Індивідуальне домашнє завдання №1

Варіант №28

1 Завдання

1.1. Розв'язати диференційні рівняння першого порядку

a) $\sqrt{x}dy = y(3 + \sqrt{x})dx$

b) $y^{`} = \frac{y}{x}\ln\frac{y}{x}$

c) $y^{`} + \frac{y}{3+x} = \ln 5x$

d) $y^{`} - xy - y^3xe^{-x^2} = 0$

1.2. Розв'язати диференційні рівняння вищих порядків

a) $y^{``} - (y^{`})^2 + y^{`}(y-1) = 0; \quad y(0) = 2; \ y^{`}(0) = 2$

b) $y^{``} - 4y = 4x \cdot e^{2x}$

c) $y^{``} + 2y^{`} + y = 9e^{-x} + x; \qquad y(0) = 1; y^{`}(0) = 2$

1.3. Розв'язати систему диференційних рівнянь

$$\begin{cases} x^{`} = 3x - 4y + 2t \\ y^{`} = x + y + t \end{cases}$$

2 Виконання

1.1 a) $\sqrt{x}dy = y(3 + \sqrt{x})dx \mid : y$

$$\frac{\sqrt{x}dy}{y} = 3 + \sqrt{x}dx \mid : \sqrt{x} \mid \ \frac{dy}{y} = \left(\frac{3}{\sqrt{x}} + 1\right)dx \mid \ \int \frac{1}{y}dy^{(1)} = \int \left(\frac{3}{\sqrt{x}} + 1\right)dx^{(2)};$$

$(1)\int \frac{1}{y}dy = \ln|y|$; $(2)\int \left(\frac{3}{\sqrt{x}}+1\right)dx = 3\int \frac{dx}{\sqrt{x}} + \int dx = 3*\int x^{-\frac{1}{2}}dx + x + C$

$$= 3*\frac{x^{-\frac{1}{2}+\frac{2}{2}}}{-\frac{1}{2}+\frac{2}{2}} + x + C = 3*2\sqrt{x} + x + C = x + 6\sqrt{x} + C$$

$\ln|y| = x + 6\sqrt{x} + C \mid e^{\ln(a)} = a$ **Відповідь:** $\mathbf{y = Ce^{x+6\sqrt{x}}}$

1.1 b) $y` = \frac{y}{x}\ln\frac{y}{x}$; $\left[\left[\frac{y}{x} = u\right]; y = ux;\ y` = u`x + u^{(1)};\right]$

$(1)\ u`x + u = \frac{xdu}{dx} + \frac{udx}{dx} = \frac{xdu + udx}{dx}$

$\frac{xdu + udx}{dx} = u\ln u \mid xdu + udx = u\ln u\,dx \mid xdu = (u\ln u - u)dx;$

$\frac{du}{u\ln u - u} = \frac{dx}{x} \mid \int \frac{1}{u\ln u - u}du^{(2)} = \int \frac{1}{x}dx^{(3)}$

$(2)\int \frac{1}{u\ln u - u}du = \int \frac{du}{u(\ln u - 1)} \mid \left[d(\ln u) = \frac{du}{u}\right]$

$\int \frac{d(\ln u)}{\ln u - 1} \mid [d(\ln u - 1) = d(\ln u)] \mid \int \frac{d(\ln u - 1)}{\ln u - 1} = \ln|\ln u - 1| + C$

$(3)\int \frac{1}{x}dx = \ln|x| + C$

$\ln|\ln u - 1| = \ln|x| + C \mid [\ln a + \ln b = \ln ab]$

$\ln|\ln u - 1| = \ln|xC| \mid \left[e^{\ln(a)} = a\right] \mid \ln u - 1 = xC \mid \ln\frac{y}{x} - 1 = xC \mid \left[e^{\ln(a)} = a\right]$

$\frac{y}{x} * e^{-1} = e^{Cx} \mid \frac{y}{ex} = e^{Cx} \mid \frac{y}{ex} = e^{Cx} \mid$

Відповідь: $y = xe^{Cx+1}$

1.1 c) $y` + \frac{y}{3+x} = \ln 5x$; $[y = uv; y` = u`v + uv`]$

$$u^{`}v + uv^{`} + \frac{uv}{3+x} = \ln 5x \;\; | \;\; u^{`}v + u\left(v^{`} + \frac{v}{3+x}^{(1)}\right) = \ln 5x \quad (0next)$$

$$(1)\; v^{`} + \frac{v}{3+x} = 0 \;|\; \frac{dv}{dx} = -\frac{v}{3+x} \;|\; \frac{dv}{v} = -\frac{dx}{3+x} \;|\; \int \frac{dv}{v}^{(2)} = -\int \frac{dx}{x+3}^{(3)} ;= (1next)$$

$$(2)\; \int \frac{dv}{v} = \ln|v|\,; \quad (3) - \int \frac{dx}{x+3}\; [d(x+3) = dx] \;|\; -\int \frac{d(x+3)}{x+3} = -\ln|x+3|$$

$$(1next)\; \ln|v| = -\ln|x+3| \;\; [e^{\ln(a)} = a] \;|\; \mathbf{v} = \frac{\mathbf{1}}{\mathbf{x+3}}$$

$$(0next)\;\; u^{`}v + u\left(v^{`} + \frac{v}{3+x}\right) = \ln 5x \;|\; \left[\mathbf{v} = \frac{\mathbf{1}}{\mathbf{x+3}},\; v^{`} + \frac{v}{3+x} = 0\right]$$

$$u^{`}v = \ln 5x \;|\; \frac{u^{`}}{3+x} = \ln 5x \;|\; u^{`} = (x+3)\ln 5x;$$

$$du = (x+3)\ln 5x\, dx \;|\; \int du = \int (x+3)\ln 5x\, dx^{(4)} \;;= (0next)$$

$$(4) \int (x+3)\ln 5x\, dx = \int (x+3)(\ln x + \ln 5)dx$$

$$= \int (x\ln x + x\ln 5 + 3\ln x + 3\ln 5)dx$$

$$= \int x\ln x\, dx + \int x\ln 5\, dx + \int 3\ln x\, dx + \int 3\ln 5\, dx$$

$$= \int x\ln x\, dx^{(5)} + \ln 5 \int x dx + 3\int \ln x\, dx^{(6)} + 3\ln 5 \int dx = (4next)$$

$$(5) \int x\ln x\, dx \begin{bmatrix} u = \ln x & du = \frac{1}{x}dx \\ dv = x\,dx & v = \frac{x^2}{2} \end{bmatrix} \;|\; \frac{\ln x\, x^2}{2} - \int \frac{x^2}{2} * \frac{1}{x}dx = \frac{\ln x\, x^2}{2} - \frac{1}{2} * \frac{x^2}{2}$$

$$= \frac{\ln x\, x^2}{2} - \frac{x^2}{4}$$

$$(6)\;\; 3\int \ln x\, dx\,; \begin{bmatrix} u = \ln x & du = \frac{1}{x}dx \\ dv = dx & v = x \end{bmatrix} ;\; 3\int \ln x\, dx = 3\left(x\ln x - \int x * \frac{1}{x}dx\right)$$

$$= 3(x\ln x - x)$$

$$(4next) = \left(\frac{x^2 \ln x}{2} - \frac{x^2}{4}\right) + \frac{\ln 5\, x^2}{2} + 3(x\ln x - x) + 3x\ln 5 + C$$

$$= \frac{2x^2 \ln x}{4} - \frac{x^2}{4} + \frac{2\ln 5\, x^2}{4} + \frac{12(x\ln x - x)}{4} + \frac{12(x\ln 5)}{4} + \frac{4C}{4}$$

$$(0next)\ \left[u = \frac{y}{v};\ \ v = \frac{1}{x+3}\right]$$

$$y = \frac{2x^2 \ln x - x^2 + 2\ln 5\, x^2 + 12(x\ln x - x) + 12(x\ln 5) + 4c}{4(x+3)}$$

$$y = \frac{2x^2 \ln x - x^2 + 2\ln 5\, x^2 + 12x\ln x - 12x + 12x\ln 5 + 4c}{4(x+3)}$$

$$y = \frac{(2x^2 + \ 12x)\ln x + (2\ln 5 - 1)x^2 + (12\ln 5 - 12)x + 4c}{4(x+3)}$$

$$y = \frac{2(x^2 + 6x)\ln x}{4(x+3)} + \frac{(2\ln 5 - 1)x^2}{4(x+3)} + \frac{4((3\ln 5 - 3)x + C)}{4(x+3)}$$

Відповідь: $y = \frac{(x^2+6x)\ln x}{2x+6} + \frac{(2\ln 5-1)x^2}{4(x+3)} + \frac{(3\ln 5-3)x+C}{x+3}$

<u>1.1 d)</u> $y` - xy - y^3xe^{-x^2} = 0$

$$y` - xy - y^3xe^{-x^2} = 0 \mid y` - xy - \frac{y^3xe}{x^2} = 0 \mid y` - xy = \frac{y^3x}{e^{x^2}} \ |:y^3\ ;$$

$$\frac{y`}{y^3} - \frac{x}{y^2} = \frac{x}{e^{x^2}} \mid \left[\begin{array}{l|l} p = \frac{1}{y^2};^{(1)} & y = \frac{1}{\sqrt{p}}^{\ (2)} \\ p` = -\frac{2y`}{y^3} & y` = -\frac{p`y^3}{2} \end{array}\right]$$

$$(1)\ \ p = \frac{1}{y^2} \mid p` = (y^{-2})` = -2 * y^{-3} * y` = \ -\frac{2y`}{y^3}$$

$$(2)\ y = \frac{1}{\sqrt{p}} \mid y` = \left(p^{-\frac{1}{2}}\right)` = -\frac{1}{2} * p^{-\frac{3}{2}} * p` = \ -\frac{u`}{2u^{\frac{3}{2}}} = a \mid \left[p = \frac{1}{y^2}\right]$$

$$\left[p^{\frac{3}{2}} = \sqrt{p^3} = \sqrt{\left(\frac{1}{y^2}\right)^3} = \sqrt{\frac{1}{y^6}} = \sqrt{y^{-6}} = y^{-3};\right] \mid a = -\frac{u^{`}}{2p^{\frac{3}{2}}} = -\frac{u^{`}y^3}{2}$$

$$-\frac{p^{`}y^3}{y^3 2} - px = \frac{x}{e^{x^2}} \;\mid : -\frac{1}{2} \mid p^{`} + 2px = -\frac{2x}{e^{x^2}} \mid \begin{bmatrix} p = uv \\ p^{`} = u^{`}v + uv^{`} \end{bmatrix}$$

$$u^{`}v + uv^{`} + 2uvx = -\frac{2x}{e^{x^2}} \mid u^{`}v + u(v^{`} + 2vx) = -\frac{2x}{e^{x^2}}$$

$$v^{`} + 2vx = 0;\;\; v^{`} = -2vx \mid \frac{dv}{dx} = -2vx \mid \frac{dv}{v} = -2xdx$$

$$\int \frac{dv}{v} = -2\int xdx \mid \ln|v| = -2\frac{x^2}{2} \mid \ln|v| = -x^2 \mid v = \frac{1}{e^{x^2}}$$

$$u^{`}v + u(v^{`} + 2vx) = -\frac{2x}{e^{x^2}} \mid [v^{`} + 2vx = 0;\;\; v = \frac{1}{e^{x^2}}]$$

$$\frac{u^{`}}{e^{x^2}} = -\frac{2x}{e^{x^2}} \Big| : \frac{1}{e^{x^2}} \mid u^{`} = -2x \mid du = -2xdx \mid u = -2 * \frac{x^2}{2} + C;$$

$$u = C - x^2 \mid p = uv = \frac{C - x^2}{e^{x^2}} \mid \frac{1}{y^2} = \frac{C - x^2}{e^{x^2}} \mid y^2 = \frac{e^{x^2}}{C - x^2}$$

Відповідь: $\mathbf{y = \pm\sqrt{\frac{e^{x^2}}{C-x^2}}}$

1.2 а) $y^{``} - (y^{`})^2 + y^{`}(y-1) = 0; \quad y(0) = 2; \;\; y^{`}(0) = 2$

$\mathbf{[y^{`} = p;\;\; y^{``} = p^{`}y^{`} = p^{`}p]}$

$p^{`}p - p^2 + p(y-1) = 0;$

$p^{`}p - p^2 + py - p = 0 \mid : p$

$p^{`} - p + y - 1 = 0 \mid p^{`} = p - y + 1 \mid p^{`} - p = -y + 1$

$\mathbf{[p = uv;\;\; p^{`} = u^{`}v + uv^{`}]} \mid u^{`}v + uv^{`} - uv = 1 - y \mid u^{`}v + u(v^{`} - v) = 1 - y$

$$v^{`} - v = 0 \mid v^{`} = v \;\; \frac{dv}{dy} = v \mid \frac{dv}{v} = dy \mid \int\frac{dv}{v} = \int dy \mid \ln|v| = y$$

$\underline{\mathbf{v = e^{y}}}$

$$u`e^y = 1 - y \;\; | \;\; u` = \frac{1-y}{e^y} \;\; | \;\; du = \frac{1-y}{e^y}dy;$$

$$u = \int \frac{1-y}{e^y}dy \;\; | \;\; u = \int \frac{1}{e^y}dy - \int \frac{y}{e^y}dy;$$

$$u = \int e^{-y}dy - \int ye^{-y}dy \;\; | \;\; \begin{bmatrix} a = y & da = 1 \\ db = e^{-y} & b = -\frac{1}{e^y} \end{bmatrix}$$

$$u = -\frac{1}{e^y} - \left(-\frac{y}{e^y} - \int -\frac{1}{e^{-y}}dy\right) =$$

$$= -\frac{1}{e^y} - \left(-\frac{y}{e^y} + \left(-\frac{1}{e^y}\right)\right) = -\frac{1}{e^y} + \frac{y}{e^y} + \frac{1}{e^y} = \mathbf{\frac{y}{e^y} + C}$$

$$y` = p = uv = \left(\frac{y}{e^y} + C\right)e^y \;|\; y` = y + Ce^y \;|\; 2 = 2 + Ce^2 \;|\; e^2C = 0 \;|\; \mathbf{C = 0}$$

$$\frac{dy}{dx} = y \;|\; \frac{dy}{y} = dx \;|\; \ln|y| = x + C_1 \;|\; y = C_1e^x \;|\; 2 = C_1e^0 \;|\; \mathbf{C_1 = 2};$$

Відповідь: $\mathbf{y = 2e^x}$

<u>**1.2 b)**</u> $y`` - 4y = 4x \cdot e^{2x}$

$[\mathbf{y = \tilde{y} + y^*}]$

$$y`` - 4y = 0 \;|\; k^2 - 4 = 0 \;|\; (k-2)(k+2) = 0$$

$$k_1 = 2;\; k_2 = -2$$

$$\mathbf{\tilde{y} = c_1e^{2x} + \frac{c_2}{e^{2x}}}$$

$$f(x) = 4x \cdot e^{2x} \;\; => \;\; \mathbf{z = 2 + 0j}$$

$$\mathbf{y^* = x * e^{2x} * (Ax + B)}$$

$$y`^* = \left(x * \left(e^{2x} * (Ax + B)\right)\right)` = x` * \left(e^{2x} * (Ax + B)\right) + x * \left(e^{2x} * (Ax + B)\right)` =$$

$$1 * e^{2x} * (Ax + B) + x * \left((e^{2x})` * (Ax + B) + e^{2x} * (Ax + B)`\right) =$$

$$e^{2x} * (Ax + B) + x * \left(e^{2x} * (2x)` * (Ax + B) + e^{2x} * \left(Ax` + B`\right)\right) =$$

$$e^{2x} * (Ax + B) + x * \left(e^{2x} * 2 * 1 * (Ax + B) + e^{2x} * (A * 1 + 0)\right) =$$

$$e^{2x}(Ax+B)+x(2(Ax+B)e^{2x}+Ae^{2x})$$

$$\mathbf{y^{`*}} = \mathbf{x(\ 2(Ax+B)e^{2x}+Ae^{2x}\ )+e^{2x}(Ax+B)}$$

$$y^{``*} = \left(x(2(Ax+B)e^{2x}+Ae^{2x})+e^{2x}(Ax+B)\right)^{`}$$

$$= \left(x(\ 2(Ax+B)e^{2x}+Ae^{2x}\ )\right)^{`}+\left(e^{2x}(Ax+B)\right)^{`} =$$

$$\left(x^{`}*(2(Ax+B)e^{2x}+Ae^{2x}\ )+x*(2(Ax+B)e^{2x}+Ae^{2x}\ )^{`}\right)$$

$$+\left((e^{2x})^{`}(Ax+B)+e^{2x}(Ax+B)^{`}\right) =$$

$$2(Ax+B)e^{2x}+Ae^{2x}+x*2\left((Ax+B)e^{2x}\right)^{`}+(Ae^{2x})^{`}+\left(e^{2x}*2x^{`}\right)(Ax+B)$$

$$+e^{2x}\left(Ax^{`}+B^{`}\right) =$$

$$2(Ax+B)e^{2x}+Ae^{2x}+2x\left(\left(Ax^{`}+B^{`}\right)e^{2x}+(Ax+B)(e^{2x})^{`}\right)+(Ae^{2x})^{`}$$

$$+(2e^{2x}(Ax+B)+Ae^{2x}) =$$

$$2(Ax+B)e^{2x}+Ae^{2x}+2x\left(Ae^{2x}+2e^{2x}(Ax+B)\right)+2Ae^{2x}+2e^{2x}(Ax+B)+Ae^{2x} =$$

$$2e^{2x}(Ax+B)+Ae^{2x}+2xAe^{2x}+4xe^{2x}(Ax+B)+2xAe^{2x}+2e^{2x}(Ax+B)+Ae^{2x} =$$

$$2xAe^{2x}+4xe^{2x}(Ax+B)+2xAe^{2x}+4e^{2x}(Ax+B)+2Ae^{2x} =$$

$$e^{2x}(2(Ax+B)+A+2xA+4x(Ax+B)+2xA+2(Ax+B)+A) =$$

$$e^{2x}(2Ax+2B+A+2xA+4Ax^2+4xB+2xA+2Ax+2B+A) =$$

$$e^{2x}(2Ax+2xA+2xA+2xA+4Ax^2+4xB+A+A+2B+2B) =$$

$$e^{2x}(8Ax+4Ax^2+4xB+2A+4B) = e^{2x}(4Ax^2+(8A+4B)x+2A+4B)$$

$$\mathbf{y^{``*}} = \mathbf{e^{2x}(4Ax^2+(8A+4B)x+2A+4B)}$$

$$e^{2x}(4Ax^2+(8A+4B)x+2A+4B)-4(x*e^{2x}*(Ax+B)) = 4x\cdot e^{2x}$$

$$e^{2x}(4Ax^2+(8A+4B)x+2A+4B)-4xe^{2x}(Ax+B) = 4x\cdot e^{2x}\ |:e^{2x}$$

$$4Ax^2+(8A+4B)x+2A+4B-4x(Ax+B) = 4x$$

$$4Ax^2+8xA+4xB+2A+4B-4Ax^2-4xB-4x = 0$$

$$8xA+2A+4B = 4x$$

$$\begin{cases} 8A = 4 \\ 2A + 4B = 0 \end{cases}; \begin{cases} A = \frac{1}{2} \\ 4B = -2 * \frac{1}{2} \end{cases}; \begin{cases} A = \frac{1}{2} \\ B = -\frac{1}{4} \end{cases}$$

$$y = \tilde{y} + y^* = c_1 e^{2x} + \frac{c_2}{e^{2x}} + x * e^{2x} * \left(\frac{1}{2}x - \frac{1}{4}\right)$$

Відповідь: $\mathbf{y = c_1 e^{2x} + \frac{c_2}{e^{2x}} + \frac{x^2 e^{2x}}{2} - \frac{x e^{2x}}{4}}$

<u>1.2 c)</u> $y^{``} + 2y^{`} + y = 9e^{-x} + x; \quad y(0) = 1; y^{`}(0) = 2$

$[\mathbf{y = \tilde{y} + y^*}]$

$y^{``} + 2y^{`} + y = 0$

$k^2 + 2k + 1 = 0;$

$(k + 1)^2 = 0$

$k_{1,2} = -1$

$$\mathbf{\tilde{y} = e^{-1x}(c_1 + xc_2) = \frac{c_1 + xc_2}{e^x}}$$

$$f(x) = \left(\frac{9}{e^x}\right)^{(1)} + (x)^{(2)};$$

$(1)\ y_0 = e^{-1*x}(9) \ => \ \mathbf{z = -1 + 0j}$

$$\mathbf{y_0^* = x^2 * e^{-1x} * A = \frac{Ax^2}{e^x}};$$

$$y_0^{*`} = \left(\frac{Ax^2}{e^x}\right)^{`} = \frac{(Ax^2)^{`} * e^x - Ax^2 * (e^x)^{`}}{e^{2x}} = \frac{A2xe^x - Ax^2 e^x}{e^{2x}} = \frac{e^x(2Ax - Ax^2)}{e^{2x}}$$

$$= \mathbf{\frac{2Ax - Ax^2}{e^x}}$$

$$y_0^{*``} = \left(\frac{2Ax - Ax^2}{e^x}\right)^` = \frac{(2Ax - Ax^2)^` e^x - (2Ax - Ax^2) * (e^x)^`}{e^{2x}}$$

$$= \frac{(2A - A2x)e^x - (2Ax - Ax^2)e^x}{e^{2x}} = \frac{(2A - A2x) - (2Ax - Ax^2)}{e^x}$$

$$= \mathbf{\frac{Ax^2 - 4Ax + 2A}{e^x}}$$

$$\frac{Ax^2 - 4Ax + 2A}{e^x} + 2\left(\frac{2Ax - Ax^2}{e^x}\right) + \frac{Ax^2}{e^x} = \frac{9}{e^x}$$

$$\frac{Ax^2 - 4Ax + 2A + 4Ax - 2Ax^2 + Ax^2}{e^x} = \frac{9}{e^x}$$

$$2A = 9;\ A = \frac{9}{2}$$

$$\mathbf{y_0^* = \frac{9x^2}{2e^x}};$$

$(1)\ y_1 = x\ => \mathbf{z = 0 + 0j}$

$\mathbf{y_1^* = Ax + B};\ \ \mathbf{y_1^{*`} = A};\ \mathbf{y_1^{*``} = 0}$

$0 + 2A + Ax + B = x;\quad \begin{cases} A = 1 \\ 2A + B = 0 \end{cases} \begin{cases} A = 1 \\ B = -2 \end{cases}$

$$\mathbf{y_1^* = x - 2}$$

$$\mathbf{y^* = \frac{9x^2}{2e^x} + x - 2}$$

$$\mathbf{y = \frac{c_1 + xc_2}{e^x} + \frac{9x^2}{2e^x} + x - 2}$$

$$y` = \left(\frac{c_1 + xc_2}{e^x}\right)` + \left(\frac{9x^2}{2e^x}\right)` + x` + (-2)`$$

$$= \frac{(c_1 + xc_2)` * e^x - (c_1 + xc_2) * (e^x)`}{e^{2x}} + \frac{(9x^2)` * 2e^x - 9x^2 * (2e^x)`}{(2e^x)^2} + 1$$

$$= \frac{c_2 e^x - (c_1 + xc_2)e^x}{e^{2x}} + \frac{9 * 2 * x * 2e^x - 9x^2 * 2e^x}{4e^{2x}} + 1$$

$$= \frac{e^x\left(c_2 - (c_1 + xc_2)\right)}{e^{2x}} + \frac{2e^x(18x - 9x^2)}{4e^{2x}} + 1$$

$$= \frac{c_2 - c_1 - xc_2}{e^x} + \frac{18x - 9x^2}{2e^x} + 1 = \mathbf{\frac{c_2 - c_1 - xc_2}{e^x} + \frac{9x}{e^x} - \frac{9x^2}{2e^x} + 1}$$

$$\begin{cases} \dfrac{c_1 + xc_2}{e^x} + \dfrac{9x^2}{2e^x} + x - 2 = y \\ \dfrac{c_2 - c_1 - xc_2}{e^x} + \dfrac{9x}{e^x} - \dfrac{9x^2}{2e^x} + 1 = y` \end{cases} \quad \begin{cases} \dfrac{c_1 + 0c_2}{e^0} + \dfrac{90^2}{2e^0} + 0 - 2 = 1 \\ \dfrac{c_2 - c_1 - 0c_2}{e^0} + \dfrac{90}{e^0} - \dfrac{90^2}{2e^0} + 1 = 2 \end{cases}$$

$$\begin{cases} c_1 - 2 = 1 \\ c_2 - c_1 + 1 = 2 \end{cases} \quad \begin{cases} c_1 = 3 \\ c_2 = 2 - 1 + 3 \end{cases} \quad \begin{cases} c_1 = 3 \\ c_2 = 4 \end{cases}$$

Відповідь: $\mathbf{y = \frac{9x^2}{2e^x} + \frac{4x+3}{e^x} + x - 2}$

1.3 $\begin{cases} x` = 3x - 4y + 2t \\ y` = x + y + t \end{cases}; \quad y(t)$

$3x - 4y + 2t = x` \;|\; -4y = x` - 3x - 2t \;|\; \mathbf{y = -\frac{x`}{4} + \frac{3x}{4} + \frac{t}{2}}$

$x`` = 3x` - 4y` + 2t` \;|\; x`` = 3x` - 4\left(x - \frac{x`}{4} + \frac{3x}{4} + \frac{t}{2} + t\right) + 2t`$

$x`` = 3x` - 4x + x` - 3x - 2t - 4t + 2t; \;\; x`` = 4x` - 7x - 4t$

$\mathbf{x`` - 4x` + 7x = -4t}$

$\mathbf{x = \tilde{x} + x^*}$

$x`` - 4x` + 7x = 0$

$k^2 - 4k + 7 = 0$

$$D = 16 - 4*1*7 = -12; \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm 3j$$

$$\mathbf{\tilde{x} = e^{2t}\left(c_1 \cos\left(\sqrt{3}t\right) + c_2 \sin\left(\sqrt{3}t\right)\right)}$$

$$f(t) = -4t \Rightarrow z = 0$$

$$\mathbf{x^* = A; \ x^` = 0; x^{``} = 0}$$

$$0 - 4*0 + 7*A = -4t; \ 7A = -4t; \ A = \frac{-4t}{7}$$

$$\mathbf{x^* = \frac{-4t}{7}}$$

$$\mathbf{x = e^{2t}\left(c_1 \cos\left(\sqrt{3}t\right) + c_2 \sin\left(\sqrt{3}t\right)\right) - \frac{4t}{7}}$$

$$x^` = \left(e^{2t}\left(c_1 \cos\left(\sqrt{3}t\right) + c_2 \sin\left(\sqrt{3}t\right)\right) - \frac{4t}{7}\right)^` = \left(e^{2t}\left(c_1 \cos\left(\sqrt{3}t\right) + c_2 \sin\left(\sqrt{3}t\right)\right)\right)^` - \frac{4}{7}t^`$$

$$= (e^{2t})^`\left(c_1 \cos\left(\sqrt{3}t\right) + c_2 \sin\left(\sqrt{3}t\right)\right) + e^{2t}\left(c_1 \cos\left(\sqrt{3}t\right) + c_2 \sin\left(\sqrt{3}t\right)\right)^` - \frac{4}{7}t^`$$

$$= 2e^{2t}\left(c_1 \cos\left(\sqrt{3}t\right) + c_2 \sin\left(\sqrt{3}t\right)\right) + e^{2t}\left(c_1\left(\cos\left(\sqrt{3}t\right)\right)^` + c_2\left(\sin\left(\sqrt{3}t\right)\right)^`\right) - \frac{4}{7} =$$

$$\mathbf{= 2e^{2t}\left(c_1 \cos\left(\sqrt{3}t\right) + c_2 \sin\left(\sqrt{3}t\right)\right) + e^{2t}\left(-c_1\sqrt{3} \sin\left(\sqrt{3}t\right) + c_2\sqrt{3} \cos\left(\sqrt{3}t\right)\right) - \frac{4}{7}}$$

$$y = -\frac{2e^{2t}\left(c_1 \cos\left(\sqrt{3}t\right) + c_2 \sin\left(\sqrt{3}t\right)\right) + e^{2t}\left(-c_1\sqrt{3} \sin\left(\sqrt{3}t\right) + c_2\sqrt{3} \cos\left(\sqrt{3}t\right)\right) - \frac{4}{7}}{4} + \frac{3\left(e^{2t}\left(c_1 \cos\left(\sqrt{3}t\right) + c_2 \sin\left(\sqrt{3}t\right)\right) - \frac{4t}{7}\right)}{4} + \frac{t}{2}$$

$$= \frac{-2e^{2t}\left(c_1 \cos\left(\sqrt{3}t\right) + c_2 \sin\left(\sqrt{3}t\right)\right) - e^{2t}\left(-c_1\sqrt{3} \sin\left(\sqrt{3}t\right) + c_2\sqrt{3} \cos\left(\sqrt{3}t\right)\right)}{4} + \frac{1}{7} + \frac{3e^{2t}\left(c_1 \cos\left(\sqrt{3}t\right) + c_2 \sin\left(\sqrt{3}t\right)\right)}{4} - \frac{3*4t}{4*7} + \frac{t}{2} =$$

$$= \frac{-e^{2t}\left(c_1\cos\left(\sqrt{3}t\right) + c_2\sin\left(\sqrt{3}t\right)\right)}{2} - \frac{e^{2t}\left(c_2\sqrt{3}\cos\left(\sqrt{3}t\right) - c_1\sqrt{3}\sin\left(\sqrt{3}t\right)\right)}{4} + \frac{1}{7}$$

$$+ \frac{3e^{2t}\left(c_1\cos\left(\sqrt{3}t\right) + c_2\sin\left(\sqrt{3}t\right)\right)}{4} - \frac{12t}{28} + \frac{14t}{14*2} =$$

$$= \frac{-e^{2t}\left(c_1\cos\left(\sqrt{3}t\right) + c_2\sin\left(\sqrt{3}t\right)\right)}{2} - \frac{e^{2t}\left(c_2\sqrt{3}\cos\left(\sqrt{3}t\right) - c_1\sqrt{3}\sin\left(\sqrt{3}t\right)\right)}{4} + \frac{1}{7}$$

$$+ \frac{3e^{2t}\left(c_1\cos\left(\sqrt{3}t\right) + c_2\sin\left(\sqrt{3}t\right)\right)}{4} - \frac{12t}{28} + \frac{14t}{14*2} =$$

$$-\frac{1}{2}e^{2t}\left(c_1\cos\left(\sqrt{3}t\right) + c_2\sin\left(\sqrt{3}t\right)\right) - \frac{1}{4}e^{2t}\left(c_2\sqrt{3}\cos\left(\sqrt{3}t\right) - c_1\sqrt{3}\sin\left(\sqrt{3}t\right)\right) + \frac{1}{7}$$

$$+ \frac{3}{4}e^{2t}\left(c_1\cos\left(\sqrt{3}t\right) + c_2\sin\left(\sqrt{3}t\right)\right) + \frac{2t}{28}$$

Відповідь:

$$\mathbf{x = e^{2t}\left(c_1\cos\left(\sqrt{3}t\right) + c_2\sin\left(\sqrt{3}t\right)\right) - \frac{4t}{7}}$$

$$\mathbf{y = -\frac{1}{2}e^{2t}\left(c_1\cos\left(\sqrt{3}t\right) + c_2\sin\left(\sqrt{3}t\right)\right) - \frac{1}{4}e^{2t}\left(c_2\sqrt{3}\cos\left(\sqrt{3}t\right) - c_1\sqrt{3}\sin\left(\sqrt{3}t\right)\right)}$$

$$\mathbf{+ \frac{1}{7} + \frac{3}{4}e^{2t}\left(c_1\cos\left(\sqrt{3}t\right) + c_2\sin\left(\sqrt{3}t\right)\right) + \frac{2t}{28}}$$