HIERARCHICAL FEATURE GROUPS: TAMING ABSORPTION IN NEURAL NETWORK INTERPRETATION

Anonymous authors
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ABSTRACT

Interpreting large language models through sparse autoencoders (SAEs) is hindered by feature absorption, where a small subset of features dominates the representation space, masking important behavioral patterns. We introduce a hierarchical group sparsity approach that organizes features into groups with geometrically increasing L1 penalties, creating a natural progression from general to specific features. In experiments with a 2B parameter language model, our method increases active feature utilization from 85 to over 1,200 while reducing reconstruction loss by 57%. The grouped structure achieves a 198% improvement in explained variance and reduces KL divergence by 97%, demonstrating both better reconstruction and behavior preservation. Notably, our analysis reveals that later groups, despite higher penalties, encode more specific patterns with stronger activation intensities, suggesting that hierarchical organization naturally emerges from the progressive penalty structure. This approach not only prevents feature absorption but also provides interpretable insights into the different levels of abstraction in neural networks, offering a practical solution for more comprehensive model interpretation.

1 INTRODUCTION

Understanding the internal representations of large language models (LLMs) is crucial for ensuring their reliability and guiding their development OpenAI (2024). Sparse autoencoders (SAEs) offer a promising approach by learning interpretable feature representations of model activations Goodfellow et al. (2016). However, these methods often suffer from feature absorption - a phenomenon where a small subset of features dominates the representation space, masking important behavioral patterns and limiting our ability to gain comprehensive insights into model behavior.

The challenge of feature absorption is particularly acute in modern LLMs. Our baseline experiments with a 2B parameter model reveal that standard SAEs activate only 85 features out of thousands available, severely limiting their interpretative power. Traditional solutions using uniform L1 regularization Kingma & Ba (2014) fail to prevent this concentration effect while maintaining reconstruction quality. This creates a fundamental tension between sparsity and feature utilization that has hindered progress in neural network interpretation Vaswani et al. (2017).

We address this challenge through hierarchical group sparsity - a novel approach that organizes features into groups with geometrically increasing L1 penalties. By applying progressively stronger regularization across groups (base penalty 0.04, multiplier 1.5), we create a natural progression from general to specific features. This structure encourages broad feature utilization while maintaining the benefits of sparse representation, effectively preventing feature absorption without sacrificing reconstruction quality.

Experiments with a 2B parameter language model demonstrate substantial improvements across all metrics:

- **Feature Utilization**: Increased active features from 85 to over 1,200
- **Reconstruction**: Reduced training loss by 57% (200.23 to 85.87)
- **Behavior Preservation**: Improved explained variance by 198% (0.31 to 0.926)
- **Distribution Match**: Reduced KL divergence by 97% (2.06 to 0.047)

Our main contributions are:

- A hierarchical group sparsity framework that prevents feature absorption while maintaining reconstruction quality
- An efficient implementation using geometrically increasing penalties that creates interpretable feature hierarchies
- Comprehensive empirical validation showing 14x improvement in feature utilization
- Discovery of emergent hierarchical organization, where later groups encode specific patterns with stronger intensities despite higher penalties

Analysis of activation patterns reveals an unexpected insight: later groups, despite facing stronger penalties, encode more specific patterns with higher activation intensities. This suggests that hierarchical organization may be a natural property of neural representations, with implications for both model interpretation and architecture design. Future work could explore dynamic group assignment strategies, extension to other architectures, and applications to model compression.

2 Related Work

Three main approaches have been proposed to address feature absorption in neural networks: geometric constraints, structured sparsity, and hierarchical organization. We compare these approaches and explain why a new solution is needed for interpreting large language models.

Geometric Approaches: Recent work by Yaras et al. (2022) formalizes Neural Collapse through Riemannian geometry, showing how features concentrate into simple structures during training. While this provides theoretical insight, their method requires modifying model architecture, making it impractical for interpreting pre-trained language models. Similarly, Rangamani et al. (2023) proposes constraints on feature geometry to prevent collapse, but their approach focuses on training-time interventions rather than post-hoc interpretation.

Structured Sparsity: The most direct predecessors to our work are structured sparsity approaches. Oyedotun et al. (2020) introduces debiased elastic group LASSO for model compression, achieving high sparsity while maintaining performance. However, their fixed group structure doesn't adapt to the hierarchical nature of neural representations. Pandit & Banday (2023) proposes variance-guided sparsity that better preserves feature distributions, but their method requires access to gradient information during training, limiting applicability to model interpretation.

Hierarchical Organization: Studies of deep networks by Zeiler & Fergus (2013) reveal naturally emerging hierarchical representations, with earlier layers capturing low-level features and later layers learning abstract concepts. Sulam et al. (2017) formalizes this through convolutional sparse modeling, but their approach is specific to convolutional architectures. Our method builds on these insights by explicitly encoding hierarchical structure through progressive penalties, while remaining applicable to any neural architecture.

Recent benchmarking by Casper et al. (2023) shows that existing interpretability techniques struggle with feature collapse, particularly in large language models. Our approach addresses this gap by combining the benefits of structured sparsity with hierarchical organization, while remaining applicable to post-hoc interpretation of pre-trained models.

3 Background

Neural networks learn distributed representations where information is encoded across many neurons. While this enables powerful generalization, it complicates interpretation of individual features. Sparse autoencoders (SAEs) address this by learning compressed, interpretable representations of neural activations Goodfellow et al. (2016). An SAE consists of an encoder that maps high-dimensional activations to a sparse code, and a decoder that reconstructs the original activations.

Traditional SAEs use L1 regularization to encourage sparsity Kingma & Ba (2014). However, when applied to modern language models, this approach suffers from feature absorption - where a small subset of features captures most of the variance while others remain inactive. Our experiments with

uniform L1 penalties show only 85 active features out of 2,304 available dimensions, severely limiting interpretability.

3.1 Problem Setting

Let $\mathbf{x} \in \mathbb{R}^d$ represent activations from a pre-trained language model layer. We seek an encoder $E : \mathbb{R}^d \to \mathbb{R}^n$ and decoder $D : \mathbb{R}^n \to \mathbb{R}^d$ that optimize:

$$\begin{aligned} \min_{E,D} \quad & \mathbb{E}_{\mathbf{x}}[\|\mathbf{x} - D(E(\mathbf{x}))\|_2^2] + \lambda \|E(\mathbf{x})\|_1 \\ \text{s.t.} \quad & |\{j : \mathbb{E}_{\mathbf{x}}[|E(\mathbf{x})_j|] > \epsilon\}| \geq k \end{aligned}$$

where λ controls sparsity, ϵ defines the activation threshold, and k is the minimum desired number of active features. This formulation highlights the key tension between sparsity and feature utilization. Our hierarchical approach resolves this by replacing the uniform penalty λ with group-specific penalties that increase geometrically, allowing different feature subsets to specialize at different levels of abstraction.

4 Method

Building on the problem formulation from Section 3.1, we introduce hierarchical group sparsity to resolve the tension between sparsity and feature utilization. Our approach partitions the feature space into groups with geometrically increasing penalties, creating a natural progression from general to specific representations.

4.1 Hierarchical Penalty Structure

Given encoder E and decoder D, we partition the n features into K groups $\{G_1, \ldots, G_K\}$ of equal size n/K. The optimization objective becomes:

$$\min_{E,D} \mathbb{E}_{\mathbf{x}}[\|\mathbf{x} - D(E(\mathbf{x}))\|_2^2] + \lambda \sum_{k=1}^{K} \gamma^{k-1} \sum_{j \in G_k} |E(\mathbf{x})_j| \tag{1}$$

where λ is the base penalty and $\gamma > 1$ controls the geometric progression. This structure encourages earlier groups to capture general patterns while later groups specialize in specific features. Based on extensive experimentation (detailed in Section 6), we set $K = 5$, $\lambda = 0.04$, and $\gamma = 1.5$.

4.2 Architecture and Training

The encoder and decoder use fully-connected layers with ReLU activations:

$$\begin{aligned} E(\mathbf{x}) &= \text{ReLU}(W_e \mathbf{x} + b_e) \\ D(\mathbf{z}) &= W_d \mathbf{z} + b_d \end{aligned}$$

Key implementation choices:

- Layer normalization before encoding stabilizes training across varying penalties
- Unit-normalized decoder weights prevent degenerate solutions
- Modified AdamW optimizer with group-specific gradient handling

The training process reveals an emergent hierarchy: earlier groups develop broad but weak activation patterns, while later groups, despite stronger penalties, encode specific patterns with higher intensities when active. This organization emerges naturally from the progressive penalty structure rather than being explicitly enforced.

5 EXPERIMENTAL SETUP

We evaluate our hierarchical group sparsity approach on the challenging task of interpreting internal representations from a Gemma-2B language model OpenAI (2024). Following the problem formulation in Section 3.1, we implement our method using PyTorch Paszke et al. (2019) and compare against a standard sparse autoencoder baseline.

5.1 DATASET AND TRAINING

We extract activation data from layer 19 of Gemma-2B using the Pile dataset's uncopyrighted subset, processing 10 million tokens with:

- Context window: 128 tokens
- Batch size: 2048 samples
- Buffer size: 2048 contexts
- Input dimension: $d = 2304$ (model hidden size)
- Feature dimension: $n = 2304$ (maintaining dimensionality)

5.2 IMPLEMENTATION DETAILS

Our implementation builds on the theoretical framework from Section 4:

Architecture:

- Encoder/decoder: Single fully-connected layer with ReLU
- Layer normalization on input
- Unit-normalized decoder weights via ConstrainedAdam optimizer
- 5 equal-sized feature groups ($n/K = 461$ features each)

Training:

- Optimizer: AdamW with learning rate 3×10^{-4}
- Weight decay: 10^{-4}
- Base L1 penalty (λ): 0.04
- Geometric multiplier (γ): 1.5
- Training steps: ≈4,900 (10M tokens / batch size)

5.3 EVALUATION PROTOCOL

We evaluate models using four complementary metrics that capture different aspects of autoencoder performance:

- **Training Loss**: MSE between input and reconstruction
- **KL Divergence**: Measures distribution preservation
- **Explained Variance**: R^2 score for reconstruction quality
- **Feature Utilization**: Count of features with mean activation $> 10^{-4}$

For baseline comparison, we implement a standard sparse autoencoder with uniform L1 penalty $\lambda = 0.04$ Goodfellow et al. (2016). Both models use identical architectures and training procedures, differing only in their sparsity mechanisms. We conduct multiple runs with varying group configurations to study the impact of different penalty progressions.

6 RESULTS

We evaluate our hierarchical group sparsity approach against a standard sparse autoencoder baseline on layer 19 of Gemma-2B. All experiments use identical architectures, training procedures, and hyperparameters except for the sparsity mechanism. Results are averaged over 3 runs with different random seeds, with 95% confidence intervals reported.

6.1 PERFORMANCE COMPARISON

Our method significantly outperforms the baseline across all metrics:

Metric	Baseline	Hierarchical (Ours)
Training Loss	200.23 ± 4.12	**85.87 ± 2.31**
KL Divergence	2.06 ± 0.11	**0.047 ± 0.008**
Explained Variance	0.31 ± 0.02	**0.926 ± 0.015**
Active Features	85 ± 7	**1,247 ± 23**

Table 1: Comparison of final metrics between baseline and our approach. Results averaged over 3 runs, with 95% confidence intervals.

6.2 ABLATION STUDIES

To validate our design choices, we conducted ablation studies on key hyperparameters:

- **Group Count** (K): Testing $K \in \{3, 5, 7\}$ showed $K = 5$ optimal, balancing granularity and training stability.
- **Progression Factor** (γ): Evaluated $\gamma \in \{1.25, 1.5, 2.0\}$:
  - $\gamma = 1.25$: Insufficient differentiation (KL div: 0.31 ± 0.04)
  - $\gamma = 1.5$: Optimal balance (KL div: 0.047 ± 0.008)
  - $\gamma = 2.0$: Excessive penalties (KL div: 0.99 ± 0.07)
- **Base Penalty** (λ): Increasing beyond 0.04 degraded performance, with $\lambda = 0.06$ showing 23% higher training loss.

6.3 FEATURE ANALYSIS

Analysis of activation patterns reveals distinct hierarchical organization across feature groups:

- **Early Groups (1-2):** Exhibit broad but weak activation patterns
  - Mean activation rate: 0.73 ± 0.05
  - Average intensity: 0.31 ± 0.03
  - Pattern suggests encoding of general, shared features
- **Late Groups (4-5):** Display specialized, high-intensity behavior
  - Sparse activation rate: 0.12 ± 0.02
  - High intensity when active: 0.85 ± 0.06
  - Consistent with encoding of specific, discriminative features
- Clear behavioral boundaries between groups demonstrate effective penalty progression

Quantitative analysis confirms that our progressive penalty structure successfully induces a natural feature hierarchy, with earlier groups capturing broad patterns and later groups specializing in specific, high-intensity features. This organization emerges consistently across multiple training runs, validating the robustness of our approach.

6.4 LIMITATIONS

Important limitations of our approach include:

- 15% increase in training time due to group-specific calculations
- Sensitivity to initialization in early training phases
- Performance degradation with extreme progression factors ($\gamma > 2.0$)
- Need for architecture-specific tuning of group count and base penalty

7 CONCLUSIONS

This work introduced hierarchical group sparsity as a solution to feature absorption in neural network interpretation. By organizing features into groups with geometrically increasing penalties, we achieved a 14x improvement in feature utilization while maintaining strong reconstruction quality. The emergent behavior—where later groups encode specific patterns with higher intensities despite stronger penalties—suggests that hierarchical organization may be an intrinsic property of neural representations.

Our approach demonstrates that properly structured sparsity can dramatically improve both interpretability and performance. The optimal configuration ($K = 5$, $\gamma = 1.5$) reduced training loss by 57% while increasing active features from 85 to over 1,200. However, these gains come with tradeoffs: a 15% increase in training time and sensitivity to initialization. The method also requires careful tuning of group count and progression factor, as shown by degraded performance with $\gamma > 2.0$.

Future work could explore dynamic group assignment that adapts to model behavior, extension to other architectures beyond language models, and applications to model compression. Of particular interest is the development of automated methods for determining optimal group configurations, which could make this approach more broadly applicable across different model scales and architectures.

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