Numerical Methods 2023
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Deadline: Feb 27, 2023

Instructions: you must send a zipped folder with your codes, named lastname_code_ps2.zip (e.g. santos_code_ps2.zip). You must also send a PDF file with information about your solutions named lastname_answers_ps2.pdf (e.g. santos_answers_ps2.pdf). You must send these files to me and the TA (emails on the wiki).

For this problem set, you will have to solve the RBC model using different techniques for the value function iteration. The model is very standard. Here, I'll give you a brief description. For more details, see for example Cooley and Prescott (1995).

Preferences

The households have preferences given by:

$$U(c) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where

$$u(c_t) = \frac{c^{1-\mu} - 1}{1-\mu}$$

and $\beta = \frac{1}{1+\zeta}$.

Technology

There is a representative firm with the following production function:

$$Y_t = z_t F(K_t, N_t) = z_t K_t^{\alpha} N_t^{1-\alpha},$$

where Y_t is output, K_t is the capital stock, N_t is labor and z_t is the TFP, which is stochastic. The capital stock depreciates at rate δ.

For z_t, assume an AR(1) stochastic process in logs such that:

$$\log z_t = \rho \log z_{t-1} + \epsilon_t,$$

with $\epsilon_t \sim N(0, \sigma^2)$.

Equilibrium

Note that the first welfare theorem holds for this economy. Hence, you can solve the planner's problem to find the equilibrium allocation.

Calibration

We need to set values for the parameters. Use $\beta = 0.987$, a standard value. The coefficient of relative risk a version is $\mu = 2$,also standard. For the production function, use $\alpha = 1/3$, which implies a ratio of labor income to total income of $2/3$, consistent with the data. Use a depreciation rate of $\delta = 0.012$. For the stochastic process for the TFP shocks, use the values in Cooley and Prescott (1995): $\rho = 0.95$ and $\sigma = 0.007$.

Exercises

1. Write the planner's problem in recursive form.
2. For now, assume no uncertainty, i.e., $\sigma = 0$. Derive the Euler equation and find the steady state capital stock k_{ss}.
3. From now on, use the full model with uncertainty. Solve this problem in the computer using value function iteration (VFI).[1] For this, you will need to discretize your state variables. For the TFP shock, use Tauchen's (1986) method with 7 grid points. For the grid for the capital stock, use 500 linearly spaced points in $[0.75k_{ss}, 1.25k_{ss}]$. I strongly recommend that you do not use the brute force grid search method to find the policy function. For this and the following exercises, provide evidence for your solution: figures for the value and/or policy function, running time, Euler errors, etc.
4. For this exercise, redo the previous item using the accelerator. That is, only perform the maximization part for a few iterations (10% of them, for example). Compare these results with the previous ones.
5. Now, redo everything using multigrid. First, solve the problem using a grid with 100 points, then 500 and, finally, 5000. For each successive grid, use the previous solution as your initial guess (you will need to interpolate). Compare your results.

[1] You are free to use any language. Though I'd like you to use a faster language, Matlab is enough for this problem set.

6. Now, solve the problem using the endogenous grid method. Compare your results.

References

[1] Cooley, T. and E. Prescott. 1995. "Economic Growth and Business Cycles," in Cooley (ed.) Frontiers of Business Cycle Research.

[2] Tauchen, G. 1986. "Finite state markov-chain approximations to univariate and vector autoregressions," Economics Letters.