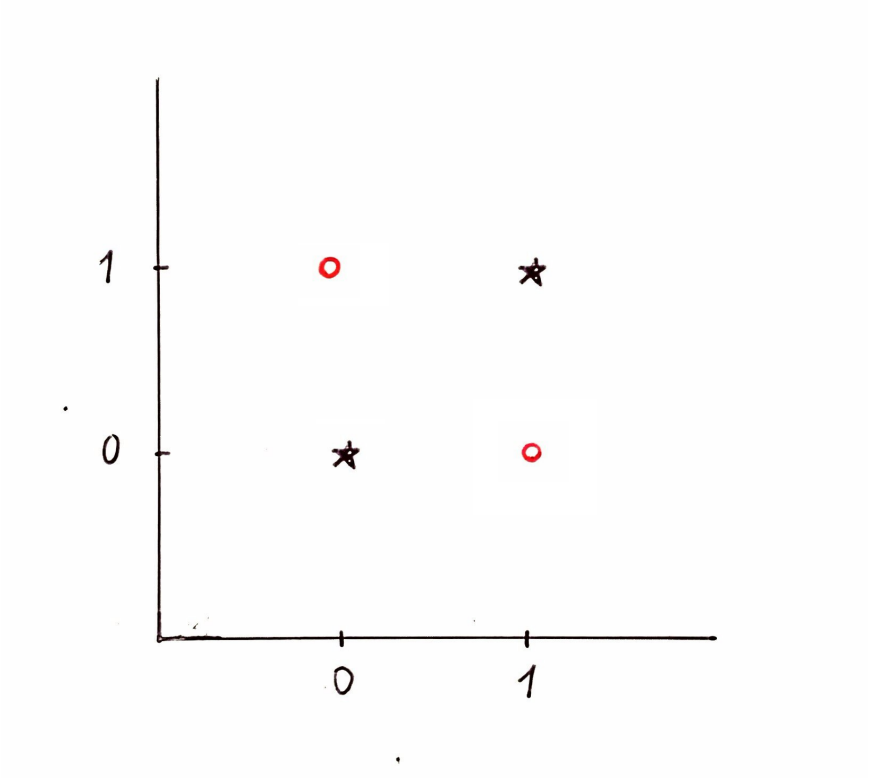


It is clearly not linearly separable.

# Advantages and limitations of linear models

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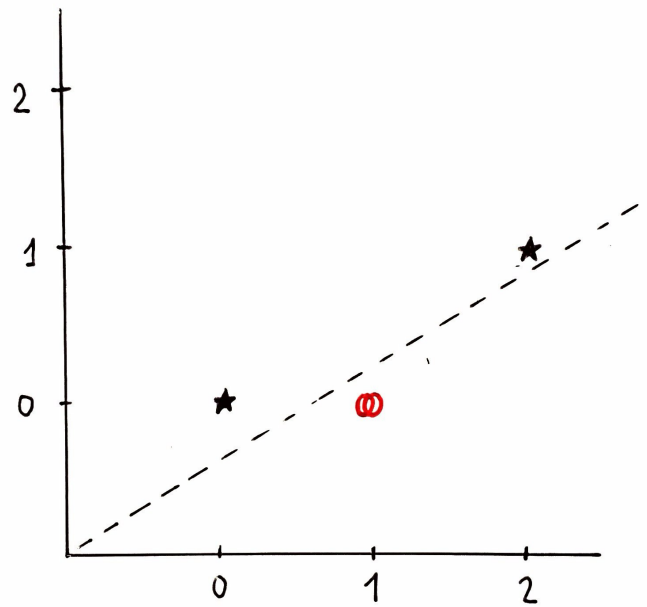
For example, $\phi(x_1, x_2) = [x_1 + x_2, x_1 \times x_2]$ maps the instances to another representation and makes the XOR problem linearly separable:



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$$\phi(\boldsymbol{x}) = g(\boldsymbol{x}\boldsymbol{W}' + \boldsymbol{b}') \tag{8}$$

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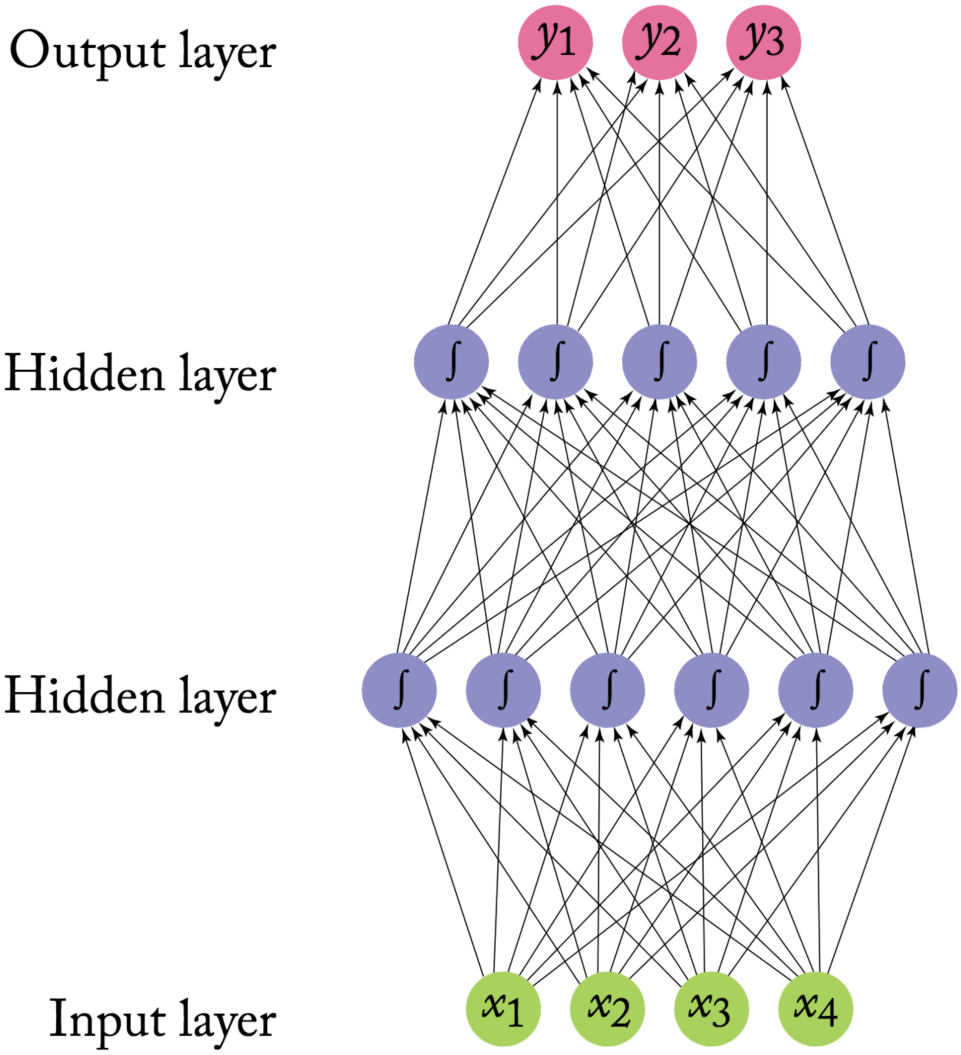
- ...where $g$ is a non-linear activation function, and $\boldsymbol{W}', \boldsymbol{b}'$ are its trainable parameters.
- The equation above defines a simple multi-layer perceptron (MLP): a neural model.

# Contents

# Going deeply non-linear: multi-layered perceptrons




A simple feed-forward neural network (perceptron) with 2 hidden layers.

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Important: **neural networks with hidden layers can theoretically approximate any function** [Leshno et al., 1993].

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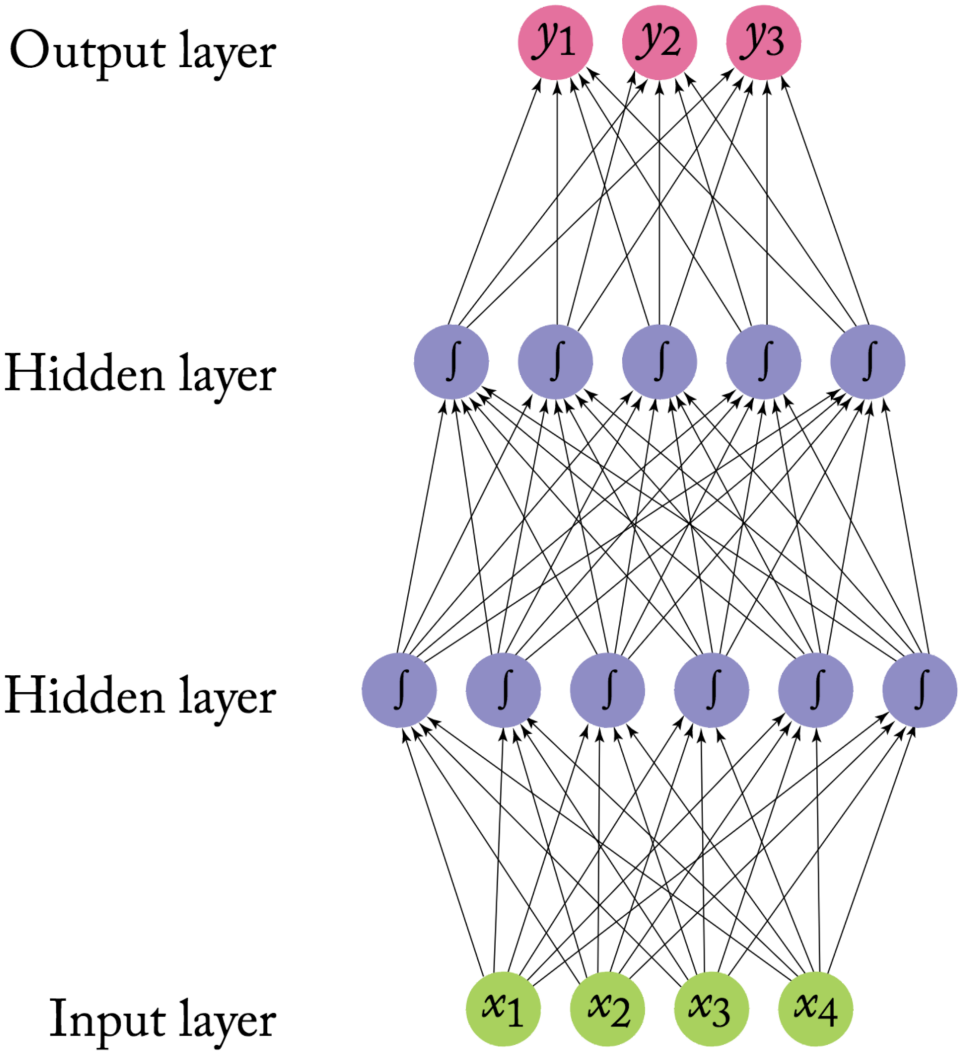


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‘Machines of this character can behave in a very complicated manner when the number of units is large.’

*Alan Turing, ‘Intelligent Machinery’*

[Turing, 1948]

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  - vectors,
  - matrices,
  - sequential algebraic operations on them.

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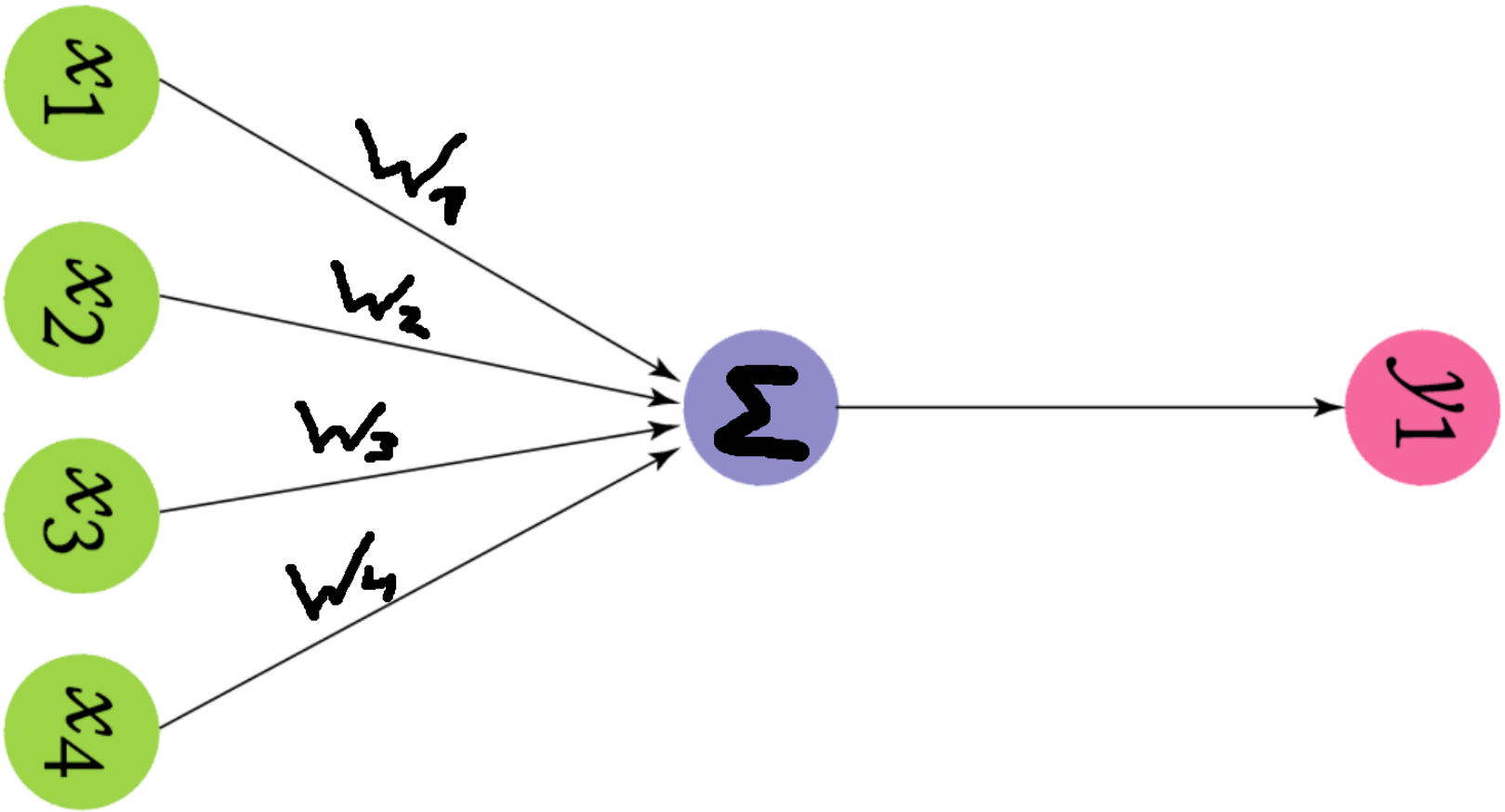

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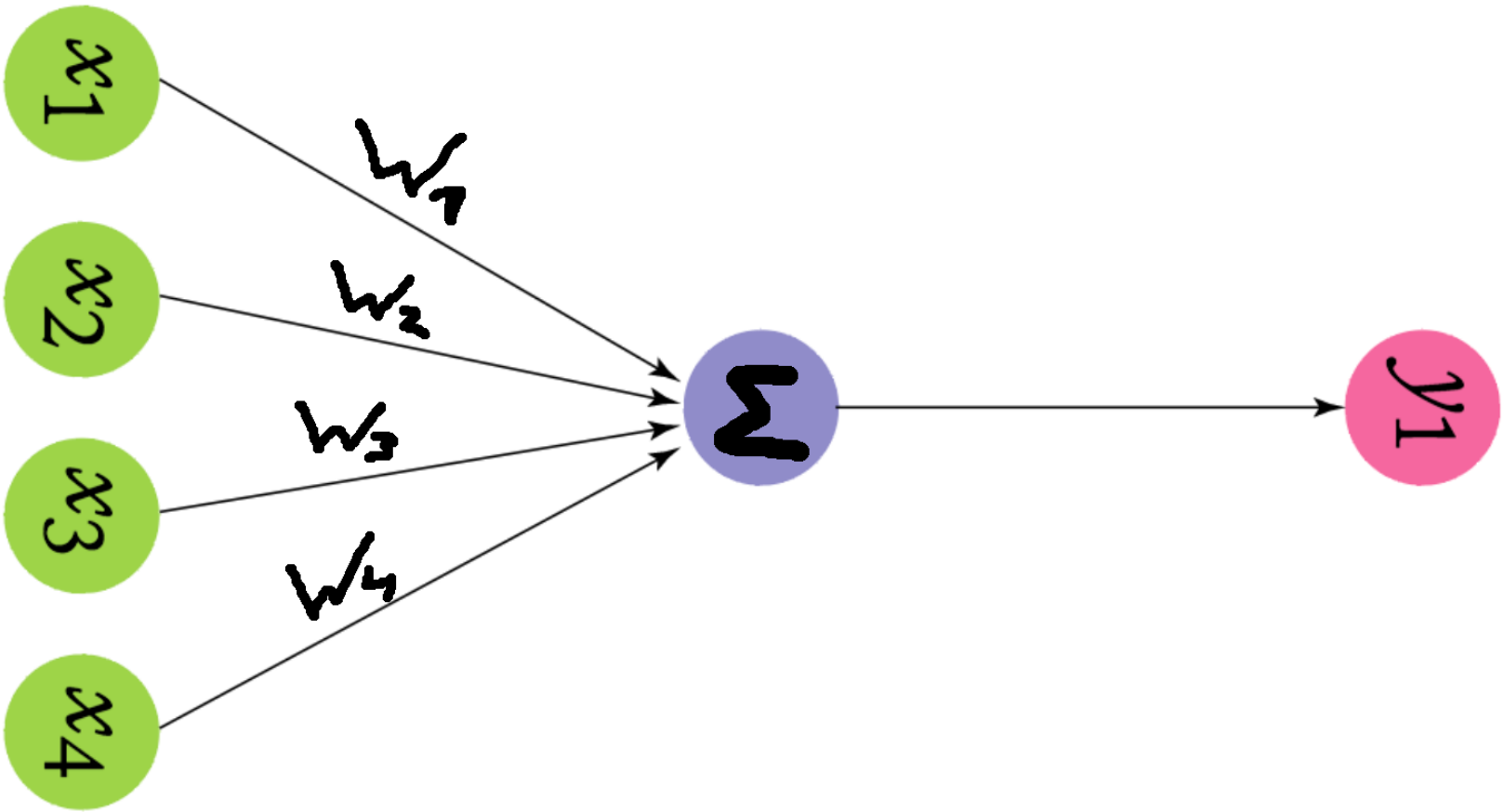


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- with one neuron $\Sigma$ as a computational unit.

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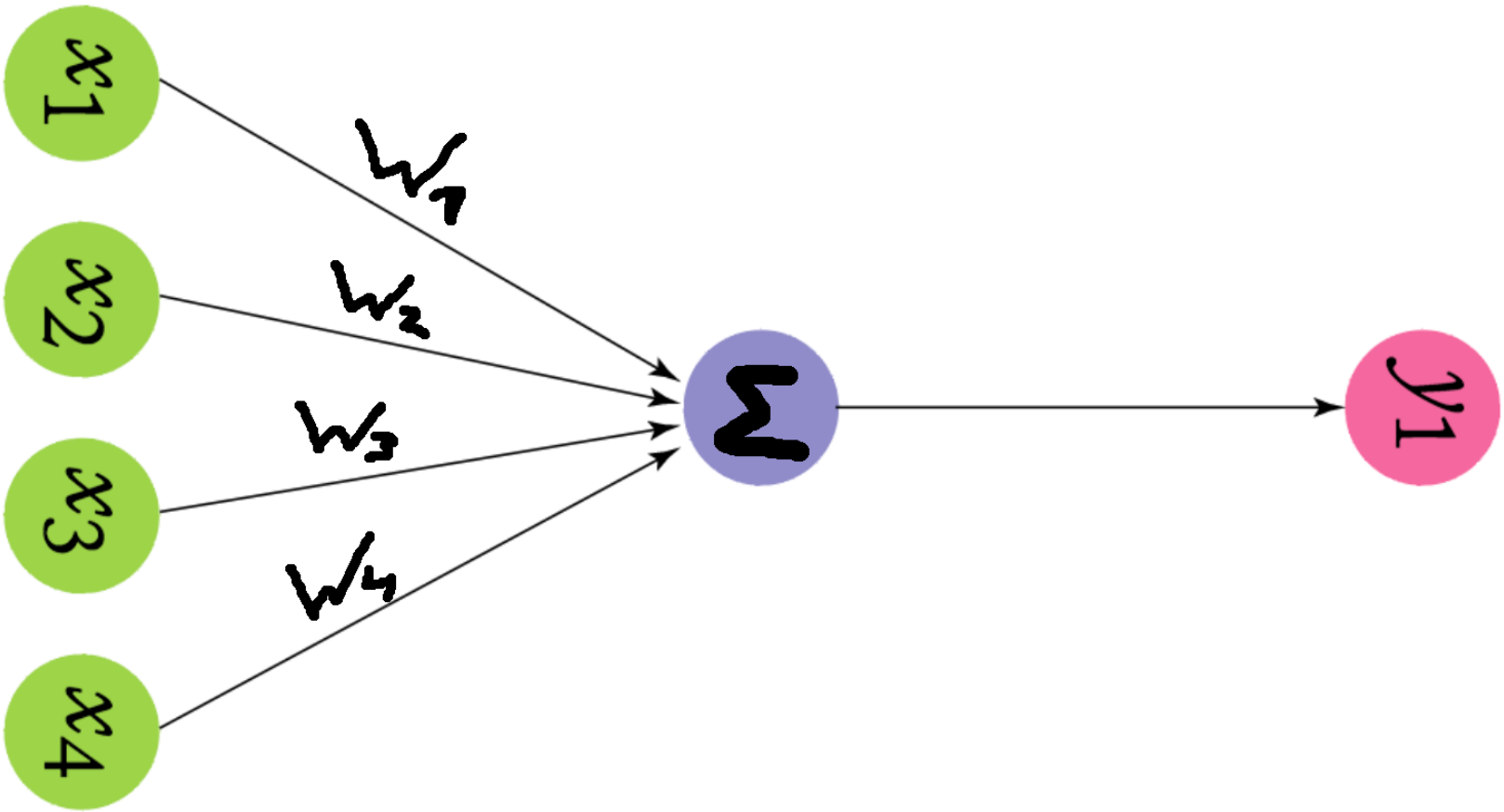


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- $\Sigma$ takes 4 input values and returns their weighted sum as output value.

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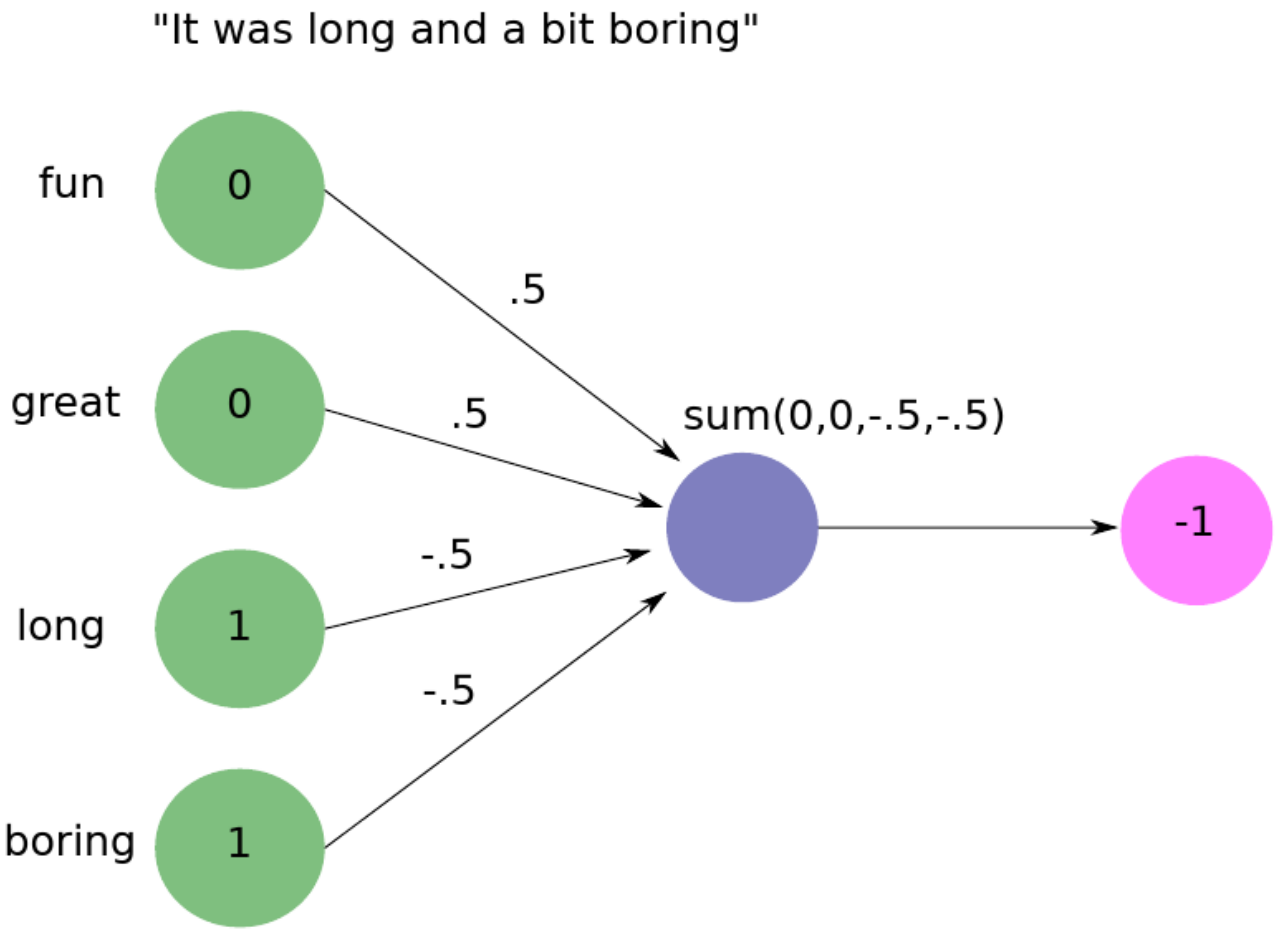


Sentiment analysis with only one neuron and binary bag-of-words.

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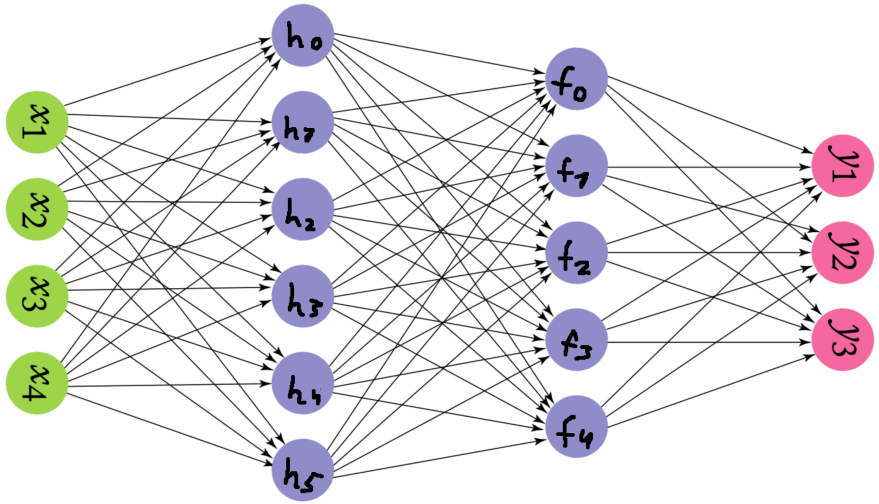

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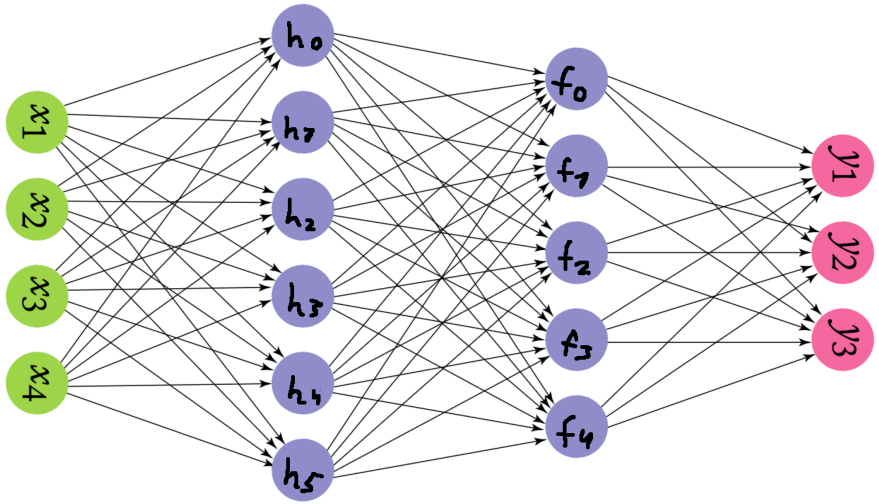


## Looks much like a 'deep neural network'

Stacked linear classifier with multiple computational units at each layer:

$$h = \boldsymbol{x}\boldsymbol{W}^{\mathbf{0}} \tag{10}$$

$$f = \boldsymbol{h}\boldsymbol{W}^{\mathbf{1}} \tag{11}$$

$$\hat{\boldsymbol{y}} = \boldsymbol{f}\boldsymbol{W}^{\mathbf{2}} \tag{12}$$

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# Going deeply non-linear: multi-layered perceptrons

- But any stack of linear classifiers is still a linear classifier :-(
- Still can't handle XOR and other non-linear problems.

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- Hyperbolic tangent:

$$\tanh(\boldsymbol{x}) = \frac{e^{2\boldsymbol{x}} - 1}{e^{2\boldsymbol{x}} + 1} \rightarrow [-1, 1]$$

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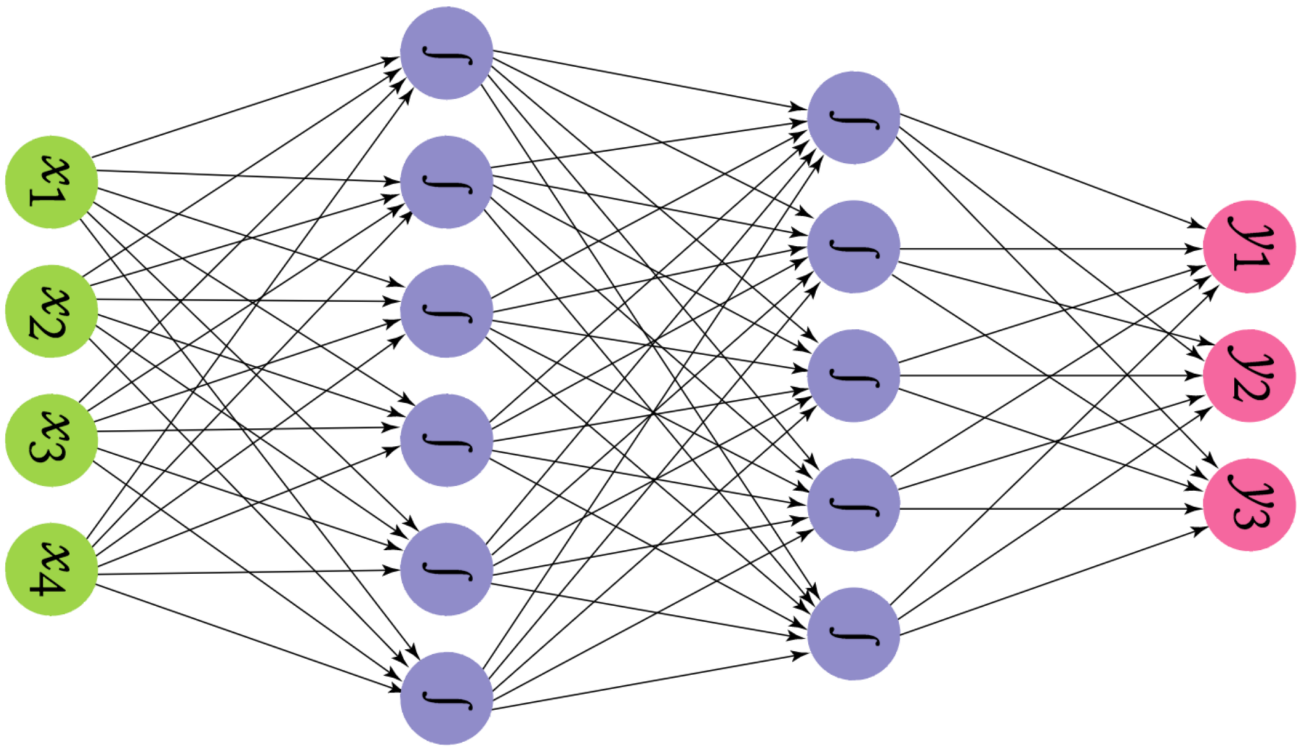


The only difference between this deep neural network and a stack of linear classifiers: non-linearities ($\int$) between linear transformations.

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- The whole system is trained end-to-end.

$$\phi(\boldsymbol{x}) = g(\boldsymbol{x}\boldsymbol{W}') \tag{13}$$

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3. all layers in between (hidden layers) contain hidden units, which form hidden representations of the input data.

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  - ...and reflect this in its hidden representations.

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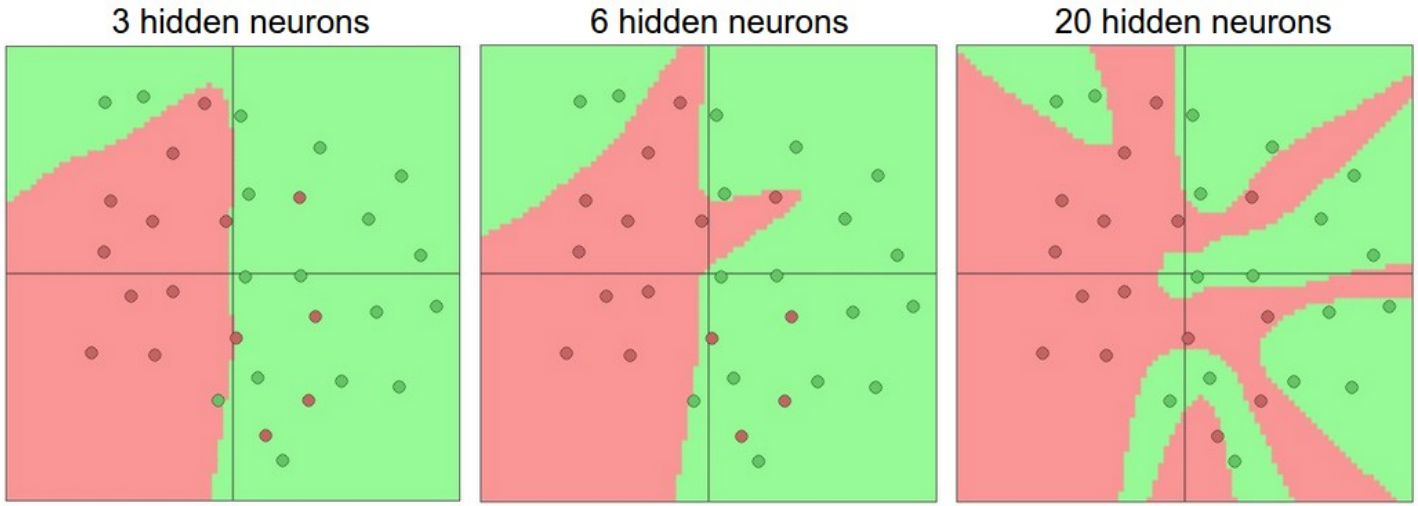
# Model size

The deeper and wider the model is, the more capacity (parameters) it has, the more complex functions it can approximate.

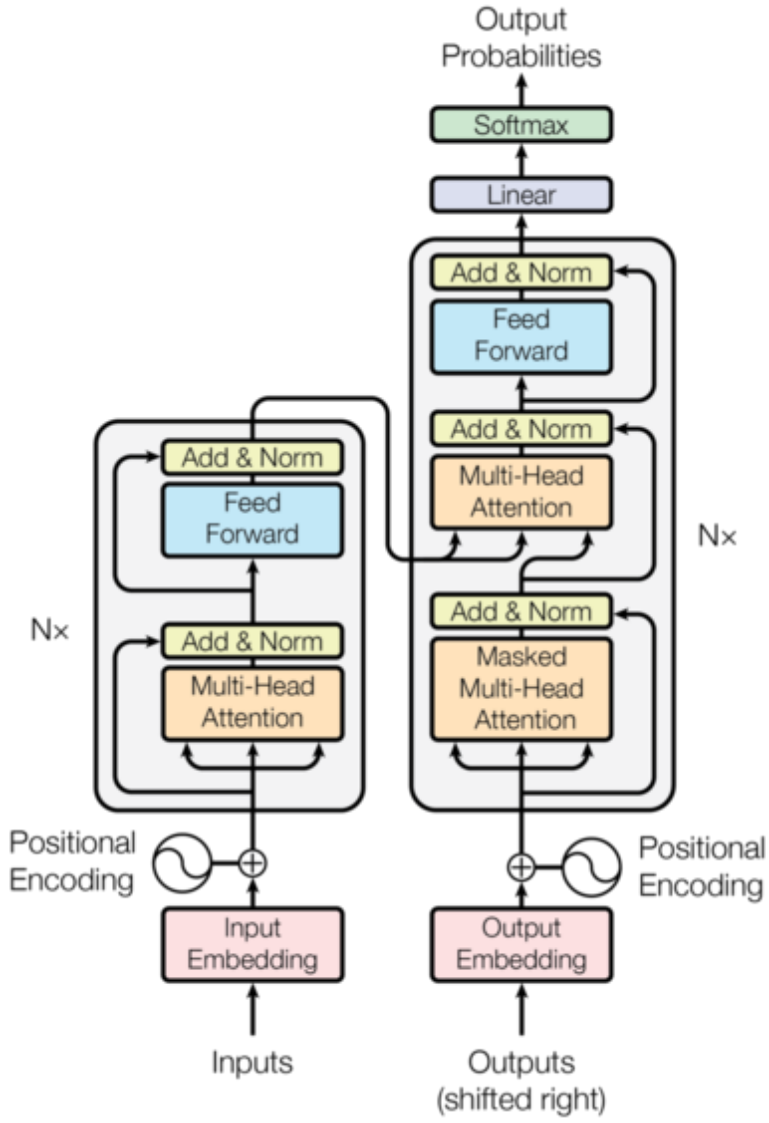


# Contents

# The One

Almost all modern large language models are invariably based in a variation of feed-forward neural network: Transformer architecture.

- BER**T**, RoBER**T**a, BAR**T**
- GP**T** variants
- Text-to-Text Transfer **T**ransformer (T5)
- PaLM, BARD, LaMDA, LLaMA bla bla bla
- Used for text-to-image in DALL-E, Stable Diffusion etc..



# Attention is all you need

Attention: the way for the model to learn relative importance of tokens/words in different positions of the input.

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## Transformer revolution

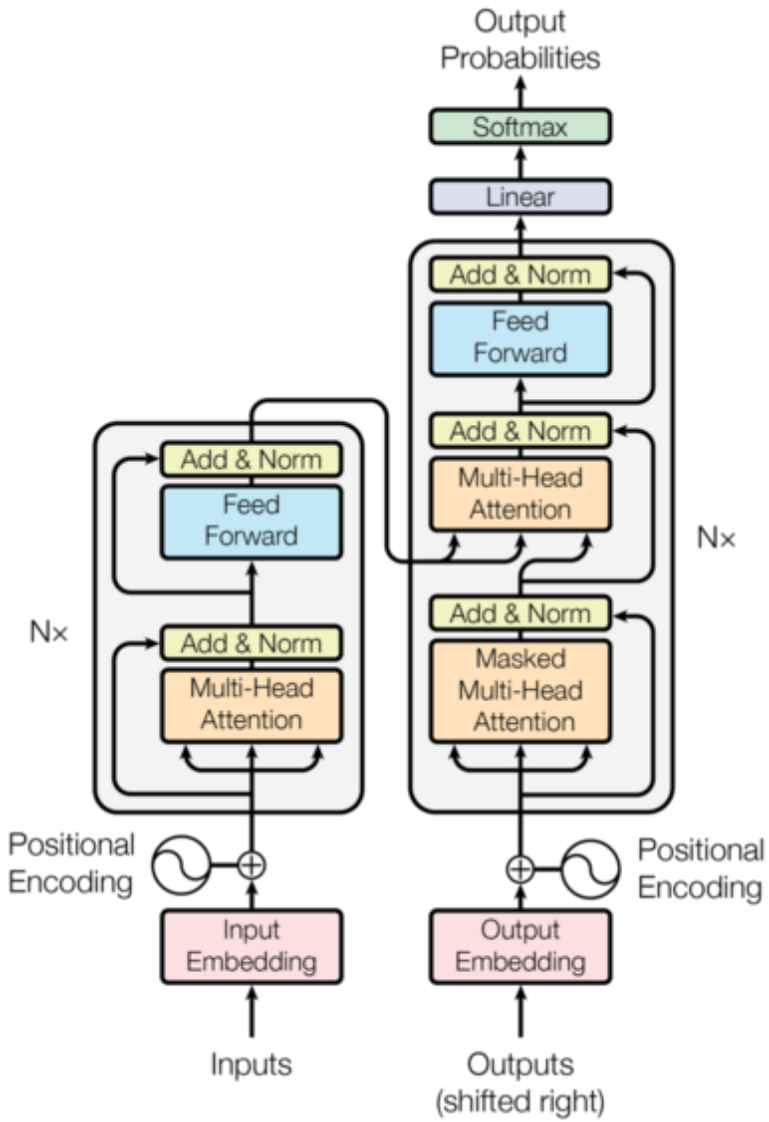
"The dominant sequence transduction models are based on complex recurrent or convolutional neural networks in an encoder and decoder configuration. The best performing models also connect the encoder and decoder through an attention mechanism. We propose a novel, simple network architecture, the Transformer, based solely on attention mechanisms, dispensing with recurrence and convolutions entirely."

– [Vaswani et al., 2017]

– Per 28.02.2023: cited 66,7k times

This simplification made Transformer a very parallelizable architecture. It makes very good use of modern computational hardware (GPUs and TPUs).

# The Transformer, at a glance



# The Transformer, at a glance

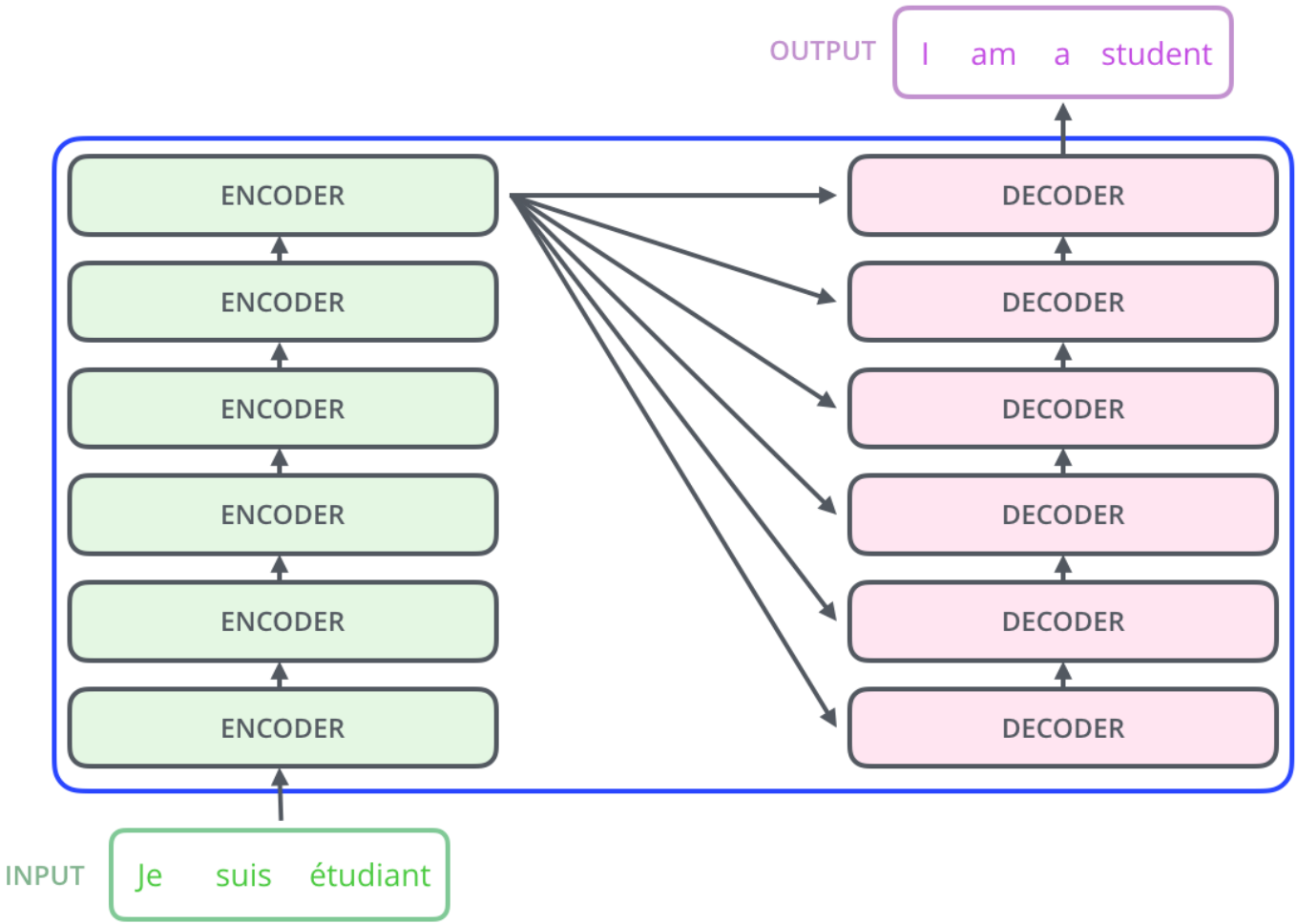



Figure from Jay Alammar's, *The Illustrated Transformer*[1]

[1] https://jalammar.github.io/illustrated-transformer/

# Applications of Attention: high level explanation

Three different applications of attention in the Transformer:

1. In the Encoder-Decoder way: we use the 'hidden state' from the encoders and do some operation with the state in the decoder
2. Encoder Self-Attention: each position in the encoder can attend to previous positions
3. Decoder Self-Attention: each position can attend to all previous positions up to that point (autoregressive via masking)

Those are also the three types of modern large language models (LLMs).
Today we are mostly talking about decoder-only models.

# So how good is it?

Transformer turned out to be extremely good in machine translation:

Table 2: The Transformer achieves better BLEU scores than previous state-of-the-art models on the English-to-German and English-to-French newstest2014 tests at a fraction of the training cost.

| Model | BLEU | | Training Cost (FLOPs) | |
|---|---|---|---|---|
| | EN-DE | EN-FR | EN-DE | EN-FR |
| ByteNet [18] | 23.75 | | | |
| Deep-Att + PosUnk [39] | | 39.2 | | $1.0 \cdot 10^{20}$ |
| GNMT + RL [38] | 24.6 | 39.92 | $2.3 \cdot 10^{19}$ | $1.4 \cdot 10^{20}$ |
| ConvS2S [9] | 25.16 | 40.46 | $9.6 \cdot 10^{18}$ | $1.5 \cdot 10^{20}$ |
| MoE [32] | 26.03 | 40.56 | $2.0 \cdot 10^{19}$ | $1.2 \cdot 10^{20}$ |
| Deep-Att + PosUnk Ensemble [39] | | 40.4 | | $8.0 \cdot 10^{20}$ |
| GNMT + RL Ensemble [38] | 26.30 | 41.16 | $1.8 \cdot 10^{20}$ | $1.1 \cdot 10^{21}$ |
| ConvS2S Ensemble [9] | 26.36 | **41.29** | $7.7 \cdot 10^{19}$ | $1.2 \cdot 10^{21}$ |
| Transformer (base model) | 27.3 | 38.1 | **$3.3 \cdot 10^{18}$** | |
| Transformer (big) | **28.4** | **41.8** | $2.3 \cdot 10^{19}$ | |

'A small fraction of the training cost'

[Vaswani et al., 2017]

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'A small fraction of the training cost'

[Vaswani et al., 2017]

...not long after Transformer became the dominant architecture for language models, NLP models in general, and 'artificial intelligence' overall.

# Technicalities again

## Materials for the hands-on session

- Slides, code, instructions:
  `https://github.com/ltgoslo/nldl_llm_tutorial`
- You are allocated Linux virtual machines for the hands-on session
- Connect via `ssh` and run Python; can also use Jupyter Lab.
- Find the ip address, username and key file for your virtual machine (VM) in the spreadsheet below:

`https://docs.google.com/spreadsheets/d/13A-iyiBb2tmhm9RLMJp1wndfgm0Bt6HackY2miT5yeY`

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