

$$\begin{array}{c} K \\ \downarrow f \\ K' \end{array}$$

$$\begin{array}{ccc} GL_n(K) & \xrightarrow{\det_K} & K^\times \\ GL_n(f) \downarrow & & \downarrow f^\times \\ GL_n(K') & \xrightarrow{\det_{K'}} & K'^\times \end{array}$$