Gaussian Process for FEM Reduction Applied to Fatigue life Estimation of Non-linear Model

(Bibliography report)

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***Abstract*—In engineering design problems, advanced simulations are time consuming and computationally expensive. As a cheap replacement of the expensive simulations, surrogate models have been widely accepted as an efficient tool of computational engineering design. Gaussian Process Regression (GPR) method is used in this article as a replacement of Finite Element Simulation to predict the fatigue life of a non-linear model. Singular Value Decomposition (SVD) method is also used in this paper to reduce the code.**

I. Introduction

Engineering design requires engineers to precise decisions out of numerous analysis. These engaged analysis could spend month time by dedicated teams. The 'Surrogate model' approach is proposed as an approximate model that could greatly reduce both time and cost in solving engineering problems.

II. Problem statement

This project aims to construct a surrogate model of Finite Element simulation applied to fatigue life estimation of a non-linear model. As shown in Figure 1, the life cycle of a simple non-linear component can be reduced to a linear combination of loads,F_x,F_y. The model is non linear due to contact with the red pin in the middle of the structure. Each load combination should be executed to analyze the life cycle due to the non-linearity. Since the Finite Element Analysis can be very expensive and the number of load combination can be of 5000 different loads, this approach can be prohibitive. As a replacement of ABAQUS simulation, this surrogate model is constructed so as to bridge the gap between accuracy and complexity.

III. Methods

A. Gaussian Process Regression

The type of surrogate model used in this project is Gaussain Process Regression (also called Kriging model). Kriging

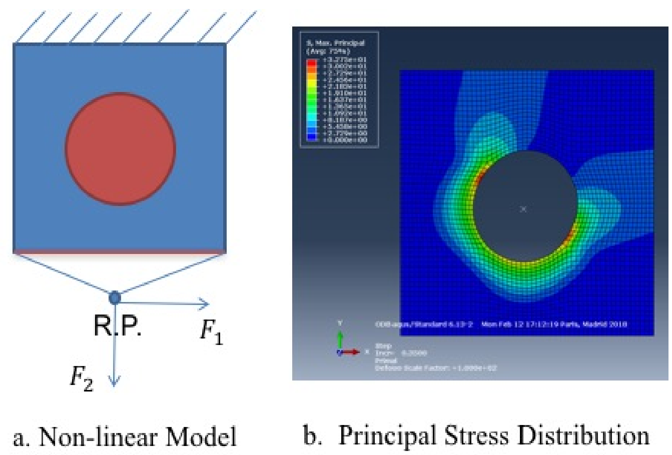

Fig. 1. Non-linear model of a small aircraft component

method [1] supposes that responses of data points are correlated and its values depend on the 'distance' between input points.

$$cor[Y(x^{(i)},)Y(x^{(l)}] = exp(-\sum_{j=1}^{K-1}\theta_j|x_j^{(i)} - x_j^{(l)}|^{p_j}) \tag{1}$$

Equation 1 indicates that the correlation of the responses in two points $x^{(i)}$, $x^{(l)}$ depend on the absolute distance between the sample points and the parameters θ_j and p_j. Kriging method also supposes that the predicted output is a linear combination of observed responses. w_i are the weights and its reflect the structural "proximity" of samples to the estimation location.

$$\hat{y}(x) = \sum_{i=0}^{n} w_i y_i \tag{2}$$

C E Rasmussen et. al. [2] give another understanding of Gaussian process. It starts with the prior distribution over functions specified by a particular Gaussian process.This prior is taken to represent our prior beliefs over the kinds of functions we expect to observe, before seeing any data. Samples from a function will be normally distributed, where the covariance between any two samples is the covariance function (or kernel) of the Gaussian process evaluated at the spatial location of two points. A set of values is then observed, each value associated with a spatial location. Now, a new value can be predicted at any new spatial location, by combining the Gaussian prior with a Gaussian likelihood function for each of the observed values. The resulting posterior distribution is also Gaussian, with a mean and covariance that can be simply computed from.

B. Applications in solving engineering problems

H.Xu et. al. [3] have proposed a supervised learning-enhanced cokriging method to solve multi-material structural design problems. The proposed method is compared with several existing mixed-variable Surrogate model methods to understand their pros and cons. These methods include Neural Network (NN) regression, Classification and Regression Tree (CART) and Gaussian Process (GP). By applying all these methods to optimize material selection and structure design of coil spring, Gaussian Process Regression is found to have a better accuracy and robustness comparing to other surrogate model methods when there is a small number of training points.

A.Chiplunkar et.al. [4]–[6] have proposed to build better Gaussian Process (GP) regression models by integrating the prior knowledge of Aircraft design with experimental data. For example, by using spectral mixture kernels, surrogates models are built for structural dynamic experiments and dynamic parameters such as modal frequency are automatically identified. Relationships between multiple outputs and two main methods of scaling Gaussian Process samples are also discussed in their article.

S.Suram et.al. [7] have constructed a reduced order model for a hydraulic mixing nozzle using proper orthogonal decomposition (single value decomposition). Flow structures are compared for the original and optimized shape nozzles to study the effect of changing the nozzle's shape on the flow characteristics. Inspired by their study, single value decomposition can also be used in our project to get the reduced order model and reduce the code.

IV. First results and future work

A. Planning

The whole project planning can be summarized as follows:

1. Read the ABAQUS output information to MATLAB.

2. Construct a surrogate model to predict the maximum principle stress for each load combination.

- Input: Loads: F_x,F_y.
- Output: Maximum Principle Stress σ_{max}

3. Optimize the surrogate model to predict the maximum principle stress and its position for each load combination

- Input: Node coordinates: x,y, Loads: F_x,F_y.
- Output: Max Principle Stress σ_{max}, Position: x_{max}, y_{max}.

4. Construct a surrogate model to predict the principle stress distribution on each node for each load combination.

- Input: Node coordinates: x,y, Loads: F_x,F_y.
- Output: Principle Stress on each node: σ.

5. Use singular value decomposition (SVD) based learning algorithm to reduce the code.

B. First Result

Until now I have finished the first step of my project: writing the Matlab code to read the ABAQUS output information to MATLAB. The principle stress σ on each node with its coordinate x,y for each load combination have been read and storied in Matlab matrix.

Acknowledgment

I would like to thank my tutor Dr. Joseph Morlier, who gave me this opportunity to work on this project and also help me in the the academic side. My thanks also go to Dr. Simone Coniglio, who has presented the whole subject of this project to me and has told me how to finish my work step by step. Last but not least, I would like to express thanks to Dr. Pierre-jean Barjhoux with his kindly help with my questions in Matlab.

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