Risk Returns characteristics

Table 1 reveals that Amazon (ticker: AMZN) gives the best return while also carrying the greatest risk, therefore is suited to investors who are more risk tolerant and desire to maximise returns regardless of the costs. The Sharpe ratio, which assesses the stock's performance after adjusting for risk, was calculated using a 2% risk-free rate. Visa (ticker: V) offers the best risk-return trade off as it has the largest Sharpe ratio. On the other hand, Exxon Mobile (ticker: XOM) and Goldman Sachs (ticker: GS) have a standard deviation of 24.64 and 29.36%, respectively, thus leading to significantly lower Sharpe ratios than other stocks. Moreover, V and Nike (ticker: NKE) had a negative skew of -0.23 and -0.29, respectively, indicating that extremely poor returns occur more frequently than favourable returns.

Table 1: Risk Return Characteristics

	Annualised Mean (%)	Annualised Standard Deviation (%)	Annualised Sharpe Ratio	Skewness
XOM	10.59	24.64	0.35	0.39
NKE	19.29	24.21	0.71	-0.29
AMZN	24.03	30.31	0.73	0.30
GS	11.32	29.36	0.32	0.10
V	20.20	20.28	0.90	-0.23

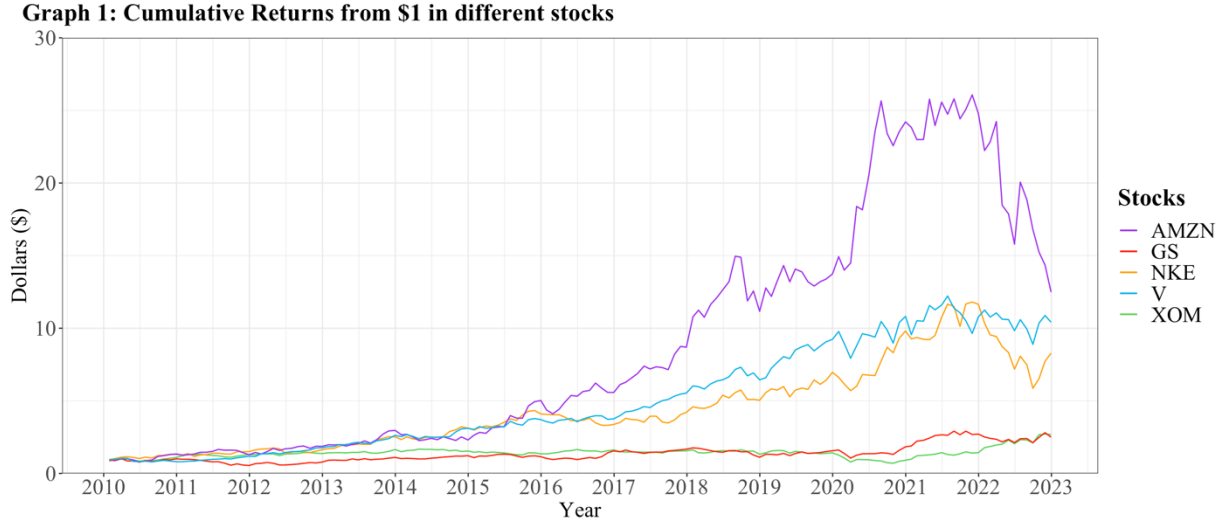
Table 2 shows the annualised variance-covariance matrix which demonstrates that all the stocks have positive covariances with one another, indicating that all the stocks' returns move in the same direction.

Table 2: Variance and Covaraince Matrix

	XOM	NKE	AMZN	GS	V
XOM	0.0603385	0.0140790	0.0130058	0.0368956	0.0228019
NKE	0.0140790	0.0582311	0.0245780	0.0301430	0.0214941
AMZN	0.0130058	0.0245780	0.0912727	0.0273279	0.0253983
GS	0.0368956	0.0301430	0.0273279	0.0856542	0.0252950
V	0.0228019	0.0214941	0.0253983	0.0252950	0.0408600

According to Graph 1, Amazon had the largest cumulative return in the end, followed by V, NKE, XOM, and GS. After reaching a peak of almost $25 in 2020 to 2022, Amazon's return substantially decreased. GS and XOM offered the lowest cumulative return throughout the years, as can clearly be observed.

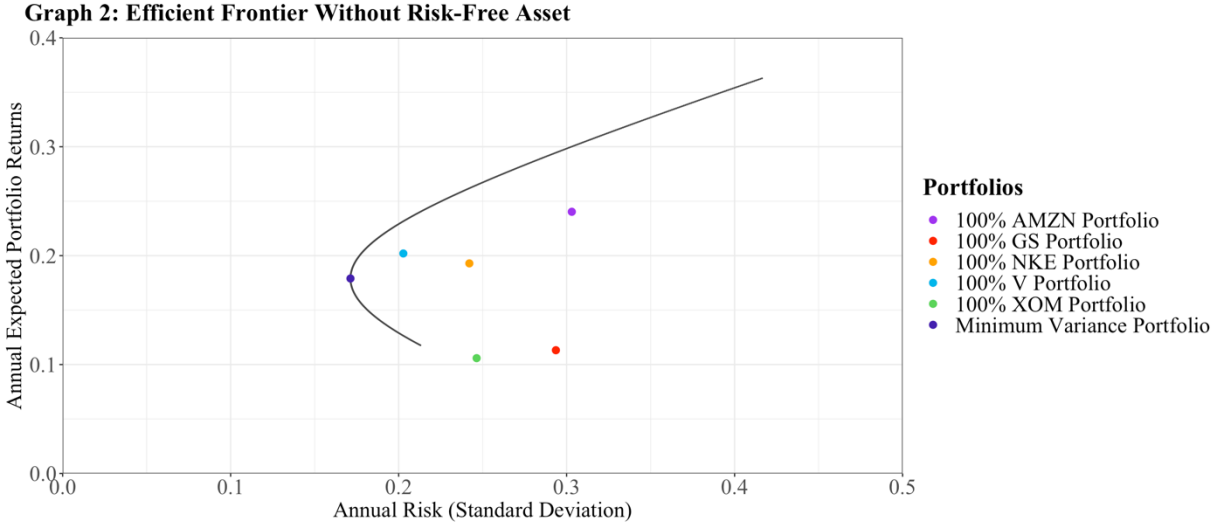
Graph 1: Cumulative Returns from $1 in different stocks



Efficient frontier without risk-free asset

The investment opportunity set in the absence of a risk-free asset is the region inside the curved line, known as the minimum variance frontier, shown in Graph 2. The upward-sloping portion of the minimum variance frontier, which begins at the minimum variance portfolio, is known as the efficient frontier. Accordingly, any portfolio on the efficient frontier offers the best risk-return combination for a rational investor, with the minimum variance portfolio offering the lowest risk. The efficient frontier is suitable for any investors regardless of their risk aversiveness as there are plenty of options for each level of risk. High risk tolerant investors will prefer to be on the right side of the efficient frontier since the increased risk correlates with higher returns. Here, the minimum variance portfolio is formed by putting 26.82% into XOM, 26.06% in NKE, 11.40% in AMZN, 36.64% in V and shorting 0.91% from GS. This provides an annual expected return of 17.9% with a minimal annual risk of 17.13%.

Graph 2: Efficient Frontier Without Risk-Free Asset



With the risk-free asset of 2%, the efficient frontier is the optimal capital allocation line (CAL), which graph 3 depicts with a dashed line. The CAL in Graph 3 shows a combination of different proportions of risk-free asset and the tangent portfolio. Regardless of the risk preferences, the investor will hold a proportion of the tangency portfolio and the risk-free asset.

Graph 3: Efficient Frontier With Risk-Free Asset

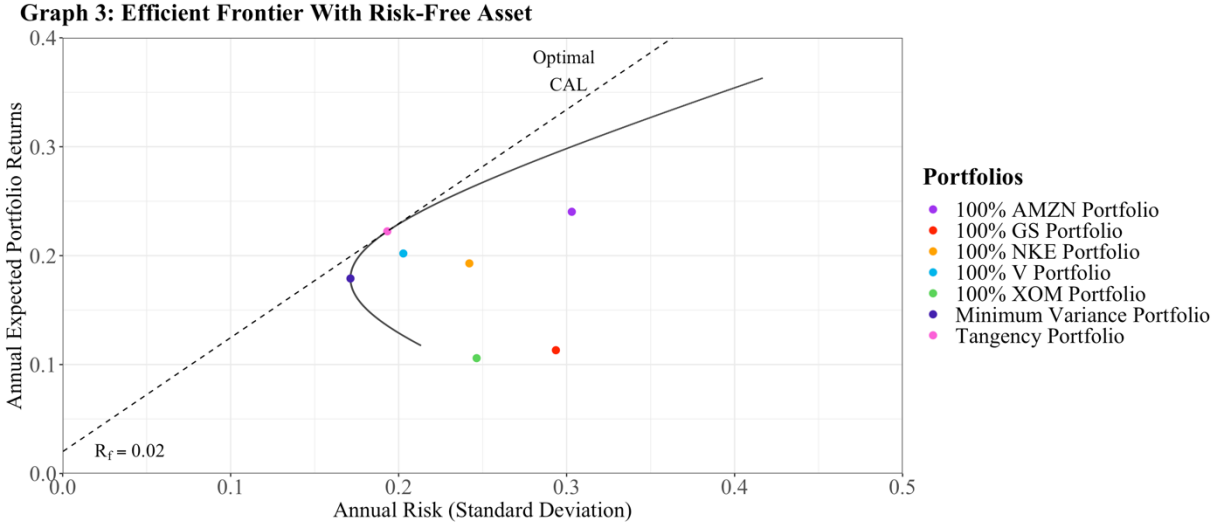


Table 3 shows an overview of the tangency portfolio that maximises the Sharpe ratio. The weights involve shorting 16.94% of GS stocks in order to invest in other assets, resulting in a 22.23% expected return, a 19.32% annual standard deviation, and a Sharpe ratio of 1.05.

Table 3: Tangent Portfolio

	Weights (%)					Risks/Returns		
	XOM	NKE	AMZN	GS	V	Expected Return (%)	Standard Deviation (%)	Sharpe Ratio
Tangency Portfolio	1.28	30.57	24.28	-16.94	60.81	22.23	19.32	1.05

Portfolio Diversification

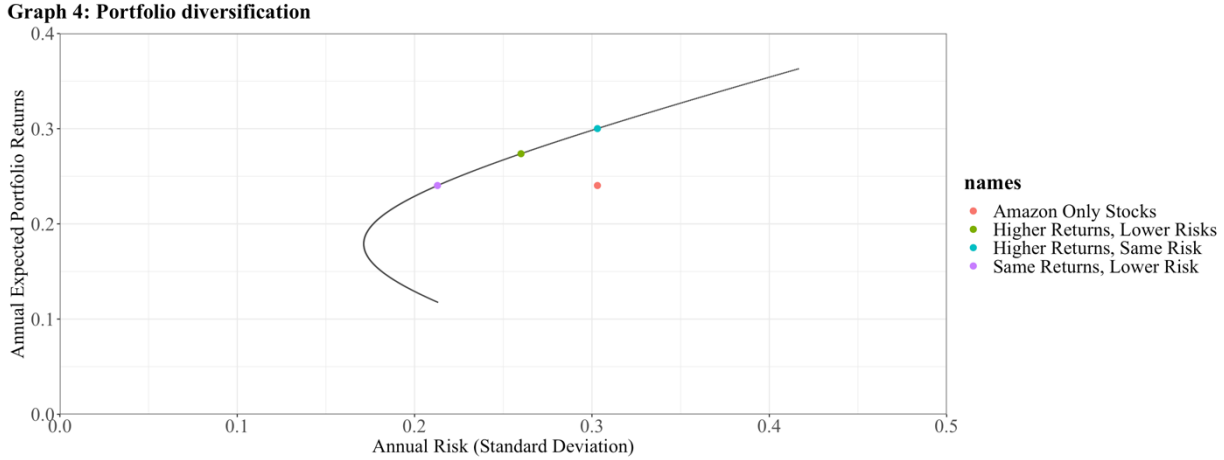
According to Graph 4, owning a portfolio consisting solely of Amazon stocks is extremely inefficient since it is substantially below the efficient frontier curve, resulting in a lower return with increased risks when compared to many other portfolios on the efficient frontier line. As a result, it is preferable for Jennifer to diversify her portfolio by combining assets that offer varying levels of returns and risk. This helps shields Jennifer from any losses due non-systematic risks by establishing a portfolio containing several risky assets.

Table 4 depicts the many situations in which a portfolio with a combination of shorting and diversification might achieve the same level of expected return, if not better, while minimising risks. It can be seen in the first row that investing only in Amazon stocks results in the worst risk-adjusted performance, with a 0.73 Sharpe ratio. Ideally, the diversified portfolios should be on efficient frontier curve to maximise the returns and risk trade-off, if Jennifer is not interest in burrowing and leveraging the risk-free asset, which is depicted in Graph 4.

Table 4: Types of Portfolios

	Weights (%)					Risks/Returns		
	XOM	NKE	AMZN	GS	V	Expected Return (%)	Standard Deviation (%)	Sharpe Ratio
Only Amazon Stocks	0.00	0.00	100.00	0.00	0.00	24.03	30.31	0.73
Same Risk, Higher Returns	-9.31	32.43	29.62	-23.58	70.83	24.03	21.29	1.03
Lower Risk, Same Returns	-44.59	38.69	47.43	-45.74	104.21	30.01	30.31	0.92
Lower Risk, Higher Returns	-29.05	35.96	39.61	-35.96	89.44	27.37	26.00	0.98

Graph 4: Portfolio diversification



Investors Utility

Given Tom and Amy's risk aversion coefficients of 8 and 12, respectively, and the tangent portfolio as the optimal risky portfolio, the optimal risky portfolio weight is as follows:

$$w_p^* = \frac{r_p - r_f}{A\sigma_p^2}$$

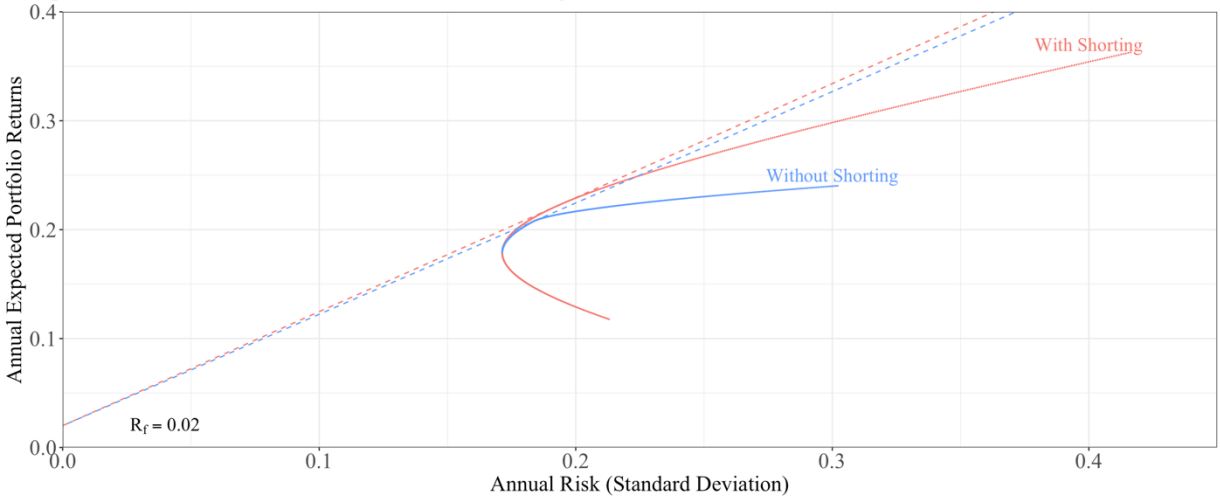
As a result, Tom and Amy should put 67.72 and 45.15% of their money in the tangent portfolio and the remainder in risk-free assets. Table 5 shows the suggested money allocation for each investor's $2 million, as well as the risk and reward characteristics. Indeed, the Sharpe ratio remained constant among investors since each of the weightings assigned is proportionate to the tangent portfolio, therefore sitting on the same CAL line. Visually, it is comparable to the notion depicted in graph 5 below, where Tom and Amy's portfolio is located to the left of the tangent portfolio on the optimal CAL.

Table 5: Investors Risk Aversion

	Money Allocation ($)						Risks/Returns		
	XOM	NKE	AMZN	GS	V	Risk-Free	Expected Return (%)	Standard Deviation (%)	Sharpe Ratio
Tom	17304.32	413990.02	328890.70	-229407.03	823645.23	645576.76	0.16	0.13	1.05
Amy	11536.21	275993.35	219260.47	-152938.02	549096.82	1097051.17	0.11	0.09	1.05

These portfolios are not reachable without shorting since the original tangency portfolio requires shorting to get the optimal Sharpe ratio. Graph 5 depicts the risk-return trade-off between the tangent portfolio with and without shorting. The Sharpe ratios differ across the portfolios because the portfolio with shorting has its optimum CAL intersecting at a steeper slope, resulting in a greater Sharpe ratio than the portfolio without shorting.

Graph 5: Efficient Frontier with and without shorting



Part 2

The suggestions of Blackrock and Charles contradict the portfolio theory. There are two reasons for this: the weights of bonds and stocks among various risk-averse individuals are not proportional, and none of the portfolios for aggressive investors has the process of burrowing risk-free assets to leverage more investments in the tangent portfolio.

The first reason that the suggested portfolio weights are flawed is that it demonstrates that each portfolio's stock and bond weights are not proportional to each other. This implies that not all risky portfolios are the most efficient at each level of risk, as it is not related to the tangent portfolio. The tangent portfolio is the most efficient mix of bonds and stocks and has the greatest Sharpe ratio of any viable portfolio set not including T-bills. Therefore, when determining the most efficient risky portfolio, it may be viewed as a mutual fund of the two risky assets, with stocks and bonds assigned according to the tangent portfolio weights.

Table 6 shows the various bond and stock ratios that are in the portfolio the investors chose; thus, it can be seen from the uneven distribution of weights from both Blackrock and Charles Schwab between

their conservative, moderate, and aggressive portfolios that not all of the portfolios are on the optimal CAL line, which is what a rational investor would want.

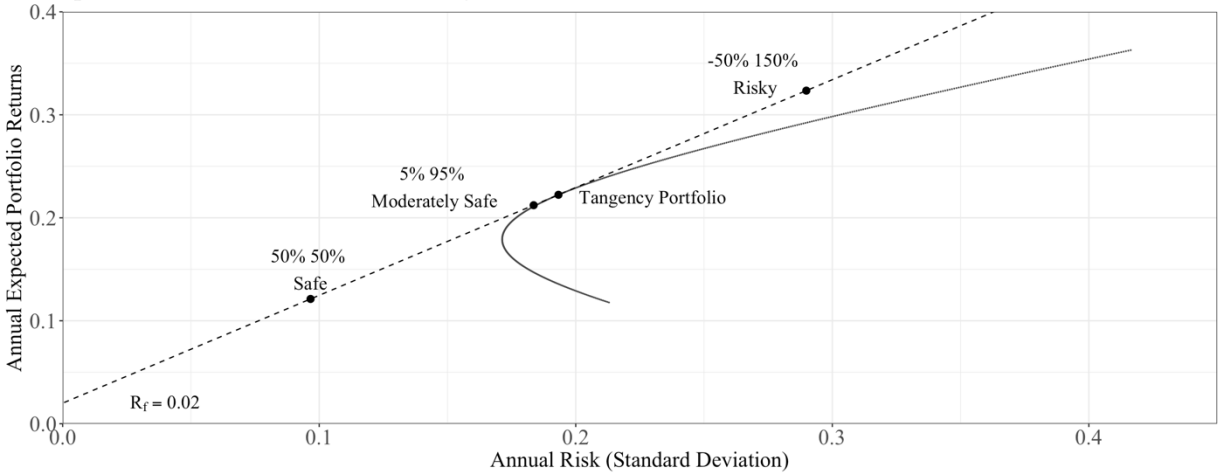
Table 6: Weight distribution in risky portfolio

	Bond weight in risky portfolio (%)	Equity weight in risky portfolio (%)
Blackrock		
Conservative	60.00	40.00
Moderate	50.00	50.00
Aggressive	32.58	68.42
Charles Schwab		
Conservative	43.75	56.25
Moderate	42.11	57.89
Aggressive	21.05	78.95

Furthermore, if an investor is particularly risk averse, such as a conservative investor, a portfolio with very low volatility will be more enticing to them. This means they will give T-bills more weight and the tangent portfolio relatively little weight. In this case, Blackrock has a stronger advice for conservative investors, as 50% of the weight is assigned to T-bills, compared to 20% of the weight advised by Charles Schwab.

However, when dealing with moderate to aggressive investors, the weights suggested by both were too safe since if an investor has a high-risk tolerance, they will prefer a bigger return regardless of volatility. This involves borrowing T-bills and reinvesting the proceeds in the tangent portfolio. According to graph 6, even the most aggressive investment of 5% in T-bills and 95% in the tangency portfolio is safe in comparison to the tangency portfolio. A scenario was created in graph 6 for riskier investment, where the portfolio burrows 50% of T-bills and use it to leverage greater investment of 150% in the tangent portfolio, which would yield higher returns. For more moderate investors, it would be feasible to recommend them to burrow less to leverage, as long as it suits their risk preferences. Therefore, the portfolios advised by Blackrock and Charles Schwab do not correspond to all risk-averse investors based on the modern, since most of their portfolios are on the safe side.

Graph 6: Two fund theroem between risky and risk-free assets



*Graph is under the assumption that the Blackrock and Charles Schwab portfolios uses the optimal risky portfolio