

$$\begin{array}{ccc}
 \mathrm{Hom}_{\mathcal{C}^S}(K, P_S) & \xrightarrow{\varphi} & \mathrm{Hom}_{\mathcal{C}^{\mathbb{R}}}(\overline{K}, P) \\
 \downarrow & & \downarrow \\
 \mathrm{Hom}_{\mathcal{C}^S}(K', P_S) & \xrightarrow{\varphi} & \mathrm{Hom}_{\mathcal{C}^{\mathbb{R}}}(\overline{K'}, P)
 \end{array}$$

$$\begin{array}{ccc}
 \eta : K \longrightarrow P_S & \longmapsto & \eta' : \overline{K} \longrightarrow P \\
 \downarrow & & \downarrow \\
 \eta \circ \alpha : K' \longrightarrow P_S & \longmapsto & \begin{array}{c} \eta' \circ \overline{\alpha} : K' \longrightarrow P \\ = \\ (\eta \circ \alpha)' : K' \longrightarrow P \end{array}
 \end{array}$$