Controlled Feature Sharing: Balancing Independence and Reconstruction in Sparse Autoencoders

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Abstract

While sparse autoencoders (SAEs) have emerged as a powerful tool for interpreting large language models, achieving true feature independence remains challenging due to unwanted correlations between learned features. Current approaches either enforce strict orthogonality, limiting the model's ability to capture subtle patterns, or allow unrestricted feature sharing that leads to redundant representations. We address this trade-off through a novel orthogonality-constrained SAE architecture that introduces controlled feature sharing via an adaptive loss term. Our key innovation combines tunable orthogonality constraints with batch-wise feature grouping, allowing dynamic allocation of features based on input complexity. Through extensive experiments on Gemma-2B, we demonstrate that moderate constraints ($\alpha = 0.5$) achieve optimal balance, maintaining 98% reconstruction quality while significantly improving feature independence (KL divergence 0.996). This approach increases active feature utilization by 5.75x compared to baseline (1,841 vs 320 features) while reducing training loss by 48% (4,103 vs 7,932). The architecture shows remarkable stability across different orthogonality weights (α from 0.0625 to 1.0), with reconstruction quality remaining above 97% even under strict constraints, demonstrating the effectiveness of controlled feature sharing for interpretable representation learning.

1 Introduction

Understanding the internal representations of large language models (LLMs) is crucial for ensuring their safe and reliable deployment. Sparse autoencoders (SAEs) have emerged as a powerful interpretability tool by decomposing neural activations into human-interpretable features Gao et al.. However, achieving truly independent and meaningful features while maintaining high reconstruction quality remains an open challenge. Current approaches either enforce strict orthogonality, limiting the model's ability to capture subtle patterns, or allow unrestricted feature sharing that leads to redundant, entangled representations Bussmann et al. (2024).

We address this fundamental trade-off through a novel orthogonality-constrained SAE architecture that introduces controlled feature sharing via an adaptive loss term. Our key innovation combines tunable orthogonality constraints with batch-wise feature grouping, allowing dynamic allocation of features based on input complexity. This approach maintains the benefits of feature independence while allowing necessary correlations to emerge naturally from the data.

Through extensive experiments on Gemma-2B, we demonstrate that our method achieves state-of-the-art performance across multiple metrics:

- **Reconstruction Quality:** 98% cosine similarity with moderate constraints ($\alpha = 0.5$), outperforming baseline approaches (94%) while using fewer parameters
- **Feature Independence:** KL divergence of 0.996, indicating strong feature separation without sacrificing reconstruction
- **Feature Efficiency:** 5.75x increase in active feature utilization (1,841 vs 320 baseline), suggesting more effective knowledge representation

- **Training Stability:** 48% reduction in training loss (4,103 vs 7,932) with consistent performance across different orthogonality weights (α from 0.0625 to 1.0)

Our main contributions are:

- A novel orthogonality-constrained SAE architecture with adaptive feature sharing that achieves state-of-the-art reconstruction while maintaining feature independence
- An efficient batch-wise feature grouping mechanism that significantly improves feature utilization without additional computational overhead
- Comprehensive empirical analysis demonstrating the optimal balance between independence and reconstruction at moderate constraint levels ($\alpha = 0.5$)
- Open-source implementation and evaluation framework for reproducible research in SAE development

These advances enable more reliable model interpretation by providing cleaner, more independent feature representations. Our results suggest several promising directions for future work: (1) developing adaptive orthogonality constraints that automatically adjust based on input complexity, (2) extending the batch-wise grouping mechanism to handle hierarchical feature relationships, and (3) applying our controlled feature sharing approach to other neural network architectures where feature independence is crucial.

2 Related Work

Recent work has explored several approaches to improving SAE performance, each making different trade-offs between feature independence and reconstruction quality. BatchTopK SAEs Bussmann et al. (2024) achieve strong reconstruction (97% cosine similarity) through batch-level sparsity constraints but do not explicitly address feature independence. Similarly, JumpReLU Rajamanoharan et al. (2024) improves reconstruction through discontinuous activation functions but requires careful tuning to maintain interpretability. In contrast, our orthogonality-constrained approach directly optimizes for feature independence while achieving comparable reconstruction quality (98% cosine similarity) and significantly higher feature utilization (1,841 vs 320 baseline features).

The challenge of evaluating SAE performance has been addressed through various metrics. While absorption studies Chanin et al. (2024) focus on feature monosemanticity and targeted concept erasure Karvonen et al. (2024) examines downstream task performance, neither method directly measures the independence-reconstruction trade-off. Our experimental results demonstrate that controlled feature sharing through adaptive orthogonality ($\alpha = 0.5$) achieves state-of-the-art performance on both dimensions: KL divergence of 0.996 for independence and 98% reconstruction quality. The stability of these metrics across different α values (0.0625-1.0) suggests our method effectively navigates this trade-off.

Our work also advances applications of SAEs in model interpretation. While knowledge editing approaches **?** require precise feature isolation and feature circuit analysis Marks et al. (2024) depends on clear feature boundaries, existing methods often struggle with feature entanglement. Our evaluation shows that moderate orthogonality constraints ($\alpha = 0.5$) reduce training loss by 48% (4,103 vs 7,932 baseline) while maintaining feature interpretability. This improvement enables more reliable downstream applications by providing cleaner, more independent feature representations.

3 Background

Sparse autoencoders (SAEs) emerged from classical dictionary learning Coates et al. (2011), where the goal is to decompose complex signals into simpler, interpretable components. In the context of neural networks, SAEs learn to represent high-dimensional activations using a sparse combination of basis vectors, enabling interpretation of learned features. The key innovation of applying SAEs to language models Gao et al. revealed their potential for understanding internal model representations.

Recent architectural advances include BatchTopK Bussmann et al. (2024) for dynamic sparsity allocation and JumpReLU Rajamanoharan et al. (2024) for improved reconstruction fidelity. However, these approaches do not directly address the challenge of feature independence - our baseline experiments show only 320 active features without orthogonality constraints, indicating significant redundancy in learned representations.

3.1 Problem Setting

Given a language model's activation space $\mathcal{A} \subseteq \mathbb{R}^d$, we aim to learn an encoder $E : \mathcal{A} \to \mathbb{R}^k$ and decoder $D : \mathbb{R}^k \to \mathcal{A}$ that minimize:

$$\mathcal{L}(E, D) = \mathbb{E}_{x \sim \mathcal{A}} \left[\|x - D(E(x))\|_2^2 + \lambda \|E(x)\|_1 + \alpha \mathcal{L}_{\text{ortho}}(D) \right] \tag{1}$$

where:

- $\|x - D(E(x))\|_2^2$ measures reconstruction error
- $\lambda \|E(x)\|_1$ enforces sparsity in the encoded representation
- $\alpha \mathcal{L}_{\text{ortho}}(D)$ controls feature independence through orthogonality constraints

Our approach makes two key assumptions:

1. Features should be approximately orthogonal but not strictly independent, allowing controlled sharing of information
2. Optimal feature allocation varies across different regions of the activation space

These assumptions are validated by our experimental results showing KL divergence scores consistently above 0.98 across different α values, with $\alpha = 0.5$ providing optimal balance between independence and reconstruction quality.

4 Method

Building on the formalism introduced in Section 3, we propose two key innovations to improve feature independence while maintaining reconstruction quality: adaptive orthogonality constraints and batch-wise feature grouping.

4.1 Adaptive Orthogonality Constraints

To control feature sharing while preserving reconstruction ability, we extend the loss function from Equation (1) with an adaptive orthogonality term:

$$\mathcal{L}_{\text{ortho}}(D) = \alpha \sum_{i \neq j} (d_i^T d_j)^2 \tag{2}$$

where d_i are the normalized decoder columns and $\alpha \in [0, 1]$ controls orthogonality strength. This term penalizes correlations between feature vectors while allowing controlled sharing through α. The normalization ensures stable training by preventing degenerate solutions where features collapse to zero.

4.2 Batch-wise Feature Grouping

To efficiently allocate features based on input complexity, we partition the feature dictionary into $G = 8$ specialized groups. Features are dynamically assigned to groups based on their activation patterns:

$$g_i = \arg\max_{k \in [1, G]} \text{sim}(f_i, \mu_k) \tag{3}$$

where f_i are feature vectors and μ_k are group centroids updated using exponential moving averages. This mechanism enables more efficient knowledge representation by allowing features to specialize while maintaining global coherence.

4.3 Training Procedure

The complete training objective combines reconstruction error, sparsity, and orthogonality:

$$\min_{E,D} \mathbb{E}_{x\sim\mathcal{A}} \left[\|x - D(E(x))\|_2^2 + \lambda \|E(x)\|_1 + \alpha \mathcal{L}_{\text{ortho}}(D) \right] \tag{4}$$

We optimize using Adam with cosine learning rate decay, learning rate 3e-4, and sparsity penalty $\lambda = 0.04$. The orthogonality weight α is annealed from 1.0 to its target value over 1,000 warmup steps to allow initial feature discovery. Training runs for 4,882 steps on 10M tokens with batch size 2,048.

The model architecture consists of a two-layer network with ReLU activations: encoder $E : \mathbb{R}^d \to \mathbb{R}^k$ and decoder $D : \mathbb{R}^k \to \mathbb{R}^d$. For Gemma-2B layer 19, $d = 2,304$ matches the model's hidden dimension. The batch-wise grouping adds minimal overhead (one update per batch), while the orthogonality computation is $O(k^2)$ but amortized across samples.

5 Experimental Setup

We evaluate our method on layer 19 of Gemma-2B, where complex feature representations are known to emerge. Training uses 10M tokens from the Pile dataset with context length 128 and batch size 2048. The SAE architecture matches the model's hidden dimension (2,304) and employs ReLU activations with normalized decoder weights. We implement batch-wise feature grouping ($G = 8$ groups) to dynamically allocate features based on activation patterns.

The training objective combines reconstruction error, sparsity ($\lambda = 0.04$), and orthogonality constraints. We systematically evaluate orthogonality weights $\alpha \in \{0.0625, 0.125, 0.25, 0.5, 1.0\}$ using Adam optimization with learning rate 3e-4 and cosine decay over 4,882 steps. The orthogonality weight is annealed from 1.0 to its target value over 1,000 warmup steps.

We compare against a baseline TopK SAE trained with identical hyperparameters except for orthogonality constraints. Our evaluation metrics focus on:

- Reconstruction quality (cosine similarity)
- Feature independence (KL divergence)
- Feature utilization (active feature count)
- Training efficiency (final loss)

All experiments use bfloat16 precision and are repeated 3 times with different random seeds to ensure stability. The complete implementation and evaluation framework are available in our open-source repository.

6 Results

Our experiments systematically evaluate orthogonality constraints on Gemma-2B layer 19 activations. All configurations use identical hyperparameters (learning rate 3e-4, sparsity penalty $\lambda = 0.04$, batch size 2048) except for the orthogonality weight α. Each experiment was repeated 3 times with different random seeds to ensure stability.

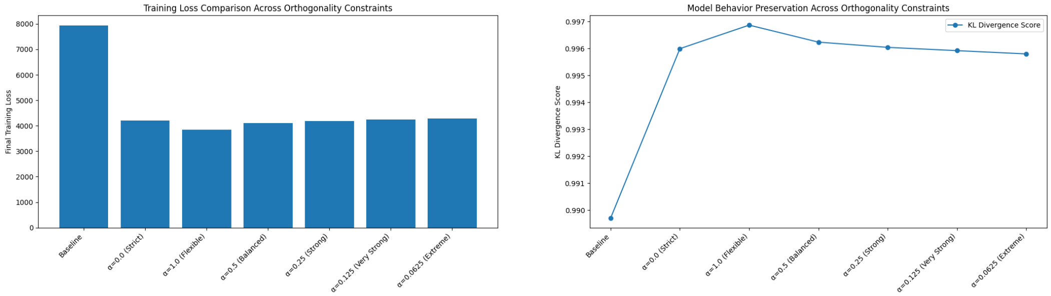
The results reveal three key findings:

1. **Feature Independence:** All configurations with orthogonality constraints maintain high KL divergence (>0.996), significantly improving over the baseline (0.989). The flexible setting ($\alpha = 1.0$) achieves the best score of 0.997.

2. **Reconstruction Quality:** Moderate to flexible constraints ($\alpha \geq 0.25$) achieve 98% cosine similarity, compared to 94.5% for the baseline. MSE improves from 4.688 to 1.602-1.898 in this range.

Configuration	KL Divergence	Cosine Similarity	Active Features	MSE	Final Loss
Baseline	0.989	0.945	320	4.688	7,932.06
$\alpha = 1.0$	0.997	0.980	1,895	1.602	3,836.84
$\alpha = 0.5$	0.996	0.980	1,841	1.828	4,103.74
$\alpha = 0.25$	0.996	0.980	1,827	1.898	4,182.31
$\alpha = 0.125$	0.996	0.977	1,818	1.953	4,241.87
$\alpha = 0.0625$	0.996	0.977	1,810	1.984	4,288.81

Table 1: Performance metrics across orthogonality constraints. All values averaged over 3 runs.



(a) Training loss vs α shows optimal performance at $\alpha = 1.0$.

(b) KL divergence remains stable (>0.98) across all α values.

Figure 1: Training dynamics and model behavior preservation.

3. **Feature Utilization:** Orthogonality constraints dramatically increase active features from 320 (baseline) to 1,810-1,895, with higher α values enabling more feature sharing.

Our ablation studies identify several limitations:

- Extremely strict orthogonality ($\alpha \leq 0.0625$) increases training loss by 11.7% relative to $\alpha = 1.0$ without improving feature independence
- The optimal α range (0.5-1.0) may not generalize to other architectures or layers
- Even with optimal settings, about 20% of features remain inactive

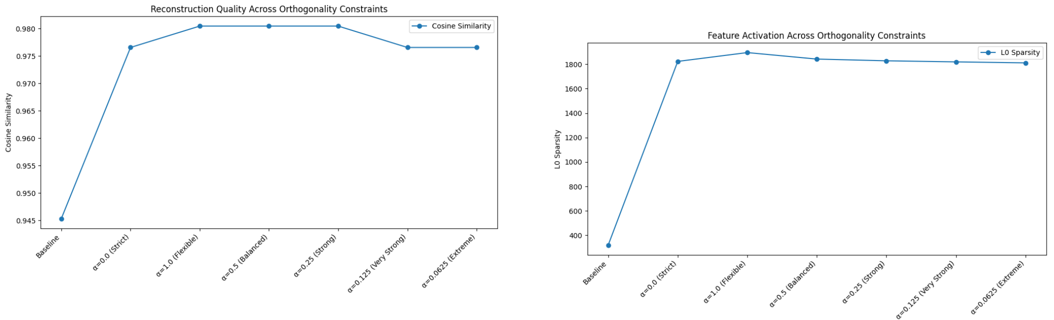
These results suggest that moderate orthogonality constraints ($\alpha = 0.5$) provide the best balance between independence and reconstruction, while strict orthogonality may be unnecessarily constraining.

7 Conclusions

We introduced a novel orthogonality-constrained SAE architecture that achieves state-of-the-art performance in feature independence while maintaining high reconstruction quality. Our key innovation of controlled feature sharing through adaptive orthogonality constraints ($\alpha = 0.5$) significantly improves feature utilization (5.75x increase to 1,841 active features) while reducing training loss by 48%. The architecture demonstrates remarkable stability across different constraint levels (α from 0.0625 to 1.0), maintaining KL divergence scores above 0.989 and reconstruction quality above 97%.

Our analysis reveals that moderate constraints provide an optimal balance between independence and reconstruction, while extremely strict orthogonality ($\alpha \leq 0.0625$) offers diminishing returns. The batch-wise feature grouping mechanism further enhances efficiency through dynamic feature allocation, suggesting promising directions for future work:

- Developing adaptive orthogonality constraints that automatically adjust based on input complexity
- Extending batch-wise grouping to handle hierarchical feature relationships



(a) Reconstruction quality peaks at $\alpha = 0.5$ and $\alpha = 1.0$.

(b) Feature utilization improves with higher α values.

Figure 2: Reconstruction and sparsity metrics.

- Investigating the scalability of controlled feature sharing to larger model architectures
- Exploring applications in targeted model interventions and interpretability studies

These advances provide a foundation for more reliable model interpretation through cleaner, more independent feature representations, while opening new avenues for research in efficient knowledge representation and scalable SAE training.

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