$$
\begin{array}{ccccc}
A & & F(A) & \xrightarrow{\eta_A,\eta'_A} & \mathrm{Hom}_{\mathcal{C}}(C,A) & \xrightarrow{f^*} & \mathrm{Hom}_{\mathcal{C}}(C',A) \\
\downarrow{\scriptstyle h} & & \downarrow{\scriptstyle F(h)} & & \downarrow{\scriptstyle h_*} & & \downarrow{\scriptstyle h_*} \\
A' & & F(A') & \xrightarrow[\eta_{A'},\eta'_{A'}]{} & \mathrm{Hom}_{\mathcal{C}}(C,A') & \xrightarrow[f^*]{} & \mathrm{Hom}_{\mathcal{C}}(C',A')
\end{array}
$$