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Multi-Product Multi-Period Fixed Charge Transportation Problem: an Ant Colony Optimization Approach

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Abstract: Most of the practical applications of a transportation network, in addition to the variable cost, there incurs a fixed charge. This work formulates a Multi-Stage Multi-Period Fixed Charge Transportation Problem for a multi-product scenario. The problem is modeled using an optimization modeling tool, 'A Mathematical Programming Language' and its solution is obtained in BONMIN solver. The exact algorithms mostly require longer computational time to find an optimal solution for large problem size that are in practice. In these operational problems, in which process speed is as important as the solution quality, an Ant Colony Optimization based heuristic is proposed. Finally, the solution obtained from proposed heuristic is compared with that of exact methods using randomly generated data sets. The comparative analysis shows the competitiveness of the proposed heuristic.

Keywords: Modeling; Transportation Logistics; Multi-Stage Multi-Period Fixed Charge Transportation Problem; Ant Colony Optimization based heuristic.

1. INTRODUCTION

A supply chain is a system of suppliers, manufacturers, and distribution facilities that convert raw materials into usable finished products through multiple manufacturing stages and transports them to the clients through one or more distribution stages. In the classical case of the transportation problem, shipping cost proportional to the volume of goods transported is only considered with a focus to minimize the total transportation cost. But in the practical scenario, a fixed cost is also incurred whenever a transportation route is opened between a plant and customer.

The Fixed Charge Transportation Problem (FCTP) is a variant of the well-known Transportation Problem and occurs when both fixed and variable costs are present simultaneously. Two kinds of costs are considered in an FCTP namely, (i) a variable cost that varies linearly with the amount transported from the point of supply to the point of destination, and (ii) a fixed cost that is incurred whenever a non-zero quantity is transported between the point of supply and point of destination. When an FCTP is applied to a single period, it is called a Single Period Fixed Charge Problem. In order to take the benefit of lot-sizing, transportation choices are extended for more than one period. Such problems are called Multi-Period Fixed Charge Transportation Problem (MPFCTP).

As the demand fluctuates from period to period in a planning horizon, the MPFCTP considers an inventory and two types of inventory models, lost sales and back order. If a shortage happens in a period, the backorder will be carried over at the customer side or is treated as lost sales. In both cases, it incurs a penalty cost. The excess inventory in a period is an additional supply available for the subsequent periods. Similarly, if there is an excess production in any period, the excess is either stored with the manufacturer or the customer. In the real world situation, a supply chain network has to deal with a wide variety of products which will be similar or entirely dissimilar. This work focuses on the extension of an MPFCTP to a multi-product scenario and is known as a Multi-Product Multi-Period Fixed Charge Transportation Problem.

The remainder of the paper is organized as follows: Section 2 presents a review of relevant literature and provides a description in section 3. Section 4 explains the formulation of the mathematical model; Section 5 develops the solution methodology, and Section 6 designs the computational study. This paper concludes in section 7.

2. REVIEW OF LITERATURE

The introduction of fixed costs to transportation problems makes the objective function concave and discontinuous. In practice, the above facts confirm the difficulty to solve problems with fixed cost than the problems with linear costs. These problems were formulated as mixed-integer programming problem and solved by methods such as branch-and-bound (Barr et al., 1981) and cutting plane (Steinberg, 1970), but proved to be generally inefficient and

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computationally expensive, particularly for large sized problems that are in practice.

In the literature, there are many works reporting about the single stage FCTP that focus on the minimization of total transportation cost (Lotfi and Tavakkoli-Moghaddam, 2013; El-Sherbiny et al., 2013; Kowalski et al., 2014). As an extension, Panicker et al. (2013) presented a two-stage FCTP in which multiple manufacturers serves goods to a set of customers through a set of distribution centre with infinite capacity, but still, the model dealt with a single period and a single product. Similarly, Molla-Alizadeh-Zavardehi et al., (2011) extend the FCTP to a two-stage supply chain problem considering the potential distribution centres with fixed capacity for each distribution centre to be opened, and the model minimizes the total cost by opening required number of distribution centres to fulfil the demands of the customers. Hong et al. (2018) also consider a distribution-allocation problem in a two-stage supply chain with fixed costs for transportation route and facilities.

In practice, fixed cost problems are much more difficult to solve than a linear problem. If problems are too large to be solved by using exact methods, suitable heuristics or meta-heuristic approaches are to be used. Many approaches like genetic algorithm (Raj and Rajendran,2012), simulated annealing algorithm (Jawahar et al. 2012), artificial immune and genetic algorithm (Molla-Alizadeh-Zavardehi et al., 2011), simplex-based simulated annealing (Yaghini, et al., 2012) and ant colony optimization (Panicker et al., 2013) are applied in solving complex cases of FCTP. The literature reports the suitability of ant colony optimization algorithms to solve the network problems and continues to be a successful methodology for combinatorial optimisation problems. In the literature, ant colony optimization approach has been adopted for various optimisation problems such as the travelling salesman problem (Dorigo and Gambardella, 1997), job-shop scheduling problem (Girish and Jawahar. 2009), and quadratic assignment problem (Stützle and Dorigo, 1999). Indeed, the work of Panicker et al., (2013) reports a comparative analysis between ant colony optimization heuristic approach and genetic algorithm based heuristic and reveals the efficiency of the former to solve a two-stage FCTP.

Considering the economies of scale, it is beneficial to plan for more than one period, and thus an MPFCTP is formulated. Though Jawahar et al. (2012) modelled an MPFCTP, the authors considered a single-stage supply chain considering inventory and back order conditions for a single product. The excess production is considered as inventory at either supplier side or customer side, whichever minimizes the total cost and the backorder is kept at customer side. Tiwari et al. (2012) formulate and solve a single source multi-product, multi-stage supply chain network problem, but the model considers only variable transportation cost.

Therefore, the review of literature reveals that there is no formulation for a distribution-allocation problem considering fixed cost for a route, inventory, and back-orders in a multi-stage multi-period and multi-product scenario. In this regard, this work is directed in the above stated direction.

3. PROBLEM DEFINITION

This work considers a three echelon supply chain as depicted in Figure 1 as a Multi-Period Fixed Charge Transportation Problem for a Multi-product scenario.

3.1 Problem Statement

The distribution network model consists of a single manufacturer, m distribution centre and n customers. An allocation planning is to be prepared to serve f number of products in the above stated supply chain network for a planning horizon with t periods. Excess production in one period is stored as inventory, which can be placed either at manufacturer's side or distribution centre's side for the subsequent periods. The unmet demand of a particular period will be added to the next period demand as back order quantity at customers' side.

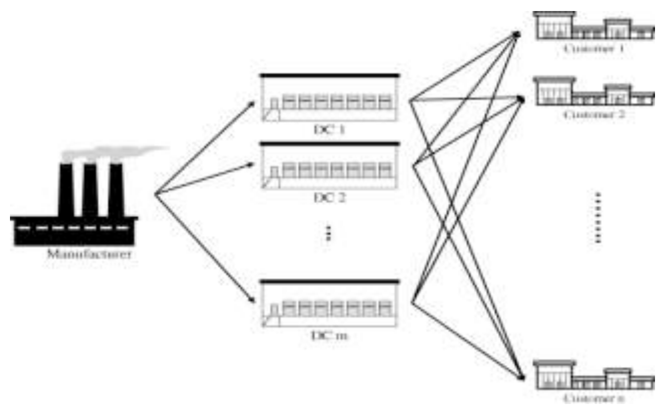

Figure 1: Schematic representation of the supply chain configuration under study

3.2 Problem Environment

The distribution network for an MPFCTP considered in this paper is stated as follows:

- There is a single manufacturer to supply f products to n customers through m distribution centres in T planning periods.
- The manufacturer produces P units of products, f =1, 2…Q in each period t =1, 2...T and there is a demand D at each customer j= 1, 2...n in each period t=1, 2...T.
- The excess amount produced in period t will be carried over to the subsequent period t +1. The excess amount can be held either at manufacturer or distribution centre at an inventory holding cost. The inventory holding cost depends on the type of product and the place being held i.e. either manufacturer or distribution centre.
- The shortage in production for the period t is booked as an extra demand for the next period t+1. The backorders at the customer is satisfied with a backorder penalty cost and varies with the type of products and customers.
- The inventory in the beginning of the period and the amount of backorder are known quantities.

4. MATHEMATICAL MODEL

Indices

- i – number of distribution centres
- j – number of customers
- f – number of products
- t – period

Parameters

- P_f^t production quantity at the manufacturer in period t for the product f
- D_{jf}^t demand of the customer j in period t for the product f
- CS_{if} unit transportation cost from manufacturer to the distribution centre i for the product f
- FCS_i fixed cost incurred for operating the route from the manufacturer to the distribution centre i
- C_{ijf} unit transportation cost from the distribution centre i to the customer j for the product f
- FC_{ijf} fixed cost incurred for operating the route from the distribution centre i to the customer j for the product f
- SH_{if} inventory holding cost per unit per period at the distribution centre i for the product f
- BC_{jf} penalty cost per unit backorder quantity per period at j^{th} customer's location for the product f
- SI^0_{if} initial inventory held at the distribution centre i for the product f
- BL^0_{jf} initial backorder quantity at the customer j for the product f

Variables

- X^t_{ijf} quantity shipped from the distribution centre i to the customer j in the period t for the product f
- S_{if}^t quantity shipped from the manufacturer to the distribution centre i in the period t for the product f
- MI_f^t inventory held at the manufacturer in the period t for the product f
- SI^t_{if} inventory held at distribution centre's location i in period t for product f
- BL^t_{jf} backorder quantity at customer j in period t for product f
- k^t_{ijf} binary variable that represents fixed transportation cost between distribution centre i and customer j
- y^t_{if} binary variable that represents fixed transportation cost between manufacturer and distribution centre

Objective Function

Minimize Total Transportation cost, TC=

$$\sum_{i=1}^{m}\sum_{f=1}^{Q}\sum_{t=1}^{T}\left(CS_{if}\times S^t_{if}+FCS_i\times y^t_{if}\right)+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{f=1}^{Q}\sum_{t=1}^{T}\left(C_{ijf}\times X^t_{ijf}+FC_{ij}\times k^t_{ijf}\right)+\sum_{f=1}^{Q}\sum_{t=1}^{T-1}\left(MH_f\times MI^t_f\right)+\sum_{i=1}^{m}\sum_{f}^{Q}\sum_{t=1}^{T-1}\left(SH_{if}\times SI^t_{if}\right)+\sum_{j=1}^{n}\sum_{f=1}^{Q}\sum_{t=1}^{T-1}\left(BC_{jf}\times BL^t_{jf}\right) \quad \ldots (1)$$

Constraints. The above objective function is subjected to the following constraints:

$$P^t_f + MI^{t-1}_f = S^t_{if} \qquad \forall i, f, t \quad (2)$$

As per the constraint (2), the cumulative production of the manufacturer at a particular period t is equal to the quantity shipped from manufacturer to the distribution period.

$$S^t_{if} + SI^{t-1}_{if} = \sum_{j=1}^{n} X^t_{ijf} + SI^t_{if} \qquad \forall i, f, t \quad (3)$$

In Constraint (3), there must be a material balance at each of the distribution centre between any two successive time intervals for each product.

$$D^t_{jf} + BL^{t-1}_{jf} = \sum_{i=1}^{m} X^t_{ijf} + BL^{t-1}_{jf} \qquad \forall j, f, t \quad (4)$$

Constraint (4) provides a material balance at the customers' side between any two successive time intervals for each product.

$$k^t_{ijf} = \begin{cases} k^t_{ijf} = 1, & if\ X^t_{ijf} > 0 \\ k^t_{ijf} = 0, & if\ X^t_{ijf} = 0 \end{cases} \quad (5)$$

Constraint (5) represents the binary variable associated with fixed charge between distribution centre and customers.

$$y^t_{if} = \begin{cases} y^t_{if} = 1, & if\ S^t_{if} > 0 \\ y^t_{if} = 0, & if\ S^t_{if} = 0 \end{cases} \quad (6)$$

Constraint (6) represents the binary variable associated with fixed charge between manufacturer and distribution centre.

$$S_{if}^{t} \geq 0 \text{ and integer } \forall_i \; i = 1...m,$$
$$\forall_f f = 1...Q, \; and \;\; \forall_t t = 1...T \qquad (7)$$
$$X_{ijf}^{t} \geq 0 \text{ and integer } \forall_i \; i = 1...m,$$
$$\forall_f f = 1..Q, \forall_j j = 1...n, \; and \;\; \forall_t \; t = 1...T \quad (8)$$
$$MI_f^{t} \geq 0 \text{ and integer}$$
$$\forall_f f = 1...Q, and \;\; \forall_t t = 1...T (9)$$
$$SI_{if}^{t} \geq 0 \text{ and integer } \forall_i i = 1...m$$
$$\forall_f f = 1..Q, \; and \;\; \forall_t t = 1...T \qquad (10)$$
$$BL_j^{t} \geq 0 \text{ and integer } \forall_j j = 1...n,$$
$$\forall_f f = 1...Q, and \;\; \forall_t \; t = 1...T \qquad (11)$$

The above constraints (7-11) represents the non-negativity and integrality of the decision variables.

5. SOLUTION METHODOLOGY

The problem is modelled using an optimization modeling tool, 'A Mathematical Programming Language' (AMPL) and the model is solved using BONMIN solver for integer problems. MPMPFCTP is difficult to solve using exact methods because of the presence of a fixed charge which causes a discontinuity in the objective function and is non-polynomial (NP) hard in nature. In line with the findings of Barr et al., (1981) and Steinberg (1970), exact methods are not computationally efficient. The ability of Ant Colony Optimization (ACO) based heuristic has been proved in the literature for network based problems (Tang et al. 2013, Panicker et al., 2013). Therefore, an Ant Colony Optimization (ACO) based meta-heuristic approach is developed for solving the MPMPFCTP.

5.1 Ant *Colony Optimization*

Ant Colony Optimization (ACO) is a meta-heuristic approach inspired by the behaviour of real ants and their communication and coordination by the principle of pheromone deposition. Ants live in colonies and they communicate between each other by using a chemical called pheromone. This type of communication helps them in finding the shortest path from their nest to the food source. This natural behaviour of ants is used to find the best solution in an optimization problem.

To solve an optimization problem using ACO, the problem is to be denoted by nodes and edges. Primarily definite amount of pheromone is deposited on each edge. Then ants are made to move from one node to another based on a probability rule as given in equation (12).

$$\Pr_{i,j}^{t} = \frac{(\tau_{i,j})^{\alpha}\left(\dfrac{1}{C_{i,j}}\right)^{\beta}}{\sum_{i,j}(\tau_{i,j})^{\alpha}\left(\dfrac{1}{C_{i,j}}\right)^{\beta}} \quad \forall t \qquad (12)$$

Equation (12) provides the probability to choose the next node. The values of parameters α and β determine the relative importance of pheromone and heuristic information respectively. After each ant of the colony complete their trip from the nest to the food source, the pheromone density is to be updated locally and globally which is equivalent to deposition and evaporation of pheromone in the natural system.

The proposed ACO based heuristic has two stages; the first stage allocates the products from the manufacturer to the distribution centre considering the maximum capacity of the distribution centre. After the allocation to distribution centre second stage starts, it is the allocation from the distribution centre to satisfy the demands of each customer. Each product is considered separately and the final cost of each product is added to get the total cost of the supply chain.

The proposed two-stage ACO is explained in four steps. The first step gives the actual demand of customer and production capacity at the manufacturer. The second step presents the calculation of equivalent cost matrix, and the third step explains the calculation of the cumulative probability matrix. Finally, the fourth step provides the illustration of the iteration process. The first three steps are described in detail as follows:

Step-1: **Calculation of actual production and demand matrix**

Before starting the allocation, actual production and demand matrix is calculated. For each product, initial inventory held at the manufacturer is added to the respective production at the first period to get cumulative production of that product. This cumulative production acts as quantity used to meet the demand of customers. Likewise, initial backorder is added to the first-period demand to get the actual demand of each customer for a particular product. This is taken as the demand to be met.

Step-2: **Calculation of the equivalent cost matrix**

Equivalent cost (C^f_{ij}) from manufacturer to customer j through distribution centre i is calculated by considering fixed and variable cost of transportation from manufacturer to i^{th} distribution centre, fixed and variable cost of transportation from i^{th} distribution centre to j^{th} customer, inventory cost for the inventory held at manufacturer, inventory cost for the inventory held at i^{th} distribution centre and backorder cost for the backorder occurred at j^{th} customer for f^{th} product.

$$\begin{aligned}&\text{Equivalent cost}(C^{f}_{ij}) = [(\text{fixed cost} + \text{variable cost}) \text{ from} \\ &\qquad \text{manufacturer to distribution centre}] + \\ &[(\text{fixed cost} + \text{variable cost}) \text{ from distribution centre} \\ &\text{to customer}] + [\text{inventory holding cost at} \\ &\text{manufacturer}] + [\text{inventory holding cost at} \\ &\text{distribution centre}] + [\text{back order cost at customer}] \quad (13)\end{aligned}$$

Initially equivalent cost is calculated by assuming that quantity equivalent to distribution centre's capacity is transported from the manufacturer to the distribution centre. The demand of each customer is satisfied fully through a particular distribution centre. The total equivalent cost is calculated by considering all products. The equivalent cost matrix is calculated using equation (13) by adding up the variable cost, the inventory holding cost and the back order cost of all products. But the fixed cost is considered only once when a route is opened.

Step-3: **Calculation of the probability matrix**

As mentioned above, equation (12) is used to calculate the probability matrix. The equivalent cost matrix from Step-2 is used to calculate the probability matrix. In equation (12), Pr^t_{ij} is the probability of allocating from manufacturer to j^{th} customer through i^{th} distribution centre at period t. Fine tuning of parameters is done using Taguchi experimental design method. The value of parameters considered in ACO, α= 0.2 and β= 0.9, where α and β are positive real parameters. Initially, the pheromone concentration τ_{ij}^t is assumed to be 0.5 at the start of the iteration process. After calculating the probability matrix, the cumulative probability matrix is formed by taking cumulative sum of probability in each period for each customer from the probability matrix.

Step-4: **Iteration**

There are two separate iterations for this ACO based heuristic. First one is to allocate products from manufacturer to distribution centre and second is to allocate products from distribution centre to customers.

In the first phase of iteration starts with the first product at first period. An array is randomly generated whose size is equal to the total number of distribution centre, and then the allocation is made from manufacturer to distribution centre. Quantity corresponding to the maximum capacity of the distribution centre is shipped from the manufacturer to the distribution centre, if production at manufacturer is more than the capacity of distribution centre else the quantity available at the manufacturer is shipped. This process continues till the production at the manufacturer is finished or till the maximum capacity of all distribution centre is attained. This iteration is continued for all products. The second stage of iteration from the distribution centre to customer starts after allocation of products to a distribution centre. This stage of iteration also starts with Product-1 at Timeperiod-1. Monte-Carlo simulation method is used to select distribution centre DC_i and to allocate products to the selected customer C_j by considering the cumulative probability matrix obtained from step 3.

Iteration enters from period t to period $t+1$, if either demand of all customers in that period t is satisfied or the capacity at all distribution centre in period t is exhausted. Then the unmet demand considered as backorder at each customer C_j at period t and is added to the demand of the next period $t+1$, to make the total demand of customer C_j as $D^{t+1}{}_j$. The excess capacity considered as inventory, at each distribution centre DC_i at period t is added to the capacity in the next period $t+1$ to make the total capacity of distribution centre DC_i as $P^{t+1}{}_i$.

The capacity matrix, inventory matrix, demand matrix and backorder matrix is updated after period. This process is continued till the period reaches its maximum value T. Then the next product is selected to allocate it to the customer. The same procedure is continued for all products till all the products are considered for allocation. After each ant completes its path, the total cost of the supply chain is found out. Total cost comprises of fixed and variable cost of transportation from the Manufacturer to the customers through distribution centre, inventory cost and backorder cost. The number of ants is incremented, and the above-mentioned allocation process is continued. The total cost is determined separately, after each ant complete its path. The number of the ant is incremented till all the M ants of the colony complete its path. Once M ants complete their path iteration is incremented.

Pheromone value is updated before entering into the next iteration process, to mimic the process of pheromone deposition by ants and its evaporation with time. The pheromone updating process is done by subtracting an amount $\Delta\tau$ from the entire path and adding a small fraction to the shortest path to represent the increased concentration of pheromone. By this process, the concentration of pheromone will be slightly more in the shortest path, i.e. ant with the minimum total cost. With the new pheromone value, the shortest path corresponding to the route with minimum total cost is more probable for getting selected. After updating the pheromone value, the iteration is incremented and the above mentioned allocation process is carried out. Each iteration process is followed by a pheromone updating process. The iteration process terminates when it reaches the specified maximum value. Then minimum cost of all the iteration is taken as the best total cost.

6. COMPUTATIONAL STUDY

The problem stated above is modelled using an optimization modeling tool, 'A Mathematical Programming Language' (AMPL) and solved using BONMIN solver as an integer problem. The objective of the model is to minimize the total cost to determine the size of the shipments considering the backorder quantity and the inventory at each period, and the transportation cost.

The computational study is designed using randomly generated data pertaining to different supply chain configuration. The number of customers is varied between 08 and 16. The number of products is varied from two to five. For each supply chain configuration, cost data, demand data and production data are also generated randomly. These problem instances are also used to solve Ant Colony Optimization based methodology. The difference between the solution obtained using proposed heuristic and that of exact method is compared in terms of the percentage variation using the equation (14):

$$\%\ \text{Variation} = \frac{\text{ACO solution} - \text{AMPL solution}}{\text{AMPL solution}} \times 100 \quad (14)$$

The results of comparative study are tabulated in Table 1.

Table 1: Computational results of various problem instances

Sl. No.	Problem Size (mxnxtxf)	AMPL		ACO		% Variation in solution
		Solution	Time taken (in sec.)	Solution	Time taken (in sec.)	
1	3x8x3x2	360968	58243.10	382026	44.28	5.834
2	3x6x3x2	333929	54018.94	340975	27.28	2.111
3	4x12x4x2	952759	117963.59	980105	169.50	2.871
4	3x6x3x2	312307	101190.00	328169	27.66	5.079
5	3x14x3x2	798703	1071783.05	826423	131.92	3.471
6	3x8x2x3	645512	791468.53	654798	105.45	1.439
7	3x10x3x3	707113	98580.79	746195	74.56	5.527
8	3x10x4x3	1076132*	127874	1135146	115.65	5.484
9	3x14x3x3	940647*	87864.53	970665	108.01	3.192
10	3x16x5x3	1687449*	100676.57	1738791	1442.19	3.043
11	3x8x3x4	1282431*	99206.74	1364882	371.98	6.430
12	3x12x3x4	1346872*	112539.54	1470831	511.14	9.204
13	3x16x3x4	1398932*	94226.96	1478296	936.79	5.674
14	3x16x4x5	1728250*	1562418.67	1856346	1246.56	7.412
15	3x12x4x5	1623421*	1373465.35	1738567	987.23	7.093

m-number of distribution centres; n-number of customers; t-number of time periods; f-number of products; *- indicates interrupted solution (non-optimal solution)

7. CONCLUSION

A simple model for MPFCTP is developed for a multi-product scenario. The computational study reveals that using exact methods though the solution obtained is optimal, the computational time taken for solving a mathematical model is high. As the problem size increases, the exact methods are not very efficient, so an Ant Colony Optimization method is proposed to solve the model. A computational study is conducted for both methods, and results from the exact method and ACO are compared. It is understood that ACO gives a near optimal solution in much lesser time than exact methods. In addition to this, the work can be extended to include more number of manufacturers and transportation between distribution centres can be considered to include real-world problems. It may be interesting to hybridize the proposed heuristic.

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