CS258 Information Theory Homework 1

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Exercise 1 *Coin flips* A fair coin is flipped until the first head occurs. Let X denote the number of flips required.

1. Find the entropy $H(X)$ in bits. The following expressions may be useful:

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}, \quad \sum_{n=0}^{\infty} nr^n = \frac{r}{(1-r)^2} \tag{1}$$

2. A random variable X is drawn according to this distribution. Find an "efficient" sequence of yes–no questions of the form, "Is X contained in the set S?" Compare $H(X)$ to the expected number of questions required to determine X.

Solution. □

Exercise 2 *Zero conditional entropy* Show that if $H(Y|X) = 0$, then Y is a function of X [i.e., for all x with $p(x) > 0$, there is only one possible value of y with $p(x, y) > 0$].

Proof. □

Exercise 3 *Coin weighing* Suppose that one has n coins, among which there may or may not be one counterfeit coin. If there is a counterfeit coin, it may be either heavier or lighter than the other coins. The coins are to be weighed by a balance.

1. Find an upper bound on the number of coins n so that k weighings will find the counterfeit coin (if any) and correctly declare it to be heavier or lighter.

2. (Difficult) What is the coin-weighing strategy for $k = 3$ weighings and 12 coins?

Solution. □