Numerical Methods 2023
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Deadline: March 17, 2023

Instructions: you must send a zipped folder with your codes, named lastname_code_ps2.zip (e.g. santos_code_ps2.zip). You must also send a PDF file with information about your solutions named lastname_answers_ps2.pdf (e.g. santos_answers_ps2.pdf). You must send these files to me and the TA (emails on the wiki).

For this problem set, you will have to solve the RBC model using different projection methods. The model is very standard. Here, I'll give you a brief description. For more details, see for example Cooley and Prescott (1995).

Preferences

The households have preferences given by:

$$U(c) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where

$$u(c_t) = \frac{c^{1-\mu} - 1}{1-\mu}$$

and $\beta = \frac{1}{1+\zeta}$.

Technology

There is a representative firm with the following production function:

$$Y_t = z_t F(K_t, N_t) = z_t K_t^{\alpha} N_t^{1-\alpha},$$

where Y_t is output, K_t is the capital stock, N_t is labor and z_t is the TFP, which is stochastic. The capital stock depreciates at rate δ.

For z_t, assume an AR(1) stochastic process in logs such that:

$$\log z_t = \rho \log z_{t-1} + \epsilon_t,$$

with $\epsilon_t \sim N(0, \sigma^2)$.

Equilibrium

Note that the first welfare theorem holds for this economy. Hence, you can solve the planner's problem to find the equilibrium allocation.

Calibration

We need to set values for the parameters. Use $\beta = 0.987$, a standard value. The coefficient of relative risk a version is $\mu = 2$,also standard. For the production function, use $\alpha = 1/3$, which implies a ratio of labor income to total income of $2/3$, consistent with the data. Use a depreciation rate of $\delta = 0.012$. For the stochastic process for the TFP shocks, use the values in Cooley and Prescott (1995): $\rho = 0.95$ and $\sigma = 0.007$.

Exercises

1. For this problem set, you must use projection methods. For this, solve the model using a global projection method. In particular, use Chebyshev polynomials and the collocation method to solve the model. Provide evidence for your solution: figures for the value/policy functions, running times, Euler errors, etc.

2. Now, use again a projection method: the finite elements method. Divide the space in several elements. To solve this, use both the collocation and the Galerkin methods.[1] Again, provide evidence!

References

[1] Cooley, T. and E. Prescott. 1995. "Economic Growth and Business Cycles," in Cooley (ed.) Frontiers of Business Cycle Research.

[2] Judd, K. 1998. "Numerical Methods in Economics".

[1] To use Galerkin's method, you will have to compute numerical integrals. For this, you should use quadratures. For more on this, you can refer to Grey Gordon's slides or Judd (1998).