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To: ME 5554 / AOE 5754 / ECE 5754 Students

From: Dr. Steve Southward

Subject: Phase I Development Program (i.e. your Midterm Project)

The design engineers at Precision 3D Measuring Inc. have just developed a prototype 3D Coordinate Measuring Machine (CMM). In order to keep the costs low, the engineers have significantly reduced the amount of structural support in the frame, which unfortunately increases the compliance (i.e. reduces the stiffness) at the measurement probe. High compliance is generally not acceptable in a precision measurement system.

Background
A sketch of the prototype coordinate measurement machine is shown in Figure 1. This particular design is essentially a 3-axis gantry robot where:

- Motor M_1 with **input** u_1 applies control forces in the y direction (red)
- Motor M_2 with **input** u_2 applies control forces in the x direction (green)
- Motor M_3 with **input** u_3 applies control forces in the z direction (blue)

The prototype CMM uses a local GPS sensor to provide direct real-time **output** measurements of the probe tip location given by coordinates (x, y, z).

Recognizing that the frame in this design has a lot more compliance than what a traditional CMM frame has, the design engineers have included two additional “Proof Mass” actuators near the probe tip, where:

- Motor M_4 with **input** u_4 applies inertial control forces in the y direction (orange)
- Motor M_5 with **input** u_5 applies inertial control forces in the x direction (purple)

The original design strategy was to use motors 1, 2, and 3 for coarse position control of the probe tip, and then use motors 4 and 5 for reducing any oscillation or overshoot caused by the frame compliance. The engineers at Precision 3D Measuring Inc. have failed to produce an acceptable solution using this design strategy.

You have been asked to determine whether a State-Space feedback control solution can be used to minimize the effects of compliance in order to achieve the desired probe positioning accuracy. To accomplish this, you must design a multi-input multi-output control system using full state feedback.

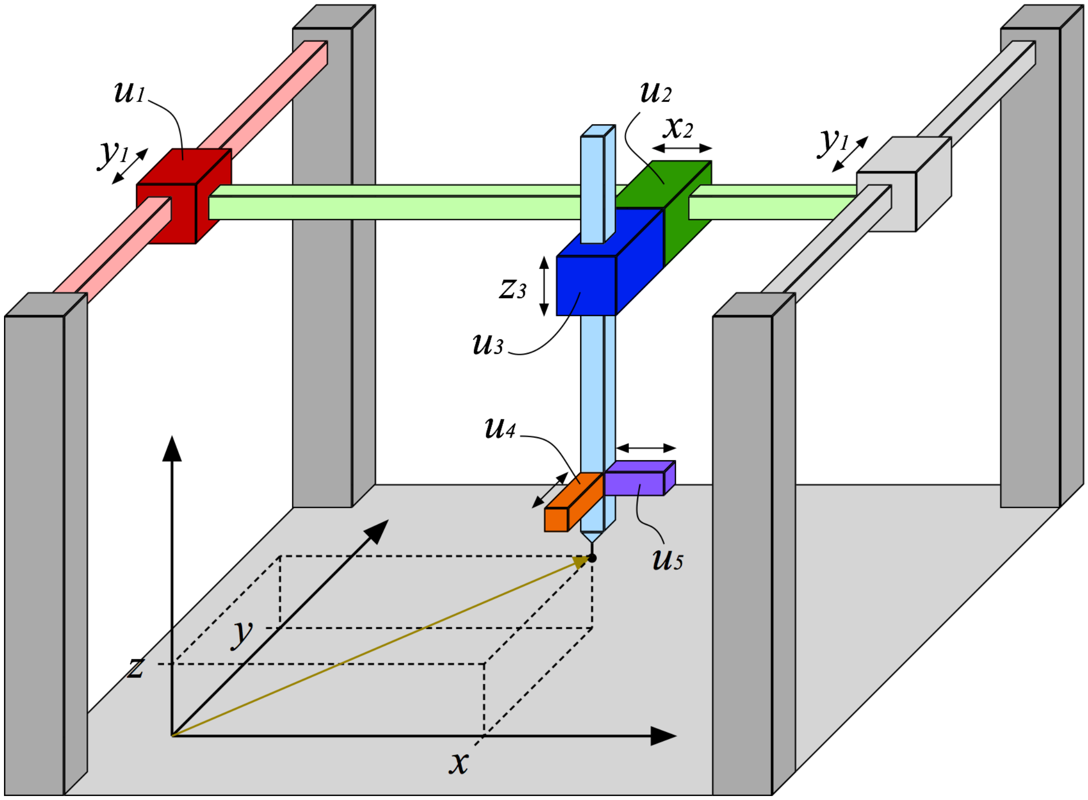

Figure 1. Prototype 3D Coordinate Measurement System.

Equations of Motion

The equations of motion for the prototype CMM have already been derived and are given by the following equations:

$$\dot{p}_1 = \alpha_1 u_1 - \left(\frac{b_y}{M_y}\right) p_1 - \dot{p}_4 \qquad \dot{p}_5 = \alpha_2 u_2 - \left(\frac{b_x}{M_x}\right) p_5 - \dot{p}_8$$

$$\dot{q}_2 = \left(\frac{1}{M_y}\right) p_1 - \left(\frac{1}{m_4}\right) p_4 - \dot{q}_3 \qquad \dot{q}_6 = \left(\frac{1}{M_x}\right) p_5 - \left(\frac{1}{m_5}\right) p_8 - \dot{q}_7$$

$$\dot{q}_3 = \left(\frac{1}{b_4}\right)\left(\alpha_4 u_4 + \dot{p}_4 - k_4 q_3\right) \qquad \dot{q}_7 = \left(\frac{1}{b_5}\right)\left(\alpha_5 u_5 + \dot{p}_8 - k_5 q_7\right)$$

$$\dot{p}_4 = k_y q_2 \qquad \dot{p}_8 = k_x q_6$$

$$\dot{y}_1 = \left(\frac{1}{M_y}\right) p_1 \qquad \dot{x}_2 = \left(\frac{1}{M_x}\right) p_5$$

$$y = y_1 - q_2 \qquad x = x_2 - q_6$$

$$M_z \ddot{z} = -b_z \dot{z} + \alpha_3 u_3$$

Problem 1a. [10 pts.] Using the differential and algebraic equations of motion, construct a complete mathematical State-Space model for this dynamic system, i.e. write out an expression for the State and Output equations. DO **NOT** SUBSTITUTE NUMERICAL VALUES FOR ANY PARAMETERS!!!!

Problem 1b. [8 pts.] Using the numerical values below, define an LTI object representation of your state-space model in Matlab. You must define the ***statename***, ***inputname***, and ***outputname*** properties for the LTI object using appropriate short (preferably two-character) signal names.

$$M_y = 150kg, \quad M_x = 100kg, \quad M_z = 50kg, \quad b_y = 40\tfrac{Ns}{m}, \quad b_x = 50\tfrac{Ns}{m}, \quad b_z = 10\tfrac{Ns}{m}, \quad k_x = 0.2\tfrac{N}{m}$$

$$m_4 = 20kg, \quad b_4 = 2.51\tfrac{Ns}{m}, \quad k_4 = 7.89\tfrac{N}{m}, \quad m_5 = 20kg, \quad b_5 = 3.77\tfrac{Ns}{m}, \quad k_5 = 7.89\tfrac{N}{m}, \quad k_y = 0.1\tfrac{N}{m}$$

$$\alpha_1 = 0.05\tfrac{N}{V}, \quad \alpha_2 = 0.1\tfrac{N}{V}, \quad \alpha_3 = 0.1\tfrac{N}{V}, \quad \alpha_4 = 0.3\tfrac{N}{V}, \quad \alpha_5 = 0.5\tfrac{N}{V}$$

Problem 1c. [2 pts.] Use the MINREAL function in Matlab to demonstrate whether your LTI state-space model is a minimum realization or not.

Problem 2. [10 pts.] Demonstrate that this open-loop system is Completely Controllable. Also determine whether this system is Completely Controllable using any **subset** of control inputs. If the system is completely controllable using any subset, find all possible subsets of control inputs where this is true. Justify your result.

Problem 3. [10 pts.] Compute the open loop poles of this system, the natural frequencies with units of Hz, and the damping ratios for each eigenvalue. Tabulate your results in ascending order with respect to the natural frequency magnitude.

Problem 4. [10 pts.] Simulate the open-loop response of the system for three seconds, assuming all initial states are zero except for the non-zero IC's defined below. Plot only the **outputs** in the same axes on a single figure. Properly annotate your figure with axis labels, grid lines, and a legend.

$$y_1(0) = 0.5 \qquad p_1(0) = 300 \qquad z(0) = 0.7$$

$$x_2(0) = 0.6 \qquad p_5(0) = -150 \qquad \dot{z}(0) = -0.09348$$

Problem 5. [20 pts.] Design a full-state feedback controller that meets the following performance requirements:

- Each output must be within $\pm 1cm$ after 8 seconds
- $|u_1| \le 1000, \quad |u_2| \le 1000, \quad |u_3| \le 200, \quad |u_4| \le 100, \quad |u_5| \le 100$

Simulate the closed-loop response of the system for ten seconds, assuming all initial states are zero except for the non-zero IC's defined below.

$$y_1(0) = 1 \qquad x_2(0) = 1 \qquad z(0) = 1$$

Document a minimum of five design iterations that demonstrate that you attempted to achieve the requirements. There is no grade penalty for violating some of the requirements unless you do not demonstrate that you made an earnest attempt, and that you documented some progress toward achieving the requirements. In other words, your design iterations must show sufficient improvement.

Your documentation of each iteration should at least include:

- The desired closed-loop poles you chose
- A description of whether you meet the requirement for each output
- A description of whether you meet the requirement for each control signal

For your FINAL design ONLY, plot a single figure window where all outputs are on separate axes (subplots) on the left side of the figure, and all control signals are on separate axes (subplots) on the right side of the figure. Properly annotate each axes with axis labels and grid lines.