Strategic Generation With Conjectured Transmission Price Responses in a Mixed Transmission Pricing System—Part I: Formulation

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***Abstract*—The conjectured supply function (CSF) model calculates an oligopolistic equilibrium among competing generating companies (GenCos), presuming that GenCos anticipate that rival firms will react to price increases by expanding their sales at an assumed rate. The CSF model is generalized here to include each generator's conjectures concerning how the price of transmission services (point-to-point service and constrained interfaces) will be affected by the amount of those services that the generator demands. This generalization reflects the market reality that large producers will anticipate that they can favorably affect transmission prices by their actions. The model simulates oligopolistic competition among generators while simultaneously representing a mixed transmission pricing system. This mixed system includes fixed transmission tariffs, congestion-based pricing of physical transmission constraints (represented as a linearized dc load flow), and auctions of interface capacity in a path-based pricing system. Pricing inefficiencies, such as export fees and no credit for counterflows, can be simulated. The model is formulated as a linear mixed complementarity problem, which enables very large market models to be solved. In the second paper of this two-paper series, the capabilities of the model are illustrated with an application to northwest Europe, where transmission pricing is based on such a mélange of approaches.**

***Index Terms*—Complementarity, electricity competition, electricity generation, market models, strategic pricing, transmission pricing.**

I. Introduction

MANY game-theoretic price models have appeared in the literature that explicitly consider strategic market actions by electricity generators who possess market power. The models differ from one another in several ways [10], [11], [19].

- The types of markets simulated (bilateral/decentralized vs. central auction/POOLCO/market splitting).
- Network representation, including no transmission constraints, transportation (or "path-based") pricing models, dc linearizations, and full ac load flow.
- The generator's strategic variables (e.g., bids, production, sales, and purchases of transmission services) and type of game (Cournot, Bertrand, and supply function equilibria models being most popular[1]).
- Solution methodology (iteration among players/ Gauss–Seidel, payoff matrices, closed form solution, mathematical programs with equilibrium constraints, quadratic programming, and complementarity solvers, among other methods).

Generally, however, when models have been used to analyze the effects of transmission constraints [e.g., [3], [4], [6], [9], and [11]], they have usually assumed that transmission is efficiently allocated and priced through some sort of congestion pricing scheme, such as locational marginal pricing (LMP) or flowgate pricing (FG) [7], [23]. By "efficient," we mean that the market for transmission is complete (each firm that buys transmission services has the same marginal valuation of comparable services [5]), and the price of services is set to maximize the value of services provided, rather than being fixed by regulation. The LMP model of transmission pricing is used by the transmission system operators (TSOs) in the northeastern U.S. power markets, and is a central element of the U.S. Federal Energy Regulatory Commission's (FERC's) proposed Wholesale Power Market Platform [35].

Efficient pricing implies, for example, that a price of zero is what a TSO should charge for any point-to-point transmission service that would not alter the amount of flow on any fully used transmission line (or other transmission constraint). This is because generation would not have to be redispatched to accommodate that flow, so there is no cost for providing the service. (Of course, system losses or the amount of ancillary services required might be affected, which could by themselves imply a nonzero price for transmission service; however, we focus here on congestion.) Efficient pricing also implies symmetric pricing: a party requesting a transmission service that decreases flow on a congested transmission line by X will receive a payment *from* the TSO. Further, this payment equals the payment made *to* the TSO by a transaction that instead increases that

[1]In a Cournot game, each GenCo is modeled as choosing its optimal quantity (megawatt hour output or sales) assuming that its rivals will not change their quantities. In contrast, a Bertrand game involves competition in prices, in which each GenCo is represented as choosing its offering price(s) anticipating no response in the prices offered by others. The supply function equilibrium (SFE) game can be viewed as a generalization of both the Bertrand and Cournot models, in that each GenCo offers an nondecreasing bid schedule (price-quantity pairs), and is modeled as if it chooses its schedule assuming that rival GenCos would not alter their bids. If a solution exists among the GenCos in which no one believes it can improve its profit unilaterally, this is termed a Nash equilibrium. Standard game theory and industrial economics texts provide complete definitions of these and other types of games [e.g., [13]].

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same flow by X. This is called "flow netting" and is satisfied by pure LMP and FG pricing.

But transmission services in many jurisdictions are not priced in this manner. FERC has described several such inefficiencies [34], such as the "pancaking" of administratively-imposed transmission tariffs by TSOs that lie along an imaginary "path" that a transaction takes from the point of injection to the point of withdrawal. Below are listed some inefficient features of transmission management in the European Union (EU). (see [1], [5], [15], [25], [31] for summaries and critiques of EU congestion management and transmission pricing methods.)

- For transactions occurring within a zone controlled by a single TSO, transmission services are free or are assessed a "postage stamp" rate. (With the exception of Germany, TSO areas coincide with country borders in the EU.) This is even though such a transaction may aggravate or relieve congestion both within and outside of the region. As an example, internal congestion within Germany impacts the amount of exports that are possible to the Netherlands [17], but transmission fees within Germany do not reflect whether a transaction affects that congestion. When congestion problems occur within a zone, the TSO responds by redispatching generation, either by accepting "inc" and "dec" bids from generators in the balancing market (which creates locational price differences a la LMP) or by directly ordering generation changes.
- In contrast, congestion between the Netherlands and its neighbors (Germany and Belgium) is managed by auctioning off aggregate transmission capability on paths between countries, rather than capacity of individual congested facilities. (The French-Belgian interface is instead allocated by an administrative procedure.) Boucher and Smeers [5] describe some of the problems that can arise from auctions of such aggregations, and from inconsistencies between within- and between-country congestion management systems. For instance, although it is the net flow through an interface that matters, these auctions generally give no credit for counterflows. A justification offered is that transmission rights are options that might or might not be exercised, rather than firm commitments to flow power, and so the TSO must be able to accommodate the net flows whether or not all transmission rights are actually exercised. As a result of this policy, suppliers who are downstream of a constraint (i.e., in higher priced zones) do not have an efficient incentive to sell power upstream (in the lower cost zones), as they do not get revenue from relieving congestion.
- Transactions between countries are charged the same export fee, irrespective of their effect on congestion. (In the EU, the fee was called a network access charge, but it only applied to exports to other countries.)

The resulting inefficiencies include a failure of transactions whose true benefits exceed their costs to take place, while other transactions with negative net benefits are consummated. Market power may also be enhanced. In the case of the last two features listed above (export fees and no credit for counterflows), firms outside a particular country may be discouraged from competing in that market, increasing local market power. Besides transferring income from consumers to strategic suppliers, such diminished competition can have two distinct impacts on economic efficiency: *allocative* (pricing) *inefficiencies* and *production inefficiencies*.[2]

Market power can affect not only the prices that consumers pay, but also the revenues received by TSOs. In particular, suppliers may manipulate output to diminish payments in transmission capacity auctions. Oren [28] shows how Cournot producers who are upstream of transmission congestion may deliberately decrease their output in order to decongest lines and decrease the amount they pay the TSO. That is, by backing off its output slightly, a generator can slightly reduce the flow on an interface that was congested, and the interface's price would fall to zero. In a LMP market, the upstream nodal price (at the generator) would then increase, in turn increasing the generator's net revenue. Evidence that this may occur in northwest Europe is discussed in our companion paper [22].

A purpose of this paper is to present a model for analyzing generator market power on networks when a mix of transmission pricing policies are used. Such policies can include the following.

- Congestion pricing of networks, in which load flows are approximated by the linearized dc model [32]. This represents LMP or FG type pricing, as used in the northeastern U.S. TSOs, and also balancing market mechanisms that induce spatial differences in prices.[3]
- Auctions of capacity along administratively predetermined paths, where these paths can differ from the actual power flows. This practice may soon disappear in the U.S., thanks to the FERC Wholesale Power Market Platform, but is common in Europe.
- Export fees or other fixed (per megawatt hour) network use tariffs that have no necessary relationship to congestion costs. This fee rate is exogenous to our model; however, more general formulations allow tariffs to be set endogenously to recover a prespecified fixed transmission cost [36].

Modeling these policies makes it possible to analyze how inefficient transmission pricing policy may exacerbate (or perhaps ameliorate) the efficiency distortions caused by market power.

[2]An example of an allocative inefficiency is when a firm withholds capacity to raise price p above marginal cost *MC*; then, if demand is at all responsive to price, some consumers will wind up not buying some power whose value (consumer willingness to pay) nevertheless exceeds *MC*. Thus, some socially beneficial consumption does not take place. An illustration of a productive inefficiency is when there is a mix of large and small firms. The larger ones have an incentive to restrict output so their $MC < p$; as a consequence, their marginal costs will be less than for competitive firms whose $MC \sim p$. Therefore, costly power from competitive firms will partially replace inexpensive capacity withdrawn by firms exercising market power.

[3]However, we do not represent the artificial arbitrage opportunities that arise in multisettlement systems when forward markets impose a single price or zonal pricing, while balancing markets implicitly or explicitly result in more fine-grained nodal pricing. These types of inconsistencies characterize many power markets. An example is in The Netherlands where the Amsterdam Power Exchange provides a single day-ahead price but Tennet (the TSO) uses an "inc-dec" system to clear within-country congestion in the real-time market. Where there is significant congestion, this can lead to phenomena such as the infamous "dec" game, in which providers deliberately overcommit to sell day-ahead and produce congestion, which they are then paid in real-time to relieve by backing down their generation. The result is revenue without production.

TABLE I
COMPARISON OF ALTERNATIVE TRANSMISSION MARKET MODEL FORMULATIONS

Market Model	Formulation of GenCo Optimization	Formulation of Equilibrium Model	Solution Technique	Equilibrium Characteristics under General Conditions
MCP (Transmission Price Taker) [11,19]	Convex NLP: MAX Profit, subject to *exogenous* price of transmission services	MCP	MCP Solver (such as PATH [12])	Exists; prices & profits unique [27]
MPEC/EPEC [4,6,21]	MPEC: MAX Profit, s.t. *endogenous* transmission prices as an (exact) function of generation, sales, and/or bids (by putting TSO KKTs in constraints)	EPEC	Iteration among MPECs solved by NLP solvers or specialized MPEC methods	May not exist; if exists, there may exist several distinct price equilibria
MCP (Conjectured Price Response) (This paper)	Convex NLP: MAX Profit, s.t. *endogenous* transmission prices as an (inexact) first-order function of generation and sales (using assumed slope)	MCP	MCP Solver	Exists; prices & profits unique

The second purpose of this paper is to explicitly document how a network-constrained market equilibrium model formulated as a mixed complementarity problem (MCP) can be derived from models of generator, TSO, and arbitrager behavior.[4] The intent is to help model developers who wish to use this powerful market modeling approach by showing each step in the derivation. Previous papers that present related models [11], [19] only summarized some of the derivations.

Besides its representation of inefficient transmission pricing, another distinctive feature of the model of this paper is that it includes generator conjectures about how their demand for transmission services will alter the prices of those services. Previous complementarity-based models of competition on power networks were formulated assuming that generators were price takers relative to transmission prices. As discussed in [11], this assumption made possible the solution of large-scale models (e.g., of the eastern interconnection of North America [18]) using efficient complementarity solvers (e.g., PATH [12]). The complementarity formulation also facilitated the establishment of existence and uniqueness properties [27]. In contrast, models in which a generator correctly anticipates how the costs of transmission will change in response to its generation decisions require that the first-order conditions for the TSO be imbedded in the generator's constraint set [6], [21]. This type of model, called a mathematical program with equilibrium constraints (MPEC), accounts for how changes in generation decisions can cause the TSO's transmission constraints to shift from binding to nonbinding (and vice versa), thereby changing transmission prices. However, embedding the TSO's Karush-Kuhn-Tucker (KKT) conditions in the GenCo's optimization problem causes that problem to have a nonconvex feasible region, and for that reason, such a model is generally difficult to compute for large systems [26]. If each generator is solving such an MPEC, the problem of obtaining an equilibrium among such MPECs is called an equilibrium problem with equilibrium constraints (EPEC) [10]. Because the MPEC problem is generally nonconvex, such an equilibrium might not exist or there might be multiple equilibria [3], [20], [21].[5]

As summarized in Table I, the model of this paper attempts to approach the realism of the MPEC/EPEC-based approach while retaining the computational ease of MCP models. It does this by allowing the generator to believe that the cost of transmitting power between two points or the price resulting from auctions of transmission interfaces can be affected by the generator's demand for transmission services. This belief is captured in the form of a *conjectured transmission price response*, an exogenous parameter expressing the derivative of the price with respect to the amount of services demanded. This conjecture is the GenCo's belief about the first-order approximation of the transmission price around its equilibrium. In contrast to the MPEC-based approach, in which the generator correctly and *endogenously* calculates the transmission price response (by including the TSO's first-order conditions in the GenCo constraint set), the conjectured price response is an *exogenous* assumption representing the modeler's judgment about how each generator might anticipate that the price will change. Very large generators might be assumed to expect a large response, while small ones might anticipate no response (the latter being the assumption of previous MCP models). Since "true" conjectures held by generators are difficult or impossible to estimate,[6] the conjectured price response is best used parametrically to show how a generator who considers its effect on transmission prices will behave differently from a generator who does not.[7]

This paper presents the model formulation. The next section defines notation, while Section III documents the basic profit maximization problem for each market player. The first-order conditions for those problems plus the market clearing conditions yield a market equilibrium problem formulated as a mixed complementarity problem (Section IV). The Appendix

[4]A complementarity problem is a problem of the following general form: Find vector X such that $X \geq 0$, $F(X) \leq 0$, and $X^T F(X) = 0$, where $F(\,)$ is a vector-valued function of the same dimension as X and T is the transpose operator. The term "complementarity" applies because either x_i or $f_i(X)$ can be positive, but not both, where x_i is the i^{th} element of X and $f_i(X)$ is the i^{th} element of $F(X)$. An MCP is more general: find vectors X and Y, such that $X \geq 0, F(X,Y) \leq 0, X^T F(X,Y) = 0$, and $G(X,Y) = 0$, where $G(\,)$ is a vector-valued function of the same length as Y. See the Appendix for a simple example of a complementarity formulation of a market equilibrium problem, and a contrast with other formulations.

[5]See the Appendix for an example of an MPEC, and a comparison with complementarity problems.

[6]Garcia *et al.* [14] show how GenCo conjectures regarding reactions of rival suppliers to power prices can be estimated from market data. A similar approach can be used to obtain transmission price conjectures, based on information on energy and transmission prices, loads, and generator outputs and costs.

[7]In [21], the results of a two-node EPEC model in which two GenCos correctly anticipate transmission price changes are compared with a two-node MCP model in which the two generators are price-takers with respect to transmission; the results are sometimes the same, but often are not. For instance, there exists no equilibrium in the EPEC model for some values of transmission capacity when the generators have the same costs and are located at different nodes, but an equilibrium always exists for the MCP model.

provides background on complementarity and MPEC models along with simple examples. A companion paper [22] presents some illustrative results of the model for the Belgium-Netherlands market. There we describe the effects of some alternative representations of generator behavior, and the interaction of market power with transmission market design and generation market structure.

II. Model Notation

Variables are designated as lower case latin letters (primal variables) or lower case greek letters (LaGrange or dual variables), while coefficients are given in upper case. Dual variables are not defined in this subsection, but are instead introduced within parentheses to the right of their constraints in the models of Section III. Indices and their sets are represented by lower and upper case latin letters, respectively.

A. Indices and Sets

A distinction between this paper's model and previous complementarity models is its inclusion of two separate representations of power flows, each with its own indices (defined in this subsection) and variables. One is the linearized dc power flow, as in [19], whose flows satisfy the dc model's analogues of Kirchhoff's laws and are constrained by thermal line ratings and other physical limitations. The second representation is a simplified accounting system that tracks interregional power transfers and their more-or-less fictional "paths" for the purpose of administering auctions of a few aggregate power system interfaces. In an ideal world, the two systems would be equivalent; but in practice, they can diverge significantly [5]. The accounting system is based on single (rather than parallel) paths and often uses very conservative values for interface capacity in order to ensure actual flow feasibility under a range of conditions. Counterflows might not be given credit, which can result in binding constraints in both directions. Consequently, the path-based commercial system can be more of a constraint upon interregional transfers than the actual physical system. Both transmission systems are considered simultaneously in the model, as either or both can be binding in any particular simulation, as is the case in actual markets. Network nodes (or aggregations of buses) i are used in the dc load flow, while aggregations of nodes used in the simplified path-based accounting system are designated by country c.[8,9]

In the actual implementation of the model, all variables and relevant coefficients are indexed by time period t (summer peak period, winter shoulder period, etc.). However, we omit t here for brevity.

$c \in C$	Set of countries used in path-based transmission model.
$c(i)$	Country where transmission node i is located.
$f \in F$	Set of generation firms.
$h \in H(f,i)$	Set of generating units owned by f at i.
$i \in I$	Set of nodes in linearized dc transmission network.
$i \in I^A$	Set of nodes in dc transmission network subject to arbitrage.
$i \in I_c$	Set of nodes in transmission network within country c.
$k \in K$	Set of constrained flowgates (used in the dc load flow-based model of transmission).
$m \in M$	Set of constrained interfaces with net transfer capabilities (used in the path-based model of transmission).

B. Primal Variables

If a variable x has an asterisk (x^*), this indicates that the variable is exogenous to the generation firms and TSO, but endogenous to the market. An example can be a price variable p; a price-taking firm naïvely views it as fixed (p^*), even though the full market model equilibriates price to order to equate supply with demand. For several primal variables, both x and x^* appear in the generator's profit-maximization model. Such a pair $\{x, x^*\}$ is used to model how the generator conjectures that the variable will change if some other particular variable (say, u) changes. Then x^* is the (exogenous to the firm) equilibrium level, while x represents the value that the firm conjectures that the variable will move to if the generator changes u from its equilibrium level u^*. That is, $(x - x^*) = \mathrm{A}(u - u^*)$, where A is a constant in a first-order approximation. For instance, as in [11], sales s_{-fi} at node i by firm f's rivals are assumed by f to increase if price p_i increases from its equilibrium value

$$(s_{-fi} - s^*_{-fi}) = SFC_{-fi}\,(p_i - p^*_i)$$

where $SFC_{-fi} > 0$ is the "supply function conjecture" that f holds concerning its rivals' sales at that node. The model of this paper not only has such conjectured relationships for rival suppliers, but also for prices of transmission, as explained in the following.

Generator's physical variables

g_{fih}	MW generation by unit h owned by firm f at node i.
s_{fi}, s^*_{fi}	MW sales by firm f at node i.
s_{-fi}, s^*_{-fi}	MW sales by firms other than f at node i.
$t_{fcc'}$	MW power sales by f in country c' assigned to generation in country c. These flows are subject to the path-based transmission constraints.

Arbitrager's variables

a_i, a^*_i	MW transferred by arbitrager from the hub to node i. The choice of hub node is arbitrary in the linearized dc load flow formulation used in this model. That is, the load flows resulting from a power injection at one node and an equal withdrawal at another do not depend on the location of the hub through which the transaction is routed.

[8]Superposition of path-based and dc load flow constraints in the same competitive market model was proposed but not implemented by FERC [33].

[9]The aggregations used for the path-based accounting system are called "countries" because the model was first applied to the EU where the simplified accounting system is used only for intercountry flows. In other applications, for instance in North America, "c" could instead designate separate control areas whose interfaces are allocated by a path-based market process.

ap_c Net MW purchased by arbitragers in country c and transferred to other countries.

as_c Net MW sold by arbitragers in country c and transferred from other countries.

$t^a_{cc'}$ MW power sales by the arbitrager in country c' assigned to arbitrager purchases in country c. These are subjected to path-based transmission constraints.

TSO's variables

y_i MW of transmission services provided by TSO from hub node to i (subject to the dc load flow constraints).

z_m MW of flow through constrained interface m (subject to the path-based transmission constraints).

Price variables

p^*_i Euro/MWh price of energy at i.

w^*_i Euro/MWh price of transmission services from hub to i (associated with congestion in the linearized dc transmission system).

w_{fi} Euro/MWh price of transmission services from hub to i, conjectured by generator f.

wt^*_m Euro/MWh price of transfer capability for interface m in the path-based transmission system.

wt_{fm} Euro/MWh price of transfer capability for interface m, conjectured by f.

C. Coefficients and Functions

Generator's coefficients/functions

$C_{fih}(g_{fih})$ Total cost, in Euro per period, of generation for unit h owned by f at i. This function should be convex. For a linear mixed complementarity model (LMCP), this function should also be quadratic or affine.

CO_{fi} An index defining whether firm f behaves competitively $(CO_{fi} = 0)$ or strategically $(CO_{fi} = 1)$ at node i.

G_{fih} MW capacity of generator fih.

MC_{fih} Euro/MWh marginal cost of generation for unit h owned by f at i, if an affine $C_{fih}(g_{fih})$ is assumed.

SFC_{-fi} Slope of conjectured rival supply function for f at i, MW/(Euro/MWh) [(GEN5)]. Note that:

- if $SFC_{-fi} = 0$, then the Cournot model results (generators believe that rivals will not alter the amounts they supply even if price changes);
- if $SFC_{-fi} \rightarrow \infty$, then the perfect competition model results (a small price change elicits a huge supply response; also called the Bertrand conjecture).

WC_{fi} Slope of conjectured transmission price response function for f at i (Euro/MWh)/(MW) ((GEN6), associated with dc load flow congestion-based prices).

WCT_{fm} Slope for firm f's conjectured transfer capability price response function for interface m (Euro/MWh)/(MW) [(GEN7), associated with path-based transmission prices].

TSO's coefficients

$PTDF_{ik}$ Power transmission distribution factor MW/MW describing the MW increase in flow through flowgate k (in dc load flow) resulting from a 1 MW increase in power transferred from the (arbitrary) hub to node i.

PTC_m Net MW transfer capability for interface constraint m (in path-based transmission system).

$PTCU_{cc'm}$ MW of transfer capability in interface m consumed by a 1 MW transfer from country c to country c' (based on administratively defined path from c to c').

T_k Upper MW limit for flow through flowgate k (in dc load flow).

Market coefficients/functions

$EFEE_{cc'}$ Fixed Euro/MWh export fee for sales from country c to country c'. This can also be used to model grid access charges if all exports from c (including "exports" to c itself) are charged the same fee.

$P_i(\Sigma_{f\in F}s_{fi}+a_i)$ Euro/MWh inverse demand function at i. (Note that in this paper a_i is understood to be omitted from this expression for a given i if arbitragers do not have access to a node.) For i that border neighboring markets, the function should be adjusted to account for net imports/exports and their elasticities. The function should also be strictly decreasing, doubly continuous, and such that revenue is a concave function. For a LMCP, this function should also be affine:

$$P_i(\Sigma_{f\in F}s_{fi} + a_i) = P^o_i - \left(\frac{P^o_i}{Q^o_i}\right)(\Sigma_{f\in F}s_{fi} + a_i),$$

where P^o_i is the price intercept of the demand function and Q^o_i is the quantity intercept. If the affine function passes through point $\{P_i^1, Q_i^1\}$, and has an elasticity of $\varepsilon_i < 0$ at that point, then

$$P_i^o = \left(1 - \frac{1}{\varepsilon_i}\right) P_i^1$$
$$\frac{P_i^o}{Q_i^o} = -\frac{P_i^1}{(Q_i^1 \varepsilon_i)}.$$

III. Profit Maximization Problems and Market Clearing Conditions

Before introducing the models for individual market participants below, we discuss the principles underlying the path-based transmission pricing simulation.

To represent the path-based transmission system, we create an accounting system for each firm in which nonnegative flows from one country to another are defined by a simple transportation model for finding least-cost paths. Nonnegativity is required because export fees are applied only to positive transfers from one country to another, and because nonnegative flows are required to correctly account for (and, depending on the pricing policy, exclude) counterflows from path-based interface constraints. (Counterflows can be omitted by dropping terms with negative $PTCU_{cc'm}$ coefficients from the transfer capability terms in conditions (GEN1), (GEN7), (A1), and (MC3)) A similar accounting system is also defined for the arbitrager's model. These intercountry flows are then constrained by transfer capabilities via (MC3) and (TSO4).

We could formulate the path-based transportation model in two distinct ways that differ in the commodity that is traded:[10]

1) Generators can buy separate *portions of constrained interfaces* (e.g., someone selling power to Belgium from Germany might be required to buy portions of both the German-Dutch interface and the Dutch-Belgian interface, if the transmission path passes through the Netherlands).
2) Generators can buy *bundled country-to-country transmission services* from the TSO, and the TSO determines how much of each constrained interface is consumed (e.g., a German producer selling in Belgium buys transmission service from Germany to Belgium, and the TSO calculates how much of the relevant interfaces are required).

The two are mathematically equivalent if $WCT_{fm} = 0$; we choose the former representation because it resembles the interface auction used for the Dutch interfaces.

A. Generator Model for $f \in F$

The structure of the GenCo model can be summarized as follows. A GenCo chooses its generation and sales in order to maximize its profit, equaling revenue from sales minus the cost of generation and transmission services. Constraints on the decision variables include: equations relating sales and generation at different locations to path-based transmission services; capacity constraints on generation; and equations representing the GenCo's conjectures about how energy prices affect rivals' sales and how demands for transmission services affect path-based and congestion-based transmission prices. A more detailed explanation is provided following the formulation

$$\begin{aligned}
\text{MAX } & \Sigma_{i\in I}\left[(1-CO_{fi})p_i^* \right.\\
&\qquad \left. + CO_{fi}P_i\left(s_{fi}+s_{-fi}+a_i^*\right) - w_{fi}\right]s_{fi}\\
&- \Sigma_{i\in I}\Sigma_{h\in H(f,i)}\left[C_{fih}(g_{fih}) - w_{fi}g_{fih}\right]\\
&- \Sigma_{c\in C}\Sigma_{c'\in C}(EFEE_{cc'}\\
&\qquad + \Sigma_{m\in M} wt_{fm}PTCU_{cc'm})t_{fcc'} \qquad \text{(GEN1)}
\end{aligned}$$

subject to :

$$\Sigma_{i\in I^c}s_{fi} - \Sigma_{c'\in C}t_{fc'c} = 0 \quad \left(\theta_{fc}^S\right)\ c\in C \qquad \text{(GEN2)}$$

$$\begin{aligned}
&- \Sigma_{i\in I^c}\Sigma_{h\in H(f,i)}g_{fih}\\
&\quad + \Sigma_{c'\in C}t_{fcc'} = 0 \quad \left(\theta_{fc}^G\right)\ \ c\in C \qquad \text{(GEN3)}
\end{aligned}$$

$$g_{fih} \le G_{fih} \qquad (\mu_{fih})\ \ i\in I,\ h\in H(f,i) \qquad \text{(GEN4)}$$

$$\begin{aligned}
&s_{-fi} - \left\{s_{-fi}^* + SFC_{-fi}\left[P_i\left(s_{fi}+s_{-fi}+a_i^*\right) - p_i^*\right]\right\} = 0\\
&\qquad (\beta_{fi})\ \ i\in I \qquad \text{(GEN5)}
\end{aligned}$$

$$\begin{aligned}
&- w_{fi} + \left\{w_i^* + WC_{fi}\left[\left(s_{fi} - \Sigma_{h\in H(f,i)}g_{fih}\right)\right.\right.\\
&\qquad \left.\left. - \left(s_{fi}^* - \Sigma_{h\in H(f,i)}g_{fih}^*\right)\right]\right\} = 0\\
&\qquad (\gamma_{fi})\ \ i\in I \qquad \text{(GEN6)}
\end{aligned}$$

$$\begin{aligned}
&- wt_{fm} + \left\{wt_m^* + WCT_{fm}\left[\Sigma_{c\in C}\Sigma_{c'\in C}PTCU_{cc'm}\right.\right.\\
&\qquad \left.\left. \times\left(t_{fcc'} - t_{fcc'}^*\right)\right]\right\} = 0\\
&\qquad (\zeta_{fm})\ \ m\in \mathrm{M} \qquad \text{(GEN7)}
\end{aligned}$$

$$\forall g_{fih},\ s_{fi},\ t_{fcc'} \ge 0.$$

As noted earlier, the decision variables in this profit maximization model (as in the TSO and arbitrager models below) are any x without an asterisk; in contrast, those variables x^* with an asterisk are exogenous to the optimization (but endogenous to the market model as a whole). In this formulation, three types of prices are endogenous: energy prices at locations where $CO_{fi} = 1$ (function $P_i(s_{fi} + s_{-fi} + a_i^*)$), the price w_{fi} of transmission services from the hub node to node i, and the price of path-based transmission interfaces wt_{fm}. If the firm is strictly a price-taker for energy ($CO_{fi} = 0$ for all i), then only transmission prices are endogenous. The exogenous variables include arbitrage (consistent with our Cournot assumption), and the equilibrium prices and quantities that are used in the conjectural (GEN5)–(GEN7), explained below.

The first two constraints are energy balances. (GEN2) defines the sum of firm f's energy sales in a country c as equaling the power imported by f (including "imports" from the same country c). (GEN3) defines the sum of exports by f from c (including "exports" to the same country) as the sum of that firm's output in that country. Next, (GEN4), is a generation capacity constraint. (Note that θ_{fc}^S, θ_{fc}^G, and β_{fi} are the respective dual variables for those constraints.)

Constraints (GEN5)–(GEN7) represent firm f's conjectures regarding how rival sales and the cost of transmission will change if f shifts its sales and generation decisions from their equilibrium values. (GEN5) is the conjectured supply function (CSF) for rival generators, in which their sales are assumed

[10] Note that this modeling choice also has to be made for the dc load flow model. Supplier purchases of individual flowgate capacities correspond to the FG rights proposal, while supplier purchases of point-to-point transmission services corresponds to LMP. The two systems are equivalent under pure competition. Our model's dc load flow pricing approach corresponds to the latter system (as in [11], [19]), but the choice is arbitrary and does not affect the equilibrium energy prices or profits.

to diverge linearly from the equilibrium value if price $P_i()$ is changed from its equilibrium value p_i^*.[11] The larger SFC_{-fi} is, the greater the supply response that firm f anticipates from other firms.

Meanwhile, (GEN6) and (GEN7) represent how the firm conjectures that the price of transmission services will change if the firm changes the amount of services it demands. (GEN6) is the conjecture for the price of hub-to-node transmission in the dc load flow, in which f assumes that w_{fi} deviates from w_i^* if the firm's sales minus generation at i are changed from their equilibrium values. (GEN7) instead represents how the price of path-based interface m is conjectured to change, showing how wt_{fm} changes from wt_m^* when the firm's intercountry flows are shifted from their equilibrium values.

The above GenCo model can be made more sophisticated in several ways. For instance, it is easy to add a term in the objective function to account for the desirability of increasing market share. This would be done by adding $P_{MSfi}\Sigma_{i\in I}s_{fi}$ to the generator's objective function, with P_{MSfi} representing firm f's willingness to sacrifice profit to increase sales at i (in Euro/MWh). Inclusion of such a term has been interpreted as representing a generator's concern with long-run profitability [2]. Contracting positions (forward or contracts for differences) can also be represented with appropriate terms [2], [16]. Pumped storage or hydropower can be modeled by defining storage and release variables together with constraints imposing continuity among periods. Linear unit commitment constraints (such as ramping rates) can also be imposed, but not binary constraints such as start-up costs.

B. TSO Model

The TSO efficiently allocates scarce transmission capacity to the most highly valued transmission services. Services are divided into two types: the first, y_i, are constrained by the dc load flow model (approximating actual physical flows and constraints), while the second, z_m, is the path-based model (representing the use of interfaces between countries). Under the linearized dc load flow assumption, the flow through a transmission flowgate [the left side of constraint (TSO2–3)] depends linearly upon the transmission services demanded. The path-based model is simpler; interface flows are subject only to simple upper bounds, representing the assumed available transmission capability. As noted earlier, either or both types of services can be represented in an actual implementation of the model.

Efficient allocation of transmission can be modeled by representing the TSO as a profit maximizer who is a "price taker" [10]. It chooses what services y_i and z_m to provide in order to maximize that profit (TSO1), subject to both dc load flow constraints (TSO3–4) and path-based constraints (TSO4). In other words, this formulation chooses the most valuable mix of transmission services (where "value" is based on the willingness-to-pay for transmission as revealed in the prices), subject to feasibility of the resulting dc load flow and path-based flows

$$\text{MAX} \quad \Sigma_{i\in I}{w_i}^* y_i + \Sigma_{m\in M} wt_m^* z_m \qquad \text{(TSO1)}$$

$$\text{s.t.}: \; \Sigma_{i\in I} PTDF_{ik} y_i \le T_k \quad (\lambda_k) \;\; k\in K \qquad \text{(TSO2)}$$

$$\Sigma_{i\in I} y_i = 0 \qquad (\theta) \qquad \text{(TSO3)}$$

$$z_m \le PTC_m \qquad (\psi_m) \;\; m\in M. \qquad \text{(TSO4)}$$

Note that given a set of values of transmission services y_i and z_m, the TSO has no discretion in routing them (consistent with fixed PTDFs and paths). However, if we instead used a more general TSO model with phase shifters or other FACTS devices, then some discretion would exist, at least with respect to the dc constraints (TSO2).

Another, equivalent interpretation of this model is that the TSO efficiently clears the market. This can be understood from the KKT conditions for this model (conditions (KT1)–(KT3), Section IV), which ensure that the following is true. If a transmission price is positive, it set at a level at which the demand for that constraint (flows) equals the supply (capacity). But if a transmission constraint is slack (demand $<$ supply), the price is zero.

C. Arbitrager

Where arbitragers have access to markets, they buy power in one place and resell it elsewhere if its purchase price (plus transmission service charges) is no more than the price they could sell it for. Arbitrage from country c to country c' is represented by nonnegative variable $t_{cc'}^a$. Meanwhile, arbitrage from the hub node to a node i (which can be positive or negative) is given by unrestricted variable a_i. These distinct types of arbitrage variables are required in order to correctly model flows in both the dc-based and path-based transmission pricing models. For instance, nonnegative between-country arbitrage is necessary to account for export fees or other tariffs.

Based on the above assumptions, the arbitrager's model maximizes profit (based on price differences between nodes, net of transmission payments and export fees), subject to constraints that define between-country flows based on the amounts arbitraged

$$\begin{aligned}\text{MAX} \quad & \Sigma_{i\in I^A}({p_i}^* - {w_i}^*)\,a_i \\ & - \Sigma_{c\in C}\Sigma_{c'\in C}\,(EFEE_{cc'} \\ & \qquad + \Sigma_{m\in M} {wt_m}^* PTCU_{cc'm})\, t_{cc'}^a \qquad \text{(A1)}\end{aligned}$$

$$\text{s.t.}: as_c - \Sigma_{c'\in C} t_{c'c}^a = 0 \quad \left(\theta_c^{aS}\right) \; c\in C \qquad \text{(A2)}$$

$$-ap_c + \Sigma_{c'\in C} t_{cc'}^a = 0 \quad \left(\theta_c^{aP}\right) \; c\in C \qquad \text{(A3)}$$

$$-(as_c - ap_c) + \Sigma_{i\in I^A\cap I_c} a_i = 0 \quad (\rho_c) \;\; c\in C \qquad \text{(A4)}$$

$$\forall as_c, ap_c, t_{cc'}^a \ge 0.$$

Three sets of accounting constraints are needed to determine intercountry flows due to arbitrage; these flows are required in order to use the path-based transmission rights system and to calculate intercountry tariffs. Constraint (A2) defines the arbitrager's nonnegative sales in country c as equal to the power

[11]The CSF model is not the same as the SFE model [24], which calculates a Nash equilibrium in bid (supply) function strategies; a SFE model can be viewed as an endogenous calculation of the equilibrium SFC_{-fi}. Instead, the CSF model treats SFC_{-fi} as an exogenous parameter, which has significant computational advantages and can lead to a unique solution [11], unlike the SFE model. CSF-type models have been used for similar reasons in other applications [29]. Day [11] present two CSF models: one based on each generator conjecturing that their rivals follow a (conjectured) supply function with a constant slope, and another based on the conjecture that the rivals' supply function has a constant intercept. The latter results in a nonlinear complementarity problem, even if the demand and cost functions are affine. Only the former representation of supply conjectures is presented in the present paper.

it imports to that country (including "imports" from the same country c). Meanwhile, (A3) defines purchases of power at c as equaling exports from c (including "exports" to the same c). In (A4), sales net of purchases in c equal the total amount of power the arbitrager sends from the hub to nodes i located in c.

Because the a_i variables are unrestricted, it turns out that in equilibrium, an arbitrager will force price differences among nodes it has access to within a country to equal the cost of transmission between the nodes, consistent with the LMP system of Schweppe [32] and Hogan [23]. That is, $p_i^* - p_j^* = w_i^* - w_j^*$, which follows from KKT condition (KA1) in Section IV. Thus, the arbitrager can erase price differences that are not reflected in transmission prices.

Further, if arbitragers have access to all network nodes, and in addition transmission conjectures $WC_{fi} = 0$ and only the dc flow constraints are included in the model, then the following result has been proven [27]. This model of a bilateral market with full arbitrage yields the *same* prices, profits, and supplies as a Pool-type auction market model [19] in which generators sell all their output to the Pool at the generator bus, and generators anticipate that the Pool will perform the arbitrage function of moving power among nodes so that the LMP relationship (transmission prices = nodal price differences) is maintained.[12]

D. Market Clearing and Consistency Conditions

Market clearing conditions (MC1)–(MC3) enforce equality of supply and demand for various commodities (energy and transmission services). In the overall model, these conditions serve to couple the problems of the various market players. The other conditions impose consistency on the conjectures of firms (i.e., actual equilibrium quantities equal the quantities conjectured by the firms)

$$p_i^* = P_i(\Sigma_{f\in F} s_{fi} + a_i), \quad i \in I \quad \text{(MC1)}$$
$$y_i = a_i + \Sigma_{f\in F}\left(s_{fi} - \Sigma_{h\in H(f,i)} g_{fih}\right), \quad i \in I \quad \text{(MC2)}$$
$$z_m = \Sigma_{c\in C}\Sigma_{c'\in C} PTCU_{cc'm} \times \left(t^a_{cc'} + \Sigma_{f\in F} t_{fcc'}\right), \quad m \in M \quad \text{(MC3)}$$
$$s^*_{-fi} = \Sigma_{j\in F, j\neq f} s_{ji}, \quad f \in F,\ i \in I \quad \text{(MC4)}$$
$$s^*_{fi} = s_{fi}, \quad f \in F,\ i \in I \quad \text{(MC5)}$$
$$g^*_{fih} = g_{fih}, \quad f \in F,\ i \in I,\ h \in H(f,i) \quad \text{(MC6)}$$
$$t^*_{fcc'} = t_{fcc'}, \quad f \in F,\ c \in C,\ c' \in C \quad \text{(MC7)}$$
$$a_i^* = a_i, \quad i \in I. \quad \text{(MC8)}$$

Market clearing condition (MC1) is the definition of the demand function. (MC2) says that the transmission services provided in the dc load flow model equal those demanded by arbitragers and generators. (MC3) similarly says that transmission services in the path-based model (i.e., flows through interfaces) match the flows resulting from actions by arbitragers and generators. The coefficients $PTCU_{cc'm}$ map between-country flows to particular interfaces on assumed transmission paths.

The first market consistency condition (MC4) states that, in equilibrium, the amount that f conjectures that its rivals will supply equals the amount they actually sell. This is used in the supply function conjecture (GEN5). Meanwhile, (MC5)–(MC7) impose consistency on the firm's own decisions about sales, generation, and intercountry flows: their equilibrium values must equal their conjectured values. These variables appear in the transmission price conjectures (GEN6), (GEN7). Condition (MC8) says that the arbitraged quantities assumed by generators are consistent with the amounts arbitragers actually buy and sell.[13]

IV. Market Equilibrium as a Mixed Linear Complementarity Problem

By gathering together the first-order (KKT) conditions for the above generator, TSO, and arbitrager models, and then adding the market clearing and consistency conditions (MC1–8), a mixed complementarity model can be defined. This MCP should be square, having as many variables as conditions.[14] Whether a solution to these conditions exists and is unique can be determined analytically [8].

Below is the MCP resulting from concatenating the KKTs for the optimization models in Section III along with the market clearing conditions. This MCP is a reduced version, in that we have used several of the equality conditions to eliminate a number of the variables, greatly reducing the dimensionality of the model. The eliminated variables include a_i^*, g^*_{fih}, s^*_{fi}, s^*_{-fi}, s_{-fi}, $t_{fcc'}$, w_{fi}, wt_{fm}, β_{fi}, γ_{fi}, ζ_{fm}, ψ_m, and θ. We also assume affine demand $(P_i(\Sigma_{f\in F} s_{fi} + a_i) = P_i^o - (P_i^o/Q_i^o)(\Sigma_{f\in F} s_{fi} + a_i))$ and linear cost $(C_{fih}(g_{fih}) = MC_{fih} g_{fih})$.

These eliminations are advantageous, as can be shown by counting the number of variables that are dropped. Let $|S|$ be the number of elements in set S. There exist $|F||I|$ each of eliminated variables s^*_{fi}, s^*_{-fi}, s_{-fi}, w_{fi}, β_{fi}, and γ_{fi}. In addition, there are $|I^A|$ of a_i^*; $|H| \equiv \Sigma_{f\in F}\Sigma_{i\in I}|H(f,i)|$ of g^*_{fih}; $|F||C|^2$ of $t_{fcc'}$; $|F||M|$ each of ζ_{fm} and wt_{fm}; $|M|$ of ψ_m; and one θ. Deleting these variables typically reduces the model size by more than half. For instance, if there are 20 firms, 20 nodes, 4 countries, 400 power plants, and 20 congested interfaces in the path-based transmission system, and arbitragers have access to all nodes, then 3961 variables and conditions would be eliminated. If, in addition, the number of capacity constraints in the dc load flow is 40, the size of the reduced model given below becomes 1876 variables and an equal number of conditions.

For compactness, the complementarity notation "$0 \leq x \perp f(z) \leq 0$" is used below instead of the equivalent two inequalities and single equality: "$0 \leq x$; $f(z) \leq 0$; $xf(z) = 0$." Also, $f'(z)$ is defined as the first partial derivative of $f(z)$ with respect to z. To verify model "squareness," it is helpful to associate a different variable with each condition, as we do below.

[12]An assumption of this particular Pool model is that generators anticipate that when they alter their output, the differences in LMPs among nodes will not change. Other Pool models make different assumptions (for instance, the MPEC-type models mentioned in Section I and the Appendix that correctly anticipate how the Pool will adjust prices [e.g., [4], [6], [21]]), and so their results can diverge from the fully arbitraged bilateral model of this paper [20].

[13]There are also other consistency conditions that do not have to be explicitly stated. One is that $w_{fi} = w_i^*$ for all f, i. However, it is not necessary to state that condition, as it is implied by (MC5), (MC6), and (GEN6). Two other unstated consistency conditions are: $s^*_{-fi} = s_{-fi}$ for all f, i (implied by (MC1) and (GEN5)); and $wt^*_m = wt_{fm}$ for all f, m (implicit in (MC7) and (GEN7)).

[14]See the Appendix for a simple example of this concatenation of market participant KKTs and market clearing conditions.

Elsewhere [30], it is shown that under mild conditions there exists a solution to MCPs of the general form presented below, and that the resulting prices, profits, quantities demanded, and total quantities produced by each Cournot generator are unique. (However, the generation from individual units might not be unique if a firm owns identical units.)

The coupled (simultaneous) market model is as follows.

Generator Model KKTs for $f \in F$

For each g_{fih} $(f \in F,\ i \in I,\ h \in H(f,i))$:

$$0 \le g_{fih} \perp -MC_{fih} + \left[w_i^* + WC_{fi}\left(s_{fi} - \Sigma_{j\in H(f,i)} g_{fij}\right)\right] - \mu_{fih} + \theta^G_{fc(i)} \le 0 \quad \text{(KG1)}$$

For each s_{fi} $(f \in F,\ i \in I)$:

$$0 \le s_{fi} \perp p_i^* - \frac{CO_{fi}\left(\frac{P_i^o}{Q_i^o}\right) s_{fi}}{\left[1 + \left(\frac{P_i^o}{Q_i^o}\right) SFC_{-fi}\right]} - \left[w_i^* + WC_{fi}\left(s_{fi} - \Sigma_{h\in H(f,i)} g_{fih}\right)\right] - \theta^S_{fc(i)} \le 0 \quad \text{(KG2)}$$

For each $t_{fcc'}$ $(f \in F,\ c \in C,\ c' \in C)$:

$$0 \le t_{fcc'} \perp -EFEE_{cc'} - \Sigma_{m\in M} \times \{\{wt_m^* + [WCT_{fm}\left(\Sigma_{d\in C, d'\in C} PTCU_{dd'm} t_{fdd'}\right)]\} \times PTCU_{cc'm}\} + \theta^S_{fc'} - \theta^G_{fc} \le 0 \quad \text{(KG3)}$$

For each θ^S_{fc} $(f \in F,\ c \in C)$:

$$\Sigma_{i\in I^c} s_{fi} - \Sigma_{c'\in C} t_{fc'c} = 0 \quad \text{(KG4)}$$

For each $\theta^G_{fc}(f \in F,\ c \in C)$:

$$-\Sigma_{i\in I^c}\Sigma_{h\in H(f,i)} g_{fih} + \Sigma_{c'\in C} t_{fcc'} = 0 \quad \text{(KG5)}$$

For each μ_{fih} $(f \in F,\ i \in I,\ h \in H(f,i))$:

$$0 \le \mu_{fih} \perp g_{fih} - G_{fih} \le 0. \quad \text{(KG6)}$$

TSO KKTs

For each $y_i (i \in I)$:

$$w_i^* - \Sigma_{k\in K} PTDF_{ik}\lambda_k = 0 \quad \text{(KT1)}$$

For each $\lambda_k (k \in K)$:

$$0 \le \lambda_k \perp \Sigma_{i\in I} PTDF_{ik} y_i - T_k \le 0 \quad \text{(KT2)}$$

For each $z_m (m \in M)$:

$$0 \le wt_m^* \perp z_m - PTC_m \le 0. \quad \text{(KT3)}$$

Arbitrager KKTs

For each a_i $(i \in I^A)$:

$$(p_i^* - w_i^*) - \rho_{c(i)} = 0 \quad \text{(KA1)}$$

For each as_c $(c \in C)$:

$$0 \le as_c \perp -\theta_c^{aS} + \rho_c \le 0 \quad \text{(KA2)}$$

For each ap_c $(c \in C)$:

$$0 \le ap_c \perp +\theta_c^{aP} - \rho_c \le 0 \quad \text{(KA3)}$$

For each $t^a_{cc'}$ $(c \in C,\ c' \in C)$:

$$0 \le t^a_{cc'} \perp -\left(EFEE_{cc'} + \Sigma_{m\in M} wt_m^* PTCU_{cc'm}\right) + \theta^{aS}_{c'} - \theta^{aP}_c \le 0 \quad \text{(KA4)}$$

For each θ_c^{aS} $(c \in C)$:

$$as_c - \Sigma_{c'\in C} t^a_{c'c} = 0 \quad \text{(KA5)}$$

For each θ_c^{aP} $(c \in C)$:

$$-ap_c + \Sigma_{c'\in C} t^a_{cc'} = 0 \quad \text{(KA6)}$$

For each ρ_c $(c \in C)$:

$$-(as_c - ap_c) + \Sigma_{i\in I^A \cap I_C} a_i = 0. \quad \text{(KA7)}$$

Market Clearing Conditions

For each p_i^* $(i \in I)$:

$$p_i^* = P_i^o - \left(\frac{P_i^o}{Q_i^o}\right)\left(\Sigma_{f\in F} s_{fi} + a_i\right) \quad \text{(MC1)}$$

For each w_i^* $(i \in I)$:

$$y_i = a_i + \Sigma_{f\in F}\left(s_{fi} - \Sigma_{h\in H(f,i)} g_{fih}\right) \quad \text{(MC2)}$$

For each wt_m^* $(m \in M)$:

$$z_m = \Sigma_{c\in C}\Sigma_{c'\in C} PTCU_{cc'm}\left(t^a_{cc'} + \Sigma_{f\in F} t_{fcc'}\right). \quad \text{(MC3)}$$

The model, as now implemented at ECN, is a multiperiod version of the above MCP.[15]

V. Conclusion

Network constraints can create opportunities to exercise market power, resulting in uneven benefits of power sector liberalization. We have extended the capabilities of complementarity-based models of strategic generator behavior on transmission networks in two ways. The first extension represents multiple transmission pricing systems, including inefficiencies resulting from no netting of counterflows, path-based pricing, and export fees. The second extension models how generators might expect that transmission prices will change when demands for transmission services are altered. The application of these extensions is illustrated in [22], where we examine how market power and the design and structure of power markets can interact in sometimes unexpected ways.

Appendix
Simple Examples of Complementarity and MPEC/EPEC Market Models

A. *Examples of Complementarity-Based Models*

Complementarity-based models of markets are created by first deriving the first-order conditions for profit maximization for each market participant, and then adding market clearing conditions, such as supply equaling demand. Such models are naturally formulated as MCPs because the first-order conditions for a constrained profit maximization problem can be expressed as KKT conditions, which are complementarity conditions. For instance, let an optimization problem be

$$\begin{aligned} \textbf{Problem Opt}: \quad & \underset{\{Z\}}{\text{MAX}}\, b(Z) \\ \text{subject to}: \quad & D(Z) \le 0 \\ & Z \ge 0 \end{aligned}$$

where Z is a vector of decision variables, and $b(\ )$ and $D(\ )$ are smooth functions, with $b(\ )$ being scalar and concave and

[15]The implementation is called COMPETES (Competition and Market Power in Electric Transmission and Energy Simulator). It is programmed in the modeling language AIMMS (www.paragon.nl) and uses the PATH solver [12].

$D(\,)$ being vector-valued and convex. The KKT conditions that define an optimal solution for this problem are as follows:

$$Z \geq 0, \quad \nabla_Z b - \lambda^T \nabla_Z D \leq 0, \quad Z^T(\nabla_Z b - \lambda^T \nabla_Z D) = 0$$
$$\lambda \geq 0, \quad D(Z) \leq 0, \quad \lambda^T D(Z) = 0.$$

Here, λ is the vector of LaGrange or dual multipliers for the constraints, with the same dimension as the constraint vector. Using the "perp" ($\perp$) symbol introduced in Section IV, these conditions can be more compactly stated as

$$0 \leq Z \perp \nabla_Z b - \lambda^T \nabla_Z D \leq 0$$
$$0 \leq \lambda \perp D(Z) \leq 0.$$

More general KKT conditions can be defined for optimization problems with equality constraints and unrestricted variables. An introduction to the construction of MCP-based energy market models using KKT conditions is provided in [20].

As a simple example of a MCP-based market model, consider the following model of a competitive market with two GenCos (with marginal costs MC_1 and MC_2 and capacities G_1 and G_2, respectively) and consumers whose benefit of consumption is $b(q)$. In a competitive model, each GenCo $f = 1, 2$ is assumed to be a "price taker;" that is, it treats price p^* as exogenous. The profit maximization problem for f can then be stated as follows:

$$\textbf{Problem GenCo}_\mathbf{f}: \quad \underset{\{g_f\}}{\text{MAX}}(p^* - MC_f)g_f$$
$$\text{s.t.} \quad 0 \leq g_f \leq G_f.$$

where g_f is its generation. Let μ_f be the dual multiplier for the capacity constraint. The KKT conditions for this problem are

$$0 \leq g_f \perp (p^* - MC_f) - \mu_f \leq 0$$
$$0 \leq \mu_f \perp g_f - G_f \leq 0.$$

Meanwhile, the net benefit maximization problem for consumers is

$$\textbf{Problem Con}: \quad \underset{\{q \geq 0\}}{\text{MAX}}\, b(q) - p^* q$$

where q is the amount consumed. Its KKT condition is

$$0 \leq q \perp \frac{\partial b(q)}{\partial q} \leq 0.$$

A market equilibrium can then be calculated by solving a MCP problem consisting of finding the values of all the variables that simultaneously satisfy the GenCos' and consumers' first-order (KKT) conditions plus the market clearing condition that supply = demand. Mathematically

$$\begin{aligned}
&\textbf{Problem MCP}: \\
&\quad \text{Find } \{g_1, g_2, \mu_1, \mu_2, q, p^*\} \text{ that satisfies}: \\
&\qquad 0 \leq g_f \perp (p^* - MC_f) - \mu_f \leq 0 \quad \text{for } f = 1, 2 \\
&\qquad 0 \leq \mu_f \perp g_f - G_f \leq 0 \quad \text{for } f = 1, 2 \\
&\qquad 0 \leq q \perp \frac{\partial b(q)}{\partial q} \leq 0 \\
&\qquad g_1 + g_2 = q.
\end{aligned}$$

This problem has six variables and six sets of conditions. If, for instance, we assume that $MC_f = 10$ \$MWh, $G_f = 35$ MW, $b(q) = 100q - 0.05q^2$ \$/h, then the competitive market solution is

$$\{g_1, g_2, \mu_1, \mu_2, q, p^*\} = \{35, 35, 20, 20, 70, 30\}.$$

A more general formulation of this MCP model would make price p an endogenous function of g_f (as is typical of Cournot and CSF models, including that of this paper). Another generalization would account for transmission costs. For instance, if the TSO charges w^* for transmitting power from the generator to the point of sale, then the objective of Problem GenCo_f would change to MAX $(p^* - w^* - MC_f)g_f$. In a LMP-based system, w^* would be defined by the TSO as the difference in nodal prices (calculated as shadow prices for energy at each bus in its optimal power flow). In previous MCP models of transmission-constrained power markets, the generator is assumed to view w^* as fixed (exogenous). However, in the model of this paper, the transmission price is instead an endogenous (first-order) function of the generator's sales and generation, and so is also a decision variable.

Example of MPEC Model

An example of a mathematical program with equilibrium constraints would arise in the following situation [6], [21]. Consider a LMP-based market in which a GenCo chooses values of its decision variables Z_g in order to maximize its profit $b_g(Z_g, Z_t, \lambda_t)$ subject to (1) its own constraints $D_g(Z_g, Z_t) \leq 0$, $Z_g \geq 0$ and (2) the profit- or social-welfare maximizing behavior of the TSO, who is choosing its own decision variables Z_t in order to MAX $b_t(Z_g, Z_t)$ subject to its constraints $D_t(Z_g, Z_t) \leq 0$, $Z_t \geq 0$. (Examples of the latter include the TSO problem of Section III-B or a TSO problem of maximizing the value of demand bids minus the cost of supply bids, as in [21].) Note the interdependency that arises because each party's decision variables appear in the other party's profit and constraint functions. The dual variables of the TSO's problem are arguments in the GenCo's objective because those duals determine the price of transmission services (i.e., setting that price equal to the difference between nodal duals). That the GenCo anticipates the optimal response of the TSO can then be modeled by including the TSO's KKT conditions in the GenCo's constraint set as follows:

$$\begin{aligned}
\textbf{GenCo MPEC}: &\quad \text{MAX}\, b_g(Z_g, Z_t, \lambda_t) \\
\text{s.t.}: &\quad D_g(Z_g, Z_t) \leq 0, \quad Z_g \geq 0 \\
&\quad 0 \leq Z_t \perp \nabla_{Z_t} b_t - \lambda_t^T \nabla_{Z_t} D_t \leq 0 \\
&\quad 0 \leq \lambda_t \perp D_t(Z_g, Z_t) \leq 0.
\end{aligned}$$

This type of optimization problem is hard to solve because the GenCo's feasible region is nonconvex as a result of the last two constraints, which are the TSO's complementarity conditions. (Examples of such constraints are (KT1)–(KT3), above; [20] provides a complete but simple example of such an MPEC.) Consequently, local optima for the generator are not necessarily globally optimal, and gradient-type methods may therefore converge to suboptimal points. It is impossible to write down a set of first-order conditions (KKT's) that are necessary and sufficient to define an optimal solution for this MPEC. This implies that it is also not possible to define a single MCP that defines an equilibrium among several such MPECs; instead, solutions (if any exist) to such an equilibrium problem with equilibrium constraints (EPEC) must be searched for using iterative methods [as in [6] and [21]].

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