Models of opinion formation have become popular in recent years. The basic idea is that the opinions of others will influence the opinion of individuals. The first two projects in the following explore some of the popular models.

Project 14.23 Models of opinion formation

(a) *The voter model.* On a regular lattice, assign each site the value ± 1. Choose a site (the voter) at random. The voter then adopts the same value as a randomly chosen neighbor. These two steps continue until all sites have the same value, that is, when they have reached consensus. Compute the probability of achieving a consensus of $+1$ given that the initial density of $+1$ sites is ρ_0. Use a 10×10 square lattice and make at least 20 runs at each density. Also compute the time to reach consensus as a function of the lattice size. In two dimensions this time scales as $N \ln N$, where N is the number of sites. How does the consensus time scale with N in $d = 1$ and $d = 3$ dimensions? How does it scale on a preferential attachment network (see the article by Sood and Redner)?

(b) *The relative agreement interaction model.* N individuals are initially assigned an opinion that takes on a value between 0 and 1. Choose two individuals, i and j, at random. Assume that the ith opinion O_i is greater than the jth opinion O_j. If their opinions differ by less than the parameter ϵ, then increase O_j by $(m/2)(O_i - O_j)$ and decrease O_i by the same amount, where m is another parameter. This model implements the idea that two people will influence each other only if their opinions are sufficiently close. Write a program to simulate this model. Use a `LatticeFrame` for which each cell can take on one of 256 values. The approximation of the continuum by 256 values is for visualization purposes only, and the 256 values should be sufficiently large to approximate a continuum of values. Choose $\epsilon = 10, 50$, and 100 (out of 256), and $m = 0.3$ and 0.6. To speed up the simulation include in your program the option to plot configurations only after a certain number of iterations (use `enableStepsPerDisplay(true)`). Choose $N \geq 2500$ and begin with a random set of opinions. Discuss whether a single opinion emerges and explain the magnitude of the fluctuations.

(c) *The Sznajd model.* Place individuals on a square lattice with linear dimension L and using periodic boundary conditions. Each individual has one of two opinions. At each iteration, an individual and one of the person's neighbors is chosen at random. If the two individuals have the same opinion, the opinion of the six neighbors of the pair is changed to that of the pair. The idea is that people are more likely to change their opinion to those physically near them if more than one person shares the same opinion (peer pressure). Write a program to simulate this model and show that consensus is always reached for all sites if the simulation is run for a sufficiently long time. Discuss the visual appearance of the groups of like-minded individuals. Consider initial configurations where the individuals are randomly assigned the two opinions and initial configurations where one opinion has a majority of 1%, 5%, and 10%. Choose $L \geq 50$.

(d) Generalize the Sznajd model so that an individual may be assigned one of more than two opinions. Is consensus still always reached? What happens if the individuals are not on the sites of a square lattice, but rather are the nodes of a preferential attachment network of at least 5000 nodes? ■