AI VIETNAM
AI Insight Course

Logistic Regression

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Ph.D. in Computer Science

Year 2020

Outline

- **Sigmoid function**
- From Linear to Logistic Regression
- Logistic Regression – Stochastic
- Logistic Regression – Mini-batch
- Logistic Regression – Batch

Sigmoid Function

Sigmoid function

$$y = \sigma(u) = \frac{1}{1 + e^{-u}}$$

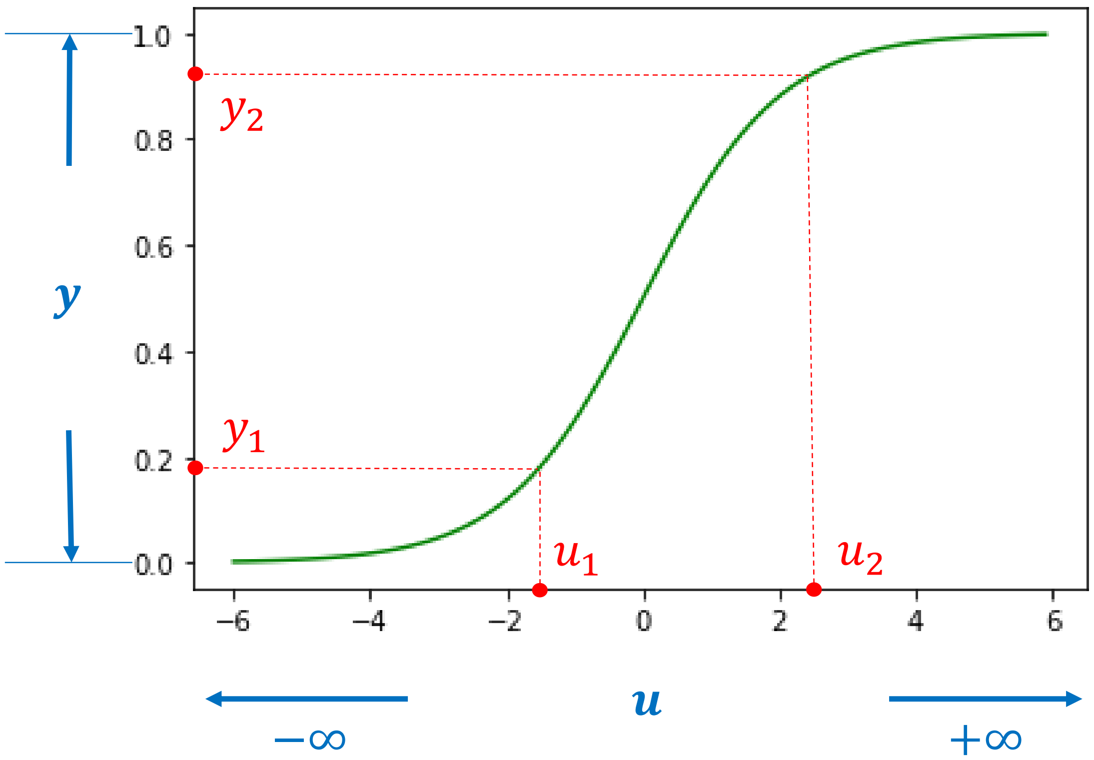
$$u \in (-\infty \quad +\infty)$$

$$y \in (0 \quad 1)$$

Property

$$\forall u_1 u_2 \in [a \quad b] \text{ và } u_1 \leq u_2$$

$$\rightarrow \sigma(u_1) \leq \sigma(u_1)$$



Sigmoid Function

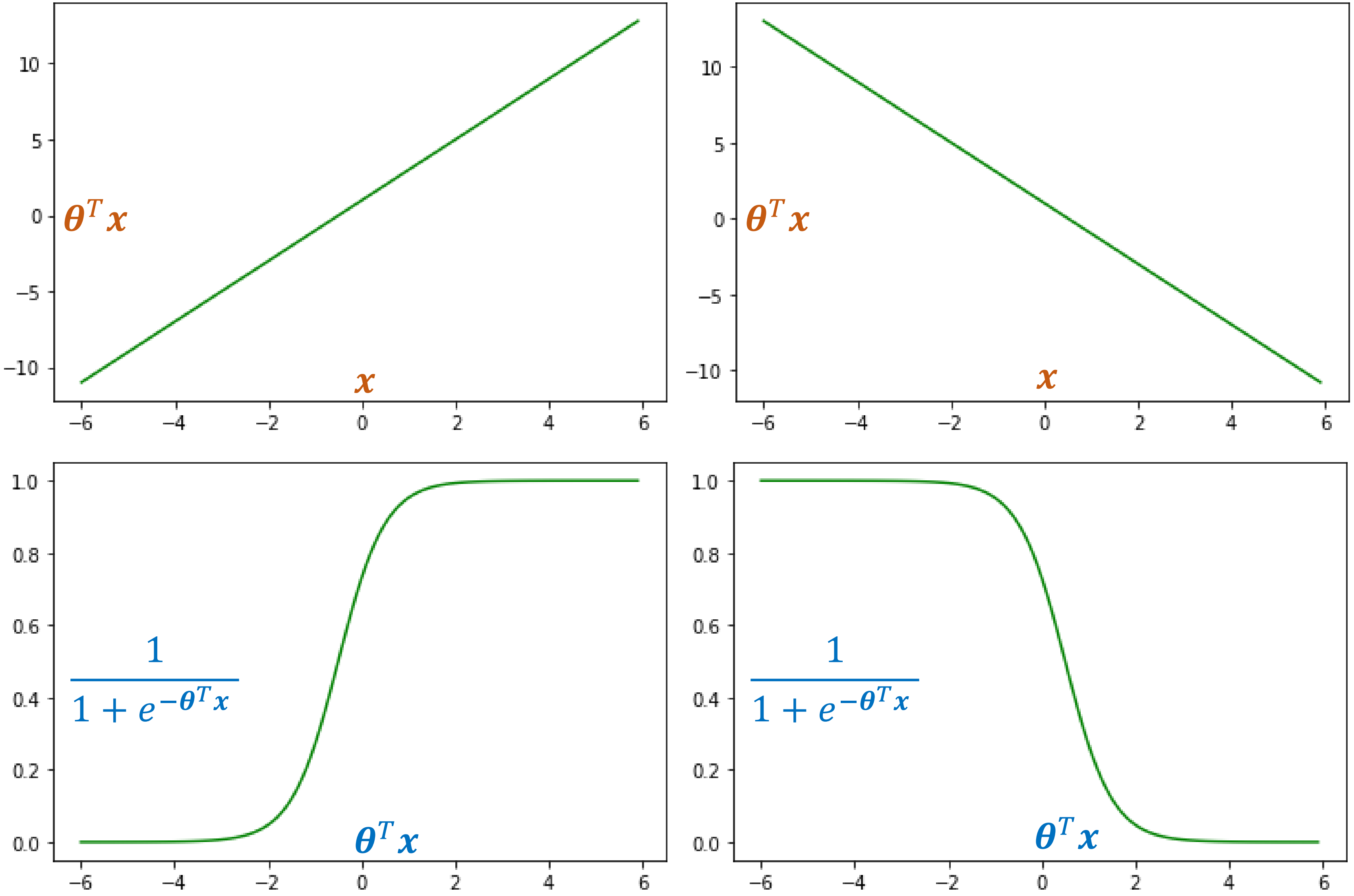
$y = \boldsymbol{\theta}^T \boldsymbol{x}$

$y \in (-\infty \quad +\infty)$

$z = \boldsymbol{\theta}^T \boldsymbol{x}$

$y = \sigma(z) = \frac{1}{1 + e^{-z}}$

$y \in (0 \quad 1)$



Sigmoid Function

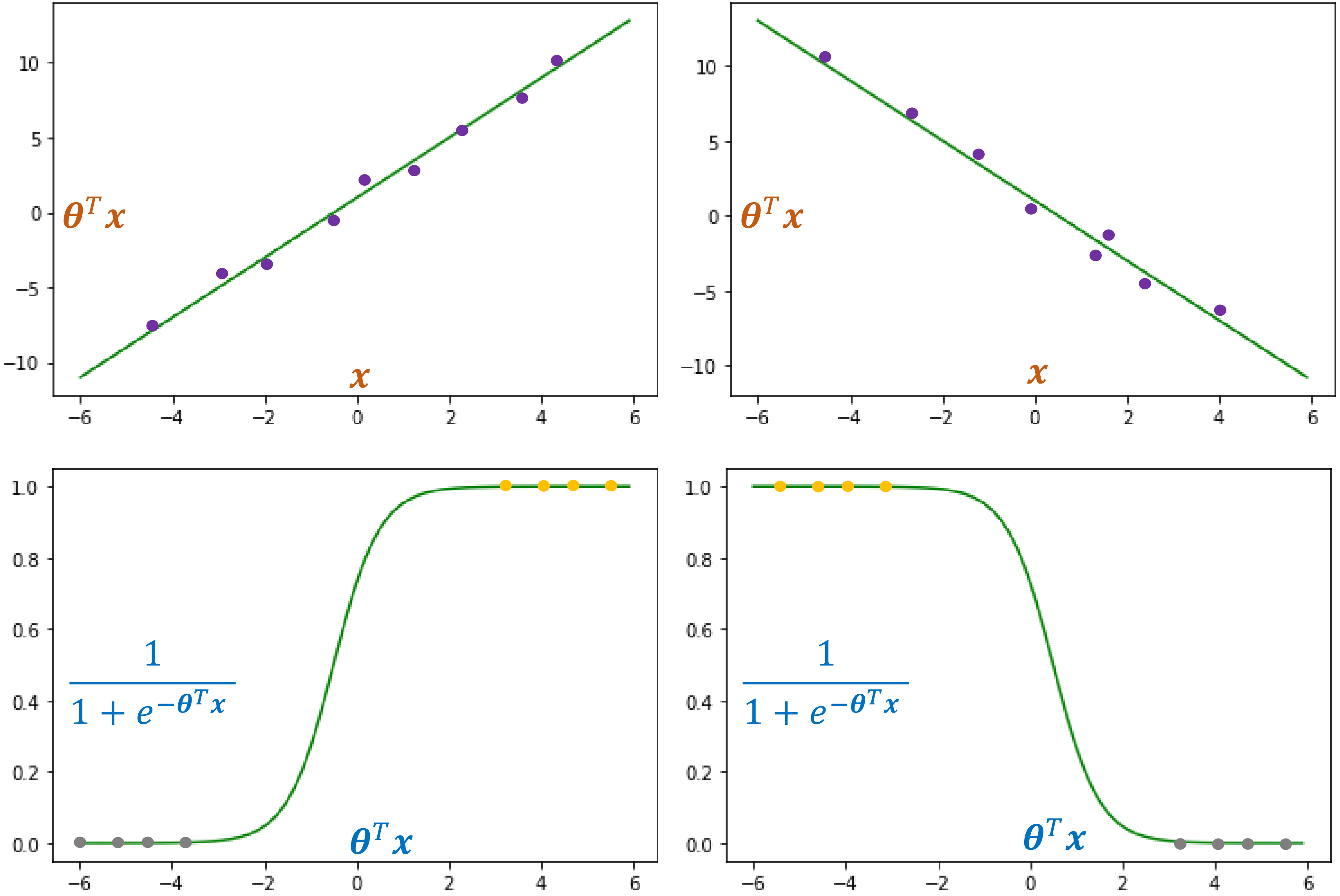
$y = \boldsymbol{\theta}^T \boldsymbol{x}$

$y \in (-\infty \quad +\infty)$

$z = \boldsymbol{\theta}^T \boldsymbol{x}$

$$y = \sigma(z) = \frac{1}{1 + e^{-z}}$$

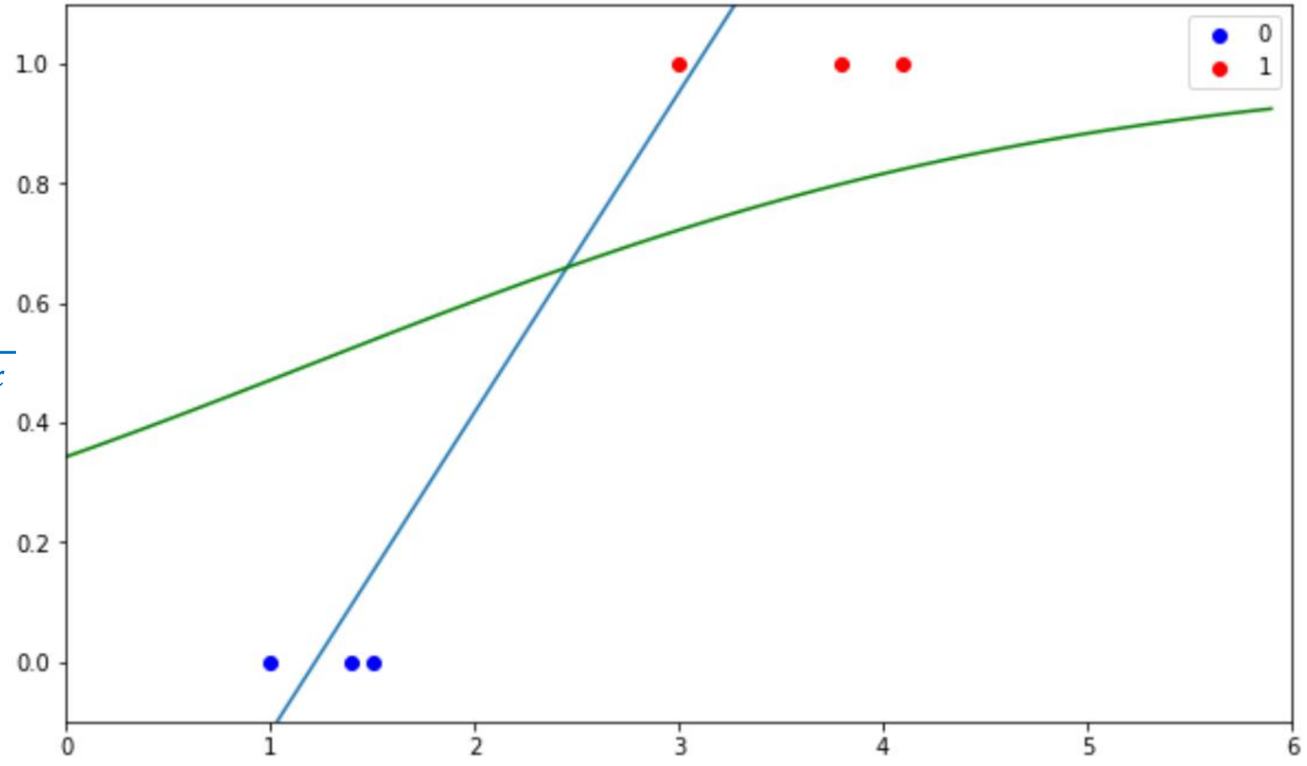
$y \in (0 \quad 1)$



Sigmoid Function

Feature	**Label**	
Petal_Length	Category	
1.4	0	Category 1
1	0	Category 1
1.5	0	Category 1
3	1	Category 2
3.8	1	Category 2
4.1	1	Category 2

$$\frac{1}{1+e^{-\boldsymbol{\theta}^T\boldsymbol{x}}}$$



z	y
0.095	0.52
-0.119	0.47
0.1485	0.53
0.951	0.72
1.379	0.79
1.5395	0.82

$$z = \boldsymbol{\theta}^T\boldsymbol{x}$$

$$y = \sigma(z) = \frac{1}{1+e^{-z}}$$

$$y \in (0 \quad 1)$$

$$z = 0.535 * x - 0.654$$

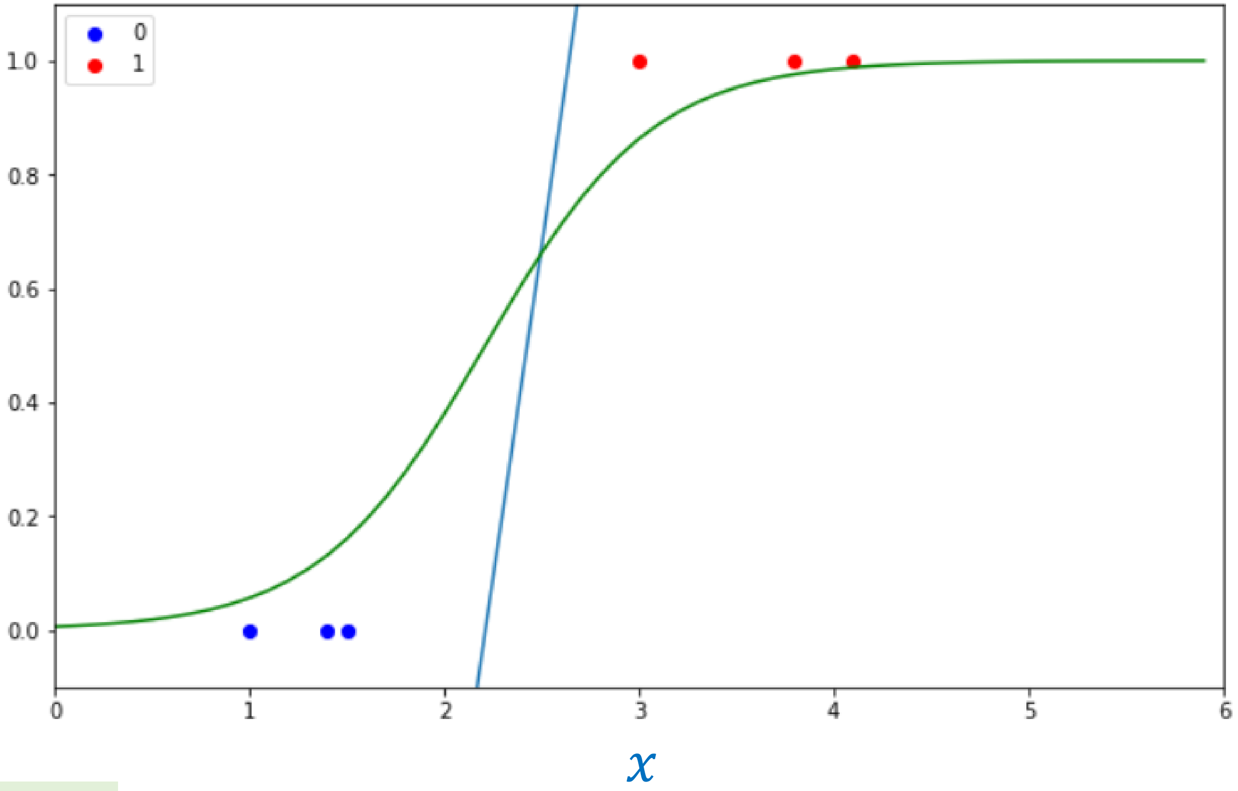
Sigmoid Function

Feature **Label**

Petal_Length	Category	
1.4	0	Category 1
1	0	
1.5	0	
3	1	Category 2
3.8	1	
4.1	1	

z	y
-1.89	0.1309
-2.82	0.0559
-1.65	0.1598
1.837	0.8625
3.701	0.9759
4.401	0.9878

$$\frac{1}{1+e^{-\boldsymbol{\theta}^T\boldsymbol{x}}}$$



$$z = \boldsymbol{\theta}^T\boldsymbol{x}$$

$$y = \sigma(z) = \frac{1}{1+e^{-z}}$$

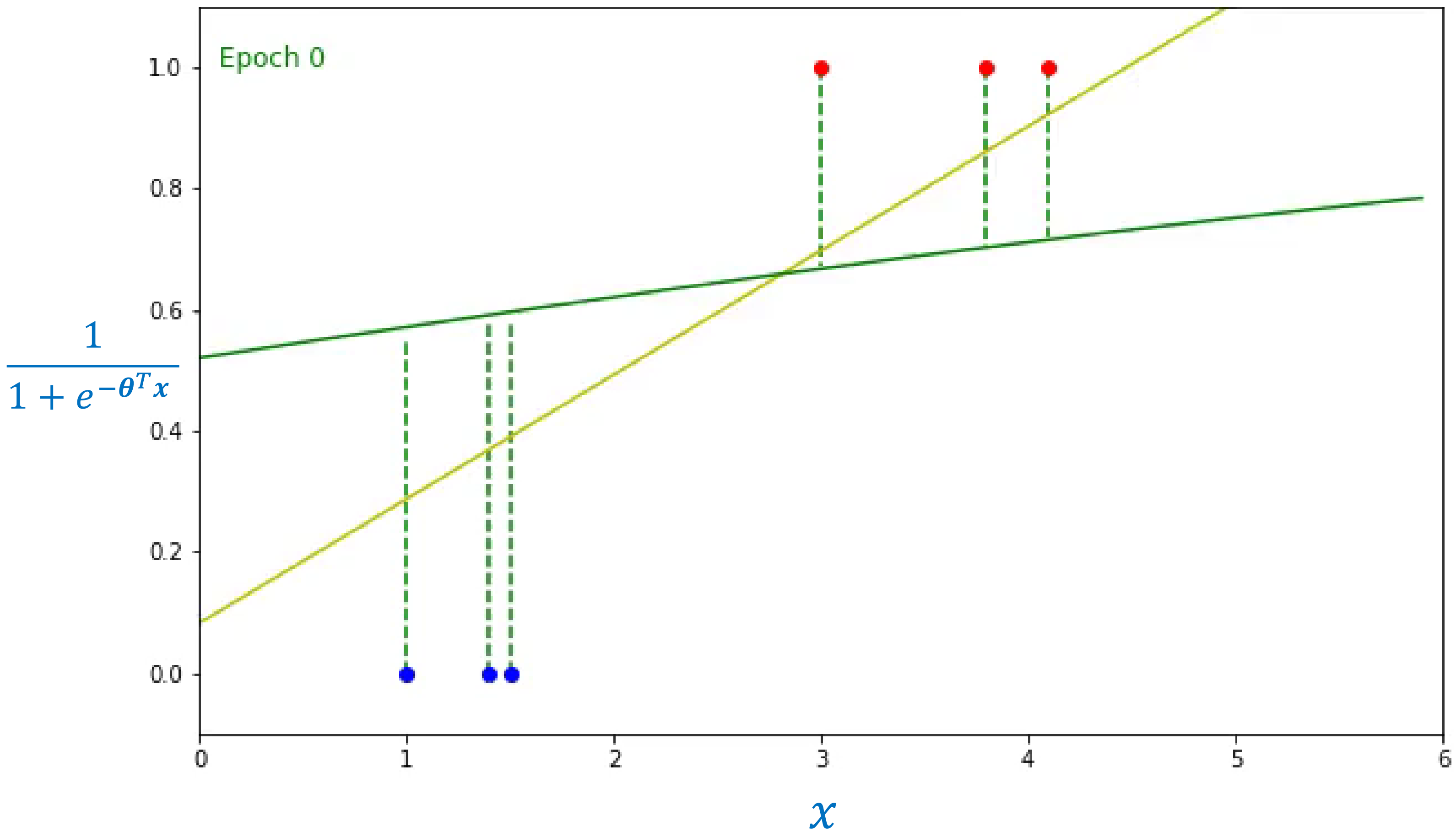
$$y \in (0 \quad 1)$$

$$z = 2.331 * x - 5.156$$

Sigmoid Function

Feature **Label**

Petal_Length	Category	
1.4	0	Category 1
1	0	
1.5	0	
3	1	Category 2
3.8	1	
4.1	1	



$$z = \boldsymbol{\theta}^T \boldsymbol{x}$$

$$y = \sigma(z) = \frac{1}{1+e^{-z}}$$

$$y \in (0 \quad 1)$$

Outline

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- **From Linear to Logistic Regression**
- Logistic Regression – Stochastic
- Logistic Regression – Mini-batch
- Logistic Regression – Batch

Idea of Logistic Regression

❖ Linear regression

Area-based House Price Data

Feature	Label
area	price
6.7	9.1
4.6	5.9
3.5	4.6
5.5	6.7

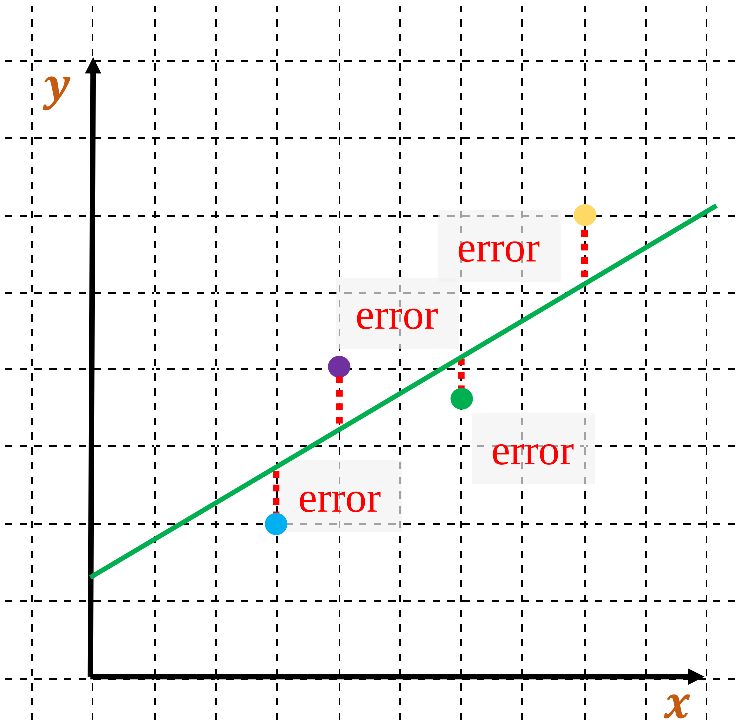
Training data

construct

$\hat{y} = \boldsymbol{\theta}^T \boldsymbol{x} = ax + b$

$\hat{y} \in (-\infty \quad +\infty)$

Model



Find the line $\hat{y} = \boldsymbol{\theta}^T \boldsymbol{x}$ that is best fit given data, then use y to predict for new data

Idea of Logistic Regression

❖ Given a new kind of data

Petal_Length	Category	
1.4	Flower A	Category 1
1	Flower A	Category 1
1.5	Flower A	Category 1
3	Flower B	Category 2
3.8	Flower B	Category 2
4.1	Flower B	Category 2

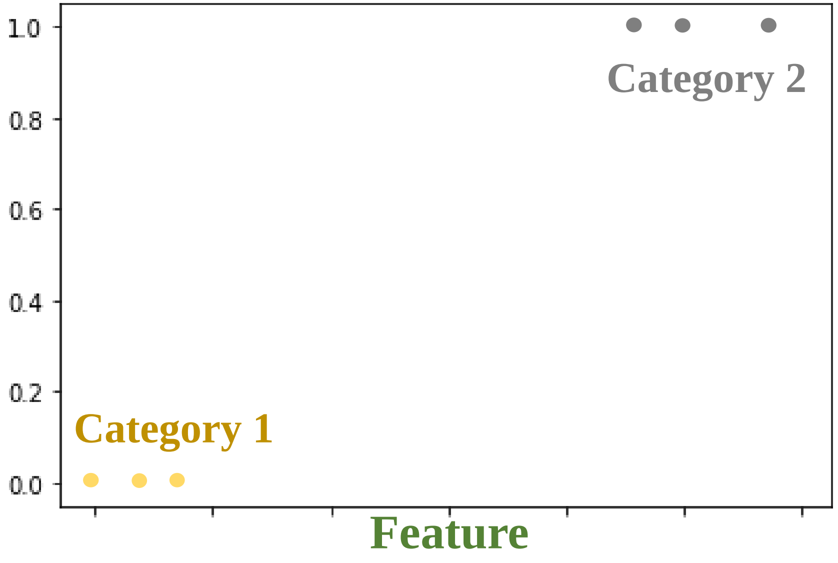
Feature: Petal_Length — Label: Category

Assign numbers to categories

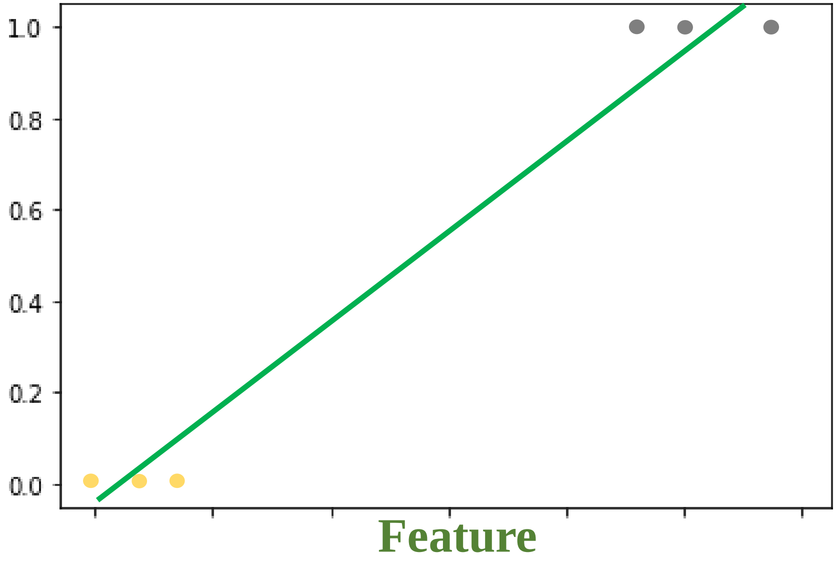
Petal_Length	Category	
1.4	0	Category 1
1	0	Category 1
1.5	0	Category 1
3	1	Category 2
3.8	1	Category 2
4.1	1	Category 2

Feature: Petal_Length — Label: Category

Plot data



A line is not suitable for this data



Idea of Logistic Regression

❖ Given a new kind of data

Feature **Label**

Petal_Length	Category	
1.4	Flower A	Category 1
1	Flower A	Category 1
1.5	Flower A	Category 1
3	Flower B	Category 2
3.8	Flower B	Category 2
4.1	Flower B	Category 2

Assign numbers to categories

Feature **Label**

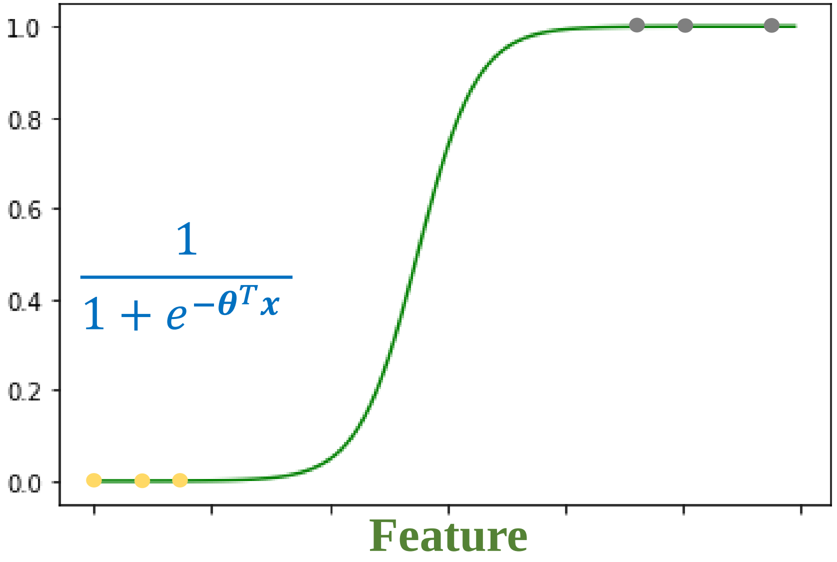
Petal_Length	Category	
1.4	0	Category 1
1	0	Category 1
1.5	0	Category 1
3	1	Category 2
3.8	1	Category 2
4.1	1	Category 2

Sigmoid function could fit the data

$$z = \boldsymbol{\theta}^T \boldsymbol{x}$$

$$\hat{y} = \sigma(z) = \frac{1}{1+e^{-z}}$$

$$\hat{y} \in (0 \quad 1)$$



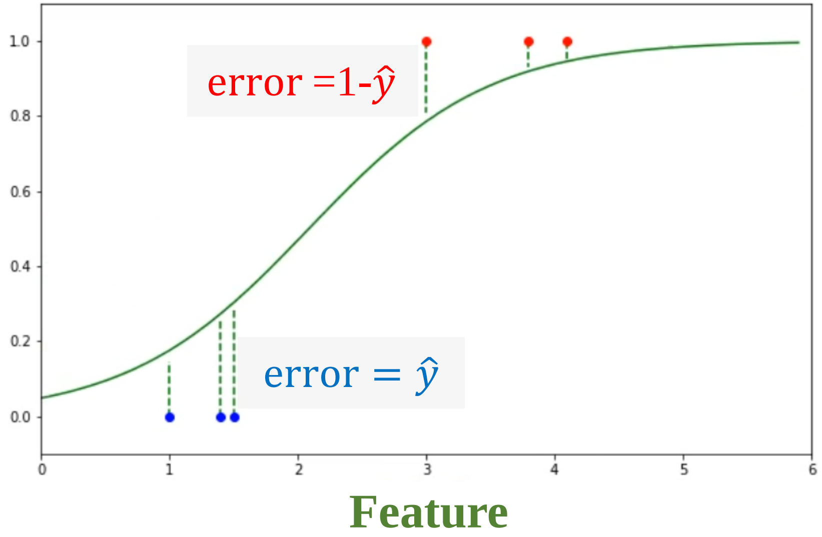
Error

if $y = 1$

$\quad \text{error} = 1 - \hat{y}$

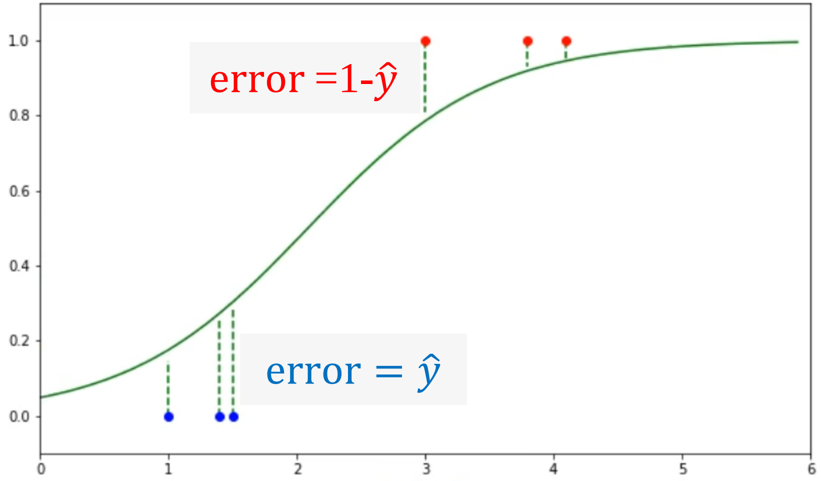
if $y = 0$

$\quad \text{error} = \hat{y}$



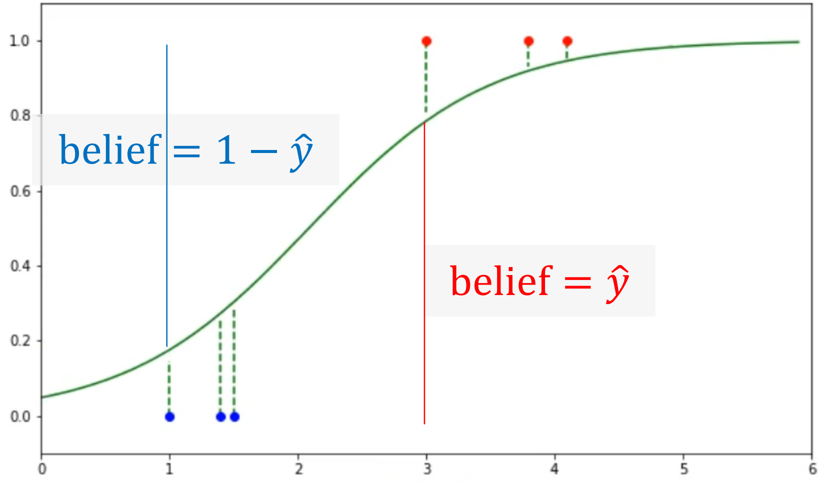
Idea of Logistic Regression

❖ Construct loss



Error

if $y_i = 1$
  error $= 1 - \hat{y}_i$
if $y_i = 0$
  error $= \hat{y}_i$



Belief

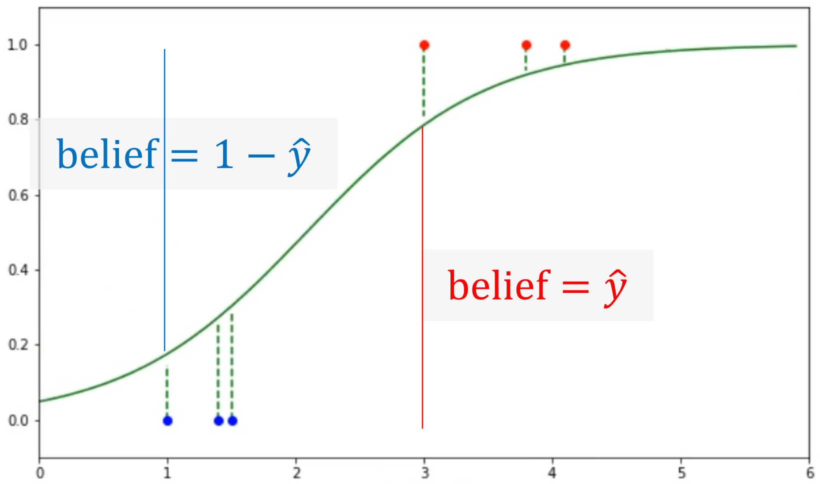
if $y_i = 1$
  belief $= \hat{y}_i$
if $y_i = 0$
  belief $= 1 - \hat{y}_i$

$$P = \hat{y}_i^{y_i}(1 - \hat{y}_i)^{1-y_i}$$

Minimize error ~ maximize belief ~ Minimize (-belief)

Idea of Logistic Regression

❖ Construct loss



Belief

if $y_i = 1$
$\quad$ belief $= \hat{y}_i$
if $y_i = 0$
$\quad$ belief $= 1 - \hat{y}_i$

$$P_i = \hat{y}_i^{y_i}(1 - \hat{y}_i)^{1-y_i}$$

$$\text{belief} = \prod_{i=1}^{n} P_i \qquad \text{since iid}$$

$$\text{log\_belief} = \sum_{i=1}^{n} \log P_i$$

$$\text{log\_belief} = \sum_{i=1}^{n} [y_i \log \hat{y}_i + (1 - y_i)\log(1 - \hat{y}_i)]$$

$$\text{loss} = -\text{log\_belief}$$
$$= -\sum_{i=1}^{n} [y_i \log \hat{y}_i + (1 - y_i)\log(1 - \hat{y}_i)]$$

$$\mathrm{L} = \frac{1}{N}\left(-y^T log(\hat{y}) - (1 - y^T) log(1 - \hat{y})\right)$$

Binary cross-entropy

Idea of Logistic Regression

❖ Construct loss

Model and Loss

$$z = \boldsymbol{\theta}^T \boldsymbol{x}$$

$$\hat{y} = \sigma(z) = \frac{1}{1 + e^{-z}}$$

$$\mathrm{L} = \frac{1}{N}[-y^T \log(\hat{y}) - (1 - y^T)\log(1 - \hat{y})]$$

Derivative

$$\frac{\partial L}{\partial \theta} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial z} \frac{\partial z}{\partial \theta}$$

$$\frac{\partial L}{\partial \hat{y}} = \frac{1}{N}\left(-\frac{y}{\hat{y}} + \frac{1 - y}{1 - \hat{y}}\right) = \frac{1}{N} \frac{\hat{y} - y}{\hat{y}(1 - \hat{y})}$$

$$\frac{\partial \hat{y}}{\partial z} = \hat{y}(1 - \hat{y})$$

$$\frac{\partial z}{\partial \theta} = x$$

$$\frac{\partial L}{\partial \theta} = \frac{1}{N} x^T (\hat{y} - y)$$

Idea of Logistic Regression

❖ Construct loss

Model and Loss

$$z = \boldsymbol{\theta}^T \boldsymbol{x}$$

$$\hat{y} = \sigma(z) = \frac{1}{1+e^{-z}}$$

$$\mathrm{L} = \frac{1}{N}[-y^T \log(\hat{y}) - (1-y^T)\log(1-\hat{y})]$$

Derivative

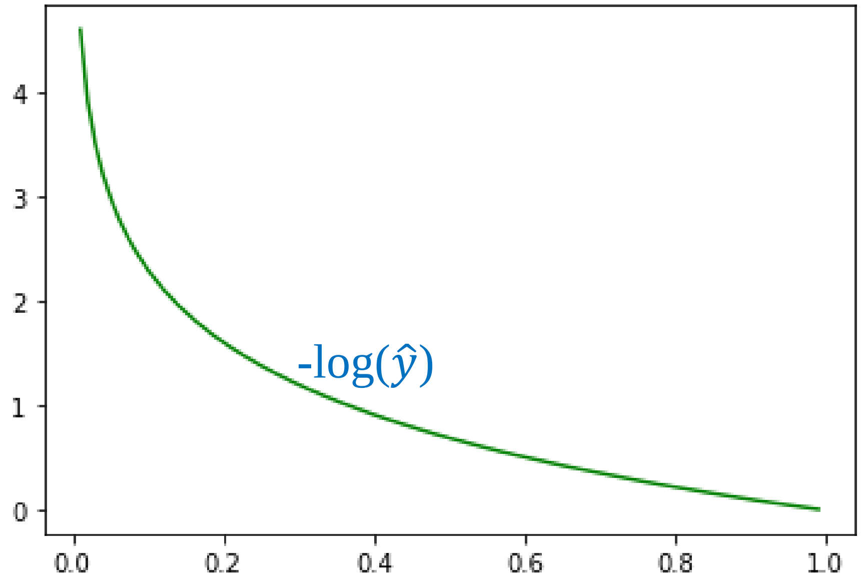
$$\frac{\partial L}{\partial \theta} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial z} \frac{\partial z}{\partial \theta}$$

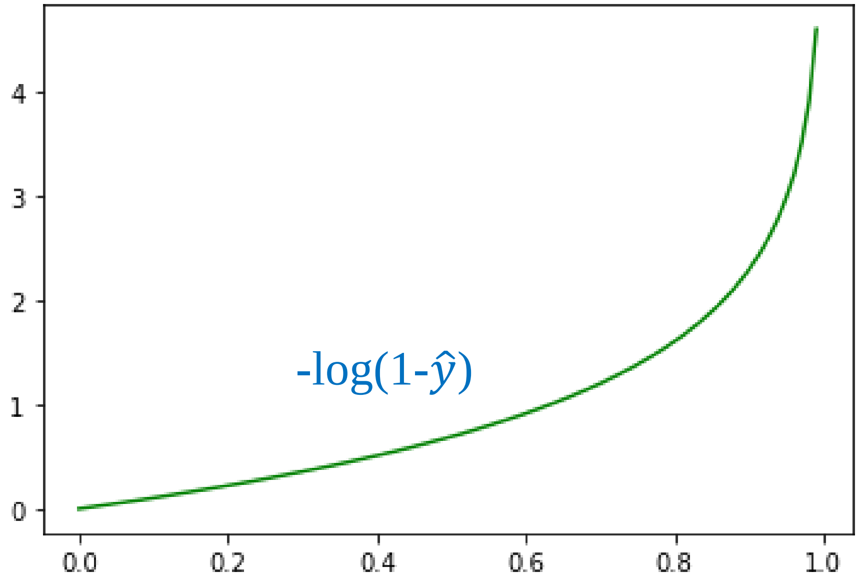
$$\frac{\partial L}{\partial \hat{y}} = \frac{1}{N}\left(-\frac{y}{\hat{y}} + \frac{1-y}{1-\hat{y}}\right) = \frac{1}{N}\frac{\hat{y}-y}{\hat{y}(1-\hat{y})}$$

$$\frac{\partial \hat{y}}{\partial z} = \hat{y}(1-\hat{y})$$

$$\frac{\partial z}{\partial \theta} = x$$

$$\frac{\partial L}{\partial \theta} = \frac{1}{N} x^T(\hat{y} - y)$$





Idea of Logistic Regression

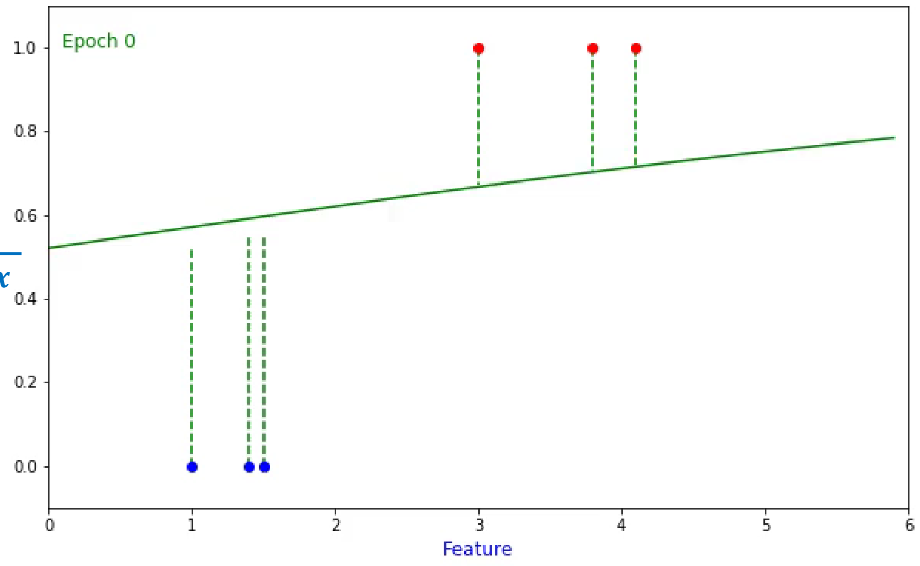
Petal_Length (Feature)	Category (Label)	
1.4	0	Category 1
1	0	Category 1
1.5	0	Category 1
3	1	Category 2
3.8	1	Category 2
4.1	1	Category 2

$$z = \boldsymbol{\theta}^T \boldsymbol{x}$$

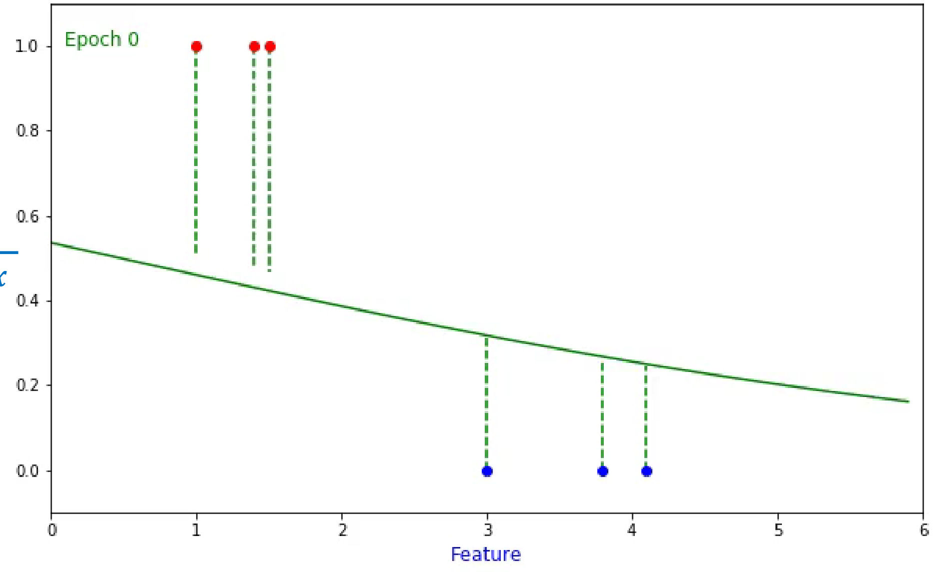
$$\hat{y} = \sigma(z) = \frac{1}{1 + e^{-z}}$$

Petal_Length (Feature)	Category (Label)	
1.4	1	Category 1
1	1	Category 1
1.5	1	Category 1
3	0	Category 2
3.8	0	Category 2
4.1	0	Category 2

$$\frac{1}{1 + e^{-\boldsymbol{\theta}^T \boldsymbol{x}}}$$



$$\frac{1}{1 + e^{-\boldsymbol{\theta}^T \boldsymbol{x}}}$$



Outline

- Sigmoid function
- From Linear to Logistic Regression
- **Logistic Regression – Stochastic**
- Logistic Regression – Mini-batch
- Logistic Regression – Batch

Logistic Regression-Stochastic

1) Pick a sample (x, y) from training data

2) Tính output $\hat{y}$

$$z = \boldsymbol{\theta}^T \boldsymbol{x}$$

$$\hat{y} = \sigma(z) = \frac{1}{1 + e^{-z}}$$

3) Tính loss

$$L(\boldsymbol{\theta}) = (-y\log\hat{y} - (1-y)\log(1-\hat{y}))$$

4) Tính đạo hàm

$$L'_{\boldsymbol{\theta}} = \mathbf{x}(\hat{y} - y)$$

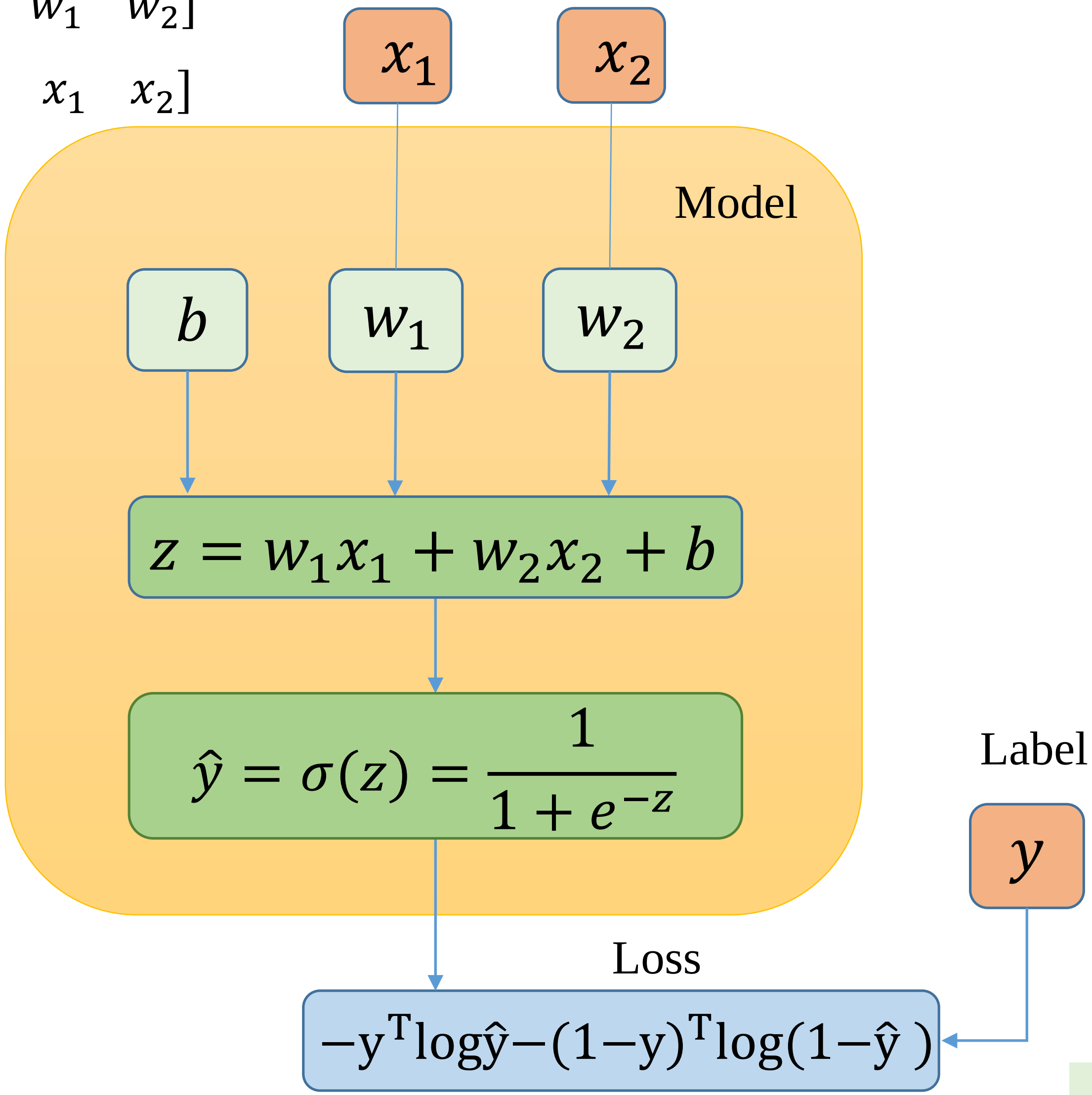
5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta L'_{\boldsymbol{\theta}}$$

η is learning rate

$$\boldsymbol{\theta}^T = [b \quad w_1 \quad w_2]$$

$$\boldsymbol{x}^T = [1 \quad x_1 \quad x_2]$$

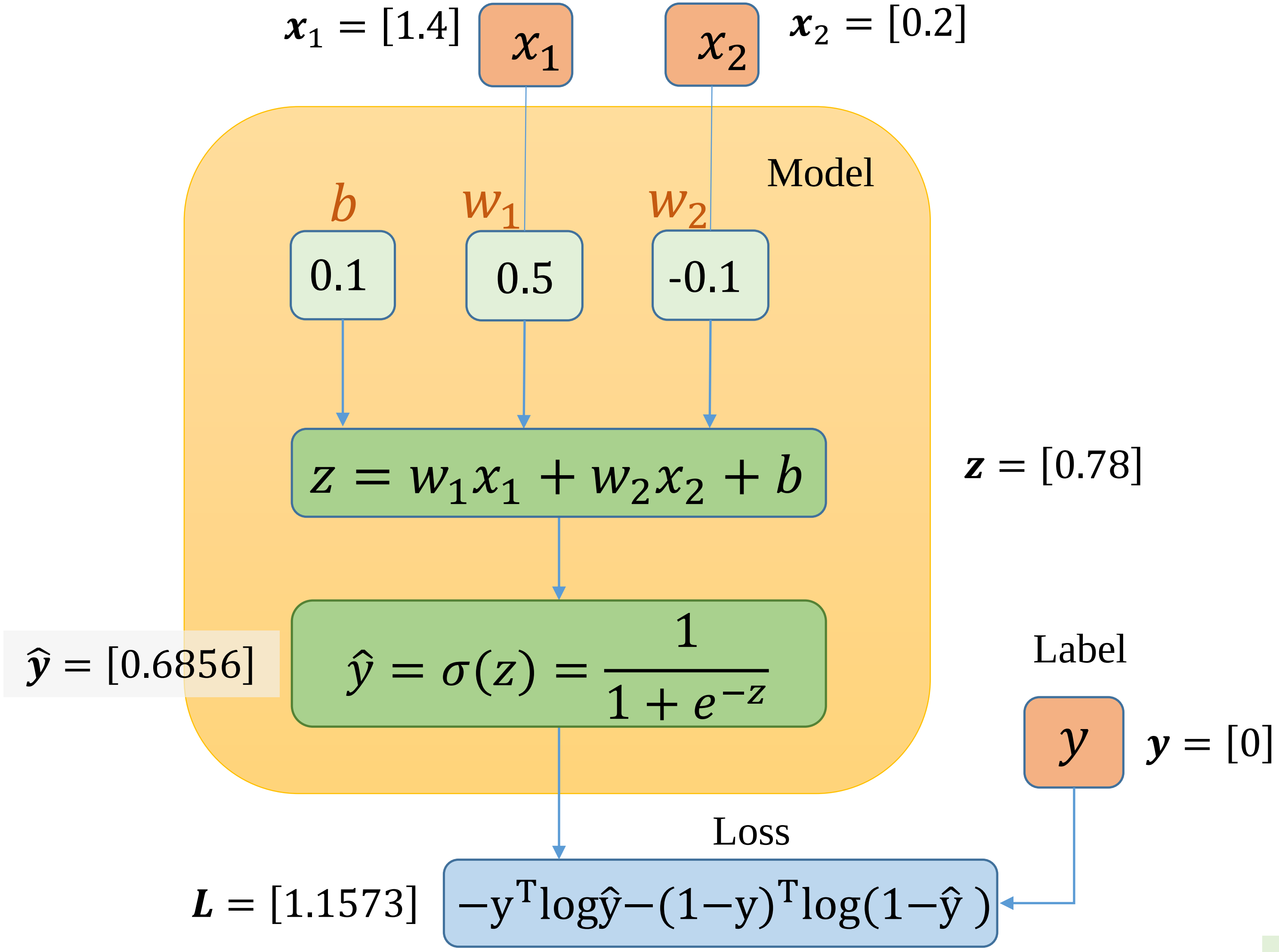


Logistic Regression-Stochastic

Dataset

Petal_Length	Petal_Width	Label
1.4	0.2	0
1.5	0.2	0
3	1.1	1
4.1	1.3	1

$$\boldsymbol{x} = \begin{bmatrix} 1 \\ 1.4 \\ 0.2 \end{bmatrix} \qquad \boldsymbol{y} = [0]$$



Logistic Regression-Stochastic

Dataset

w_1 b $\eta = 0.01$

$$\boldsymbol{x} = \begin{bmatrix} 1 \\ 1.4 \\ 0.2 \end{bmatrix} \qquad \boldsymbol{y} = [0]$$

$\eta = 0.01$

$b = 0.1 - \eta 0.6856 = 0.0931$

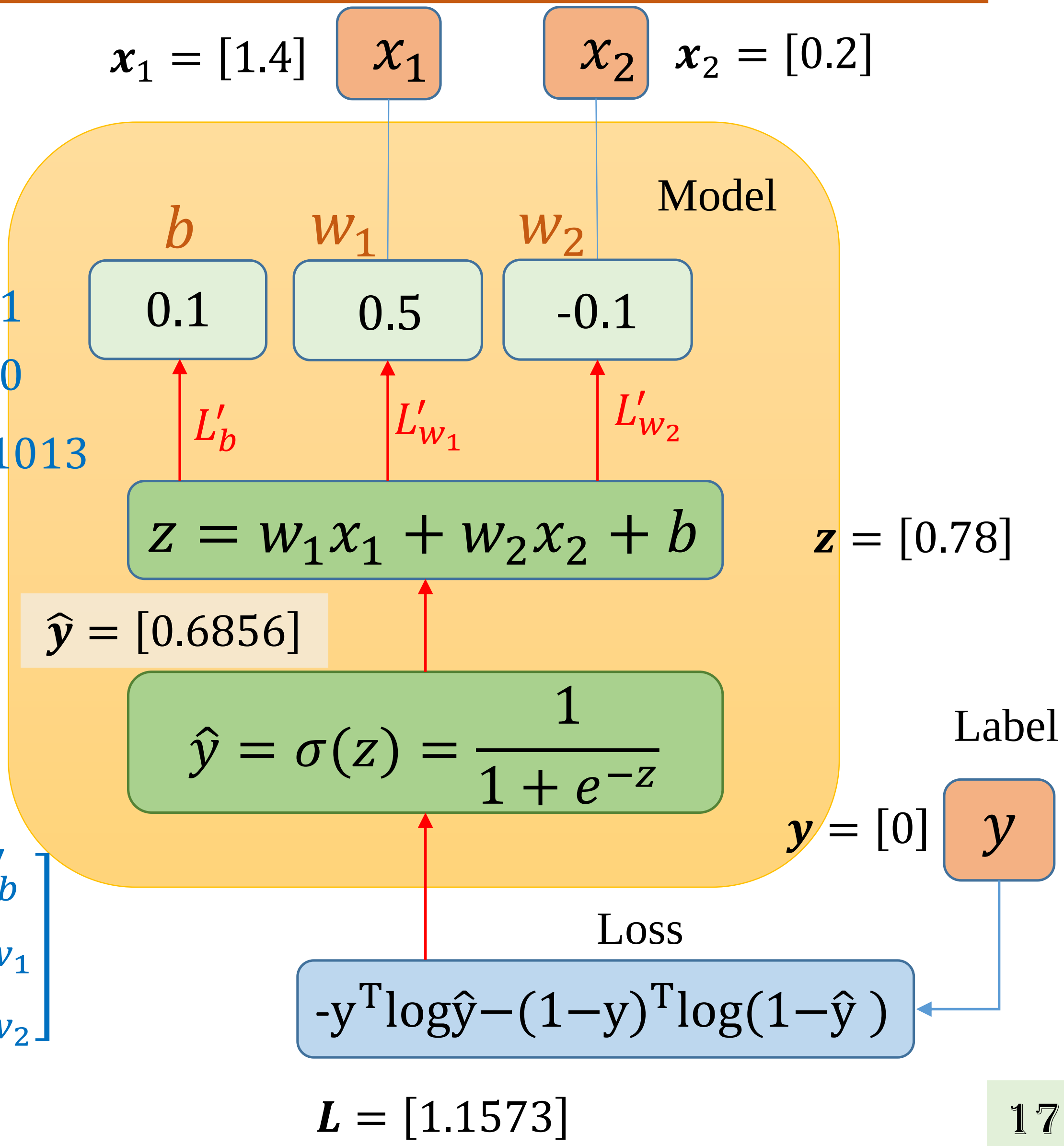
$w_1 = 0.5 - \eta 0.9598 = 0.4990$

$w_2 = -0.1 + \eta 0.1371 = -0.1013$

$$L'_{\boldsymbol{\theta}} = \mathrm{x}(\hat{\mathrm{y}} - y)$$

$$= \begin{bmatrix} 1 \\ 1.4 \\ 0.2 \end{bmatrix} [0.6856]$$

$$= \begin{bmatrix} 0.6856 \\ 0.9599 \\ -0.1371 \end{bmatrix} = \begin{bmatrix} L'_b \\ L'_{w_1} \\ L'_{w_2} \end{bmatrix}$$



Logistic Regression-Stochastic

Dataset

w_1 $\quad b \quad$ $\eta = 0.01$

$$\boldsymbol{x} = \begin{bmatrix} 1 \\ 1.4 \\ 0.2 \end{bmatrix} \qquad \boldsymbol{y} = [0]$$

$\eta = 0.01$

$b = 0.1 - \eta 0.6856 = 0.0931$

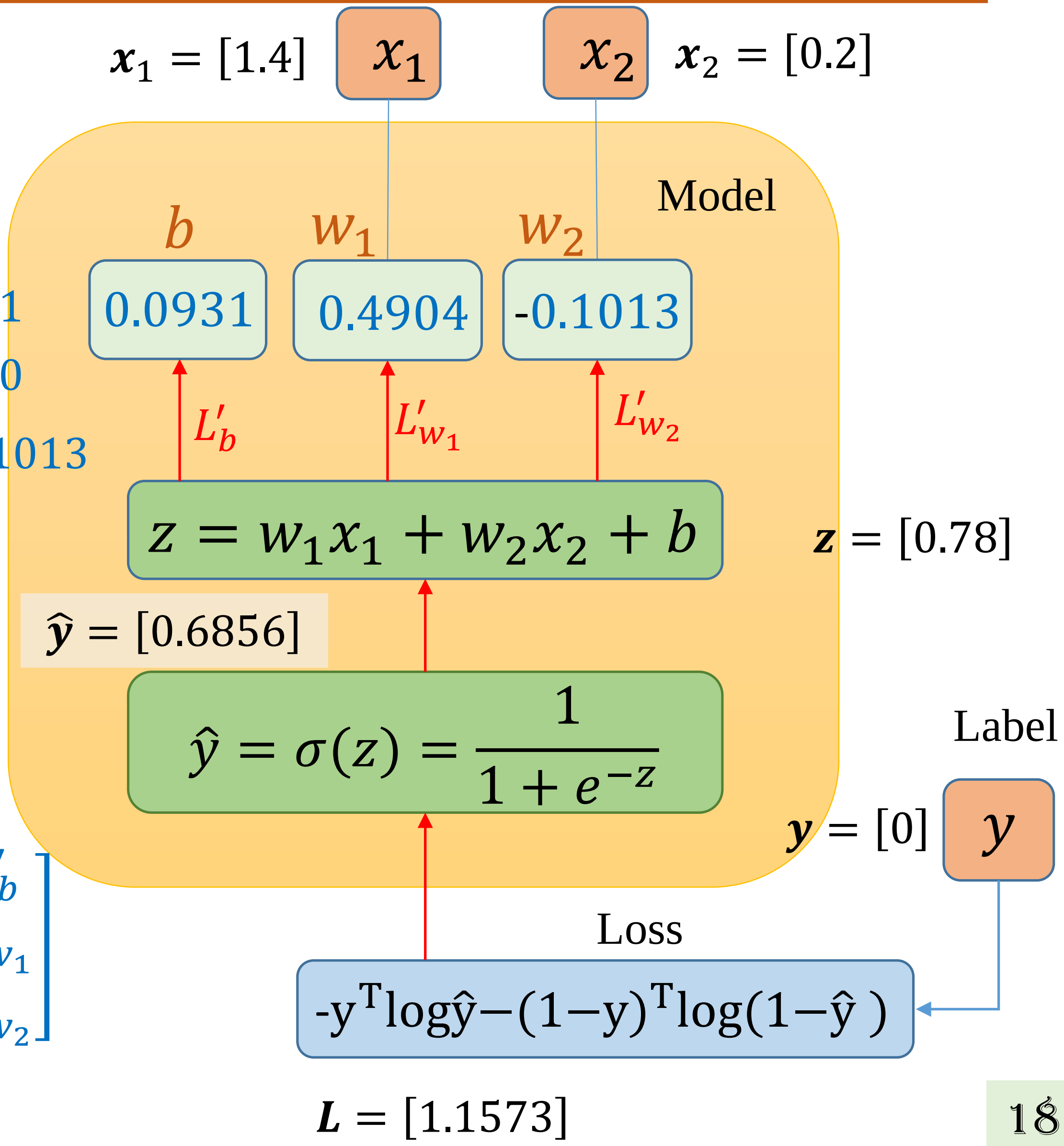
$w_1 = 0.5 - \eta 0.9598 = 0.4990$

$w_2 = -0.1 + \eta 0.1371 = -0.1013$

$$L'_{\boldsymbol{\theta}} = \mathrm{x}(\hat{\mathrm{y}} - y)$$

$$= \begin{bmatrix} 1 \\ 1.4 \\ 0.2 \end{bmatrix} [0.6856]$$

$$= \begin{bmatrix} 0.6856 \\ 0.9599 \\ -0.1371 \end{bmatrix} = \begin{bmatrix} L'_b \\ L'_{w_1} \\ L'_{w_2} \end{bmatrix}$$



Outline

- Sigmoid function
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- Logistic Regression – Stochastic
- **Logistic Regression – Mini-batch**
- Logistic Regression – Batch

Logistic Regression - Minibatch

1) Pick m samples from training data

2) Tính output $\hat{y}$

$$z = \boldsymbol{\theta}^T \boldsymbol{x}$$

$$\hat{\boldsymbol{y}} = \sigma(z) = \frac{1}{1 + e^{-z}}$$

3) Tính loss

$$L(\boldsymbol{\theta}) = \frac{1}{\mathrm{m}}\left(-\mathrm{y}^{\mathrm{T}}\log\hat{\mathrm{y}} - (1-\mathrm{y})^{\mathrm{T}}\log(1-\hat{\mathrm{y}})\right)$$

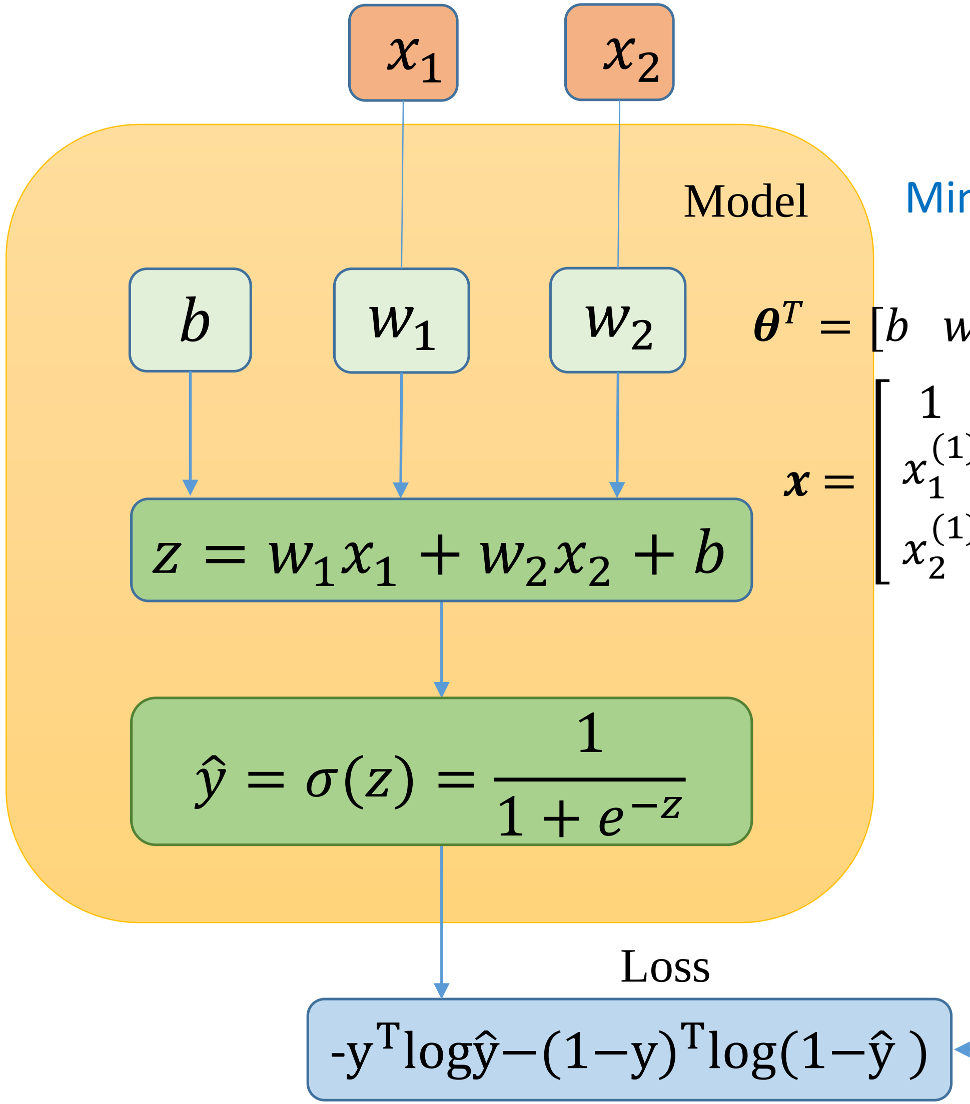
4) Tính đạo hàm

$$L'_{\boldsymbol{\theta}} = \frac{1}{\mathrm{m}}\mathrm{x}^{\mathrm{T}}(\hat{\mathrm{y}} - y)$$

5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta L'_{\boldsymbol{\theta}}$$

η is learning rate



Mini-batch m=2

$$\boldsymbol{\theta}^T = [b \quad w_1 \quad w_2]$$

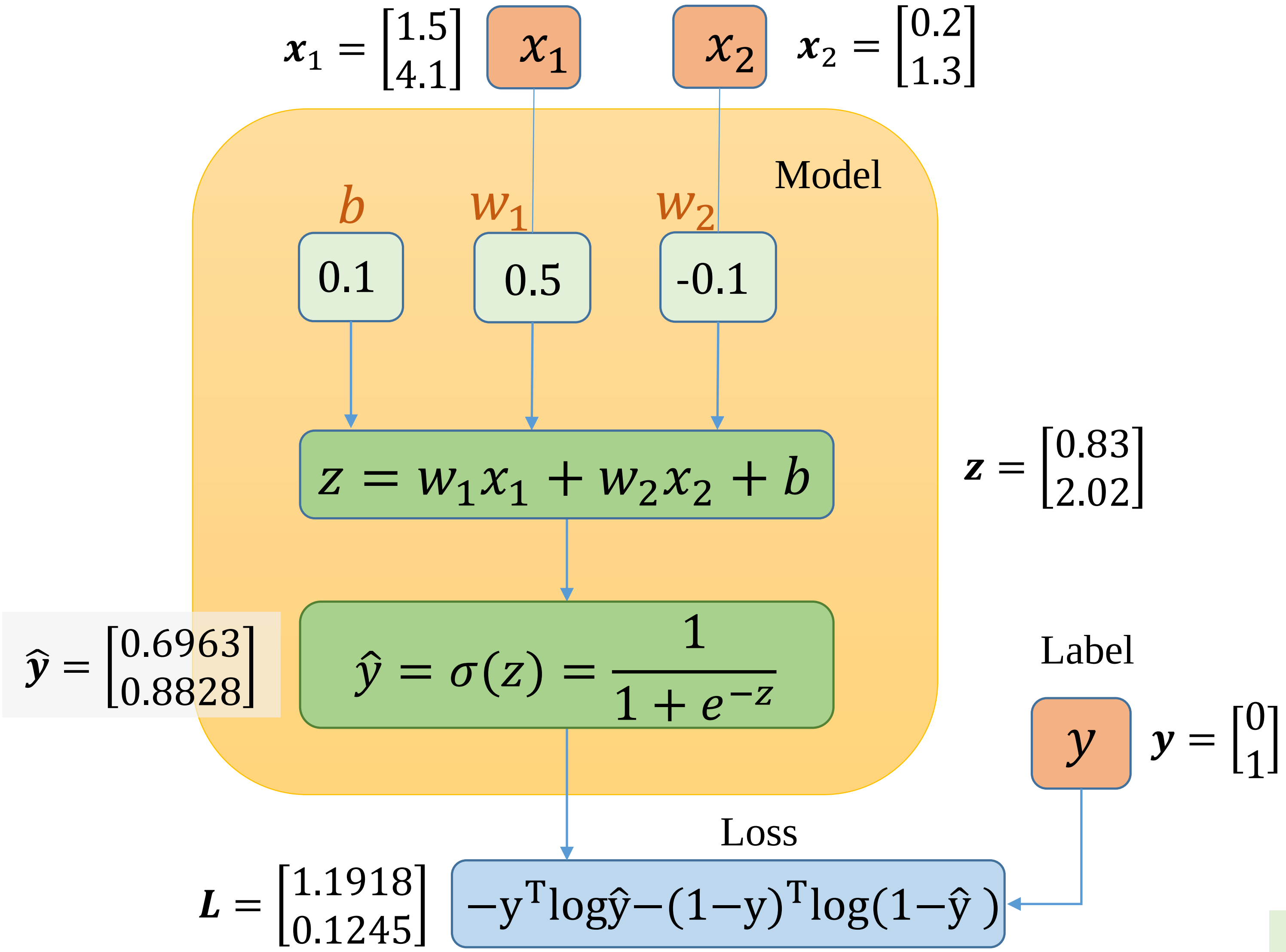
$$\boldsymbol{x} = \begin{bmatrix} 1 & 1 \\ x_1^{(1)} & x_1^{(2)} \\ x_2^{(1)} & x_2^{(2)} \end{bmatrix}$$

Logistic Regression - Minibatch

Dataset

Petal_Length	Petal_Width	Label
1.4	0.2	0
1.5	0.2	0
3	1.1	1
4.1	1.3	1

$$\boldsymbol{x} = \begin{bmatrix} 1 & 1 \\ 1.5 & 4.1 \\ 0.2 & 1.3 \end{bmatrix} \qquad \boldsymbol{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



$$b = 0.1 - \eta 0.28961 = 0.097103$$

$$w_1 = 0.5 - \eta 0.28217 = 0.49717$$

$$w_2 = -0.1 + \eta 0.0064 = -0.09993$$

Dataset

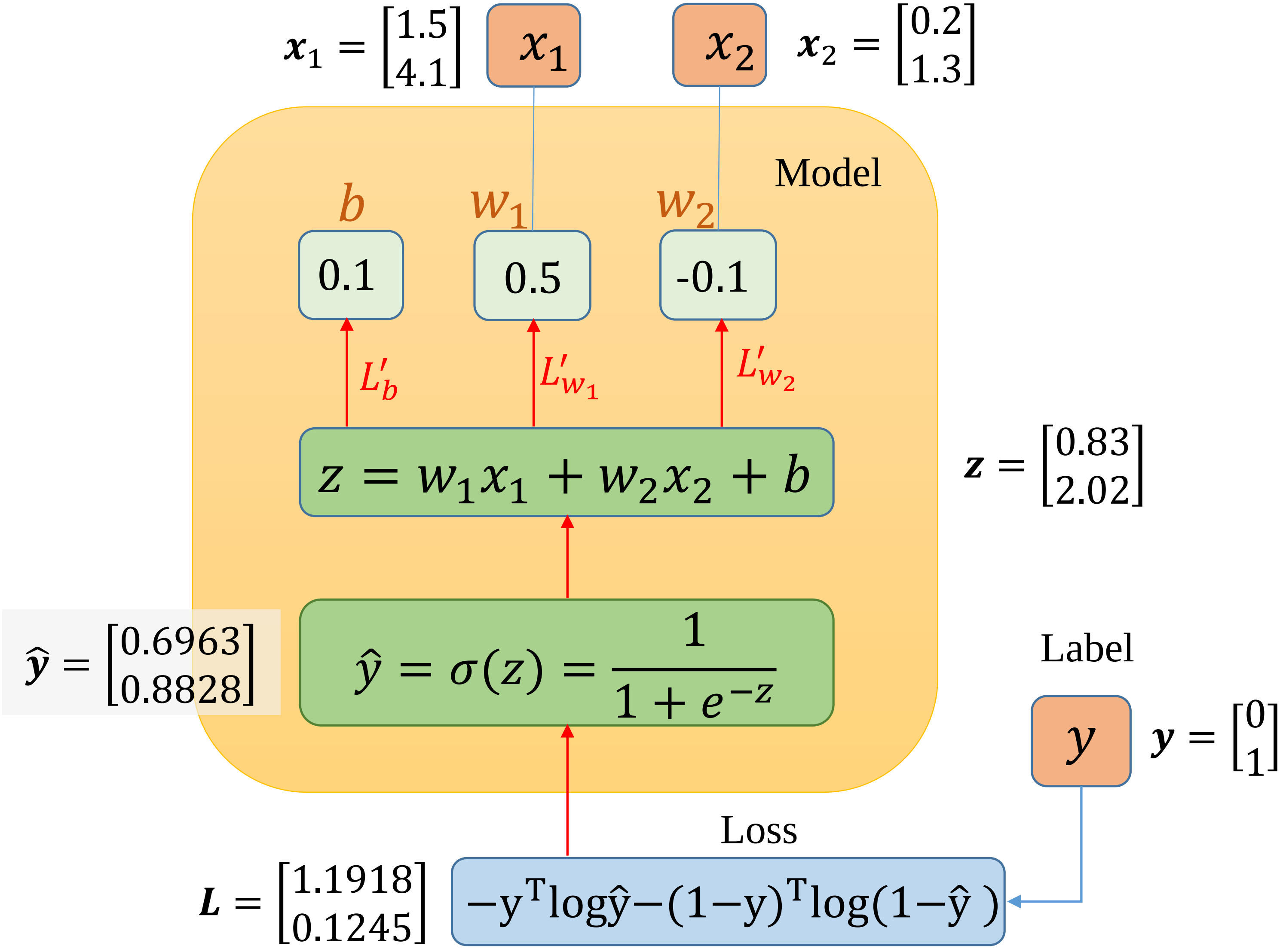
Petal_Length	Petal_Width	Label
1.4	0.2	0
1.5	0.2	0
3	1.1	1
4.1	1.3	1

$$\boldsymbol{x} = \begin{bmatrix} 1 & 1.5 & 0.2 \\ 1 & 4.1 & 1.3 \end{bmatrix} \qquad \boldsymbol{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$L'_{\boldsymbol{\theta}} = \frac{1}{\mathrm{m}} \mathrm{x}^{\mathrm{T}} (\hat{\mathrm{y}} - y)$$

$$= \frac{1}{2} \begin{bmatrix} 1.0 & 1.0 \\ 1.5 & 4.1 \\ 0.2 & 1.3 \end{bmatrix} \begin{bmatrix} 0.6963 \\ -0.1171 \end{bmatrix}$$

$$= \begin{bmatrix} 0.28961 \\ 0.28217 \\ -0.0064 \end{bmatrix} = \begin{bmatrix} L'_b \\ L'_{w_1} \\ L'_{w_2} \end{bmatrix}$$



Outline

- Sigmoid function
- From Linear to Logistic Regression
- Logistic Regression – Stochastic
- Logistic Regression – Mini-batch
- **Logistic Regression – Batch**

Logistic Regression - Batch

1) Pick all the samples from training data

2) Tính output $\hat{y}$

$$z = \boldsymbol{\theta}^T \boldsymbol{x}$$

$$\hat{\boldsymbol{y}} = \sigma(z) = \frac{1}{1 + e^{-z}}$$

3) Tính loss (binary cross-entropy)

$$L(\boldsymbol{\theta}) = \frac{1}{\mathrm{N}}\left(-\mathrm{y}^{\mathrm{T}}\log\hat{\mathrm{y}} - (1-\mathrm{y})^{\mathrm{T}}\log(1-\hat{\mathrm{y}})\right)$$

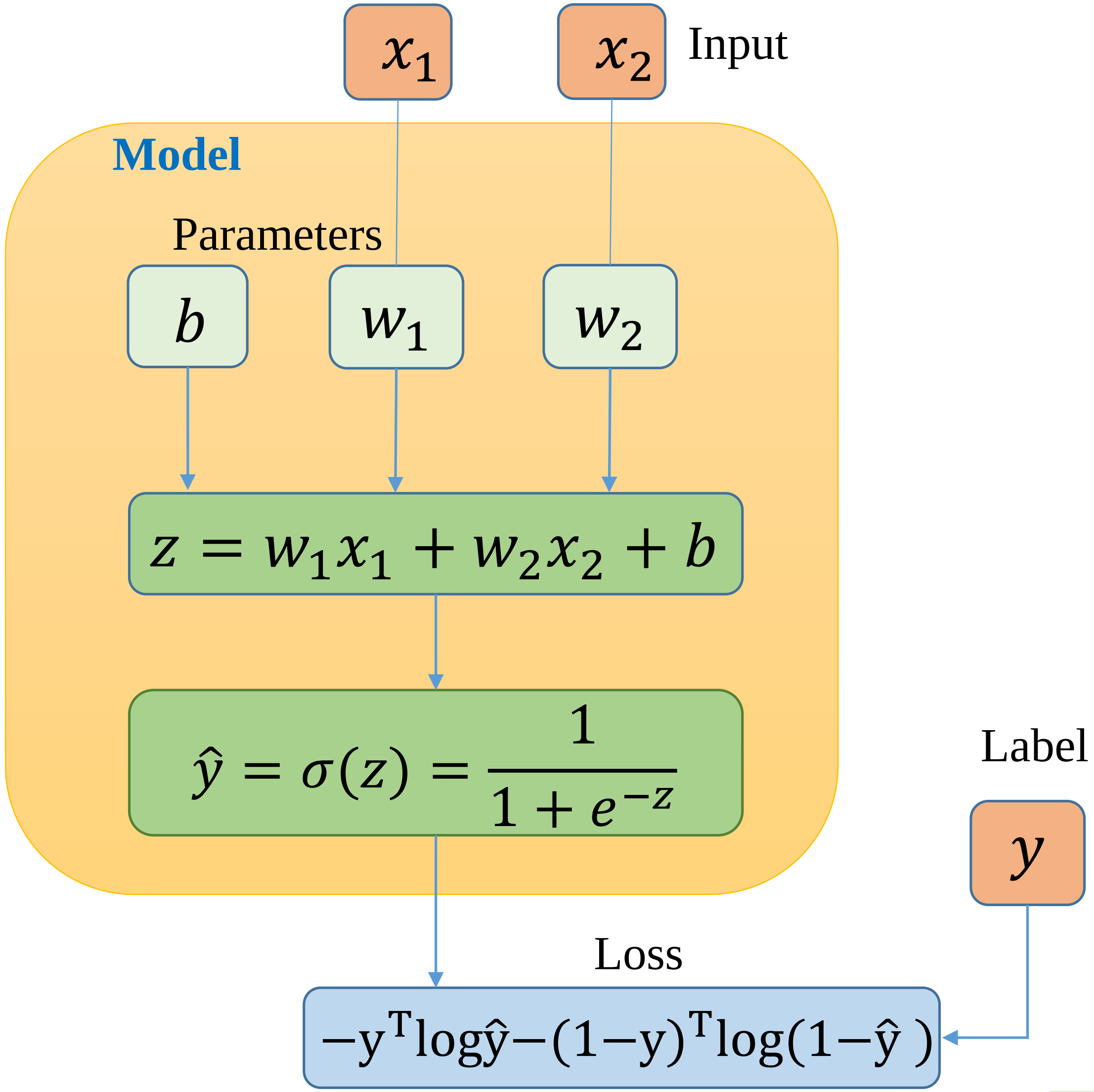
4) Tính đạo hàm

$$L'_{\boldsymbol{\theta}} = \frac{1}{\mathrm{N}}\mathrm{x}^{\mathrm{T}}(\hat{\mathrm{y}} - y)$$

5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta L'_{\boldsymbol{\theta}}$$

η is learning rate



Logistic Regression

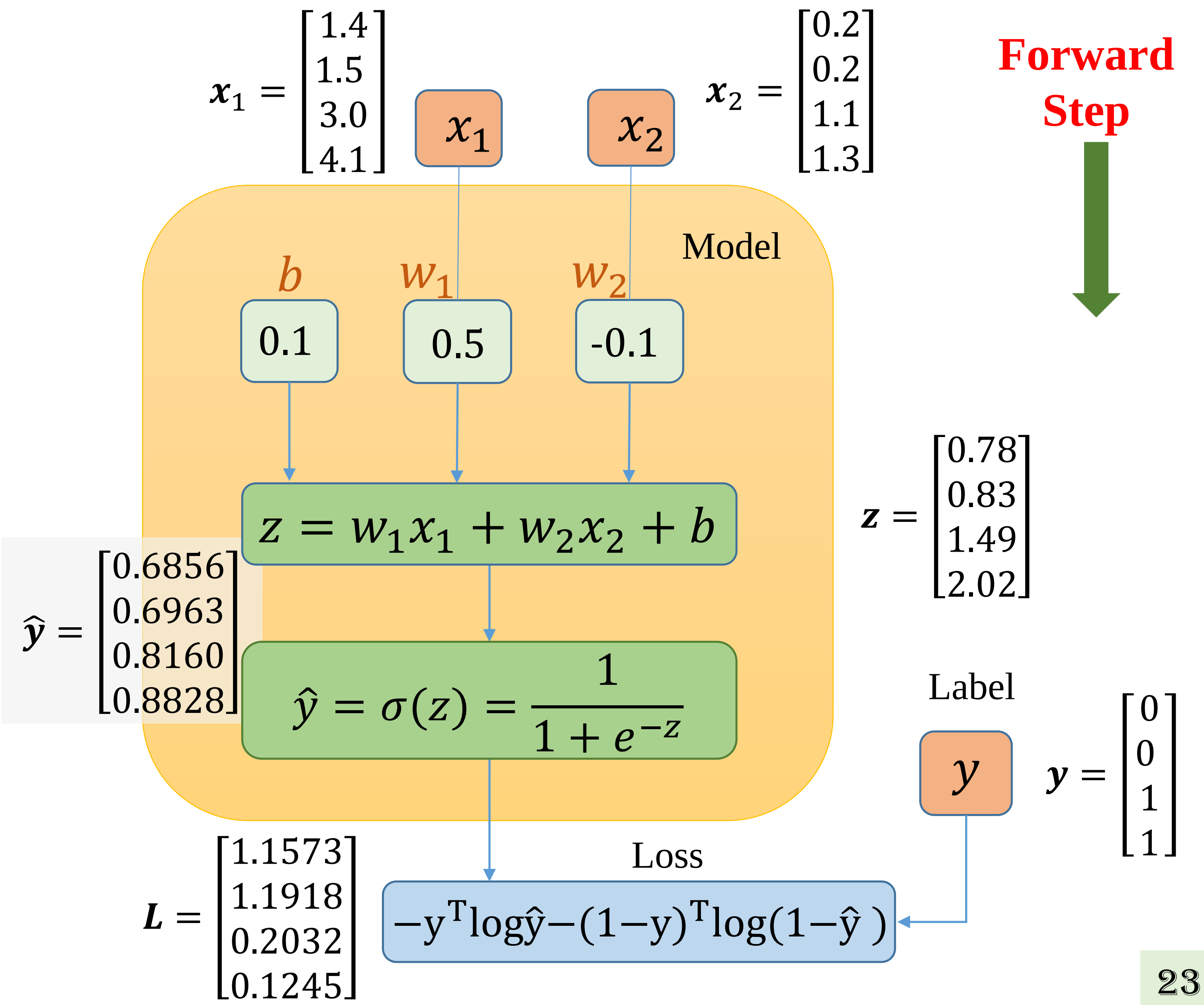
Phân loại hoa Iris dựa vào chiều dài và chiều rộng của cánh hoa

Dataset

Petal_Length	Petal_Width	Label
1.4	0.2	0
1.5	0.2	0
3	1.1	1
4.1	1.3	1

$$\boldsymbol{x} = \begin{bmatrix} 1 & 1.4 & 0.2 \\ 1 & 1.5 & 0.2 \\ 1 & 3.0 & 1.1 \\ 1 & 4.1 & 1.3 \end{bmatrix} \qquad \boldsymbol{y} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Average loss = 0.6692



Phân loại hoa Iris dựa vào chiều dài và chiều rộng của cánh hoa

Backward Step

Dataset

Petal_Length	Petal_Width	Label
1.4	0.2	0
1.5	0.2	0
3	1.1	1
4.1	1.3	1

$$\boldsymbol{x} = \begin{bmatrix} 1 & 1.4 & 0.2 \\ 1 & 1.5 & 0.2 \\ 1 & 3.0 & 1.1 \\ 1 & 4.1 & 1.3 \end{bmatrix} \qquad \boldsymbol{y} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\mathbf{x}^T = \begin{bmatrix} 1.0 & 1.0 & 1.0 & 1.0 \\ 1.4 & 1.5 & 3.0 & 4.1 \\ 0.2 & 0.2 & 1.1 & 1.3 \end{bmatrix}$$

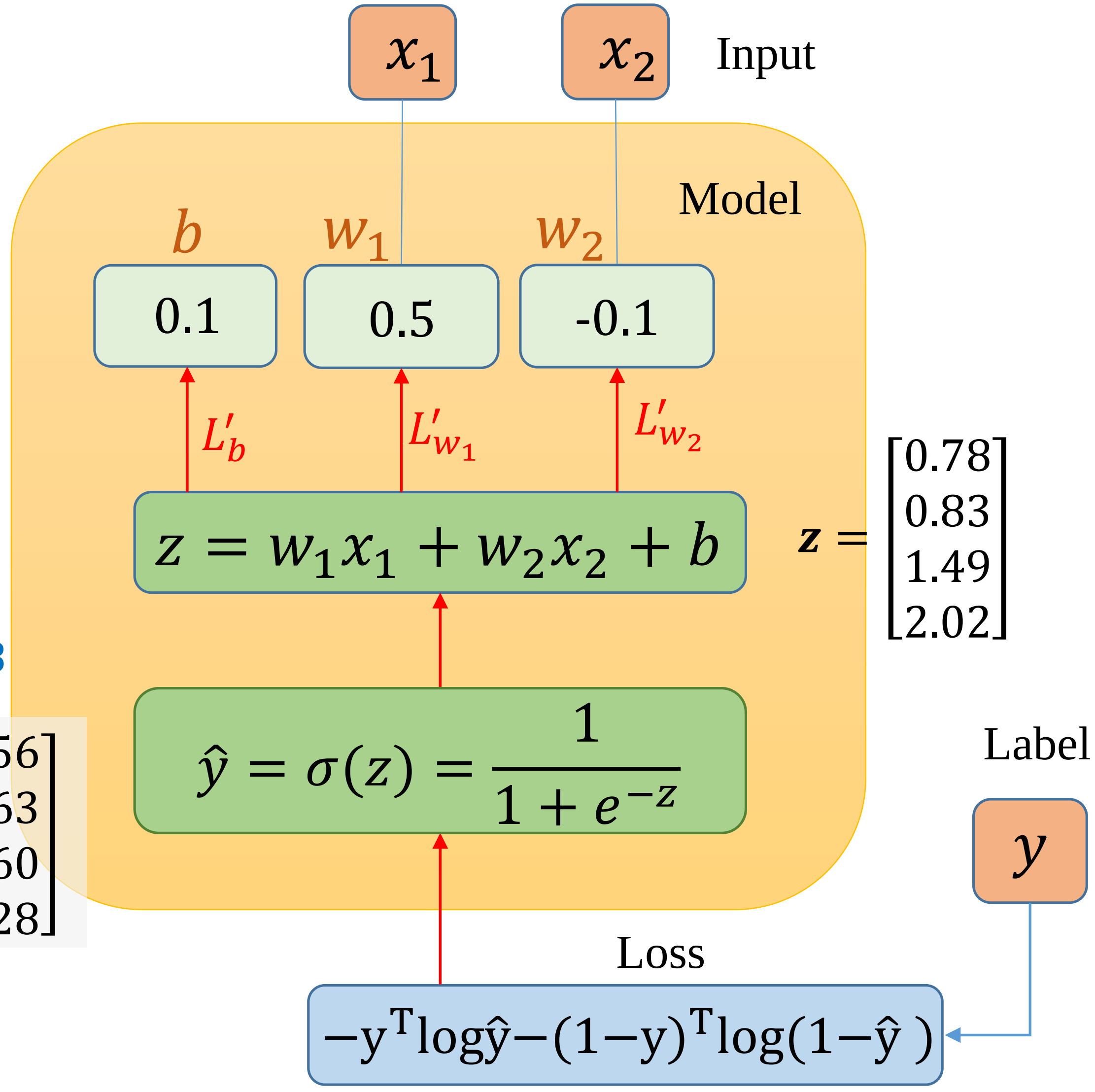
$\eta = 0.01$

$$b = 0.1 - \eta 0.2702 = 0.0972$$

$$w_1 = 0.5 - \eta 0.2431 = 0.4975$$

$$w_2 = -0.1 + \eta 0.0195 = -0.0998$$

$$\hat{\boldsymbol{y}} = \begin{bmatrix} 0.6856 \\ 0.6963 \\ 0.8160 \\ 0.8828 \end{bmatrix}$$



$$\boldsymbol{z} = \begin{bmatrix} 0.78 \\ 0.83 \\ 1.49 \\ 2.02 \end{bmatrix}$$

$$L'_{\boldsymbol{\theta}} = \frac{1}{\mathrm{N}} \mathrm{x}^T (\hat{\mathrm{y}} - y)$$

$$= \frac{1}{4} \begin{bmatrix} 1.0 & 1.0 & 1.0 & 1.0 \\ 1.4 & 1.5 & 3.0 & 4.1 \\ 0.2 & 0.2 & 1.1 & 1.3 \end{bmatrix} \begin{bmatrix} 0.6856 \\ 0.6963 \\ -0.1839 \\ -0.1171 \end{bmatrix} = \begin{bmatrix} 0.2702 \\ 0.2431 \\ -0.019 \end{bmatrix} = \begin{bmatrix} L'_b \\ L'_{w_1} \\ L'_{w_2} \end{bmatrix}$$

Phân loại hoa Iris dựa vào chiều dài và chiều rộng của cánh hoa

Dataset

Petal_Length	Petal_Width	Label
1.4	0.2	0
1.5	0.2	0
3	1.1	1
4.1	1.3	1

$$\boldsymbol{x} = \begin{bmatrix} 1 & 1.4 & 0.2 \\ 1 & 1.5 & 0.2 \\ 1 & 3.0 & 1.1 \\ 1 & 4.1 & 1.3 \end{bmatrix} \qquad \boldsymbol{y} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\mathbf{x}^{\mathrm{T}} = \begin{bmatrix} 1.0 & 1.0 & 1.0 & 1.0 \\ 1.4 & 1.5 & 3.0 & 4.1 \\ 0.2 & 0.2 & 1.1 & 1.3 \end{bmatrix}$$

Backward Step

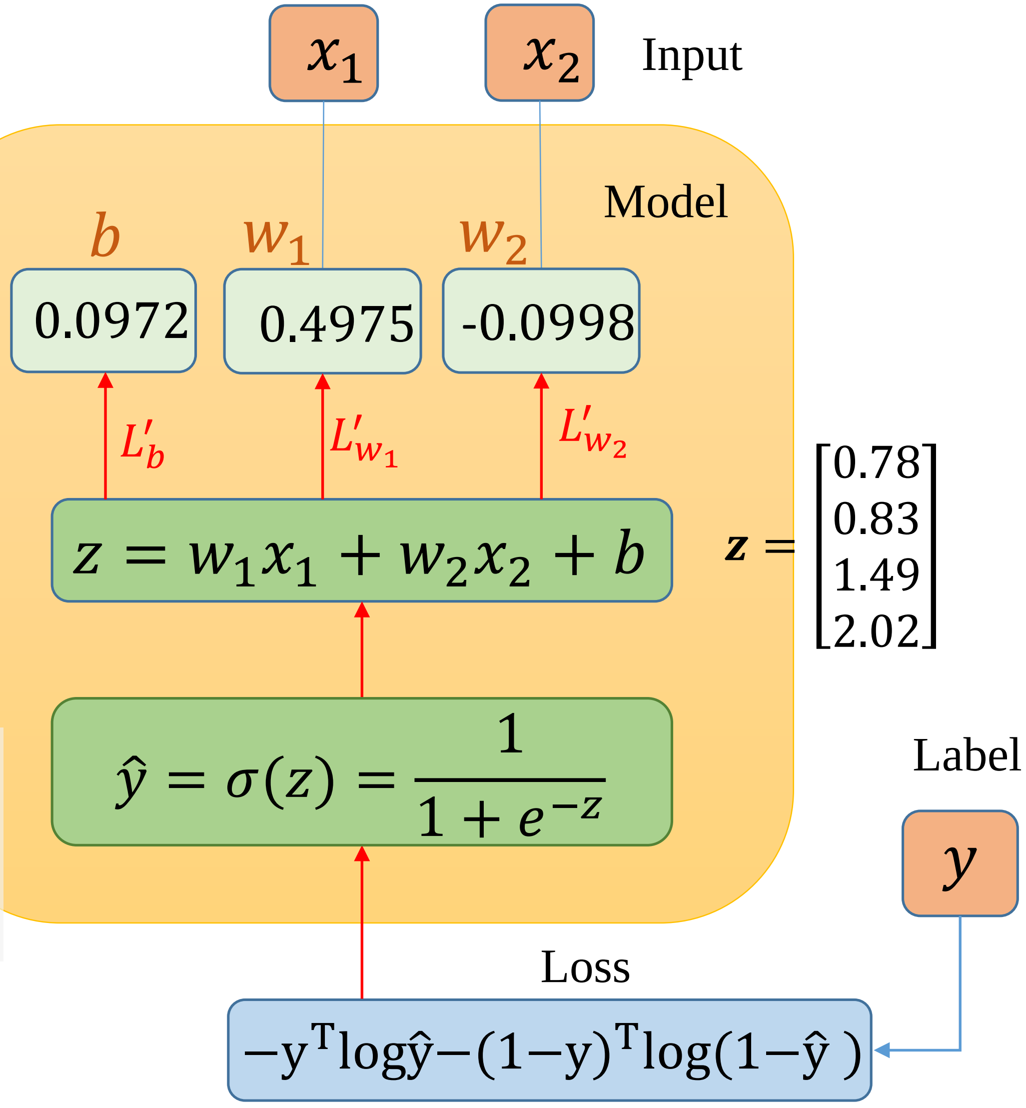
$\eta = 0.01$

$$b = 0.1 - \eta 0.2702 = 0.0972$$

$$w_1 = 0.5 - \eta 0.2431 = 0.4975$$

$$w_2 = -0.1 + \eta 0.0195 = -0.0998$$

$$\hat{\boldsymbol{y}} = \begin{bmatrix} 0.6856 \\ 0.6963 \\ 0.8160 \\ 0.8828 \end{bmatrix}$$



$$\boldsymbol{z} = \begin{bmatrix} 0.78 \\ 0.83 \\ 1.49 \\ 2.02 \end{bmatrix}$$

$$L'_{\boldsymbol{\theta}} = \frac{1}{\mathrm{N}} \mathrm{x}^{\mathrm{T}} (\hat{\mathrm{y}} - y)$$

$$= \frac{1}{4} \begin{bmatrix} 1.0 & 1.0 & 1.0 & 1.0 \\ 1.4 & 1.5 & 3.0 & 4.1 \\ 0.2 & 0.2 & 1.1 & 1.3 \end{bmatrix} \begin{bmatrix} 0.6856 \\ 0.6963 \\ -0.1839 \\ -0.1171 \end{bmatrix} = \begin{bmatrix} 0.2702 \\ 0.2431 \\ -0.019 \end{bmatrix} = \begin{bmatrix} L'_b \\ L'_{w_1} \\ L'_{w_2} \end{bmatrix}$$

Phân loại hoa Iris dựa vào chiều dài và chiều rộng của cánh hoa

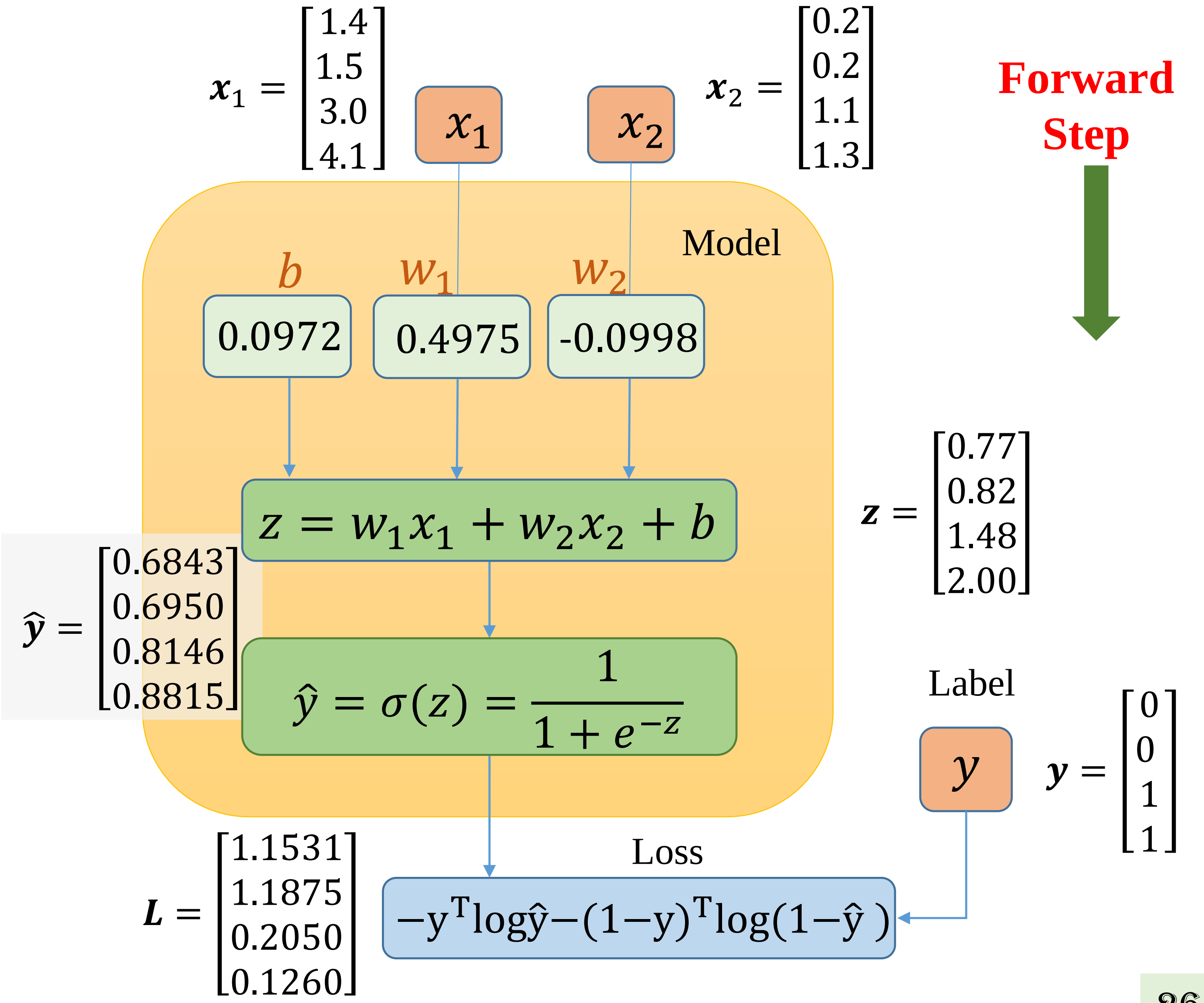
Dataset

Petal_Length	Petal_Width	Label
1.4	0.2	0
1.5	0.2	0
3	1.1	1
4.1	1.3	1

$$\boldsymbol{x} = \begin{bmatrix} 1 & 1.4 & 0.2 \\ 1 & 1.5 & 0.2 \\ 1 & 3.0 & 1.1 \\ 1 & 4.1 & 1.3 \end{bmatrix} \qquad \boldsymbol{y} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Average loss = 0.6679

Loss giảm từ 0.6692 xuống 0.6679



Logistic Regression - Question

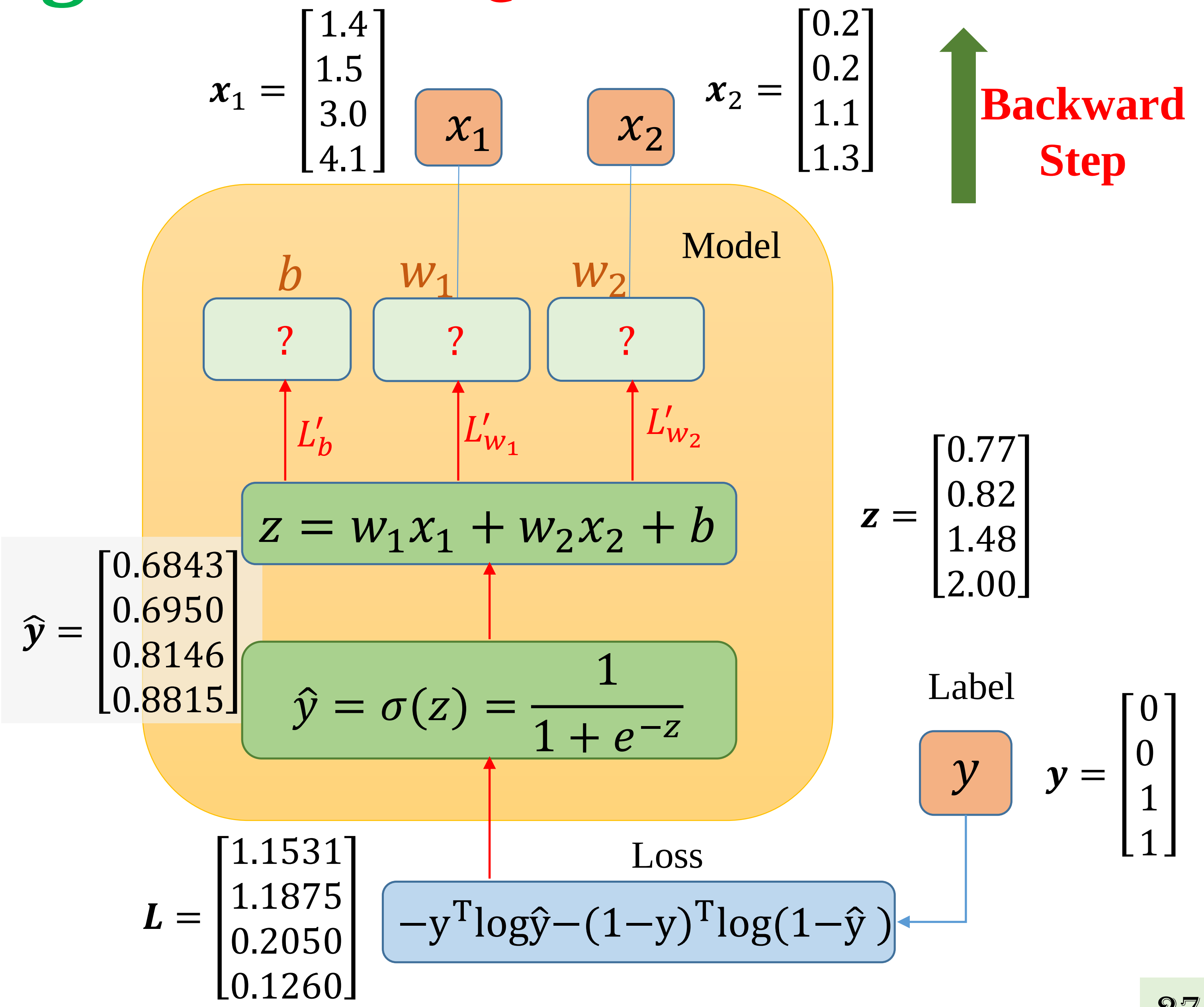
Phân loại hoa Iris dựa vào chiều dài và chiều rộng của cánh hoa

Dataset

Petal_Length	Petal_Width	Label
1.4	0.2	0
1.5	0.2	0
3	1.1	1
4.1	1.3	1

$$\boldsymbol{x} = \begin{bmatrix} 1 & 1.4 & 0.2 \\ 1 & 1.5 & 0.2 \\ 1 & 3.0 & 1.1 \\ 1 & 4.1 & 1.3 \end{bmatrix} \qquad \boldsymbol{y} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\mathbf{x}^{\mathrm{T}} = \begin{bmatrix} 1.0 & 1.0 & 1.0 & 1.0 \\ 1.4 & 1.5 & 3.0 & 4.1 \\ 0.2 & 0.2 & 1.1 & 1.3 \end{bmatrix}$$



Logistic Regression

❖ Demo

```
Python 3.7.3 (default, Apr 24 2019, 15:29:51) [MSC v.1915 64 bit (AMD64)] ::
Type "help", "copyright", "credits" or "license" for more information.
>>>
>>>
>>>
>>>
>>>
>>>
>>>
>>> for epoch in range(n_epochs):
...     sum_of_losses = 0
...     gradients = np.zeros((2,1))
...
...     for index in range(4):
...         xi = X_b[index:index+1]
...         yi = y[index:index+1]
```

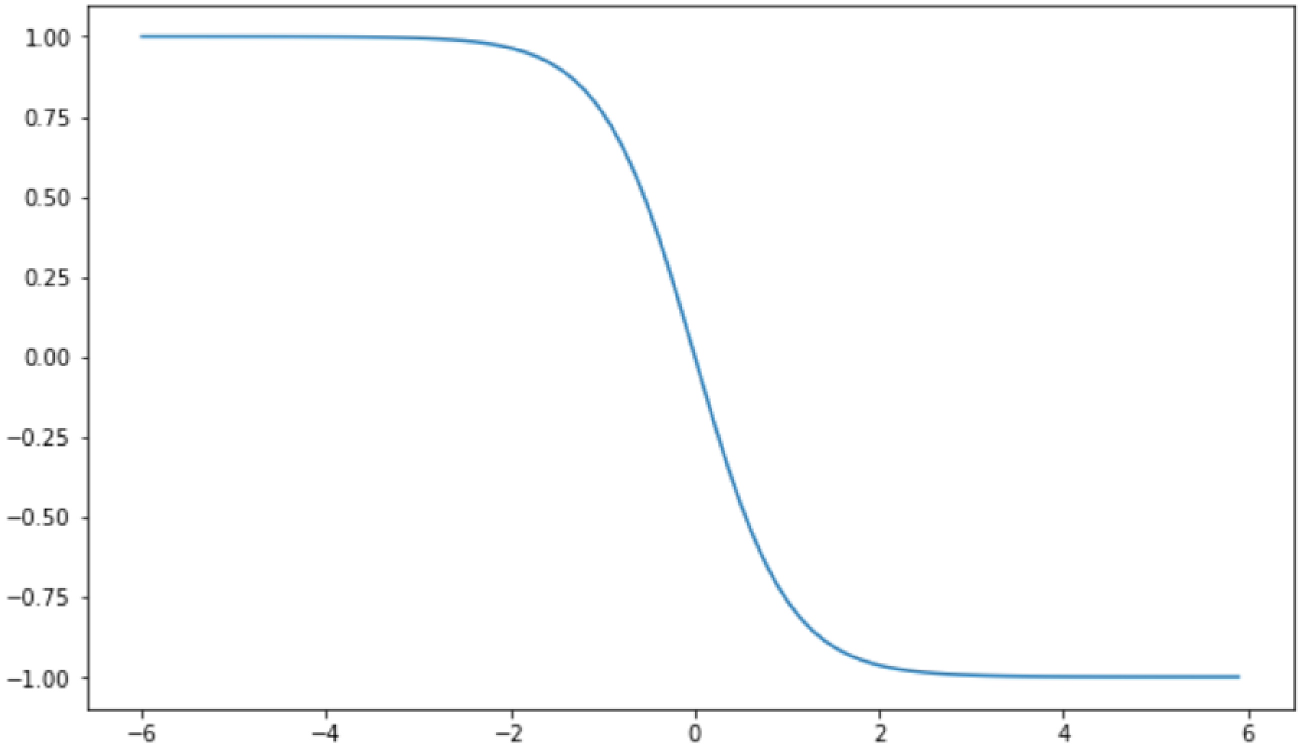
Tanh function

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

$$= 1 - \frac{2}{e^{2x} + 1}$$

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1 - e^{-2x}}{1 + e^{-2x}}$$

$$= -\frac{e^{-2x} - 1}{e^{-2x} + 1} = \frac{2}{e^{-2x} + 1} - 1$$



Tanh function

$$\tanh(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1} = 1 - \frac{2}{e^{2x} + 1}$$

$$\tanh(x) = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} = \frac{1 - e^{-2x}}{1 + e^{-2x}} = -\frac{e^{-2x} - 1}{e^{-2x} + 1} = \frac{2}{e^{-2x} + 1} - 1$$

$$tanh'(x) = \left(\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}\right)' = \frac{(e^{x} + e^{-x})(e^{x} + e^{-x}) - (e^{x} - e^{-x})(e^{x} - e^{-x})}{(e^{x} + e^{-x})^{2}}$$

$$= \frac{(e^{x} + e^{-x})^{2} - (e^{x} - e^{-x})^{2}}{(e^{x} + e^{-x})^{2}}$$

$$= 1 - \left(\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}\right)^{2} = 1 - tanh^{2}(x)$$

Tanh function

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1} = 1 - \frac{2}{e^{2x} + 1}$$

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1 - e^{-2x}}{1 + e^{-2x}} = -\frac{e^{-2x} - 1}{e^{-2x} + 1} = \frac{2}{e^{-2x} + 1} - 1$$

$$tanh'(x) = \left(\frac{2}{e^{-2x} + 1} - 1\right)' = \frac{4e^{-2x}}{(e^{-2x} + 1)^2} = 4\left(\frac{e^{-2x} + 1 - 1}{(e^{-2x} + 1)^2}\right)$$

$$= 4\left(\frac{1}{e^{-2x} + 1} - \frac{1}{(e^{-2x} + 1)^2}\right) = -\left(\frac{4}{(e^{-2x} + 1)^2} - \frac{4}{e^{-2x} + 1}\right)$$

$$= -\left(\frac{4}{(e^{-2x} + 1)^2} - \frac{4}{e^{-2x} + 1} + 1 - 1\right) = 1 - \left(\frac{2}{e^{-2x} + 1} - 1\right)^2 = 1 - tanh^2(x)$$

Logistic Regression - Tanh

❖ Construct loss

Model and Loss

$$z = \boldsymbol{\theta}^T \boldsymbol{x}$$

$$\hat{y} = tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

$$\mathrm{L} = \frac{1}{N}\left[-y^T \log(\hat{y}) - (1 - y^T)\log(1 - \hat{y})\right]$$

Derivative

$$\frac{\partial L}{\partial \theta} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial z} \frac{\partial z}{\partial \theta}$$

$$\frac{\partial L}{\partial \hat{y}} = \frac{1}{N}\left(-\frac{y}{\hat{y}} + \frac{1 - y}{1 - \hat{y}}\right) = \frac{1}{N} \frac{\hat{y} - y}{\hat{y}(1 - \hat{y})}$$

$$\frac{\partial \hat{y}}{\partial z} = 1 - \hat{y}^2$$

$$\frac{\partial z}{\partial \theta} = x$$

$$\frac{\partial L}{\partial \theta} = \frac{1}{N} x^T \frac{(\hat{y} - y)(1 + \hat{y})}{\hat{y}}$$

Logistic Regression-MSE

❖ Construct loss

Model and Loss

$$z = \boldsymbol{\theta}^T \boldsymbol{x}$$

$$\hat{y} = \sigma(z) = \frac{1}{1 + e^{-z}} \qquad \mathrm{L} = (\hat{y} - y)^2$$

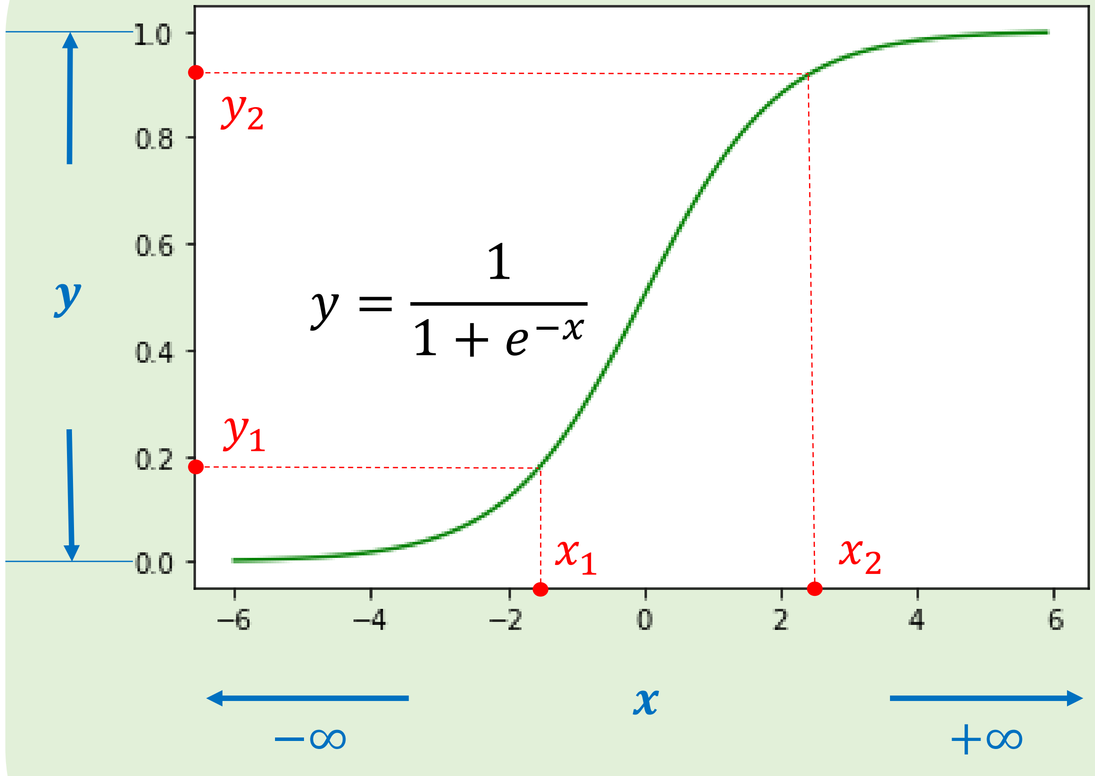
Derivative

$$\frac{\partial L}{\partial \theta} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial z} \frac{\partial z}{\partial \theta} \qquad \frac{\partial \hat{y}}{\partial z} = \hat{y}(1 - \hat{y})$$

$$\frac{\partial L}{\partial \hat{y}} = 2(\hat{y} - y) \qquad \frac{\partial z}{\partial \theta} = x$$

$$\frac{\partial L}{\partial \theta} = \frac{1}{N} 2x^T (\hat{y} - y)\hat{y}(1 - \hat{y})$$

Summary



Sigmoid function

1) Pick all the samples from training data

2) Tính output $\hat{y}$

$$z = \boldsymbol{\theta}^T \boldsymbol{x}$$

$$\hat{\boldsymbol{y}} = \sigma(z) = \frac{1}{1+e^{-z}}$$

3) Tính loss (binary cross-entropy)

$$L(\boldsymbol{\theta}) = \frac{1}{\mathrm{N}}\left(-\mathrm{y}^{\mathrm{T}}\log\hat{\mathrm{y}} - (1-\mathrm{y})^{\mathrm{T}}\log(1-\hat{\mathrm{y}})\right)$$

4) Tính đạo hàm

$$L'_{\boldsymbol{\theta}} = \frac{1}{\mathrm{N}}\mathrm{x}^{\mathrm{T}}(\hat{\mathrm{y}} - y)$$

5) Cập nhật tham số

$$\boldsymbol{\theta} = \boldsymbol{\theta} - \eta L'_{\boldsymbol{\theta}}$$

η is learning rate

Thank you!