A Vectorial and Phasor-Based Model of Programming: From Abstract Semantics to Quantum and Electrical Implementations

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Abstract

We present a representation of programming semantics in which assignments, operations, data types, and control structures are expressed as vectors and phasors within defined vector spaces. Assignments and arithmetic are modeled as affine/linear transformations; loops and decisions are represented by phasors, where iteration corresponds to angular displacement. We show how this formalism naturally translates into matrix algebra, and demonstrate mappings to quantum-circuit and electrical-circuit implementations. This unifies abstract programming constructs, linear-algebraic reasoning, and hardware realization in a single geometric framework.

Keywords: vector semantics, phasors, programming model, quantum circuits, electrical circuits, matrix algebra

1 Introduction

We propose a geometric programming model VPL (Vector-Phasor language) in which program state, data, operators and control flow are expressed as vectors or rotations (phasors) in a partitioned vector space. This view provides a compact linear-algebraic substrate for reasoning about programs and a pathway to physical implementations in quantum and electrical hardware.

2 Formal model

Let the program state be represented in homogeneous form as a column vector:

$$S = \begin{bmatrix} x \\ 1 \\ p_x \\ p_y \end{bmatrix}, \tag{1}$$

where x is the magnitude of a numeric variable, the constant 1 allows affine translations, and (p_x, p_y) encodes a control phasor in the plane.

2.1 Assignments and operators

An assignment or arithmetic operation is modeled as an affine linear operator acting on S. Example: adding a constant 2 to x is

$$A_{+2} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad S' = A_{+2}S. \tag{2}$$

2.2 Loops and decisions as phasors

Iteration and decision-making are modeled by planar rotations (phasors). A rotation by angle

theta acting on the control plane is

$$R(theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & -\sin\theta \\ 0 & 0 & \sin\theta & \cos\theta \end{bmatrix}.$$
 (3)

We adopt the convention $+90^{\circ}$ (or $+\pi/2$) represents a logical "true" phasor and -90° a logical "false" phasor; iteration corresponds to repeated application of a rotation $R(\theta)$.

3 Strings as Vectors in VPL

In the Vectorial Programming Language (VPL), strings are represented as vectors within the *String type vector space*. Each character corresponds to a basis vector, similar to ASCII encoding, but treated as a proper mathematical vector. For instance, the string

"Hello World"

is represented as

$$S = \mathbf{h} \oplus \mathbf{e} \oplus \mathbf{l} \oplus \mathbf{l} \oplus \mathbf{o} \oplus \mathbf{w} \oplus \mathbf{o} \oplus \mathbf{r} \oplus \mathbf{l} \oplus \mathbf{d}$$

where each symbol \mathbf{c} is a vector in the String subspace $\mathbb{V}_{\text{string}}$. This makes type casting (e.g. converting numbers to strings) a matter of vector translation between subspaces.

4 Data Structures as Vector Compositions

In VPL, all data structures are represented as compositions of vectors within their respective type subspaces. This provides a uniform algebraic foundation for both simple and complex structures.

4.1 Arrays and Lists

An array of n elements is a concatenation of vectors:

$$A = [a_1, a_2, \dots, a_n], \quad a_i \in \mathbb{V}_{\text{type}}.$$

Each element a_i belongs to the vector subspace corresponding to its type (e.g., numbers, strings). Lists are treated similarly, with additional operators for dynamic insertion and removal as displacements within the sequence.

4.2 Tuples and Records

Tuples are direct sums of heterogeneous vectors:

$$T = \mathbf{x} \oplus \mathbf{y} \oplus \mathbf{z}, \quad \mathbf{x} \in \mathbb{V}_{\text{int}}, \ \mathbf{y} \in \mathbb{V}_{\text{string}}, \ \mathbf{z} \in \mathbb{V}_{\text{bool}}.$$

This corresponds to records or structs in traditional programming languages. Type casting is modeled as translation between subspaces.

4.3 Trees

A tree is expressed as a recursive tensor product:

$$\mathcal{T} = \mathbf{r} \otimes (\mathcal{T}_1 \oplus \mathcal{T}_2 \oplus \cdots \oplus \mathcal{T}_k),$$

where \mathbf{r} is the root node vector, and \mathcal{T}_i are child subtrees. Traversals are phasor-controlled iterations through this structure.

4.4 Graphs

Graphs generalize trees by allowing cycles and arbitrary adjacency:

$$\mathcal{G} = \{\mathbf{v}_i\} \oplus \{E_{ij}\},\,$$

where $\mathbf{v}_i \in \mathbb{V}_{\text{nodes}}$ and edges $E_{ij} \in \mathbb{V}_{\text{relations}}$ represent connections. Graph traversal corresponds to phasor-driven exploration across edge vectors.

4.5 Implications

By defining data structures as vector compositions, VPL unifies memory representation, traversal, and operations under a single vectorial and phasor framework. This abstraction is particularly suited for parallel execution, since independent substructures (e.g., subtrees or graph components) can be processed concurrently via tensor products.

5 Parallel Execution in VPL

Parallelism in VPL arises naturally because control flow is a vector pointing to multiple instructions simultaneously. A control vector may be expressed as a superposition of instruction vectors:

$$C = \alpha \cdot I_1 + \beta \cdot I_2$$

where I_1 and I_2 are instructions and α, β weight their concurrent execution. This allows both SIMD-style data parallelism and task parallelism.

For data parallelism, an operator applied to a vector distributes over elements:

$$f([x_1, x_2, \dots, x_n]) = [f(x_1), f(x_2), \dots, f(x_n)].$$

6 Tensor Products for Concurrency

Independent tasks in VPL can be expressed as tensor products of their respective vector spaces:

$$S = \mathcal{T}_1 \otimes \mathcal{T}_2 \otimes \cdots \otimes \mathcal{T}_m$$

where each \mathcal{T}_i is a task represented as a vector in its own subspace. Execution proceeds in parallel, and synchronization corresponds to projection back into a shared subspace at measurement.

This formalism allows clear modeling of concurrent execution units (e.g. GPU threads, SIMD lanes) while keeping semantics consistent with the vectorial and phasor-based foundation of VPL.

7 Vector Space Model of Instructions

In the Vector Programming Language (VPL), control flow itself can be expressed as a vector in an *instruction space*. Instead of modeling instructions as sequential tape positions, each instruction is a basis vector in a finite-dimensional Hilbert space \mathcal{I} . The execution of a program is then represented by a control state $|C\rangle \in \mathcal{I}$ that evolves under linear operators.

7.1 Instruction Space

We define the instruction space as:

$$\mathcal{I} = \operatorname{span}\{|I_0\rangle, |I_1\rangle, \dots, |I_{N-1}\rangle\},\,$$

where each basis state corresponds to a program instruction. Execution proceeds by applying linear maps $M: \mathcal{I} \to \mathcal{I}$ that implement jumps, conditionals, and terminations.

7.2 Data Space

Program variables live in a separate vector space \mathcal{D} , whose basis represents classical values. For instance, an 8-value register uses

$$\mathcal{D} = \operatorname{span}\{|0\rangle, |1\rangle, \dots, |7\rangle\}.$$

The joint program state lies in the tensor product

$$\mathcal{J} = \mathcal{I} \otimes \mathcal{D}$$
.

7.3 Operators

Execution is defined by sparse operators acting on \mathcal{J} :

- Initialization: $M_{init} = |I_1\rangle\langle I_0| \otimes I_{\mathcal{D}}$.
- Comparison (Projection): $M_{cmp} = |I_{true}\rangle\langle I_1| \otimes P + |I_{false}\rangle\langle I_1| \otimes (I-P)$, where P is a projector selecting valid data states.
- Increment (Shift): $M_{inc} = |I_{next}\rangle\langle I_k|\otimes S_{+c}$, with S_{+c} a shift matrix adding a constant c.
- Jump: $M_{jump} = |I_j\rangle\langle I_k| \otimes I_{\mathcal{D}}$.
- Halt: $M_{halt} = |I_{halt}\rangle\langle I_{halt}| \otimes I_{\mathcal{D}}$.

The full program operator is the sum of all instruction clauses:

$$M = M_{init} + M_{cmp} + M_{inc} + M_{jump} + M_{halt}.$$

7.4 Why Control Instructions Are Vectors

In the classical model of computation, control flow is an abstract sequencing mechanism. In VPL, however, we treat every instruction—including control instructions such as if, while, and goto—as basis vectors in the same instruction space. This is justified by the following:

- 1. **Uniform Representation:** By encoding both arithmetic and control operations as vectors, we avoid a dual system where some operations live in data space and others in an external controller. All computation reduces to state evolution in a single linear space.
- 2. **Linear Composition:** Control instructions transform execution by redirecting program flow. Representing them as vectors allows these redirections to be modeled as linear maps between basis states. For example, a conditional jump is simply a projection onto one subspace (true branch) or its orthogonal complement (false branch).
- 3. **Tensor Product with Data:** Once instructions are vectors, they can be tensored with data vectors. This makes the joint system $\mathcal{J} = \mathcal{I} \otimes \mathcal{D}$ capture both "what is the current instruction" and "what is the current data" in a unified state.

- 4. **Geometric Interpretation:** Treating control instructions as phasors (rotations in the instruction space) aligns with the intuition that loops and conditionals correspond to angular displacements. A while loop is then a repeated rotation until a measurement (projection) halts it. In this convention, a decision is encoded as an angular displacement:
 - +90° corresponds to **true** (continue execution),
 - -90° corresponds to **false** (halt or exit branch).

This makes conditionals and loops interpretable as rotations in the complex plane, with branching behavior emerging from angular phase selection.

Thus, control instructions are not external directives but intrinsic vectors within \mathcal{I} , ensuring that the entire program, including flow control, remains representable as linear algebra.

8 Worked example

8.1 Phasor Update in Loop Execution with Matrix Formalism

Consider the VPL loop:

```
X = 0;
phasor(theta=90°) while (X < 5) { X = X + 2; }
measure(X)</pre>
```

In VPL, the loop counter is represented by the angle of a phasor. Each successful iteration advances the phasor by $+90^{\circ}$, while a failed condition forces the phasor to -90° , indicating termination.

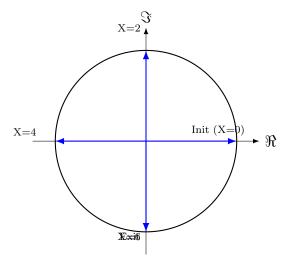
The update rule can be expressed as:

$$\theta_{k+1} = \begin{cases} \theta_k + 90^\circ, & \text{if } X < 5\\ -90^\circ, & \text{if } X \ge 5 \end{cases}$$

Matrix Representation of Operations

In VPL, each operation can be represented as a matrix acting on vector states.

Initialization. Let $|X\rangle$ be the state vector for variable X. Initialization to zero is given by the reset operator:



Phasor rotation corresponds to loop iterations

Figure 1: Phasor update for the while (X < 5) loop. Each iteration advances the phasor by 90°. On exit, the phasor points to -90°.

Increment. The update $X \mapsto X + 2$ is expressed as a shift operator:

so that

$$|X_{k+1}\rangle = S_{+2}|X_k\rangle$$

Condition. The comparison X < 5 can be represented as a projection operator:

$$P_{<5} = \text{diag}(1, 1, 1, 1, 1, 0, 0, \dots)$$

This operator filters the state vector, allowing the loop phasor to continue only if X is in the subspace $\{0, 1, 2, 3, 4\}$.

Measurement. Finally, measuring X corresponds to projecting onto the computational basis:

$$\langle X|: \{|0\rangle, |1\rangle, |2\rangle, \dots\} \to \mathbb{N}$$

This shows that a simple VPL loop can be equivalently expressed as: 1. A **phasor rotation** (control). 2. A **matrix-vector transformation** (arithmetic + condition). 3. A **measurement operator** (final readout).

Expanded View: Comparisons in VPL

In classical programming, a comparison such as X < 5 evaluates to a Boolean value {true, false}. In VPL, the same comparison is interpreted as a **phasor and a projection operator**:

1. **Projection in vector space.** Each variable X is represented as a state vector $|X\rangle$ in the type-specific vector space. The comparison X < 5 defines a subspace $\mathcal{H}_{<5} = \text{span}\{|0\rangle, |1\rangle, |2\rangle, |3\rangle, |4\rangle\}$. The projection operator is:

$$P_{<5} = diag(1, 1, 1, 1, 1, 0, 0, 0, \dots)$$

Applying $P_{<5}$ to the state either preserves $|X\rangle$ (if X<5) or annihilates it (if $X\geq 5$):

$$|X\rangle \mapsto P_{<5}|X\rangle$$

2. **Phasor interpretation.** The outcome of the projection is encoded as a phasor orientation:

if
$$P_{<5}|X\rangle = |X\rangle$$
, $\theta \mapsto \theta + 90^{\circ}$
if $P_{<5}|X\rangle = 0$, $\theta \mapsto -90^{\circ}$

Thus, the phasor simultaneously carries both the control-flow decision (continue or exit) and the iteration count.

3. Contrast with classical logic. While a Boolean comparison produces a scalar value in {0,1}, the VPL comparison is a *vector-space filter*, which not only decides the loop continuation but also preserves type-compatibility and control state in the same mathematical object.

This view highlights the dual role of comparisons in VPL: they are both *logical predicates* and *vector operators*. It is this reinterpretation that allows decision-making to be unified with loop phasors, making VPL particularly suitable for parallel execution, quantum modeling, and hardware mapping.

Matrix Compilation of a VPL Loop

The VPL program

can be compiled into a sequence of matrix operations acting on the state vector $|X\rangle$.

Initialization. The variable X is encoded as a basis state in the integer vector space:

$$|X\rangle \in \mathcal{H}_{\text{int}} = \text{span}\{|0\rangle, |1\rangle, |2\rangle, \dots\}.$$

Initialization to zero corresponds to the reset operator

$$R|X\rangle = |0\rangle.$$

Increment operator. The assignment X = X + 2 is a displacement in the integer space, represented by a shift matrix:

$$S_{+2}|n\rangle = |n+2\rangle.$$

Explicitly, the action on the first few basis states is:

$$S_{+2} = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ 1 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

Comparison operator. The loop condition X < 5 is not a Boolean, but a projection onto the subspace

$$\mathcal{H}_{<5} = \operatorname{span}\{|0\rangle, |1\rangle, |2\rangle, |3\rangle, |4\rangle\}.$$

This is implemented by the projection matrix

$$P_{<5} = \text{diag}(1, 1, 1, 1, 1, 0, 0, 0, \dots).$$

When applied:

$$P_{<5}|X\rangle = \begin{cases} |X\rangle, & X < 5\\ 0, & X \ge 5. \end{cases}$$

Phasor control. The outcome of the projection is linked to the phasor:

$$\theta_{k+1} = \begin{cases} \theta_k + 90^{\circ}, & P_{<5}|X\rangle \neq 0 \\ -90^{\circ}, & P_{<5}|X\rangle = 0. \end{cases}$$

Thus, control flow and arithmetic are unified in matrix–vector form.

Measurement. Finally, measuring X is a projection onto the computational basis, yielding a classical integer:

$$\mu(X) = \langle X|X\rangle.$$

Summary. The compiled form of the loop is therefore:

$$|X_{k+1}\rangle = P_{<5} S_{+2} |X_k\rangle,$$

iterated with phasor control until $P_{<5}|X\rangle=0$.

8.2 Example: While Loop instruction control

Consider the program:

Instruction Space. We define

$$\mathcal{I} = \operatorname{span}\{|I_0\rangle, |I_1\rangle, |I_2\rangle, |I_3\rangle, |I_4\rangle\},\$$

corresponding to initialization, comparison, increment, jump, and measurement.

Data Space. We take $\mathcal{D} = \text{span}\{|0\rangle, \dots, |7\rangle\}.$

Operators.

$$M_{init} = |I_1\rangle\langle I_0| \otimes I,$$

$$M_{cmp} = |I_2\rangle\langle I_1| \otimes P_{<5} + |I_4\rangle\langle I_1| \otimes (I - P_{<5}),$$

$$M_{inc} = |I_3\rangle\langle I_2| \otimes S_{+2},$$

$$M_{jump} = |I_1\rangle\langle I_3| \otimes I,$$

$$M_{halt} = |I_4\rangle\langle I_4| \otimes I.$$

The total operator is:

$$M = M_{init} + M_{cmp} + M_{inc} + M_{jump} + M_{halt}.$$

Execution. Starting from $|\Psi_0\rangle = |I_0\rangle \otimes |0\rangle$, iterative application of M yields:

$$|\Psi_{final}\rangle = |I_4\rangle \otimes |6\rangle.$$

Thus the loop halts with X=6, matching the classical semantics.

Interpretation. Here, the comparison operator is naturally a projection: it partitions the data subspace into a "continue" region (X < 5) and a "halt" region $(X \ge 5)$. This projection viewpoint clarifies how control flow and data interact within a unified linear-algebraic framework.

9 Mapping to quantum circuits

To obtain a quantum realization we choose an encoding. In a basis (computational) encoding, a 3-qubit register represents integer X in the range 0..7. The addition-by-2 operation becomes a modular adder unitary U_{+2} : $|x\rangle \mapsto |x+2 \pmod{8}$. The phasor state is encoded in an auxiliary qubit whose phase is rotated by $R_z(\pi/2)$ each iteration. A controlled-adder implements the conditional semantics if X < 5 then $X \leftarrow X + 2$.

10 Electrical circuit embodiment

A practical classical implementation uses well-known digital primitives:

• a 3-bit register (binary counter) storing X,

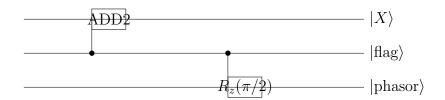


Figure 2: Quantum-circuit fragment: conditional add and phasor rotation.

- an adder stage that computes X + 2 and parallel-loads it,
- a comparator that implements the predicate (X < 5),
- a 2-bit phase register selecting one of four quadrature outputs (0°,90°,180°,270°), and
- \bullet an R-2R DAC to output analog measurements of X when desired.

VPL Construct	Electrical Circuit Component / Implementation
int X; (declare variable)	X Register (3-bit parallel load, e.g. 74HC173)
X = 0; (assignment / initialization)	Reset line wired to clear X Register $\rightarrow 000$
while (X < 5) (phasor condition)	Comparator block (74HC85 or logic decode) feeding Contro
<pre>phasor(theta=90°) (loop phase advance)</pre>	Phase Register (2-bit counter) + Phase Mux (CD4053) + C
X = X + 2; (increment by 2)	Adder block (74HC283, wired to add constant 010) feeding
Loop control / sequencing	Control FSM (clocked sequencer or MCU) issuing load and
<pre>measure(X);</pre>	Measurement block: parallel digital output or R-2R DAC (v

Table 1: Mapping between VPL constructs and electrical circuit components for the example loop.

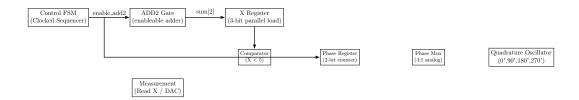


Figure 3: Scaled electrical implementation of the VPL loop. The while loop is managed by the Control FSM, with the phasor represented by phase selection of the quadrature oscillator.

11 Example: Parallel Execution in VPL

In the vector space model, parallelism arises by placing the control state into a superposition of multiple instructions. For instance, consider two loops that operate independently on different variables:

```
X = 0; while (X < 5) { X = X + 2; }
Y = 1; while (Y < 8) { Y = Y + 3; }
measure(X, Y)</pre>
```

Instruction Space. We define

$$\mathcal{I} = \operatorname{span}\{|I_X\rangle, |I_Y\rangle, |I_{halt}\rangle\},\,$$

where $|I_X\rangle$ controls the loop on X, $|I_Y\rangle$ controls the loop on Y, and $|I_{halt}\rangle$ is the shared measurement stage.

Parallel Control State. Instead of evolving $|I_X\rangle$ or $|I_Y\rangle$ alone, we prepare a superposed state:

$$|C\rangle = \alpha |I_X\rangle + \beta |I_Y\rangle,$$

with coefficients α, β controlling relative scheduling weights. This models concurrent execution of both loops.

Joint Data Space. Variables are represented as a tensor product:

$$\mathcal{D} = \operatorname{span}\{|0\rangle_X, \dots\} \otimes \operatorname{span}\{|0\rangle_Y, \dots\}.$$

Execution. Each loop uses its own comparison projector and increment operator, but acts only on its respective subspace of \mathcal{D} . For example:

$$M_X = |I_X\rangle\langle I_X| \otimes (P_{X<5}S_{+2}^{(X)} + (I - P_{X<5})),$$

$$M_Y = |I_Y\rangle\langle I_Y| \otimes (P_{Y<8}S_{+3}^{(Y)} + (I - P_{Y<8})).$$

The combined program operator is:

$$M = M_X + M_Y + M_{halt}.$$

Interpretation. At each step, both X and Y evolve in parallel, with the control phasor oscillating between $|I_X\rangle$ and $|I_Y\rangle$. This models parallel execution not as interleaving, but as true simultaneous state evolution in the tensor product space.

12 Discussion

The vectorial-phasor model yields compact algebraic representations for programs; matrix compilation demonstrates how execution can be represented as operator products acting on an initial state. While predicates and branching introduce nonlinearity (runtime-dependent operator selection), many core constructs map naturally to linear or unitary maps.

13 Conclusion

We presented a geometric programming semantics with concrete compilation into matrix operators and practical mappings to quantum and electrical implementations. This framework invites further study on richer typing partitions, reversible embeddings, and performance trade-offs for hardware embeddinents.

The Vector/Phasor Programming Language (VPL) introduces a novel way of expressing programming semantics through vectors and phasors. By treating assignments, operations, and data types as vectors, and control structures such as loops and decisions as phasors, VPL reframes computation in purely geometric terms. This provides a unified model that bridges abstract programming concepts with physical realizations in quantum systems, analog hardware, and electrical circuits.

VPL shines in several areas. As a theoretical framework, it unifies programming and linear algebra, offering an elegant geometric representation of data flow and control flow. As a compiler intermediate representation (IR), it could serve as a bridge between high-level programming languages and specialized backends such as quantum circuits, neuromorphic hardware, or GPU kernels. VPL also has clear potential as an educational tool, helping students understand that computation can be seen as trajectories in vector spaces, with loops as rotations and conditions as angular displacements. Finally, VPL is especially promising for unconventional computing paradigms, since its vector and phasor abstractions are native to quantum mechanics, signal processing, and oscillator-based analog computation.

Despite these advantages, VPL has limitations. Expressing symbolic or discrete logic requires embedding into vector spaces, which may be unintuitive or inefficient. Certain non-linear operations do not fit neatly into a purely linear or phasor-based framework, requiring approximations or extensions. Moreover, the practical use of VPL will depend on building a supporting toolchain: compilers, interpreters, and runtime systems capable of lowering vector/phasor constructs into executable code or hardware mappings.

Future work should focus on developing a compiler pipeline for VPL, where high-level constructs are translated into a vector/phasor intermediate representation and then targeted to different backends such as CPUs, GPUs, FPGAs, or quantum computers. Another research direction involves building educational platforms to help students and practitioners visualize computation as geometric motion, which could transform how programming is taught. Finally, exploring VPL as a design language for emerging hardware models—including analog, neuromorphic, and quantum computing—could provide a unified framework for algorithm design across radically different computational substrates.

Acknowledgments

Since this is an early draft... this is void.

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