ME-5554 Applied Linear System
Homework 3

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Answer Sheet

1 NE2.2 - For the following systems described by the given state equations, derive the associated transfer functions:

1. $$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + [0]u(t)$$

Transfer function H(s) is defined by:

$H(s) = C.(sI - A)^{-1}.B + D$

We have:

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & 0 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} s+3 & 0 \\ 0 & s+4 \end{bmatrix}$$

$$H(s) = \begin{bmatrix} 1 & 1 \end{bmatrix} . \begin{bmatrix} s+3 & 0 \\ 0 & s+4 \end{bmatrix}^{-1} . \begin{bmatrix} 1 \\ 1 \end{bmatrix} + [0]$$

$$H(s) = \begin{bmatrix} 1 & 1 \end{bmatrix} . \frac{1}{(s+3).(s+4)} . \begin{bmatrix} s+4 & 0 \\ 0 & s+3 \end{bmatrix} . \begin{bmatrix} 1 \\ 1 \end{bmatrix} + [0]$$

$$= \begin{bmatrix} \frac{1}{s+3} & \frac{1}{s+4} \end{bmatrix} . \begin{bmatrix} 1 \\ 1 \end{bmatrix} + [0]$$

$$= \frac{1}{s+3} + \frac{1}{s+4} = \frac{2s+7}{(s+3).(s+4)}$$

2. $$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + [0]\text{u(t)}$$

Transfer function H(s) is defined by:

$H(s) = C.(sI - A)^{-1}.B + D$

We have:

$$(sI-A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 3 & s+2 \end{bmatrix}$$

$$H(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} . \begin{bmatrix} s & -1 \\ 3 & s+2 \end{bmatrix}^{-1} . \begin{bmatrix} 0 \\ 1 \end{bmatrix} + [0]$$

$$H(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} . \frac{1}{s.(s+2)+3} . \begin{bmatrix} s+2 & 1 \\ -3 & s \end{bmatrix} . \begin{bmatrix} 0 \\ 1 \end{bmatrix} + [0]$$

$$= \frac{1}{s^2+2s+3} . \begin{bmatrix} s+2 & 1 \end{bmatrix} . \begin{bmatrix} 0 \\ 1 \end{bmatrix} + [0]$$

$$= \frac{1}{s^2+2s+3}$$

3. $$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 1 & -12 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + [0]\text{u(t)}$$

Transfer function H(s) is defined by:

$$H(s) = C.(sI-A)^{-1}.B + D$$

We have:

$$(sI-A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & -2 \\ 1 & -12 \end{bmatrix} = \begin{bmatrix} s & 2 \\ -1 & s+12 \end{bmatrix}$$

$$H(s) = \begin{bmatrix} 0 & 1 \end{bmatrix} . \begin{bmatrix} s & 2 \\ -1 & s+12 \end{bmatrix}^{-1} . \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$H(s) = \begin{bmatrix} 0 & 1 \end{bmatrix} . \frac{1}{s.(s+12)+2} . \begin{bmatrix} s+12 & -2 \\ 1 & s \end{bmatrix} . \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$= \frac{1}{s^2+12s+2} . \begin{bmatrix} 1 & s \end{bmatrix} . \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$= \frac{1}{s^2+12s+2}$$

4. $$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 5 \\ 6 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 7 & 8 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + [9]\text{u(t)}$$

Transfer function H(s) is defined by:

$H(s) = C.(sI - A)^{-1}.B + D$

We have:

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} s-1 & -2 \\ -3 & s-4 \end{bmatrix}$$

$$H(s) = \begin{bmatrix} 7 & 8 \end{bmatrix} \cdot \begin{bmatrix} s-1 & -2 \\ -3 & s-4 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 5 \\ 6 \end{bmatrix} + [9]$$

$$H(s) = \begin{bmatrix} 7 & 8 \end{bmatrix} \cdot \frac{1}{(s-1)(s-4)-6} \cdot \begin{bmatrix} s-4 & 2 \\ 3 & s-1 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 6 \end{bmatrix} + [9]$$

$$= \frac{1}{s^2 - 5s - 2} \cdot \begin{bmatrix} 7s-4 & 8s+6 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 6 \end{bmatrix} + [9]$$

$$= \frac{83s + 16}{s^2 - 5s - 2} + 9$$

$$= \frac{9s^2 + 38s - 2}{s^2 - 5s - 2}$$

2 NE2.3 - Determine the characteristic polynomial and eigenvalues for the systems represented by the following system dynamics matrices:

1. $A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$

   $$A1 = (\lambda.I - A) = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} \lambda+1 & 0 \\ 0 & \lambda+2 \end{bmatrix}$$

   Characteristic polynomial of A is defined by determinant of A1:

   $det(\lambda.I - A) = (\lambda + 1)(\lambda + 2)$

   Eigenvalues of A are defined when $det(\lambda.I - A) = 0$, so:

   $$(\lambda + 1)(\lambda + 2) = 0 \Leftrightarrow \begin{cases} \lambda_1 = -1 \\ \lambda_2 = -2 \end{cases}$$

2. $A = \begin{bmatrix} 0 & 1 \\ -10 & -20 \end{bmatrix}$

   $$A2 = (\lambda.I - A) = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -10 & -20 \end{bmatrix} = \begin{bmatrix} \lambda & -1 \\ 10 & \lambda+20 \end{bmatrix}$$

   Characteristic polynomial of A is defined by determinant of A2:

   $det(\lambda.I - A) = \lambda.(\lambda + 20) + 10 = \lambda^2 + 20\lambda + 10$

   Eigenvalues of A are defined when $det(\lambda.I - A) = 0$, so:

   $$\lambda^2 + 20\lambda + 10 = 0 \Leftrightarrow \begin{cases} \lambda_1 = -10 + 3\sqrt{10} \\ \lambda_2 = -10 - 3\sqrt{10} \end{cases}$$

3. $A = \begin{bmatrix} 0 & 1 \\ -10 & 0 \end{bmatrix}$

   $$A3 = (\lambda.I - A) = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -10 & 0 \end{bmatrix} = \begin{bmatrix} \lambda & -1 \\ 10 & \lambda \end{bmatrix}$$

   Characteristic polynomial of A is defined by determinant of A3:

   $det(\lambda.I - A) = \lambda.\lambda + 10 = \lambda^2 + 10$

Eigenvalues of A are defined when $det(\lambda.I - A) = 0$, so:

$$\lambda^2 + 10 = 0 \Leftrightarrow \begin{cases} \lambda_1 = j\sqrt{10} \\ \lambda_2 = -j\sqrt{10} \end{cases}$$

4. $A = \begin{bmatrix} 0 & 1 \\ 0 & -20 \end{bmatrix}$

$$A4 = (\lambda.I - A) = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & -20 \end{bmatrix} = \begin{bmatrix} \lambda & -1 \\ 0 & \lambda + 20 \end{bmatrix}$$

Characteristic polynomial of A is defined by determinant of A4:

$det(\lambda.I - A) = \lambda.(\lambda + 20) + 1 = \lambda^2 + 20\lambda + 1$

Eigenvalues of A are defined when $det(\lambda.I - A) = 0$, so:

$$\lambda^2 + 20\lambda + 1 = 0 \Leftrightarrow \begin{cases} \lambda_1 = -10 + 3\sqrt{11} \\ \lambda_2 = -10 - 3\sqrt{11} \end{cases}$$

3 NE2.10 - Diagonalize the following system dynamics matrices A using coordinate transformation:

1. $A = \begin{bmatrix} 0 & 1 \\ -8 & -20 \end{bmatrix}$

   $|\lambda.I - A| = det \begin{bmatrix} \lambda & -1 \\ 8 & \lambda + 20 \end{bmatrix} = \lambda.(\lambda + 20) + 8$

   Eigenvalues of A are: $\begin{cases} \lambda_1 = -10 + 2\sqrt{23} \\ \lambda_2 = -10 - 2\sqrt{23} \end{cases}$

   For $\lambda_1 = -10 + 2\sqrt{23}$ the eigenvector ν_1 is defined as $\begin{bmatrix} \nu_{11} \\ \nu_{21} \end{bmatrix}$ with:

   $[\lambda.I - A].\nu_1 = 0 \Leftrightarrow \begin{bmatrix} -10 + 2\sqrt{23} & -1 \\ 8 & 10 + 2\sqrt{23} \end{bmatrix} . \begin{bmatrix} \nu_{11} \\ \nu_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

   $\Leftrightarrow \begin{bmatrix} \nu_{11} \\ \nu_{21} \end{bmatrix} = \begin{bmatrix} 10 + 2\sqrt{23} \\ -8 \end{bmatrix}$

   For $\lambda_2 = -10 - 2\sqrt{23}$ the eigenvector ν_2 is defined as $\begin{bmatrix} \nu_{12} \\ \nu_{22} \end{bmatrix}$ with:

   $[\lambda.I - A].\nu_2 = 0 \Leftrightarrow \begin{bmatrix} -10 - 2\sqrt{23} & -1 \\ 8 & 10 - 2\sqrt{23} \end{bmatrix} . \begin{bmatrix} \nu_{12} \\ \nu_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

   $\Leftrightarrow \begin{bmatrix} \nu_{12} \\ \nu_{22} \end{bmatrix} = \begin{bmatrix} 10 - 2\sqrt{23} \\ -8 \end{bmatrix}$

   The diagonal canonical form transformation matrix T is defined base on eigenvectors ν:

   $T = \begin{bmatrix} \nu_1 & \nu_2 \end{bmatrix} = \begin{bmatrix} 10 + 2\sqrt{23} & 10 - 2\sqrt{23} \\ -8 & -8 \end{bmatrix}$

   $T^{-1} = \dfrac{1}{det(T)} . \begin{bmatrix} -8 & -10 + 2\sqrt{23} \\ 8 & 10 + 2\sqrt{23} \end{bmatrix} = \dfrac{-1}{32\sqrt{23}} . \begin{bmatrix} -8 & -10 + 2\sqrt{23} \\ 8 & 10 + 2\sqrt{23} \end{bmatrix}$

   The diagonal canonical form or matrix A is defined:

$$A_{DCF} = T^{-1}.A.T = \frac{-1}{32\sqrt{23}}.\begin{bmatrix} -8 & -10+2\sqrt{23} \\ 8 & 10+2\sqrt{23} \end{bmatrix}.\begin{bmatrix} 0 & 1 \\ -8 & -20 \end{bmatrix}.\begin{bmatrix} 10+2\sqrt{23} & 10-2\sqrt{23} \\ -8 & -8 \end{bmatrix}$$

$$= \frac{-1}{32\sqrt{23}}.\begin{bmatrix} 80-162\sqrt{23} & 192-40\sqrt{23} \\ -80-16\sqrt{23} & -192-40\sqrt{23} \end{bmatrix}.\begin{bmatrix} 10+2\sqrt{23} & 10-2\sqrt{23} \\ -8 & -8 \end{bmatrix}$$

$$= \frac{-1}{32\sqrt{23}}.\begin{bmatrix} -1472+320\sqrt{23} & 0 \\ 0 & 1472+320\sqrt{23} \end{bmatrix}$$

$$= \begin{bmatrix} 2\sqrt{23}-10 & 0 \\ 0 & -2\sqrt{23}-10 \end{bmatrix}$$

2. $A = \begin{bmatrix} 0 & 1 \\ 10 & 6 \end{bmatrix}$

$$|\lambda.I - A| = det\begin{bmatrix} \lambda & -1 \\ -10 & \lambda-6 \end{bmatrix} = \lambda.(\lambda-6) - 10 = \lambda^2 - 6\lambda - 10$$

Eigenvalues of A are: $\begin{cases} \lambda_1 = 3+\sqrt{19} \\ \lambda_2 = 3-\sqrt{19} \end{cases}$

For $\lambda_1 = 3+\sqrt{19}$ the eigenvector ν_1 is defined as $\begin{bmatrix} \nu_{11} \\ \nu_{21} \end{bmatrix}$ with:

$$[\lambda.I - A].\nu_1 = 0 \Leftrightarrow \begin{bmatrix} 3+\sqrt{19} & -1 \\ -10 & -3+\sqrt{19} \end{bmatrix}.\begin{bmatrix} \nu_{11} \\ \nu_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} \nu_{11} \\ \nu_{21} \end{bmatrix} = \begin{bmatrix} 1 \\ 3+\sqrt{19} \end{bmatrix}$$

For $\lambda_2 = 3-\sqrt{19}$ the eigenvector ν_2 is defined as $\begin{bmatrix} \nu_{12} \\ \nu_{22} \end{bmatrix}$ with:

$$[\lambda.I - A].\nu_2 = 0 \Leftrightarrow \begin{bmatrix} 3-\sqrt{19} & -1 \\ -10 & -3-\sqrt{19} \end{bmatrix}.\begin{bmatrix} \nu_{12} \\ \nu_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} \nu_{11} \\ \nu_{21} \end{bmatrix} = \begin{bmatrix} 1 \\ 3-\sqrt{19} \end{bmatrix}$$

The diagonal canonical form transformation matrix T is defined base on eigenvectors ν:

$$T = \begin{bmatrix} \nu_1 & \nu_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 3+\sqrt{19} & 3-\sqrt{19} \end{bmatrix}$$

$$T^{-1} = \frac{1}{det(T)}.\begin{bmatrix} 3-\sqrt{19} & -1 \\ -3-\sqrt{19} & 1 \end{bmatrix} = \frac{-1}{2\sqrt{19}}.\begin{bmatrix} 3-\sqrt{19} & -1 \\ -3-\sqrt{19} & 1 \end{bmatrix}$$

The diagonal canonical form or matrix A is defined:

$$A_{DCF} = T^{-1}.A.T = \frac{-1}{2\sqrt{19}}.\begin{bmatrix} 3-\sqrt{19} & -1 \\ -3-\sqrt{19} & 1 \end{bmatrix}.\begin{bmatrix} 0 & 1 \\ 10 & 6 \end{bmatrix}.\begin{bmatrix} 1 & 1 \\ 3+\sqrt{19} & 3-\sqrt{19} \end{bmatrix}$$

$$A_{DCF} = \frac{-1}{2\sqrt{19}}.\begin{bmatrix} -10 & -3-\sqrt{19} \\ 10 & 3-\sqrt{19} \end{bmatrix}.\begin{bmatrix} 1 & 1 \\ 3+\sqrt{19} & 3-\sqrt{19} \end{bmatrix}$$

$$A_{DCF} = \frac{-1}{2\sqrt{19}}\begin{bmatrix} -20 & 0 \\ 0 & 38-6\sqrt{19} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{10}{\sqrt{19}} & 0 \\ 0 & 3-\sqrt{19} \end{bmatrix}$$

3. $A = \begin{bmatrix} 0 & -10 \\ 1 & -1 \end{bmatrix}$

$$|\lambda.I - A| = det\begin{bmatrix} \lambda & 10 \\ -1 & \lambda+1 \end{bmatrix} = \lambda.(\lambda+1) + 10 = \lambda^2 + \lambda + 10$$

Eigenvalues of A are: $\begin{cases} \lambda_1 = \dfrac{-1-j\sqrt{39}}{2} \\ \lambda_2 = \dfrac{-1+j\sqrt{39}}{2} \end{cases}$

For $\lambda_1 = \dfrac{-1-j\sqrt{39}}{2}$ the eigenvector ν_1 is defined as $\begin{bmatrix} \nu_{11} \\ \nu_{21} \end{bmatrix}$ with:

$$[\lambda.I - A].\nu_1 = 0 \Leftrightarrow \begin{bmatrix} \frac{-1-j\sqrt{39}}{2} & 10 \\ -1 & \frac{1-j\sqrt{39}}{2} \end{bmatrix}.\begin{bmatrix} \nu_{11} \\ \nu_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} \nu_{11} \\ \nu_{21} \end{bmatrix} = \begin{bmatrix} \frac{1-j\sqrt{39}}{2} \\ 1 \end{bmatrix}$$

For $\lambda_2 = \dfrac{-1+j\sqrt{39}}{2}$ the eigenvector ν_2 is defined as $\begin{bmatrix} \nu_{12} \\ \nu_{22} \end{bmatrix}$ with:

$$[\lambda.I - A].\nu_2 = 0 \Leftrightarrow \begin{bmatrix} \dfrac{-1+j\sqrt{39}}{2} & 10 \\ -1 & \dfrac{1+j\sqrt{39}}{2} \end{bmatrix} . \begin{bmatrix} \nu_{12} \\ \nu_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} \nu_{12} \\ \nu_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{1+j\sqrt{39}}{2} \\ 1 \end{bmatrix}$$

The diagonal canonical form transformation matrix T is defined base on eigenvectors ν:

$$T = \begin{bmatrix} \nu_1 & \nu_2 \end{bmatrix} = \begin{bmatrix} \dfrac{1-j\sqrt{39}}{2} & \dfrac{1+j\sqrt{39}}{2} \\ 1 & 1 \end{bmatrix}$$

$$T^{-1} = \frac{1}{det(T)} . \begin{bmatrix} \dfrac{1-j\sqrt{39}}{2} & \dfrac{1+j\sqrt{39}}{2} \\ 1 & 1 \end{bmatrix} = \frac{-1}{j\sqrt{39}} . \begin{bmatrix} 1 & \dfrac{-1-j\sqrt{39}}{2} \\ -1 & \dfrac{1-j\sqrt{39}}{2} \end{bmatrix}$$

The diagonal canonical form or matrix A is defined:

$$A_{DCF} = T^{-1}.A.T = \frac{-1}{j\sqrt{39}} . \begin{bmatrix} 1 & \dfrac{-1-j\sqrt{39}}{2} \\ -1 & \dfrac{1-j\sqrt{39}}{2} \end{bmatrix} . \begin{bmatrix} 0 & -10 \\ 1 & -1 \end{bmatrix} . \begin{bmatrix} \dfrac{1-j\sqrt{39}}{2} & \dfrac{1+j\sqrt{39}}{2} \\ 1 & 1 \end{bmatrix}$$

$$A_{DCF} = \frac{-1}{j\sqrt{39}} . \begin{bmatrix} \dfrac{-1-j\sqrt{39}}{2} & \dfrac{-19+j\sqrt{39}}{2} \\ \dfrac{1-j\sqrt{39}}{2} & \dfrac{19+j\sqrt{39}}{2} \end{bmatrix} . \begin{bmatrix} \dfrac{1-j\sqrt{39}}{2} & \dfrac{1+j\sqrt{39}}{2} \\ 1 & 1 \end{bmatrix}$$

$$A_{DCF} = \frac{-1}{j\sqrt{39}} . \begin{bmatrix} \dfrac{-39+j\sqrt{39}}{2} & 0 \\ 0 & \dfrac{39+j\sqrt{39}}{2} \end{bmatrix}$$

$$A_{DCF} = \begin{bmatrix} -\dfrac{1+j\sqrt{39}}{2} & 0 \\ 0 & \dfrac{-1+j\sqrt{39}}{2} \end{bmatrix}$$

4. $A = \begin{bmatrix} 0 & 10 \\ 1 & 0 \end{bmatrix}$

$$|\lambda.I - A| = det \begin{bmatrix} \lambda & -10 \\ -1 & \lambda \end{bmatrix} = \lambda.\lambda - 10 = \lambda^2 - 10$$

Eigenvalues of A are: $\begin{cases} \lambda_1 = \sqrt{10} \\ \lambda_2 = -\sqrt{10} \end{cases}$

For $\lambda_1 = \sqrt{10}$ the eigenvector ν_1 is defined as $\begin{bmatrix} \nu_{11} \\ \nu_{21} \end{bmatrix}$ with:

$$[\lambda.I - A].\nu_1 = 0 \Leftrightarrow \begin{bmatrix} \sqrt{10} & -10 \\ -1 & \sqrt{10} \end{bmatrix} . \begin{bmatrix} \nu_{11} \\ \nu_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} \nu_{11} \\ \nu_{21} \end{bmatrix} = \begin{bmatrix} \sqrt{10} \\ 1 \end{bmatrix}$$

For $\lambda_2 = -\sqrt{10}$ the eigenvector ν_2 is defined as $\begin{bmatrix} \nu_{12} \\ \nu_{22} \end{bmatrix}$ with:

$$[\lambda.I - A].\nu_2 = 0 \Leftrightarrow \begin{bmatrix} -\sqrt{10} & -10 \\ -1 & -\sqrt{10} \end{bmatrix} . \begin{bmatrix} \nu_{12} \\ \nu_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} \nu_{12} \\ \nu_{22} \end{bmatrix} = \begin{bmatrix} -\sqrt{10} \\ 1 \end{bmatrix}$$

The diagonal canonical form transformation matrix T is defined base on eigenvectors ν:

$$T = \begin{bmatrix} \nu_1 & \nu_2 \end{bmatrix} = \begin{bmatrix} \sqrt{10} & -\sqrt{10} \\ 1 & 1 \end{bmatrix}$$

$$T^{-1} = \frac{1}{det(T)} . \begin{bmatrix} 1 & \sqrt{10} \\ -1 & \sqrt{10} \end{bmatrix} = \frac{1}{2\sqrt{10}} . \begin{bmatrix} 1 & \sqrt{10} \\ -1 & \sqrt{10} \end{bmatrix}$$

The diagonal canonical form or matrix A is defined:

$$A_{DCF} = T^{-1}.A.T = \frac{1}{2\sqrt{10}} . \begin{bmatrix} 1 & \sqrt{10} \\ -1 & \sqrt{10} \end{bmatrix} . \begin{bmatrix} 0 & 10 \\ 1 & 0 \end{bmatrix} . \begin{bmatrix} \sqrt{10} & -\sqrt{10} \\ 1 & 1 \end{bmatrix}$$

$$A_{DCF} = \frac{1}{2\sqrt{10}} . \begin{bmatrix} \sqrt{10} & 10 \\ \sqrt{10} & -10 \end{bmatrix} . \begin{bmatrix} \sqrt{10} & -\sqrt{10} \\ 1 & 1 \end{bmatrix} = \frac{1}{2\sqrt{10}} . \begin{bmatrix} 20 & 0 \\ 0 & -20 \end{bmatrix}$$

$$A_{DCF} = \begin{bmatrix} \sqrt{10} & 0 \\ 0 & -\sqrt{10} \end{bmatrix}$$