

Iso μ with casting

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1 Grammar

$typ, A, B, C, Char, Bool, ?$	$::=$	types
X		type variable
Int		int
Top		top type
$A \rightarrow B$		function type
$\mu X.A$		recursive type
exp, e, v, u, v_f	$::=$	expressions
x		variable
\top		top
i		lit
$\lambda x : A. e$		abstraction with argument annotation
$fix\ x : A. e$		fixpoint
$e_1\ e_2$		applications
$cast\ [c]e$		
$mode, m$	$::=$	modes
\oplus		positive
\ominus		negative
$castop, c$	$::=$	cast operators
id		id operator
\downarrow_A		unfold operator
\uparrow_A		castdn
$c_1 \rightarrow c_2$		arrow operator
ctx, Γ	$::=$	term context
\cdot		
$\Gamma, x : A$		
$tctx, \Delta$	$::=$	type context
\cdot		
Δ, X		

2 Subtyping

$\boxed{\vdash \Delta}$

(Well Formed Type Environment)

WFTYE-EMPTY

$\overline{\vdash \cdot}$

WFTYE-CONS

$\frac{\vdash \Delta \quad X \notin \Delta}{\vdash \Delta, X}$

3 Typing

$\Delta \vdash \Gamma$

(Well Formed Term Environment)

$$\frac{\text{WFTME-EMPTY}}{\Delta \vdash \cdot}$$

$$\frac{\text{WFTME-CONS} \quad \Delta \vdash \Gamma \quad \Delta \vdash A \quad x \notin \Gamma}{\Delta \vdash \Gamma, x : A}$$

$\Delta \vdash A$

(Well Formed Type)

$$\frac{\text{WFT-TOP}}{\Delta \vdash \text{Top}}$$

$$\frac{\text{WFT-INT}}{\Delta \vdash \text{Int}}$$

$$\frac{\text{WFT-VAR} \quad X \in \Delta}{\Delta \vdash X}$$

$$\frac{\text{WFT-ARROW} \quad \Delta \vdash A \quad \Delta \vdash B}{\Delta \vdash A \rightarrow B}$$

$$\frac{\text{WFT-REC} \quad \Delta, X \vdash A}{\Delta \vdash \mu X.A}$$

$\Delta \vdash A \hookrightarrow B : c$

(Typing Reduction Rules)

$$\frac{\text{TCAST-ID} \quad \Delta \vdash A}{\Delta \vdash A \hookrightarrow A : \text{id}}$$

$$\frac{\text{TCAST-ARROW} \quad \begin{array}{l} \Delta \vdash A_1 \hookrightarrow A_2 : c_1 \\ \Delta \vdash B_1 \hookrightarrow B_2 : c_2 \end{array}}{\Delta \vdash A_1 \rightarrow B_1 \hookrightarrow A_2 \rightarrow B_2 : c_1 \rightarrow c_2}$$

$$\frac{\text{TCAST-UNFOLD} \quad \Delta \vdash \mu X.A}{\Delta \vdash \mu X.A \hookrightarrow A[X \mapsto \mu X.A] : \downarrow_{\mu X.A}}$$

$$\frac{\text{TCAST-FOLD} \quad \Delta \vdash \mu X.A}{\Delta \vdash A[X \mapsto \mu X.A] \hookrightarrow \mu X.A : \uparrow_{\mu X.A}}$$

$\Delta; \Gamma \vdash e : A$

(Typing rules)

$$\frac{\text{TYPING-INT} \quad \vdash \Delta \quad \Delta \vdash \Gamma}{\Delta; \Gamma \vdash i : \text{Int}}$$

$$\frac{\text{TYPING-VAR} \quad \vdash \Delta \quad \Delta \vdash \Gamma \quad x : A \in \Gamma}{\Delta; \Gamma \vdash x : A}$$

$$\frac{\text{TYPING-ABS} \quad \Delta; \Gamma, x : A_1 \vdash e : A_2}{\Delta; \Gamma \vdash \lambda x : A_1. e : A_1 \rightarrow A_2}$$

$$\frac{\text{TYPING-APP} \quad \begin{array}{l} \Delta; \Gamma \vdash e_1 : A_1 \rightarrow A_2 \\ \Delta; \Gamma \vdash e_2 : A_1 \end{array}}{\Delta; \Gamma \vdash e_1 e_2 : A_2}$$

$$\frac{\text{TYPING-FIX} \quad \Delta; \Gamma, x : A \vdash e : A}{\Delta; \Gamma \vdash \text{fix } x : A. e : A}$$

$$\frac{\text{TYPING-CAST} \quad \Delta; \Gamma \vdash e : A \quad \Delta \vdash A \hookrightarrow B : c}{\Delta; \Gamma \vdash \text{cast } [c] e : B}$$

4 Semantics

value e

(Values)

$$\frac{\text{V-LIT}}{\text{value } i}$$

$$\frac{\text{V-ABS}}{\text{value } (\lambda x : A. e)}$$

$$\frac{\text{V-FOLD} \quad \text{value } e}{\text{value } (\text{cast } [\uparrow_A] e)}$$

$$\frac{\text{V-ARROW} \quad \text{value } e}{\text{value } (\text{cast } [c_1 \rightarrow c_2] e)}$$

$c_1 \sim c_2$

(DualCast)

$$\frac{\text{DCAST-ID}}{\text{id} \sim \text{id}}$$

$$\frac{\text{DCAST-ARROW} \quad \begin{array}{l} c_1 \sim c'_1 \quad c_2 \sim c'_2 \end{array}}{c_1 \rightarrow c_2 \sim c'_1 \rightarrow c'_2}$$

$$\frac{\text{DCAST-REC}}{\downarrow_A \sim \uparrow_A}$$

$$\boxed{e \hookrightarrow e'}$$

(Reduction rules)

RED-BETA

$$\frac{}{(\lambda x : A. e) e' \hookrightarrow e[x \mapsto e']}$$

RED-APPL

$$\frac{e_1 \hookrightarrow e'_1}{e_1 e_2 \hookrightarrow e'_1 e_2}$$

RED-APPR

$$\frac{\text{value } v_1 \quad e_2 \hookrightarrow e'_2}{v_1 e_2 \hookrightarrow v_1 e'_2}$$

RED-FIX

$$\frac{}{\text{fix } x : A. e \hookrightarrow e[x \mapsto \text{fix } x : A. e]}$$

RED-CAST-ARR

$$\frac{}{(\mathbf{cast} [c_1 \rightarrow c_2] e_1) e_2 \hookrightarrow \mathbf{cast} [c_2] (e_1 (\mathbf{cast} [\neg c_1] e_2))}$$

RED-CAST

$$\frac{e \hookrightarrow e'}{\mathbf{cast} [c] e \hookrightarrow \mathbf{cast} [c] e'}$$

RED-CASTELIM

$$\frac{c_1 \sim c_2 \quad \text{value } v}{\mathbf{cast} [c_1] (\mathbf{cast} [c_2] v) \hookrightarrow v}$$

RED-CASTID

$$\frac{\text{value } v}{\mathbf{cast} [\text{id}] v \hookrightarrow v}$$