



## RELATÓRIO DA AULA 8

### INTERPOLAÇÃO E POLINÔMIOS DE LAGRANGE

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1. Calcule os Polinômios de Lagrange para as seguintes funções discretas:

a.  $f(8.4)$  if  $f(8.1) = 16.94410$ ,  $f(8.3) = 17.56492$ ,  $f(8.6) = 18.50515$ ,  $f(8.7) = 18.82091$

The handwritten solution shows the calculation of the Lagrange polynomial  $P(x)$  for the function  $f(x)$  at  $x = 8.4$ . The points are  $(8.1, 16.94410)$ ,  $(8.3, 17.56492)$ ,  $(8.6, 18.50515)$ , and  $(8.7, 18.82091)$ . The Lagrange basis polynomials  $L_0, L_1, L_2, L_3$  are calculated, and the final polynomial  $P(x)$  is formed by summing the products of the function values and the basis polynomials. The final result is  $P(8.4) = 5.6657435$ .

$$P(x) = \sum_{i=0}^n f(x_i) L_i(x)$$
$$L_0 = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} = \frac{(x - 8.3)(x - 8.6)(x - 8.7)}{(8.1 - 8.3)(8.1 - 8.6)(8.1 - 8.7)} = -0.06$$
$$L_1 = \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} = \frac{(x - 8.1)(x - 8.6)(x - 8.7)}{(8.3 - 8.1)(8.3 - 8.6)(8.3 - 8.7)} = 0.024$$
$$L_2 = \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} = \frac{(x - 8.1)(x - 8.3)(x - 8.7)}{(8.6 - 8.1)(8.6 - 8.3)(8.6 - 8.7)} = 0.015$$
$$L_3 = \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} = \frac{(x - 8.1)(x - 8.3)(x - 8.6)}{(8.7 - 8.1)(8.7 - 8.3)(8.7 - 8.6)} = 0.024$$
$$P(x) = 16.94410 \left( \frac{(x - 8.3)(x - 8.6)(x - 8.7)}{-0.06} \right) + 17.56492 \left( \frac{(x - 8.1)(x - 8.6)(x - 8.7)}{0.024} \right) + 18.50515 \left( \frac{(x - 8.1)(x - 8.3)(x - 8.7)}{0.015} \right) + 18.82091 \left( \frac{(x - 8.1)(x - 8.3)(x - 8.6)}{0.024} \right)$$
$$P(8.4) = 16.94410(-0.1) + 17.56492(0.025) + 18.50515(-0.06) + 18.82091(-0.26) \rightarrow P(8.4) = -2.69441 + 0.439123 + (-1.110309) - 4.893436 \rightarrow P(8.4) = -8.258992$$

$P(8.4) = 5.6657435$



- b.  $f(-\frac{1}{3})$  if  $f(-0.75) = -0.07181250$ ,  $f(-0.5) = -0.02475000$ ,  $f(-0.25) = 0.33493750$ ,  
 $f(0) = 1.10100000$

b)  $f(-\frac{1}{3})$

$x_0 = -0.75$	$y_0 = -0.07181250$	$x_2 = -0.25$	$y_2 = 0.33493750$
$x_1 = -0.5$	$y_1 = -0.02475000$	$x_3 = 0$	$y_3 = 1.1010$
$x_2 =$			

$$L_{3,0} = \frac{(x+0.5)(x+0.25)(x)}{(-0.25+0.5)(-0.25+0.25)(-0.25)} \rightarrow L(x_0) = \frac{(x+0.5)(x+0.25)(x)}{0.09375}$$

$$L_{3,1} = \frac{(x+0.75)(x+0.25)(x)}{(-0.5+0.75)(-0.5+0.25)(-0.5)} \rightarrow L(x_1) = \frac{(x+0.75)(x+0.25)(x)}{-0.0625}$$

$$L_{3,2} = \frac{(x+0.75)(x+0.5)(x)}{(-0.25+0.75)(-0.25+0.5)(-0.25)} \rightarrow L(x_2) = \frac{(x+0.75)(x+0.5)(x)}{0.125}$$

$$L_{3,3} = \frac{(x+0.75)(x+0.5)(x+0.25)}{0.75 \cdot 0.5 \cdot 0.25} \rightarrow L(x_3) = \frac{(x+0.75)(x+0.5)(x+0.25)}{0.09375}$$

$$P(x) = -0.07181250 \left( \frac{(x+0.5)(x+0.25)(x)}{0.09375} \right) + 0.02475000 \left( \frac{(x+0.75)(x+0.25)(x)}{-0.0625} \right) + 0.33493750 \left( \frac{(x+0.75)(x+0.5)(x)}{0.125} \right) + 1.1010 \left( \frac{(x+0.75)(x+0.5)(x+0.25)}{0.09375} \right)$$

Para:

$$P(0) = 0 + 0 + 0 + 1.101 = 1.1010$$

$$P(0.5) = 0 - 0.02475000 + 0 + 0 = -0.02475000$$

$$P\left(\frac{1}{3}\right) = -0.012279 + 0.0832 + 0.7976 + 3.2450$$
$$P\left(\frac{1}{3}\right) = 4.17366$$



- c.  $f(0.25)$  if  $f(0.1) = 0.62049958$ ,  $f(0.2) = -0.28398668$ ,  $f(0.3) = 0.00660095$ ,  $f(0.4) = 0.24842440$

c)  $f(0.25)$

$$\begin{array}{l} x_0 = 0.1 \quad y_0 = 0.6204 \\ x_1 = 0.2 \quad y_1 = -0.2839 \end{array} \quad \begin{array}{l} x_2 = 0.3 \quad y_2 = 0.0066 \\ x_3 = 0.4 \quad y_3 = 0.2484 \end{array}$$
$$L_{30} = \frac{(x-0.2)(x-0.3)(x-0.4)}{(0.1-0.2)(0.1-0.3)(0.1-0.4)} \rightarrow L(x) = \frac{(x-0.2)(x-0.3)(x-0.4)}{-0.006}$$
$$L_{31} = \frac{(x-0.1)(x-0.3)(x-0.4)}{(0.2-0.1)(0.2-0.3)(0.2-0.4)} \rightarrow L(x) = \frac{(x-0.1)(x-0.3)(x-0.4)}{0.002}$$
$$L_{32} = \frac{(x-0.1)(x-0.2)(x-0.4)}{(0.3-0.1)(0.3-0.2)(0.3-0.4)} \rightarrow L(x) = \frac{(x-0.1)(x-0.2)(x-0.4)}{-0.002}$$
$$L_{33} = \frac{(x-0.1)(x-0.2)(x-0.3)}{(0.4-0.1)(0.4-0.2)(0.4-0.3)} \rightarrow L(x) = \frac{(x-0.1)(x-0.2)(x-0.3)}{0.006}$$
$$P(x) = 0.6204 \left( \frac{(x-0.2)(x-0.3)(x-0.4)}{-0.006} \right) - 0.2839 \left( \frac{(x-0.1)(x-0.3)(x-0.4)}{0.002} \right) + 0.0066 \left( \frac{(x-0.1)(x-0.2)(x-0.4)}{-0.002} \right) + 0.2484 \left( \frac{(x-0.1)(x-0.2)(x-0.3)}{0.006} \right)$$

Prova:

$$P(0.1) = 0.6204 + 0 + 0 + 0 = 0.6204 //$$
$$P(0.2) = 0 + 0 + 0.0066 + 0 = 0.0066 //$$
$$P(0.25) = -0.038775 - 0.16969 + 0.003771 + 0.015525$$
$$P(0.25) = -0.21029$$



- d.  $f(0.9)$  if  $f(0.6) = -0.17694460$ ,  $f(0.7) = 0.01375227$ ,  $f(0.8) = 0.22363362$ ,  $f(1.0) = 0.65809197$

d)  $p(0.9)$

$$\begin{array}{l} x_0 = 0.6 \text{ e } y_0 = -0.17694 \\ x_1 = 0.7 \text{ e } y_1 = 0.01375 \end{array} \quad \left\{ \begin{array}{l} x_2 = 0.8 \text{ e } y_2 = 0.22363 \\ x_3 = 1 \text{ e } y_3 = 0.65809 \end{array} \right.$$

$$L_{30} = \frac{(x-0.7)(x-0.8)(x-1)}{(0.6-0.7)(0.6-0.8)(0.6-1)} \rightarrow L(x) = \frac{(x-0.7)(x-0.8)(x-1)}{-0.008}$$

$$L_{31} = \frac{(x-0.6)(x-0.8)(x-1)}{(0.7-0.6)(0.7-0.8)(0.7-1)} \rightarrow L(x) = \frac{(x-0.6)(x-0.8)(x-1)}{0.003}$$

$$L_{32} = \frac{(x-0.6)(x-0.7)(x-1)}{(0.8-0.6)(0.8-0.7)(0.8-1)} \rightarrow L(x) = \frac{(x-0.6)(x-0.7)(x-1)}{-0.004}$$

$$L_{33} = \frac{(x-0.6)(x-0.7)(x-0.8)}{(1-0.6)(1-0.7)(1-0.8)} \rightarrow L(x) = \frac{(x-0.6)(x-0.7)(x-0.8)}{0.024}$$

$$\begin{aligned} p(x) = & -0.17694 \left( \frac{(x-0.7)(x-0.8)(x-1)}{-0.008} \right) + 0.01375 \left( \frac{(x-0.6)(x-0.8)(x-1)}{0.003} \right) \\ & + 0.22363 \left( \frac{(x-0.6)(x-0.7)(x-1)}{-0.004} \right) + 0.65809 \left( \frac{(x-0.6)(x-0.7)(x-0.8)}{0.024} \right) \end{aligned}$$

Para

$$p(0.6) = -0.17694 + 0.0107 = -0.17694$$

$$p(1) = 0.0107 + 0.65809 = 0.65809$$

$$p(0.9) = -0.044235 - 0.000177 + 0.336145 + 0.164522$$
$$p(0.9) = 0.455556$$





2. Seja  $f(x) = e^x$ , para  $0 \leq x \leq 2$ : • Aproxime  $f(0.25)$  usando uma interpolação linear neste intervalo. • Aproxime  $f(0.75)$  usando uma interpolação linear neste intervalo.

2)  $e = 2,71828$        $p(x) = e^x$  para  $0 \leq x \leq 2$

$p(0,25) = 2,71828^{0,25} = 1,284025$

$p(0,75) = 2,71828^{0,75} = 2,11699$