

$$\begin{array}{ccc}
 \mathrm{Hom}_{\mathcal{C}^S}(K, P_S) & \xrightarrow{\varphi} & \mathrm{Hom}_{\mathcal{C}^{\mathbb{R}}}(\overline{K}, P) \\
 \downarrow \text{red} & & \downarrow \text{orange} \\
 \mathrm{Hom}_{\mathcal{C}^S}(K, P'_S) & \xrightarrow{\varphi} & \mathrm{Hom}_{\mathcal{C}^{\mathbb{R}}}(\overline{K}, P')
 \end{array}$$

$$\begin{array}{ccc}
 \eta : K \longrightarrow P_S & \longmapsto & \eta' : \overline{K} \longrightarrow P \\
 \downarrow \text{red} & & \downarrow \text{orange} \\
 \sigma' \circ \eta : K \longrightarrow P'_S & \longmapsto & \begin{array}{c} \sigma \circ \eta' : K \longrightarrow P' \\ = \\ (\sigma' \circ \eta)' : K \longrightarrow P' \end{array}
 \end{array}$$