Natural Frequency and Quality Factor of a Parallel RLC Circuit

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***Abstract*—This paper investigates the natural frequency and quality factor of parallel RLC circuits configured as low-pass and band-pass filters. The paper includes the construction of the two circuits on a breadboard and the measurement of their responses using an oscilloscope. Various resistor values (1.5kΩ, 510Ω, and 390Ω) are employed to evaluate their impact on the circuits' performance. Both step-response and frequency-response characterizations are performed, focusing on the phase shifts at specific frequencies. Theoretical calculations and experimental results are compared to validate the circuit designs and their expected behaviors. This work offers valuable insights into the properties and applications of parallel RLC circuits in signal processing.**

Keywords— Transfer function, natural frequency ω_n, damped natural frequency ω_d, quality factor Q, frequency response, magnitude response, phase response

I. Introduction

A. Understanding Transfer Functions

Transfer functions, represented by *H(s)*, are essential for characterizing system behavior in the frequency domain, particularly for analyzing electrical circuits. They describe the relationship between a system's input and output as a ratio of polynomials in the Laplace transform variable *s* [1]. In parallel RLC circuits, transfer functions determine the natural frequency and Q-Factor. The natural frequency shows the rate of oscillation in the absence of damping, while the Q-Factor measures the sharpness of the resonance peak [2]. By analyzing transfer functions, engineers can predict a circuit's response to various frequencies, optimize its design for specific applications, and ensure stability and efficiency.

B. Fundamentals of RLC Circuits

RLC circuits, consisting of a resistor, inductor, and capacitor, are essential components in electrical engineering and serve as the foundation for numerous applications. These circuits can be arranged in series or parallel, each configuration displaying unique characteristics. In a parallel RLC circuit, the resistor, inductor, and capacitor are connected in parallel to the same voltage source. This setup is especially important because it can achieve resonance, where the inductive and capacitive reactance cancel each other, resulting in a purely resistive impedance at the resonant frequency [3].

Understanding the basic principles of RLC circuits is vital for designing and optimizing electronic systems that require precise frequency control and signal processing. The two RLC circuits built for this paper are shown below in Fig. 1 and Fig. 2.

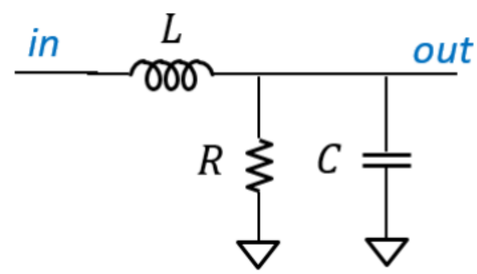

Fig. 1. Low-pass RLC circuit [1].

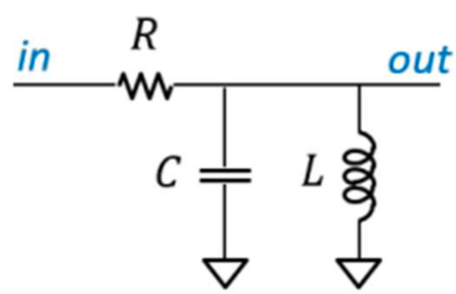

Fig. 2. Band-pass RLC circuit [1].

C. Natural Frequency and Q-Factor

The natural frequency and Quality Factor (Q-factor) are crucial parameters in the analysis and design of parallel RLC circuits. The natural frequency, often represented as ω_n, is the frequency at which the circuit naturally oscillates without an external source. It is a key property that determines the circuit's response to alternating current (AC) signals. The Q-factor measures the sharpness of the resonance peak and is defined as the ratio of the natural frequency to the bandwidth over which the circuit significantly responds [4].

A high Q-factor indicates low energy loss compared to the stored energy, resulting in a sharp resonance peak, which is ideal for applications such as filters and oscillators that require precise frequency selection. Conversely, a low Q-factor indicates higher energy dissipation and a broader resonance peak [5].

Understanding and optimizing these parameters are essential for ensuring the efficient performance of RLC circuits in various electronic and communication systems. Analyzing the natural frequency and Q-factor provides insights into the circuit's energy dynamics and stability, enabling the design of systems with improved performance and reliability.

D. Transfer Function Equations

The transfer functions for the circuits shown in Fig. 1 and Fig. 2 are expressed by (1) and (2) respectively.

$$H_{Fig.1}(s) = \frac{1}{s^2 LC + s\frac{L}{R} + 1} \tag{1}$$

$$H_{Fig.2}(s) = \frac{s\frac{L}{R}}{s^2 LC + s\frac{L}{R} + 1} \tag{2}$$

Alternatively, the transfer functions can be reformulated using the undamped natural frequency ω_n and their quality factor Q, as shown in (3) and (4). Note that the natural frequency, also known as the resonant frequency, is measured in radians per second (rad/sec), while the quality factor Q is dimensionless.

$$H_{Fig.1}(s) = \frac{{\omega_n}^2}{s^2 + \frac{\omega_n}{Q}s + {\omega_n}^2} \tag{3}$$

$$H_{Fig.2}(s) = \frac{\frac{\omega_n}{Q}s}{s^2 + \frac{\omega_n}{Q}s + {\omega_n}^2} \tag{4}$$

$$where\ \omega_n = \frac{1}{\sqrt{LC}}\left[\frac{rad}{sec}\right] and\ Q = \frac{R}{\sqrt{L/C}} \tag{5}$$

E. Materials Used

This experiment utilized three resistors (1.5kΩ, 510Ω, and 390Ω) to vary the resistance and examine its effect on the circuit's natural frequency and quality factor. A 10nF non-electrolytic capacitor and a 10mH inductor were included to create the necessary oscillatory behavior. A function generator (Agilent 33120A) provided the required electrical signals, while a two-input oscilloscope was used to observe and measure voltage signals within the circuit. The circuit was assembled on a breadboard using banana-to-grabber connectors.

II. Data

A. RLC Low-Pass Filter

Building the RLC low-pass filter required assembling the 10nF capacitor, 10mH inductor, and 1.5kΩ resistor as shown in Fig. 1. Then, a 1V step input was applied to the circuit.

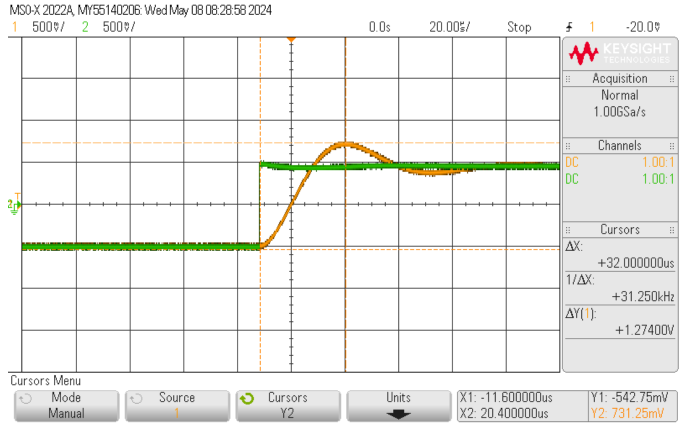

Fig. 3. Scope capture showing overshoot of underdamped RLC low-pass circuit (1.5kΩ) response (yellow) to step input (green).

In Fig. 3, the response of the circuit to a step input can be seen. Using the cursors, a time-to-first-peak value of 32μs and an overshoot voltage of 0.27V were recorded. Considering an expected voltage of 1V, the percent overshoot was recorded to be 27%. From the time-to-first-peak value, the natural frequency f_n of the circuit can be estimated at 104kHz from equation (6). Additionally, using expression (7), the percent overshoot can be calculated as 27%. This agrees with the measured value.

$$\omega_n = \frac{\pi}{t_p\sqrt{1 - \frac{1}{4Q^2}}} \tag{6}$$

$$\frac{V_{meas} - V_{in}}{V_{in}} \tag{7}$$

The time-to-first-peak and overshoot voltage indicate how quickly and by how much the circuit's response exceeds its steady-state value after a step input. A high percent overshoot suggests a more underdamped system, which oscillates more before settling.

The input signal was then changed to sinusoidal with a magnitude of 1V and the oscilloscope was used to measure phase difference. To obtain an input-output phase difference of 90º, the frequency was set to 16.7kHz.

The output signal peak-to-peak value was recorded as 2.56V. Using equation (5), the Q-Factor of the network was calculated to be 1.5. The scope capture of both input and output signals as well as measured values can be seen below in Fig. 4. As seen in the scope captures, the input-output phase difference of 90º corresponds to the circuit's natural frequency. At this frequency, the output signal lags the input signal by a quarter of a period, highlighting the phase shift introduced by the circuit.

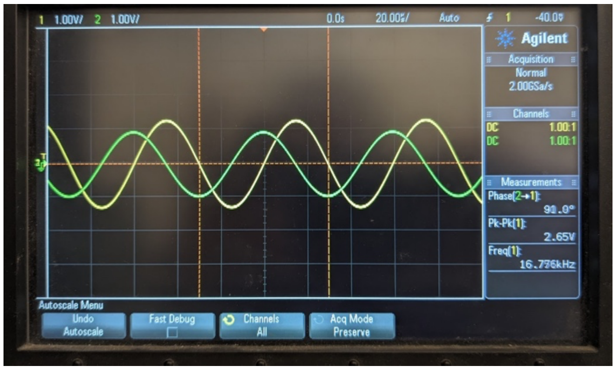

Fig. 4. Scope capture showing 2V_{pp} sine wave input (green) and output signal (yellow) lagged by ¼ period of input.

The preceding procedures and calculations were then performed on the RLC circuit with the remaining resistor values of 390Ω and 510Ω. Again, equation (5) was used to determine the quality factor. Note that no overshoot was detected with these new values, and therefore no time-to-first-peak or percent overshoot were calculated for the circuits with 510Ω and 390Ω resistor values.

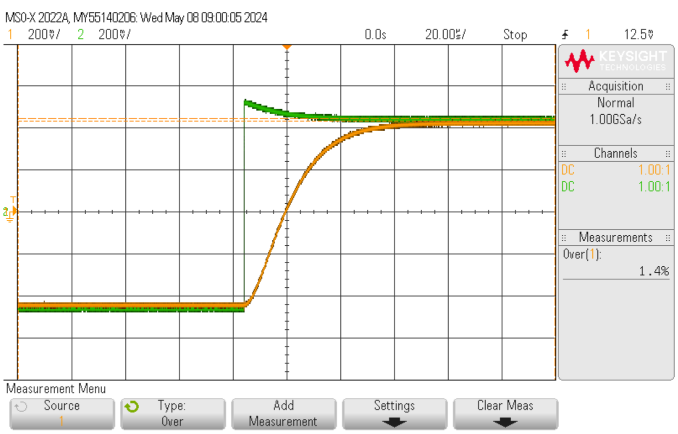

Fig. 5. Scope capture showing critically damped response (yellow) of 510Ω RLC low-pass circuit to step input (green).

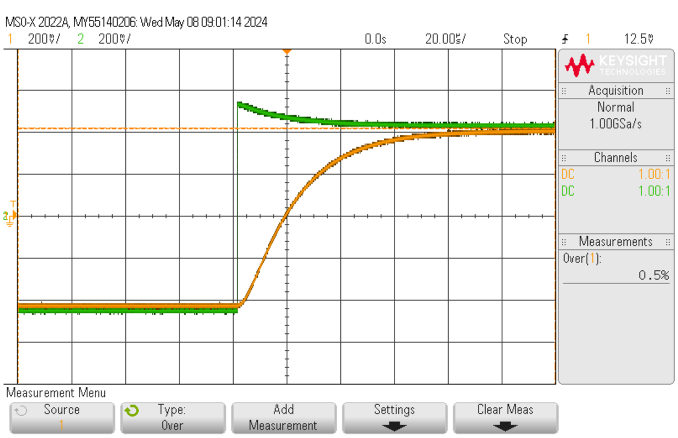

Fig. 6. Scope capture showing overdamped response (yellow) of 390Ω RLC low-pass circuit to step input (green).

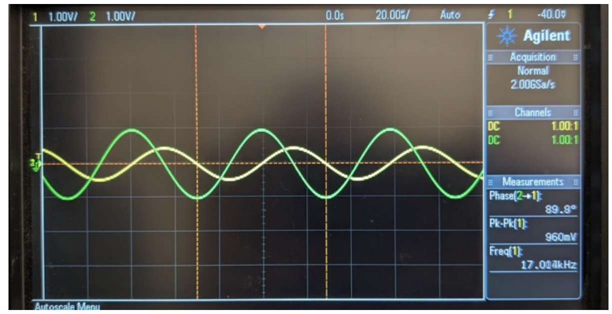

Fig. 7. Scope capture showing 960mV_{pp} sine wave input (green) and output signal (yellow) with frequency of 17kHz.

Fig. 8. Scope capture showing 800mV_{pp} sine wave input (green) and output signal (yellow) with frequency of 17.2kHz.

The data from the above scope captures along with the calculated Q values are shown below in Table I.

TABLE I. FIG 1. CIRCUIT RESISTANCE, FREQUENCY, V_{PP}, AND Q

RESISTANCE (Ω)	FREQUENCY (Hz)	V_{pp} (V)	Q
390Ω	17.2 kHz	0.8 V	0.39
510Ω	17.0 kHz	0.96 V	0.51
1.5kΩ	16.7 kHz	2.65 V	1.5

As resistance increased, both V_{pp} and Q of the network also increased. An increase in resistance reduces the damping in the circuit, leading to a higher Q factor and a sharper resonance peak [4]. This is reflected in the higher peak-to-peak voltage (Vpp) values and increased Q values with larger resistances.

B. RLC Band-Pass Filter

To build the RLC band-pass filter, the circuit shown in Fig. 4 is assembled using a 10nF capacitor, 10mH inductor, and resistor that was varied from 390Ω to 510Ω to 1.5kΩ. A square wave was then applied to the circuit with a magnitude of 1V. For each resistance, the phase difference was varied from -45° degrees to 45° in increments of 45°. It is expected that as the Q factor of the circuit increases, the window or "band" of frequencies that are passed through the circuit decreases. For the sake of simplicity, only the scope captures of the circuits with 1.5kΩ and 390Ω resistor values are shown. The Q factor for a band-pass filter can be calculated using equation (8).

$$Q_{BP} = \frac{\sqrt{\omega_L \omega_H}}{\omega_H - \omega_L} \tag{8}$$

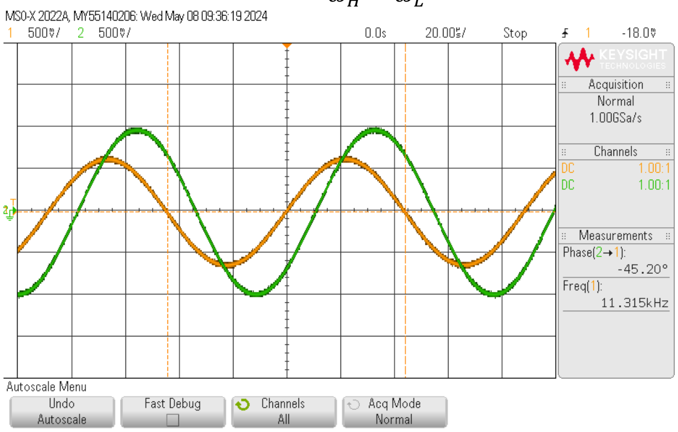

Fig. 9. Scope capture showing input (yellow) to Fig. 2 circuit with R = 1.5kΩ and -45° phase difference of response (green).

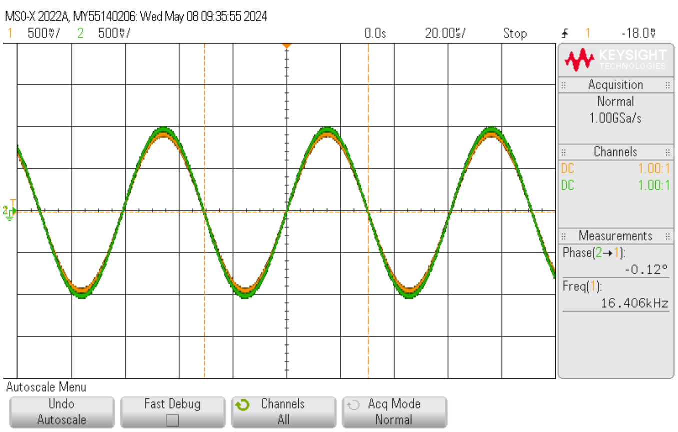

Fig. 10. Scope capture showing input (yellow) to Fig. 2 circuit with R = 1.5kΩ and 0° phase difference of response (green).

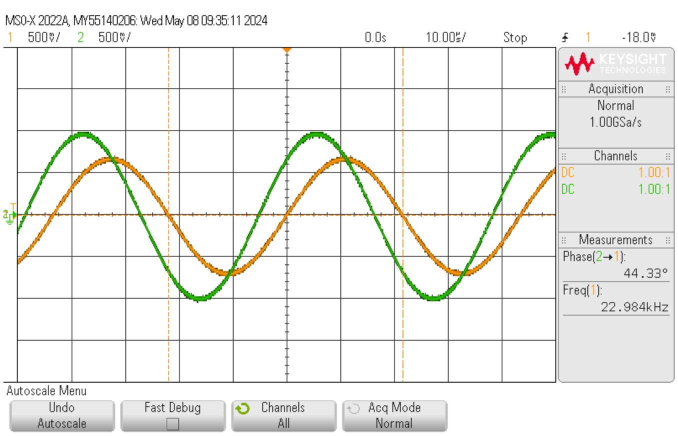

Fig. 11. Scope capture showing input (yellow) to Fig. 2 circuit with R = 1.5kΩ and 45° phase difference of response (green).

Notably, the window of frequencies allowed through this filter is 11kHz, which is determined by calculating the difference between the highest (22 kHz) and lowest (11kHz) frequencies allowed through. By using equation (8) the Q factor of this circuit can be calculated to be 1.38.

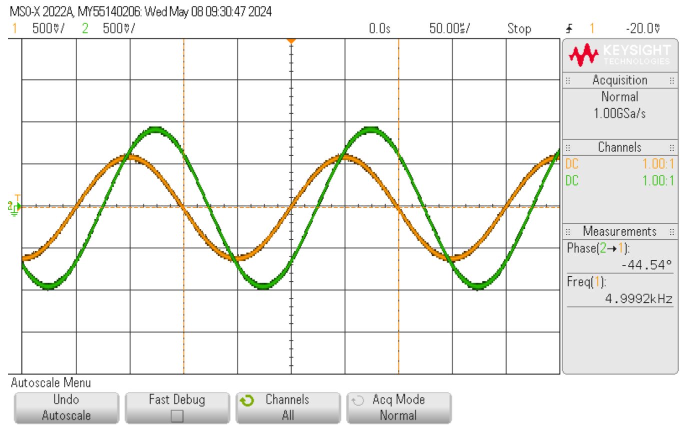

Fig. 12. Scope capture showing input (yellow) to Fig. 2 circuit with R = 390Ω and -45° phase difference of response (green).

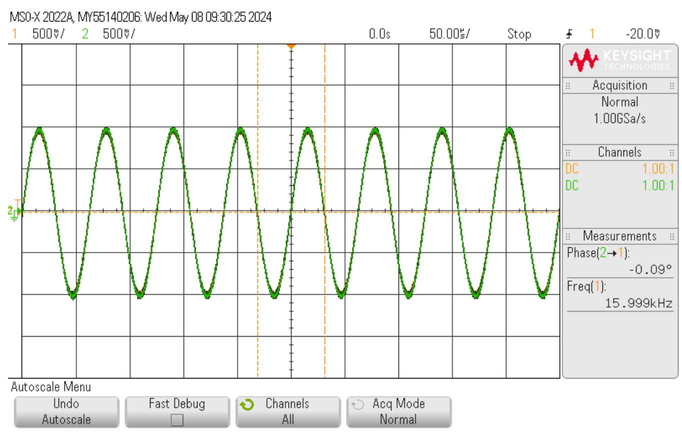

Fig. 13. Scope capture showing input (yellow) to Fig. 2 circuit with R = 390Ω and 0° phase difference of response (green).

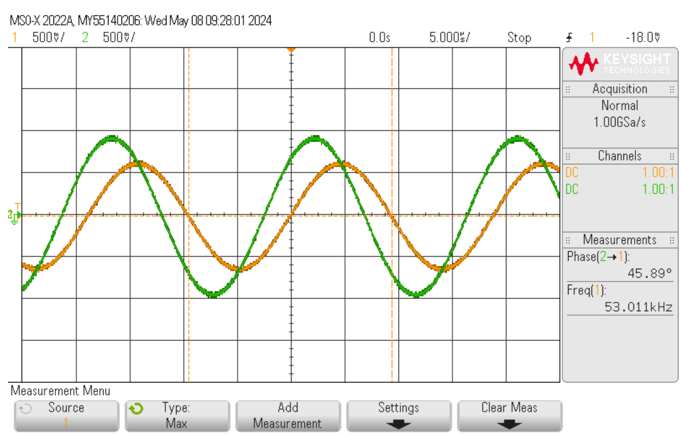

Fig. 14. Scope capture showing input (yellow) to Fig. 2 circuit with R = 390Ω and 45° phase difference of response (green).

The band-pass filter with a resistor value of 390Ω has the largest band of passed frequencies: 48kHz. This larger band corresponds with the lowest Q factor of the three circuits; 0.34. As expected, the lower the Q value of a circuit was, the higher the circuit's bandwidth.

III. Conclusion

In conclusion, this study successfully analyzed the natural frequency and quality factor of parallel RLC circuits configured as low-pass and band-pass filters. The experiments demonstrated how various components influence the circuits' behavior.

By using different resistor values (1.5kΩ, 510Ω, and 390Ω), it was observed that the resistance significantly affects the damping and, consequently, the Q-factor of the circuits. Higher resistance values resulted in higher Q-factors, indicating sharper resonance peaks and lower energy losses, which are desirable for applications requiring precise frequency selection. Conversely, lower resistance values led to broader resonance peaks and higher energy dissipation, suitable for applications where a wider frequency range is needed.

Overall, theoretical calculations and experimental results were closely aligned, validating the circuit designs and expected performance. These findings underscore the importance of selecting appropriate component values for designing efficient and reliable electronic and communication systems.

References

[1] Electrical4U. "Transfer Function of Control System." Electrical4U, 17 Apr. 2024, www.electrical4u.com/transfer-function/.

[2] "Q Factor and Bandwidth of a Resonant Circuit: Resonance: Electronics Textbook." All About Circuits, www.allaboutcircuits.com/textbook/alternating-current/chpt-6/q-and-bandwidth-resonant-circuit/. Accessed 28 May 2024.

[3] Storr, Wayne. "Parallel Resonance and Parallel RLC Resonant Circuit." Basic Electronics Tutorials, 8 Aug. 2022, www.electronics-tutorials.ws/accircuits/parallel-resonance.html.

[4] Week 6: Natural Frequency and Quality Factor of a Parallel RLC Circuit Lab Manual https://canvas.calpoly.edu/courses/131051/files/13144831?module_item_id=3505843

[5] "RLC Circuit Analysis (Series and Parallel) – Clearly Explained." Electrical4U, 7 May 2024, www.electrical4u.com/rlc-circuit/.