The Misallocation Channel of Climate Change

Evidence from Global Firm-level Microdata

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Roadmap

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Motivation for Research Question

- How does climate change affect the aggregate economy?
  - → Productivity damage as the primary effect of climate change
  - → Aggregate impact: 4°C of warming would reduce global GDP by 7-12% (Nath et al. 2023).
  - → Micro impact: temperature has large and heterogeneous effects on labor productivity.
- **But how? What are the sources and channels?**
- Macro literature on productivity and misallocation suggests that:

$$\text{TFP} = \text{Technology} - \text{Misallocation Loss}$$

- Previous literature: climate change affects technology (physical productivity).
- **This paper: climate change affects within-sector misallocation**
  - → **The misallocation channel is sizable in developed and hotter countries.**
  - → **Misallocation loss is one of the main driver of aggregate climate damage.**
  - → **Sources of climate-induced capital misallocation: firm-level heterogeneity of temperature sensitivity+ temperature volatility + damage volatility**

Contribution to Literature

- Empirical climate econometrics: propose a new methodology to derive measurable channel decomposition of climate change on aggregate TFP;
  - $\rightarrow$ This enables us to measure a new channel: misallocation.

  (Dell, Jones, and Olken 2012; Hsiang 2016; Deryugina and Hsiang 2017; Mérel and Gammans 2021; Carleton et al. 2022; Lemoine 2018)
- Value and impact of weather forecasts: we estimate the aggregate consequences of temperature forecast errors

  (Schlenker and Taylor 2021; Shrader 2023; Shrader, Bakkensen, and Lemoine 2023)
- Macroeconomic modeling of climate change: develop a heterogeneous firm model to understand the mechanisms of temperature-induced misallocation

  (Nath 2023; Nath, Ramey, and Klenow 2023; Cruz and Rossi-Hansberg 2023; Bakkensen and Barrage 2021; Casey, Fried, and Gibson 2022; Rudik et al. 2021)
- Misallocation: exploit daily temperature shocks as quasi-natural experiments to identify a new driver of misallocation

  (Hsieh and Klenow 2009; Sraer and Thesmar 2023; Bau and Matray 2023)

Detailed Comparison with Caggese et al 2023

Data

- Firm-level Data
  - → **30 European Countries:** Orbis Global Financials for Industrial Companies (covering $>$50% of EU wage bill)
  - → **India** Annual Survey of Industries (ASI)
  - → **China** National Bureau of Statistics (NBS) Above-scale Data
- Weather and Climate Data: Hourly temperature from ERA5-Land ($0.25^\circ \times 0.25^\circ$)
- Weather Forecast Data: Monthly long-range forecast from ECWMF ($1^\circ \times 1^\circ$)

Accounting Framework

We focus on misallocations within each region-sector $n = (s, r)$ ($\rightarrow$ lower bound results).

- Sectoral output Y_n follows a CES production function of J_n differentiated products,

$$Y_{nt} = \left(\sum_{i=1}^{J_n} B_{nit}^{\frac{1}{\sigma_n}} Y_{nit}^{\frac{\sigma_n - 1}{\sigma_n}} \right)^{\frac{\sigma_n}{\sigma_n - 1}}, \tag{1}$$

- B_{nit} is a firm-specific preference shifter.
- Firms produce their products with three factors: capital, labor, and material inputs.

$$Y_{nit} = A_{nit} K_{nit}^{\alpha_{Kn}} L_{nit}^{\alpha_{Ln}} M_{nit}^{\alpha_{Mn}}, \tag{2}$$

- A_{nit} is firm-level physical productivity.
- Region-sectoral aggregate TFP is: $\text{TFP}_{nt} = \frac{Y_{nt}}{K_{nt}^{\alpha_{Kn}} L_{nt}^{\alpha_{Ln}} M_{nt}^{\alpha_{Mn}}}$

Accounting Framework - Distortion Sources

Firm i's Problem and Distortions

Subject to the inverse demand and distortions, each firm i optimally chooses their quantity of inputs and price to maximize profits:

$$\max_{P_{nit},K_{nit},L_{nit},M_{nit}} \left(1-\tau^{Y}_{nit}\right) P_{nit} \underbrace{A_{nit} K_{nit}^{\alpha_{Ks}} L_{nit}^{\alpha_{Ln}} M_{nit}^{\alpha_{Mn}}}_{Y_{nit}} - \left(1+\tau^{K}_{nit}\right) R_{nt} K_{nit}$$

$$- \left(1+\tau^{L}_{nit}\right) W_{nt} L_{nit} - \left(1+\tau^{M}_{nit}\right) P^{M}_{nt} M_{nit} \tag{3}$$

$$\text{subject to : } Y_{nit} = B_{nit} Y_{nt} \left[\frac{P_{nit}}{P_{nt}}\right]^{-\sigma_n},$$

Sources of Measured Distortion

1. Price (Markup) Distortion τ^{Y}_{nit}
2. Factor Distortion τ^{F}_{nit}, $F \in \{K, L, M\}$.

General Equilibrium Given preference shifter B_{nit}, physical productivity A_{nit}, price distortions $\tau^{Y}_{nit} < 1$, factor distortions $\tau^{F}_{nit} > 0$ and total factor supply of K_{nt}, L_{nt}, M_{nt}, a general equilibrium consisting of good prices P_{nit}, factor prices, and equilibrium factor allocation $F_{nit}, F \in \{K, L, M\}$ is defined where all market clears.

Micro Effects of Climate Conditions

Climate Conditions

- Climate conditions in region r at year t: $\mathbf{T}_{rt}$, a row vector of climate sufficient statistics (e.g. realizations of daily temperature).
- History of climate conditions:$\tilde{\mathbf{T}}_{rt} = \left(\mathbf{T}_{rt}, \tilde{\mathbf{T}}_{rt-1}\right)$.

Economic Conditions

- Aggregate state of the region-sector pair: $\mathbf{X}_{nt}$
- Idiosyncratic state of a firm as $\mathbf{Z}_{nit}$
- History of realization: $\tilde{\mathbf{X}}_{nt} = \left(\mathbf{X}_{nt}, \tilde{\mathbf{X}}_{nt-1}\right)$ and $\tilde{\mathbf{Z}}_{nit} = \left(\mathbf{Z}_{nit}, \tilde{\mathbf{Z}}_{nit-1}\right)$.

All fundamentals are firm-specific functions of $\tilde{\mathbf{T}}_{rt}, \tilde{\mathbf{X}}_{nt}, \tilde{\mathbf{Z}}_{nit}$

$$A_{nit} = A_{ni}(\tilde{\mathbf{T}}_{rt}, \tilde{\mathbf{X}}_{nt}, \tilde{\mathbf{Z}}_{nit}), \quad B_{nit} = B_{ni}(\tilde{\mathbf{T}}_{rt}, \tilde{\mathbf{X}}_{nt}, \tilde{\mathbf{Z}}_{nit}),$$

$$\tau^{Y}_{nit} = \tau^{Y}_{ni}(\tilde{\mathbf{T}}_{rt}, \tilde{\mathbf{X}}_{nt}, \tilde{\mathbf{Z}}_{nit}), \quad \tau^{F}_{nit} = \tau^{F}_{ni}(\tilde{\mathbf{T}}_{rt}, \tilde{\mathbf{X}}_{nt}, \tilde{\mathbf{Z}}_{nit}), \quad \forall F \in \{K, L, M\}.$$

- Firm-specific: enabling the same climate conditions $\tilde{\mathbf{T}}_{rt}$ to differentially affect firms.
- Climate history: dynamic effect + wedges might depend on "shocks" $\Delta_{\mathbf{T}_{rt}} = \mathbf{T}_{rt} - \mathbb{E}_{t-1}(\mathbf{T}_{rt}|\tilde{\mathbf{T}}_{rt-1})$.

Log-normality

Assumption: joint log-normality

- For any set of values $\left(\tilde{\mathbf{T}}_{rt}, \tilde{\mathbf{X}}_{nt}, \left\{\tilde{\mathbf{Z}}_{nit}\right\}_i\right)$, the set of functions $\left\{B_{ni}, A_{ni}, \tau^Y_{ni}, \tau^F_{ni}\right\}$ must satisfy that their realized value $\mathbf{S}_{nit} = \left(B_{nit}, A_{nit}, 1+\tau^Y_{nit}, 1+\tau^K_{nit}, 1+\tau^L_{nit}, 1+\tau^M_{nit}\right)$, follow a joint log-normal distribution:

$$\log(\mathbf{S}_{nit}(\tilde{\mathbf{T}}_{rt}, \tilde{\mathbf{X}}_{nt}, \tilde{\mathbf{Z}}_{nit})) \sim \mathcal{N}\left(\mu^{(n)}_s(\tilde{\mathbf{T}}_{rt}, \tilde{\mathbf{X}}_{nt}), \Sigma^{(n)}_{ss}(\tilde{\mathbf{T}}_{rt}, \tilde{\mathbf{X}}_{nt})\right)$$

- $\mu^{(n)}_s$ and $\Sigma^{(n)}_{ss}$ are smooth functions of only aggregate states.
- Relevant Moments of $\tilde{\mathbf{Z}}_{nit}$ can be stacked into $\tilde{\mathbf{X}}_{nt}$.

Decomposition of TFP

Proposition: TFP Decomposition Given joint log-noramlity, each region-sector n admits an aggregate production function of the form

$$Y_{nt} = \mathrm{TFP}_{nt} K_{nt}^{\alpha_{Kn}} L_{nt}^{\alpha_{Ln}} M_{nt}^{\alpha_{Mn}},$$

where the region-sectoral aggregate Total Factor Productivity function $\mathrm{TFP}_{nt} := \mathrm{TFP}_n\left(\tilde{\mathbf{T}}_{rt}, \tilde{\mathbf{X}}_{nt}\right)$ can decomposed as following:

$$\begin{aligned}
\log \mathrm{TFP}_n(\tilde{\mathbf{T}}_{rt}, \cdot) = & \underbrace{\frac{1}{\sigma_n - 1} \log\left(\mathbb{E}_i B_{ni}(\tilde{\mathbf{T}}_{rt}, \cdot) A_{ni}(\tilde{\mathbf{T}}_{rt}, \cdot)^{\sigma_n - 1}\right)}_{\text{Efficient Technology}} - \underbrace{\frac{\sigma_n}{2} \mathrm{var}_{\log\left(1-\tau_{ni}^Y\right)}(\tilde{\mathbf{T}}_{rt}, \cdot)}_{\text{Markup Dispersion}} \\
& - \underbrace{\sum_{F \in \{K,L,M\}} \frac{\alpha_{Fn} + \alpha_{Fn}^2(\sigma_n - 1)}{2} \mathrm{var}_{\log\left(1+\tau_{ni}^F\right)}(\tilde{\mathbf{T}}_{rt}, \cdot)}_{\text{Factor Marginal Product Dispersion}} \\
& + \underbrace{\sigma_n \sum_{F \in \{K,L,M\}} \alpha_{Fn} \mathbf{cov}_{\log\left(1-\tau_{ni}^Y\right), \log\left(1+\tau_{ni}^F\right)} \left(\tilde{\mathbf{T}}_{rt}, \cdot\right)}_{\text{Markup-Factor Mixed Distortion}} \\
& - \underbrace{(\sigma_n - 1) \sum_{F \in \{K,L,M\}} \sum_{F' \neq F} \alpha_{Fn} \alpha_{F'n} \mathbf{cov}_{\log\left(1+\tau_{ni}^F\right), \log\left(1+\tau_{ni}^{F'}\right)} \left(\tilde{\mathbf{T}}_{rt}, \cdot\right)}_{\text{Factor Mixed Distortion}}
\end{aligned} \tag{4}$$

Decomposition of Climate Damage on Aggregate TFP

Lemma: First Order Effect on TFP The first order impact of climate change, represented as a collection of exogenous conditions $\tilde{\mathbf{T}}_{rt}$, on sectoral aggregate TFP can be decomposed as:

$$
\begin{aligned}
\frac{\partial \log TFP_n(\tilde{\mathbf{T}}_{rt},\cdot)}{\partial \tilde{\mathbf{T}}_{rt}} &= \frac{\partial \text{ Technology}_{nt}}{\partial \tilde{\mathbf{T}}_{rt}} - \frac{\partial \text{ Inefficiency Loss}_{nt}}{\partial \tilde{\mathbf{T}}_{rt}} \\
&= \frac{1}{\sigma_n - 1} \frac{\partial \log \left(\mathbb{E}_i B_{ni}(\tilde{\mathbf{T}}_{rt},\cdot) \left(A_{ni}(\tilde{\mathbf{T}}_{rt},\cdot) \right)^{\sigma_n - 1} \right)}{\partial \tilde{\mathbf{T}}_{rt}} - \frac{\sigma_n}{2} \frac{\partial \operatorname{var}_{\log\left(1-\tau_{ni}^{Y}\right)}(\tilde{\mathbf{T}}_{rt},\cdot)}{\partial \tilde{\mathbf{T}}_{rt}} \\
&\quad - \sum_{F\in\{K,L,M\}} \frac{\alpha_{Fn} + \alpha_{Fn}^2(\sigma_n - 1)}{2} \frac{\partial \operatorname{var}_{\log\left(1+\tau_{ni}^{F}\right)}(\tilde{\mathbf{T}}_{rt},\cdot)}{\partial \tilde{\mathbf{T}}_{rt}} \\
&\quad + \sigma_n \sum_{F\in\{K,L,M\}} \alpha_{Fn} \frac{\partial \operatorname{cov}_{\log\left(1-\tau_{ni}^{Y}\right),\log\left(1+\tau_{ni}^{F}\right)}(\tilde{\mathbf{T}}_{rt},\cdot)}{\partial \tilde{\mathbf{T}}_{rt}} \\
&\quad - (\sigma_n - 1) \sum_{F\in\{K,L,M\}} \sum_{F'\neq F} \alpha_{Fn}\alpha_{F'n} \frac{\partial \operatorname{cov}_{\log\left(1+\tau_{ni}^{F}\right),\log\left(1+\tau_{ni}^{F'}\right)}(\tilde{\mathbf{T}}_{rt},\cdot)}{\partial \tilde{\mathbf{T}}_{rt}}
\end{aligned} \tag{5}
$$

Ideally, one can have a great firm-level panel dataset to measure each sufficient statistic and identify the respective elasticity.

Special Case: only capital wedges are present

- We focus on a benchmark case where only capital wedges is present (as in David and Venky (2019) and Sraer and Thesmar (2023))
- Within each region-sector, firm's FOC yields that the Marginal Revenue Product of Capital (MRPK) of each firm i would satisfy that:

$$\mathrm{MRPK}_{nit} \propto 1 + \tau^{K}_{nit}(\tilde{\mathbf{T}}_{rt}, \cdot)$$

- And now the effect of climate conditions on aggregate TFP would be:

$$\frac{\partial \log TFP_n(\tilde{\mathbf{T}}_{rt}, \cdot)}{\partial \tilde{\mathbf{T}}_{rt}} = \frac{1}{\sigma_n - 1} \frac{\partial \log \left(\mathbb{E}_i B_{ni}(\tilde{\mathbf{T}}_{rt}, \cdot) \left(A_{ni}(\tilde{\mathbf{T}}_{rt}, \cdot) \right)^{\sigma_n - 1} \right)}{\partial \tilde{\mathbf{T}}_{rt}} - \frac{\alpha_{Kn} + \alpha^2_{Kn}(\sigma_n - 1)}{2} \frac{\partial \operatorname{var}_{mrpk_{ni}}(\tilde{\mathbf{T}}_{rt}, \cdot)}{\partial \tilde{\mathbf{T}}_{rt}} \tag{6}$$

- The (first-order) cost of climate-induced misallocation would be determined by the elasticity $\frac{\partial \operatorname{var}_{mrpk_{ni}}(\tilde{\mathbf{T}}_{rt}, \cdot)}{\partial \tilde{\mathbf{T}}_{rt}}$.
- $mrpk_{nit} \propto \log(\frac{P_{nit} Y_{nit}}{K_{nit}})$ under Cobb-Douglas technologies, which can be measured with sales over capital ratio in the data.

Identification of the Causal Elasticity on Misallocaton: $\frac{\partial \operatorname{var}_{mrpk_{ni}}(\tilde{\mathbf{T}}_{rt}, \cdot)}{\partial \tilde{\mathbf{T}}_{rt}}$

- For each region-sector $n = (r, s)$, a Taylor expansion around the observed steady state $(\overline{\operatorname{var}_{mrpk_{(s,r)i}}}, \overline{\tilde{\mathbf{T}}_r}, \overline{\tilde{\mathbf{X}}_{s,r}})$ can be written as

$$\operatorname{var}_{mrpk_{(s,r)i}}(\tilde{\mathbf{T}}_{r,t}, \tilde{\mathbf{X}}_{s,r,t}) = \overline{\operatorname{var}_{mrpk_{(s,r)i}}} + \lambda^{s,r}_{\sigma^2_{mrpk}} \cdot (\tilde{\mathbf{T}}_{r,t} - \overline{\tilde{\mathbf{T}}_r}) + \delta^{s,r}_{\sigma^2_{mrpk}} \cdot (\tilde{\mathbf{X}}_{s,r,t} - \overline{\tilde{\mathbf{X}}_{s,r}}) + H.O.T.$$
$$\approx \lambda^{s,r}_{\sigma^2_{mrpk}} \cdot \tilde{\mathbf{T}}_{r,t} + \delta^{s,r}_{\sigma^2_{mrpk}} \cdot \tilde{\mathbf{X}}_{s,r,t} + \eta_{s,r}$$

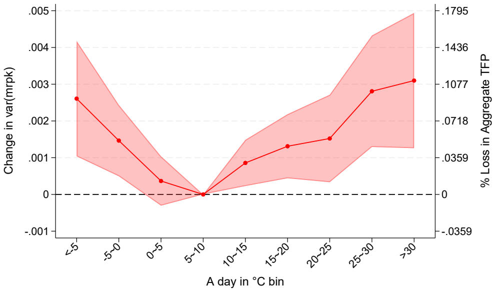
- $\eta_{s,r}$ is a sector-region specific constant.
- Think of $\frac{\partial \operatorname{var}_{mrpk_{ni}}(\tilde{\mathbf{T}}_{rt}, \cdot)}{\partial \tilde{\mathbf{T}}_{rt}} \approx \lambda^{s,r}_{\sigma^2_{mrpk}}$ as "reduced form," a local object defined with respect to observed equilibrium allocations and are not necessarily invariant to the evolving climate conditions in the long-run.
- We first estimate the average elasticity $\lambda_{\sigma^2_{mrpk}} = \mathbb{E}[\lambda^{s,r}_{\sigma^2_{mrpk}}]$ across all regions and sectors.

Identification of $\frac{\partial \, \text{var}_{mrpk_{ni}}(\tilde{\mathbf{T}}_{rt}, \cdot)}{\partial \tilde{\mathbf{T}}_{rt}}$

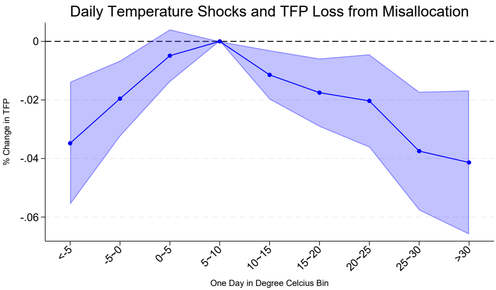
$$\text{var}_{mrpk_{(s,r),t}} = \sum_{b \in B/(5\sim 10^\circ C)} \lambda^{b}_{\sigma^2_{mrpk}} \times \text{Tbin}^{b}_{r,t} + \delta_{\sigma^2_{mrpk}} \mathbf{X}_{s,r,t} + \theta_{c(r),s,t} + \eta_{s,r} + \varepsilon_{r,s,t}.$$

- r: a NUTS-3 region in EU, a prefecture in China, a district in India.
- s denotes a 1-digit SIC sector.
- $\mathbf{T}_{r,t} = \{\text{Tbin}^{<-5^\circ C}_{r,t}, ..., \text{Tbin}^{>30^\circ C}_{r,t}\}$ as days in each temperature bins.
- $\lambda^{b}_{\sigma^2_{mrpk}}$ denotes the average elasticity of province-level daily temperature shocks on province-sector level MRPK dispersion.
- $\mathbf{X}_{s,r,t}$ is a vector of control: number of observed firms, average firm-level sales and average MRPK across firms.
- $\eta_{s,r}$: region-sector FE; $\theta_{c(r),s,t}$: country-sector-Year FE; SE clustered at region level.
- Sample: only keep the region-sector-year pairs that have more than 30 firms.

Temperature Shocks, MRPK Misallocation and TFP loss



(a) Impact on MRPK dispersion



(b) Model-implied TFP loss

If we replace a 5-10°C (41°F to 50°F) day with a hotter-than-30°C (86°F) day in a year:

- The measured MRPK dispersion will increase by about 0.003;
- The measured yearly TFP will decrease by about 0.04% through capital misallocation.

Heterogeneous Effect across Regional Long-run Climate and Levels of Development

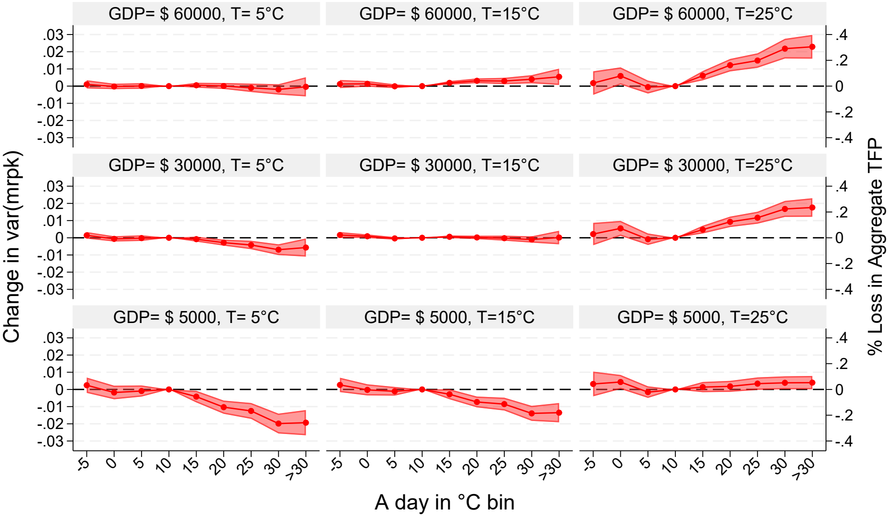
- The same hot/cold day shock is likely to have heterogeneous across region-sectors.
  Het. effect across sectors
- Effect might be ambiguous:
  - $\rightarrow$ Heat-sensitive firms in hotter region might have greater incentives to adapt.
  - $\rightarrow$ But the marginal effect of hot temperatures in already hot locations might be worse.
  - $\rightarrow$ Firms in more developed regions find it easier to cope with weather damage
  - $\rightarrow$ But firm heterogeneity is larger in developed regions.
- Following the approach of Carleton et al. (2022), we interact the long-term annual average temperature of region r and average region-level annual GDP per capita with each temperature bin:

$$
\begin{aligned}
\sigma^2_{mrpk_{s,r,t}} = & \sum_{b \in B_{/(5\sim 10^\circ C)}} \lambda^b_{\sigma^2_{mrpk}} \times \mathrm{Tbin}^b_{r,t} + \sum_{b \in B_{/(5\sim 10^\circ C)}} \lambda^{b,\bar{T}}_{\sigma^2_{mrpk}} \times \mathrm{Tbin}^b_{r,t} \times \overline{T}_r \\
& + \sum_{b \in B_{/(5\sim 10^\circ C)}} \lambda^b_{\mathrm{GDP}_{pc}} \times \mathrm{Tbin}^b_{r,t} \times \ln \overline{\mathrm{GDP}_{pc,r}} + \delta_{\sigma^2_{mrpk}} \times \tilde{\mathbf{X}}_{s,r,t} + \alpha_{c,t} + \eta_{s,r} + \varepsilon_{s,r,t},
\end{aligned} \tag{7}
$$

- Therefore, the first-order effect is region-specific:

$$
\frac{\partial \, \mathrm{var}_{mrpk_{s,r}}(\tilde{\mathbf{T}}_{rt}, \cdot)}{\partial \mathrm{Tbin}^b_{r,t}} \approx \lambda_{\sigma^2_{mrpk}} + \overline{T}_r \cdot \lambda^{\bar{T}}_{\sigma^2_{mrpk}} + \ln \overline{\mathrm{GDP}_{pc,r}} \cdot \lambda^b_{\mathrm{GDP}_{pc}}
$$

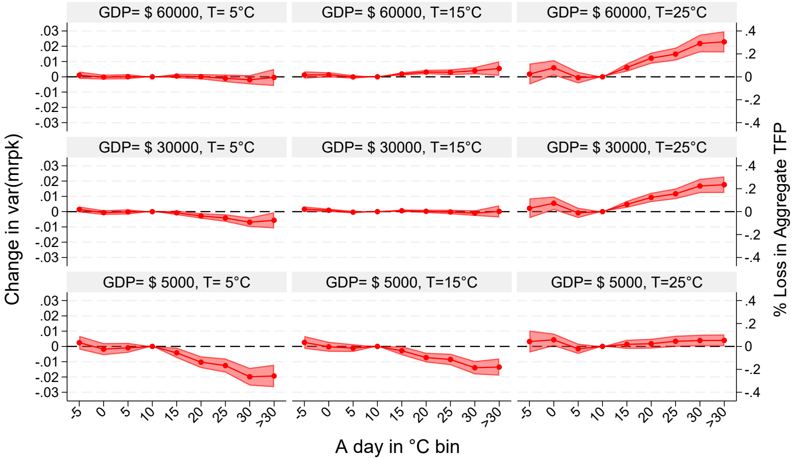
Het. Effect across national income and regional long-run climate



In terms of the misallocation channel:

- Hotter and more developed regions are more susceptible to hot day shocks.
- Cooler regions could even benefit from hot shocks.

Het. Effect across national income and regional long-run climate



In terms of the misallocation channel, an additional 30 $^\circ$ C day would lead to:

- 0.3 % TFP loss in Florida (GDP= 60,000 USD and $\overline{T}_r$=25°C)
- 0.06 % TFP loss in India (GDP= 5,000 USD and $\overline{T}_r$=25°C)
- Almost null effect in Poland and UK (GDP= 30000-40000 USD and $\overline{T}_r$=5-10°C)
- 0.08 % TFP gain in Northern China (GDP= 10,000 USD and $\overline{T}_r$=10°C)

Simple Model of Firm Dynamics

- We want to explain why both the levels and shocks of temperature matter for misallocation.
- A simple firm-dynamics model with minimal ingredients: focusing on activities within (r, s).
- Each firm i produces differentiated products of quantity Y_{it} with Cobb-Douglas technology:

$$Y_{it} = \tilde{A}_{it} K_{it}^{\tilde{\alpha}_K} N_{it}^{\tilde{\alpha}_N}, \quad \tilde{\alpha}_K + \tilde{\alpha}_N = 1,$$

- $\tilde{A}_{it}$ is the physical productivity, K_{it} is the capital input (dynamic) and N_{it} represents a composite of flexible inputs, referred to as "labor".
- The firm's product faces a constant elasticity downward-sloping demand curve with demand shifter B_{it}:

$$Y_{it} = B_{it} P_{it}^{-\sigma}.$$

- Equilibrium revenue function:

$$P_{it} Y_{it} = \hat{A}_{it} K_{it}^{\alpha_K} N_{it}^{\alpha_N}$$

where $\alpha_F = (1 - \frac{1}{\sigma})\tilde{\alpha_F}, \forall F \in \{K, N\}$ and $\hat{A}_{it} = B_{it}^{\frac{1}{\sigma}} \left(\tilde{A}_{it}\right)^{(1-\frac{1}{\sigma})}$ is the revenue-based productivity (TFPR). We will be referring to this simply as productivity.

Simple Model of Firm Dynamics

- Firms' productivity is heterogeneously impacted by temperature:

$$\hat{A}_{it} = \exp\left(\hat{\beta}_{it}(T_t - T^*)\right)\hat{Z}_{it}$$

  → $(T_t - T^*)$: deviation of temperature from the optimal temperature.
  → $\hat{Z}_{it}$: firm-specific idiosyncratic productivity.

- Two sources of heterogeneity in firm's temperature sensitivity $\hat{\beta}_{it}$:

$$\hat{\beta}_{it} = \underbrace{\hat{\beta}_i}_{\substack{\text{Persistent}\\ \text{sensitivity to}\\ \text{temperature}}} + \underbrace{\hat{\xi}_{it}}_{\substack{\text{Idiosyncratic}\\ \text{sensitivity to}\\ \text{temperature}}} + \underbrace{c(T_t - T^*)}_{\substack{\text{common state-dependent}\\ \text{sensitivity}}}$$

  → $\hat{\beta}_i \sim \mathcal{N}\left(\overline{\hat{\beta}_i}, \sigma^2_{\hat{\beta}}\right)$ is assumed to be known by the firm (depends on product characteristics and adaptability).
  → $\hat{\xi}_{it} \sim \mathcal{N}\left(0, \sigma^2_{\hat{\xi}}\right)$ is iid across firm and time, capturing damage volatility that scales with $(T_t - T^*)$.

Temperature and Productivity

- Temperature shocks and productivity components follow $\mathrm{AR}(p)$ processes:

$$(T_{t+1} - \bar{T}) = \sum_{h=1}^{p} \rho_{T,h}(T_{t+1-h} - \bar{T}) + \eta^{T}_{t+1}$$

$$\hat{z}_{it+1} = \rho_z \hat{z}_{it} + \hat{\varepsilon}_{it+1}$$

  - → Climate shocks: $\eta^{T}_{t+1} \sim \mathcal{N}\left(0, \sigma^2_{\eta}\right)$
  - → Idiosyncratic shocks: $\hat{\varepsilon}_{it+1} \sim \mathcal{N}\left(0, \sigma^2_{\hat{\varepsilon}}\right)$.

- Note: $\bar{T}$ is long-run mean temperature, need not to be optimal temperature T^*.
- Implied Forecast Errors:

$$T_{t+1} - \mathbb{E}_t(T_{t+1}) = \eta^{T}_{t+1}$$

$$\hat{\beta}_{it+1} - \mathbb{E}_t(\hat{\beta}_{it+1}) = \hat{\xi}_{it+1}$$

$$\hat{z}_{it+1} - \mathbb{E}_t(\hat{z}_{it+1}) = \hat{\varepsilon}_{it+1}$$

Temperature and TFP Volatility

Lemma: TFP Volatility, $\text{Var}(\hat{a}_{it} - \mathbb{E}_{t-1}[\hat{a}_{it}])$, defined as the cross-sectional variance of unexpected productivity shocks, can be written as:

$$\text{Var}(\hat{a}_{it} - \mathbb{E}_{t-1}[\hat{a}_{it}]) = (T_t - T^*)^2 \sigma_{\hat{\xi}}^2 + \hat{\eta}_t^{T\,2} \sigma_{\hat{\beta}}^2 + \sigma_{\hat{\varepsilon}}^2$$

- TFP Volatility reaches its minimum when:
  - $\rightarrow$ the temperature reaches its optimum $T_t = T^*$
  - $\rightarrow$ there is no unexpected change in temperature, $\eta_t^T = 0$.
- TFP volatility is dependent on the regional climate: productivity is too volatile if the region is too hot or too cold compared to T^*.

Static Input Choice - Labor

Pseudo-GE Wages:

$$W_t = \overline{W}\exp\left(\chi(T_t - T^*)\right)$$

The static labor choice solves

$$\max_{N_{it}} \exp\left(\hat{\beta}_{it}\left(T_t - T^*\right)\right)\hat{Z}_{it}K_{it}^{\alpha_K}N_{it}^{\alpha_N} - W_t N_{it}$$

Profits are given by:

$$\boxed{\Pi_{it} = GA_{it}K_{it}^{\alpha} := G\exp\left(\beta_{it}(T_t - T^*) + z_{it}\right)K_{it}^{\alpha},}$$

where

$$G := \overline{W}^{-\frac{\alpha_N}{1-\alpha_N}}\alpha_N^{\frac{\alpha_N}{1-\alpha_N}}\left(1-\alpha_N\right)$$

$$z_{it} = \frac{1}{1-\alpha_N}\hat{z}_{it}, \text{and} \quad \alpha = \frac{\alpha_K}{1-\alpha_N}$$

Firm's Investment Decision

Capital is a dynamic input and made with imperfect information about the future:

$$V\left(T_t, Z_{it}, K_{it}\right) = \max_{K_{it+1}} \quad G\exp\left(\beta_{it}(T_t - T^*) + z_{it}\right)K_{it}^{\alpha} - K_{it+1} + (1-\delta)K_{it}$$
$$+ \frac{1}{1+r}\mathbb{E}_t\left[V\left(T_{t+1}, Z_{it+1}, K_{it+1}\right)\right],$$

- Note that $M_{t+1} = \beta$ as we assume no risk aversion in this Pseudo-GE benchmark.

Euler Equation:

$$1 = \underbrace{\frac{1}{1+r}}_{\text{Discount Factor}} \left(\underbrace{\alpha G K_{it+1}^{\alpha-1}\mathbb{E}_t\left[\exp\left(z_{it+1} + \beta_{it+1}(T_{t+1} - T^*)\right)\right]}_{\text{Expected Value of Marginal Profits of Capital}} + \underbrace{(1-\delta)}_{\substack{\text{Value of} \\ \text{Undepreciated Capital}}} \right).$$

Log-linearized policy function

Log-linearization yields the investment policy function:

$$
\begin{aligned}
k_{it+1} \approx & \frac{1}{1-\alpha}\mathbb{E}_t[a_{it+1}] + k_0 \\
= & \frac{1}{1-\alpha}\left(\frac{1}{1-\alpha_N}\mathbb{E}_t[\hat{a}_{it+1}] - \frac{\alpha_N}{1-\alpha_N}\mathbb{E}_t[w_{t+1} - \overline{w}]\right) + k_0 \\
= & \frac{1}{1-\alpha}\left(\frac{1}{1-\alpha_N}\left(\mathbb{E}_t[\hat{z}_{it+1}] + \mathbb{E}_t[\hat{\beta}_{it+1}(T_{t+1} - T^*)]\right) - \frac{\alpha_N \chi}{1-\alpha_N}\mathbb{E}_t[T_{t+1} - T^*]\right) + k_0,
\end{aligned}
$$

Consider AR(1) temperature with $\rho_T > 0$, the investment policy of firm i comparing to the average firm would be:

$$
k_{it+1} - \overline{k_{it+1}} = \frac{1}{1-\alpha}\left(\rho_z z_{it} + \frac{(\hat{\beta}_i - \overline{\hat{\beta}_i})}{1-\alpha_N}\rho_T T_t\right),
$$

- Heat-loving firms, $\hat{\beta}_i > \overline{\hat{\beta}_i}$: higher temperature leads to more investment to the future.
- Heat-averse firms, $\hat{\beta}_i < \overline{\hat{\beta}_i}$: higher temperature leads to less investment to the future.

MRPK and Temperature

After all shocks are realized, we have for each firm $MPRK_{it} := \alpha_K \frac{P_{it}Y_{it}}{K_{it}}$. Relative to the average level, MRPK is higher in the firms with higher unexpected changes in productivity:

$$mrpk_{it} - \overline{mrpk_{it}} = \frac{1}{1-\alpha_N} \left\{ \underbrace{(\hat{\beta}_i - \overline{\hat{\beta}_i})\eta_t^T}_{\substack{\text{Climate}\\\text{Shock}}} + \underbrace{\hat{\xi}_{it}(T_t - T^*)}_{\substack{\text{Damage Sensitivity}\\\text{Shock}}} + \hat{\varepsilon}_{it} \right\}$$

Two kinds of firms are getting lower *mrpk* with a positive temperature shock ($\eta_t > 0$, $T_t - T^* > 0$):

- Heat-averse firms with $\hat{\beta}_i < \overline{\hat{\beta}_i}$: failed to expect the low productivity caused by the temperature shock η_t.
- Unlucky firms with $\hat{\xi}_{it} > 0$: failed to expect the low productivity caused by the damage sensitivity shock $\hat{\xi}_{it}$.

MRPK Dispersion

Proposition: MRPK Dispersion The variance of $mrpk_{it}$ across all firms in a given period is:

$$\begin{aligned}
\sigma^2_{mrpk,(r,s),t} &= \left(\frac{1}{1-\alpha_N}\right)^2 \mathrm{Var}(\hat{a}_{nit} - \mathbb{E}_{t-1}[\hat{a}_{nit}]) \\
&= \left(\frac{1}{1-\alpha_N}\right)^2 \Bigg[\underbrace{(T_{r,t} - T^*)^2 \sigma^2_{\xi,(r,s)}}_{\substack{\text{Damage Volatility} \\ \text{(Level Effect)}}} + \underbrace{{\eta^T_{r,t}}^2 \sigma^2_{\beta,(r,s)}}_{\substack{\text{Climate Volatility} \\ \text{(Shock Effect)}}} + \sigma^2_{\varepsilon,(r,s)} \Bigg]
\end{aligned} \tag{8}$$

- Similar to Asker, Collard-Wexler, and Loecker (2014), MRPK dispersion $\propto$ TFP volatility.
- Here, climate conditions endogenously generate TFP volatility.

How would climate change lead to larger misallocation?

- Larger deviation from optimal temperature: $(T_{r,t} - T^*)^2$
- Larger unexpected temperature shocks: ${\eta^T_{r,t}}^2$

We will test these channels one by one.

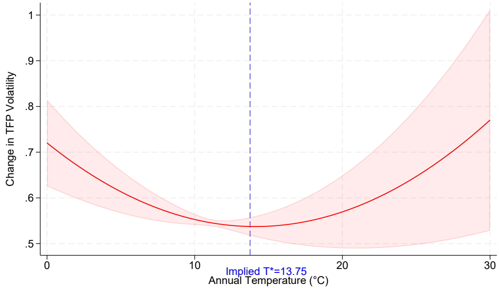
Level Effect: Temperature as endogenous volatility shock

- We proceed by testing whether firm-level TFP volatility varies non-linearly with temperature in the sector-region panel:
- We use the variance of the "first-differenced" TFP shocks, $\text{Var}_{nt}(\hat{a}_{it} - \hat{a}_{it-1})$to approximate the variability of unexpected TFPR shocks.

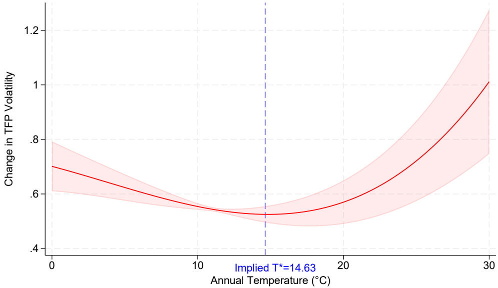
$$\text{Var}_{(s,r),t}(\hat{a}_{it} - \hat{a}_{it-1}) = \alpha + \beta f(T_{r,t}) + \eta_{s,r} + \delta_{c(r),t} + \varepsilon_{s,r,t}, \tag{9}$$

where $f(T_{r,t})$ is a polynomial of annual average temperature, $\eta_{s,r}$ and $\delta_{c(r),t}$ denotes region-sector and country-year fixed effects respectively.

(a) Second Order Estimation



(b) Third Order Estimation



Level Effect: Identifying Optimal Temperature T^*

- From theory, past productivity also depends on past temperature:

$$\text{Var}_{s,r,t}(a_{it} - a_{it-1}) = \sigma^2_{\hat{\xi}}(T_t^2 + T_{t-1}^2) - 2\sigma^2_{\hat{\xi}} T^*(T_t + T_{t-1}) + 2\sigma^2_{\hat{\xi}} T^{*2} + \sigma^2_{\hat{\beta}}(\Delta T_t)^2 + \sigma^2_{\Delta z}$$

- To exactly identify T^*, we simply need to run the regression:

$$\begin{aligned} \text{Var}_{(s,r),t}(\hat{a}_{it} - \hat{a}_{it-1}) = \alpha + \beta_1(T^2_{r,t} + T^2_{r,t-1}) + \beta_2(T_{r,t} + T_{r,t-1}) + \gamma(\Delta T_{r,t})^2 \\ + \eta_{s,r} + \delta_{c(r),t} + \varepsilon_{s,r,t}, \end{aligned}$$

  and back out $\hat{T}^* = -\frac{\hat{\beta}_2}{2\hat{\beta}_1}$.
- We show that T^* are very similar from different estimations in the next slide.

Level Effect: Identifying Optimal Temperature T^*

Table: TFP Volatility and Temperature Levels

	(1) 1st Order	(2) 2nd Order	(3) 3rd Order	(4) Model-Induced
$T_{r,t}$	-0.005319	-0.023121***	-0.012380	
	(0.004573)	(0.007536)	(0.008537)	
$T^2_{r,t}$		0.000841***	-0.000447	
		(0.000303)	(0.000679)	
$T^3_{r,t}$			0.000040**	
			(0.000018)	
$(T^2_{r,t} + T^2_{r,t-1})$				-0.021556***
				(0.005682)
$(T_{r,t} + T_{r,t-1})$				0.000882***
				(0.000216)
$(\Delta T_{r,t})^2$				-0.003604
				(0.002233)
Estimated T^*		13.75 °C	14.64°C	12.22°C
		(3.067678)	(2.173182)	(2.216646)
Region-Sector FE	Yes	Yes	Yes	Yes
Country-Year FE	Yes	Yes	Yes	Yes
Observations	113,765	113,765	113,765	113,765

Shock Effect: Climate Volatility

We use long-run temperature forecast data to examine this channel directly:

$$\sigma^2_{mrpk,(s,r),t} = \sum_{q \in \{\text{summer, winter, annual}\}} \theta_q \cdot \text{MSFE}_{q,r,t} + \gamma_1 T_{rt} + \gamma_2 T_{rt}^2 + \eta_{s,r} + \delta_{c(r),t} + \varepsilon_{s,r,t}, \tag{10}$$

- $\text{MSFE}_{q,r,t}$ represents the (demeaned) mean squared forecast errors of monthly temperature averaged over the time frame q in region r.
- For example,

$$\text{MSFE}_{\text{summer},r,t} = \frac{1}{6} \sum_{m=4}^{9} (T_{\text{m},r,t} - \mathbb{E}_{m-1} T_{m,r,t} - \text{Bias}_{m,r})^2$$

- Each coefficient θ_q corresponds to the impact of a one unit increase in MSFE in the time frame q on annual capital misallocation.

Shock Effect: Climate Volatility

Table: Temperature Forecast Errors and MRPK Dispersion

	(1)	(2)	(3)	(4)
$\text{MSFE}_{\text{annual},r,t}$	0.019114***	0.016249**		
	(0.006675)	(0.006561)		
$\text{MSFE}_{\text{summer},r,t}$			0.014908**	0.016592**
			(0.007115)	(0.007084)
$\text{MSFE}_{\text{winter},r,t}$			0.008536**	0.006096
			(0.004017)	(0.003882)
Quadratic Temperature Control	No	Yes	No	Yes
Region-Sector FE	Yes	Yes	Yes	Yes
Country-Year FE	Yes	Yes	Yes	Yes
Observations	124,065	124,065	124,065	124,065
R^2	0.876	0.876	0.876	0.876

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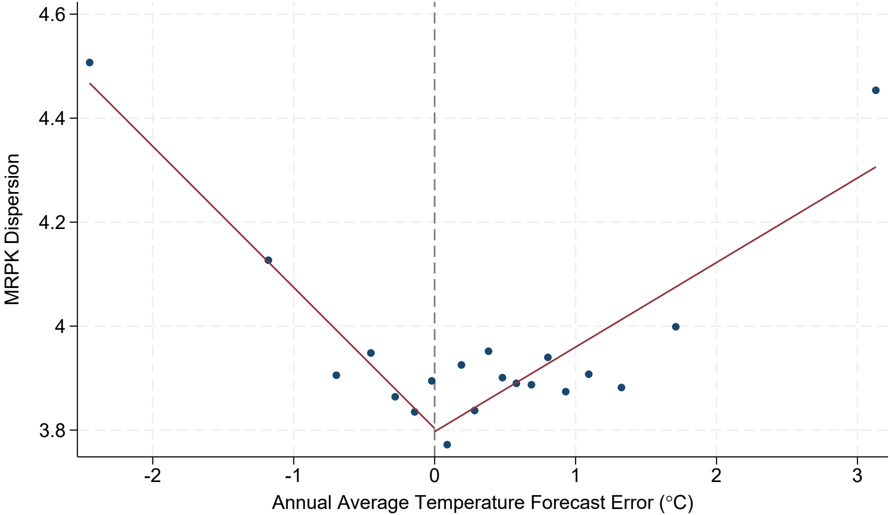
Notes: Standard errors in parentheses. We cluster standard errors at the regional level.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Shock Effect: Climate Volatility



Channel Decomposition via Model-Induced Regression

$$\mathrm{Var}(mrpk_{it+1}) = \left(\frac{1}{1-\alpha_N}\right)^2 \left[\underbrace{(T_{r,t} - T^*)^2 \sigma^2_{\xi,(r,s)}}_{\substack{\text{Damage Uncertainty}\\ \text{Channel}}} + \underbrace{\eta^{T\ 2}_{r,t}\, \sigma^2_{\beta,(r,s)}}_{\substack{\text{Climate Uncertainty}\\ \text{Channel}}} + \sigma^2_{\varepsilon,(r,s)} \right]$$

Now we estimate the following model-implied regression:

$$\mathrm{Var}_{s,r,t}(mrpk_{it}) = \beta_0 + \beta_1 (T_{r,t} - \hat{T}^*)^2 + \beta_2 \hat{\eta}^{T\ 2}_{r,t} + \delta_{s,r} + \delta_{c,s,t} + \varepsilon_{s,r,t}$$

Table: Model-induced Regressions: Level Effects and Shock Effects

	(1)	(2)
$(T_{r,t} - \hat{T}^*)^2$	0.004663^{***}	0.004621^{***}
	(0.000868)	(0.000865)
AR(10) Residuals $(\hat{\eta}^T_{r,t})^2$	0.030096^{**}	
	(0.013394)	
Annual MSFE $(\hat{\eta}^T_{r,t})^2$		0.016204^{**}
		(0.006593)
Region-Sector FE	Yes	Yes
Country-Year FE	Yes	Yes
Observations	124,065	124,065
R^2	0.976	0.976

[flushleft]

Channel Decomposition via Model-Induced Regression

$$\Delta\sigma^2_{mrpk,(s,r),t} = \underbrace{\underbrace{\left(\frac{1}{1-\alpha_N}\right)^2 \hat{\sigma}^2_{\hat{\xi},(s,r)}}_{0.0047} (T_{r,t} - T^*)^2}_{\text{Level Effect}} + \underbrace{\underbrace{\left(\frac{1}{1-\alpha_N}\right)^2 \hat{\sigma}^2_{\hat{\beta},(s,r)}}_{0.03} {\eta^T_{r,t}}^2}_{\text{Shock Effect}}, \tag{11}$$

$$\Delta \log TFP_{(s,r),t} = -\frac{\tilde{\alpha}_K + \tilde{\alpha}_K^2(\sigma-1)}{2}\Delta\sigma^2_{mrpk,(s,r),t}$$

Median Temperature Statistics	$(T_t - T^*)^2$	${\eta^T_{r,t}}^2$
Europe	4.693054	0.0708136
China	21.53677	0.0609404
India	181.7756	0.0975184

Channel Decomposition

$$\Delta\sigma^2_{mrpk,(s,r),t} = \underbrace{\underbrace{\left(\frac{1}{1-\alpha_N}\right)^2 \hat{\sigma}^2_{\hat{\xi},(s,r)}}_{0.0047} (T_{r,t} - T^*)^2}_{\text{Level Effect}} + \underbrace{\underbrace{\left(\frac{1}{1-\alpha_N}\right)^2 \hat{\sigma}^2_{\hat{\beta},(s,r)}}_{0.03} {\eta^T_{r,t}}^2}_{\text{Shock Effect}}, \tag{12}$$

$$\Delta \log TFP_{(s,r),t} = -\frac{\tilde{\alpha}_K + \tilde{\alpha}_K^2(\sigma - 1)}{2} \Delta\sigma^2_{mrpk,(s,r),t}$$

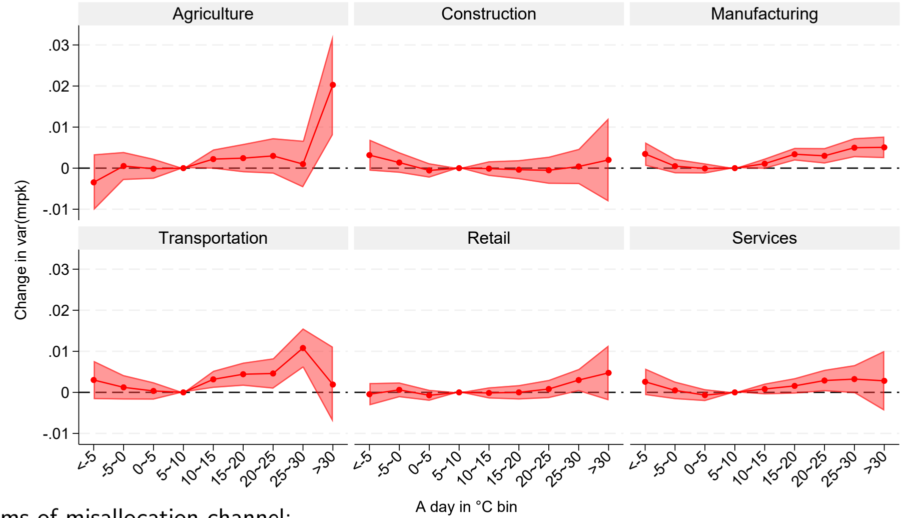
Table: Channel Contribution

Contribution to TFP loss	Level Effect	Shock Effect
Europe	0.29%	0.03%
China	1.34%	0.02%
India	11.31%	0.04%

Level effect dominates!

- Thanks!

Heterogeneous Effect across Major Sectors



In terms of misallocation channel:

- The U-shaped pattern holds for all sectors.
- Agricultural and construction sector suffer the most. (a $>30\ ^\circ$C day $\approx 0.23\%$ TFP loss)

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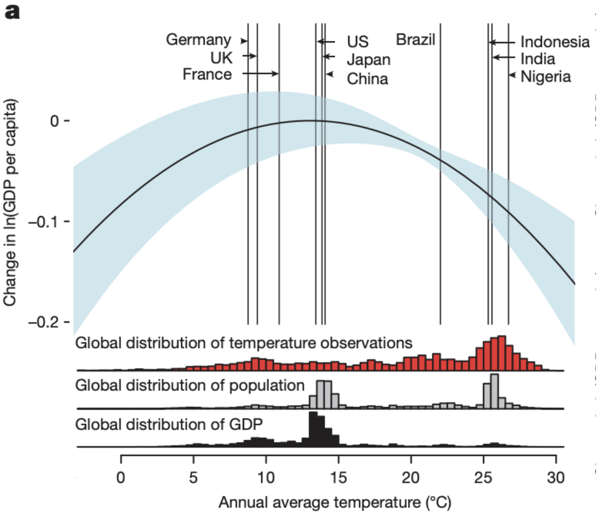
Burke, Hsiang and Miguel 2015

Their finding: ountry-level economic production is smooth, non-linear, and concave in temperature with a maximum at 13°C.



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