ME 5554 / AOE 5754 / ECE 5754: Applied Linear Systems
HOMEWORK 2

1. Place the following coupled nonlinear ODE's into first-order form. Since these equations contain nonlinear terms, you should NOT place these equations into a State-Space representation.

$$M\ddot{x}(t) + B\dot{x}(t)^2 + Kx(t) = u_1(t) + G\theta(t)$$
$$G\dot{x}(t) + L\dot{\theta}(t) = u_2(t) - R\theta(t)x(t)$$

2. Given the following State Equation and initial condition:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & -5 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \qquad \mathbf{x}_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

2a. Use the Matlab function EXPM() to compute a numerical representation for the Matrix Exponential. You will need to establish a time vector that has an appropriate time resolution and an appropriate time duration to numerically capture the time dependency of each element in the matrix exponential.

2b. Use the Symbolic Toolbox with the EXPM() function in Matlab to generate a symbolic representation for the Matrix Exponential.

2c. Numerically evaluate the symbolic representation from 2b using the same time vector from 2a.

2d. Generate an analytic representation for the Matrix Exponential using Laplace Transforms. You should do this problem by hand, but you are welcome to use the Symbolic Toolbox in Matlab to check your result.

2e. Numerically evaluate the symbolic representation from 2d using the same time vector from 2a.

2f. Use Matlab to generate a plot of all elements in the Matrix Exponential. In this plot, you must compare the numerical results from 2a, 2c, and 2e. Your subplots must be properly annotated and clearly show all three curves in the comparison. (Hint: If you did the problem correctly, the numerical results should be identical. If you plot three identical curves on top of each other, you will only be able to see the last one plotted. You need to show all three identical curves without generating a separate subplot for each case.) You should generate a different subplot or a different figure for each element of the Matrix Exponential.

3. Use any of the results from problem 2 to compute and plot the state trajectory for the Initial Value response of this dynamic system.