$$\begin{array}{cccc}
I \otimes I & I \otimes (I \otimes (v)_A) & ((u)_A \otimes I) \otimes I \\
\rho_I & & \downarrow^{\lambda_I} & & \downarrow^{\lambda_{(I \otimes (v)_A)}} & & \rho_{(u)_A} \otimes 1_I \\
I & & I \otimes (v)_A & & (u)_A \otimes I
\end{array}$$