ME 5554 - AOE 5754 - ECE 5754
Applied Linear Systems

Instructor:	**Dr. Steve Southward**
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Syllabus Review

Class Schedule

- 11:00-12:15 Tu & Th

Office Hours & Consultation

- By appointment
- Skype
- Scholar Chat
- Email: **scsouth@vt.edu**
- Telephone: **540-231-7385**

Syllabus Review

Electronic Information

- Lecture notes will be posted on Scholar prior to each class
- Grades, homework assignments, and solutions will be posted on Scholar
- Course announcements will be posted on Scholar and/or sent via email
- Do **NOT** send any hardcopy documents to my department mailbox or office
- Lectures will be recorded and the links posted on Scholar (can take up to 24 hours for posting!)
- Some lectures may be pre-recorded

Assignment Policy

- Virginia Tech Graduate Honor Code will be strictly enforced
- All graded assignments MUST be submitted electronically through Scholar
- No graded assignments will be accepted past the due date/ time
- All assignments are due by midnight of the stated due date
- Electronic submissions must be in PDF format
- No hardcopy submissions to my department mailbox or office will be accepted
- You are encouraged to work with others on all assignments except take-home exams, but your submission must be the result of your own efforts

Grading

Homework:	10%
Quiz:	15%
Midterm Exam (10/4):	25%
Midterm Project (10/28):	25%
Final Project (12/9):	25%
Optional Final Exam (12/9):	25%

- Lowest grade of ME, MP, FP, FE will be dropped
- Must qualify for optional final: >50% on all other grades

Grading Policy

- Homework solutions available on Scholar after due date
- Each student is responsible for assigning their own homework grade (subject to the VT Graduate Honor Code)
  - 2 points for attempting ALL problems and getting >80% correct
  - 1 point for attempting ALL problems and getting >40% correct
  - 0 points otherwise
- Each student is responsible for electronically submitting homework grade to instructor within 48 hours of the original due date
- Grading disputes for exams and/or projects must be electronically submitted with a written explanation.
- NO extra credit assignments

Controls Curriculum Options @ VT

FALL	SPRING
Linear Systems Theory cross-listed as ECE 5744 AOE 5744 ME 5544 Fall 2009, listed as ECE 5704, taught by Dan Stilwell Proposed official syllabus	**Linear Control Theory** cross-listed as ECE 6744 AOE 6744 ME 6544 Tentatively scheduled to be taught each spring semester. Proposed official syllabus
Applied Linear Systems cross-listed as ECE 5754 AOE 5754 ME 5554 Fall 2009, listed as ME5505, taught by Steve Southward Proposed official syllabus	**Applied Linear Control** cross-listed as AOE 5764 ME 5564 Tentatively scheduled to be taught each spring semester. Proposed official syllabus
Nonlinear Systems Theory cross-listed as ECE 5774 AOE 5774 ME 5574 Fall 2009, listed as AOE 5344, taught by Craig Woolsey. Proposed official syllabus	**Adaptive Controls Systems** cross-listed as ECE 6774 AOE 6774 ME 6574 Tentatively scheduled to be taught each spring semester. Proposed official syllabus

Course Goals

In this course you will learn how to:

- Represent multi-input-multi-output (MIMO) dynamic systems using state-space models
- Solve for the dynamic response of a linear dynamic system and relate the response to the state-space system description
- Design linear control systems using frequency domain, state estimation, and pole placement techniques

In-Class Participation is EXPECTED!!!

Course Prerequisites

Classical feedback control background:

- Block diagram manipulation & analysis
- Laplace transforms
- Root locus analysis
- Frequency response & transfer functions
- Frequency domain feedback control design

Mandatory Software:

- Matlab 8.0 R2012b (or later)
- Simulink, Control system toolbox
- Powerpoint, Word, Equation Editor
- PDF conversion software

Reference Material

Textbook:

- *Linear State-Space Control Systems,* Robert Williams & Douglas Lawrence, John Wiley & Sons, 2007

Additional References:

- Franklin, G.F., Powell, J.D., and Emami-Naeini, A., *Feedback Control of Dynamic Systems*, Addison-Wesley, Reading, MA, 1991.
- Kuo, B.C., *Automatic Control Systems*, Prentice-Hall, Englewood Cliffs, NY, 1987.
- Dorf, R.C., Bishop, R.H., *Modern Control Systems*, Addison-Wesley, Reading, MA, 1995.
- Bolton, W., *Control Engineering*, Longman, Harlow, Essex, England, 1996.
- Levine, W., *The Control Handbook*, CRC Press, 1996.
- Friedland, B., *Control System Design – An Introduction to State-Space Methods*, McGraw-Hill, 1986.

Course Outline - 1st Half

- Review Classical Feedback Control
- Review Vector/Matrix Theory
- State-Space Representations
- LTI Response & Matrix Exponential
- Transfer Functions & Eigenvalues
- Frequency-Domain Analysis
- Harmonic & Impulse Responses
- Pole Placement
- Controllability

Course Outline - 2nd Half

- Full State Feedback for MIMO
- Stabilizability
- State Estimation & Output Feedback
- Observability & Duality
- Exogenous Inputs, Integral Control
- Optimal Control (LQR/LQG)
- Robustness & Sensitivity
- Kalman Filtering
- Introduction to Discrete Time

Control System Basics

Single-Input Single-Output (SISO) System

$$u \longrightarrow \boxed{H} \longrightarrow y$$

Input u is the command (control) signal

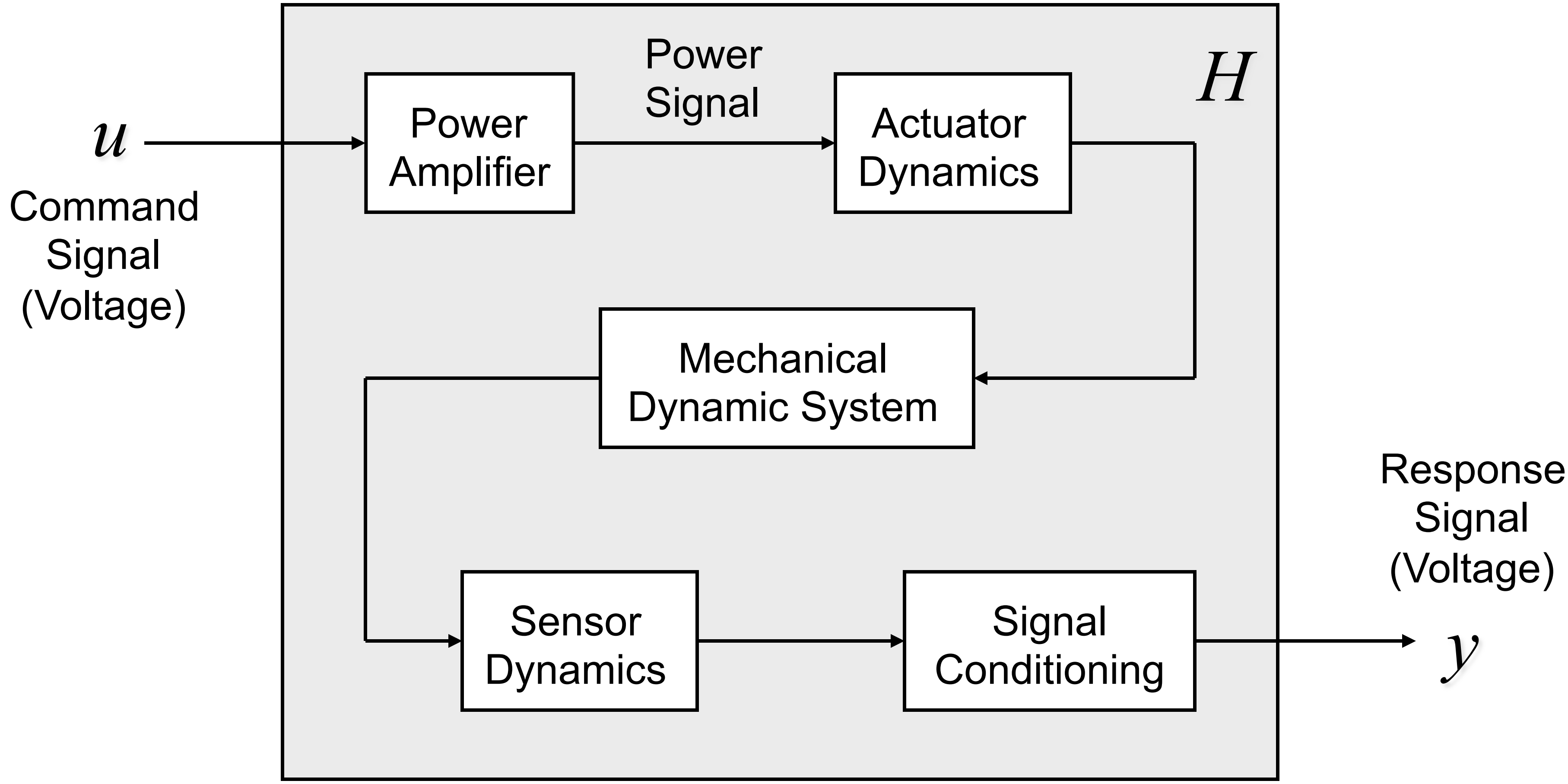
Output y is the response (measurement) signal

Process (a.k.a. "Plant") H is a dynamic system that transforms the input into the output

Control objective: Find u to generate a desired y

Control System Basics

Typical electro-mechanical plants are made up of connected dynamic subsystems:



Example Control Systems

Aircraft utilize many control systems:

Thrust Vectoring (Pitch Control)

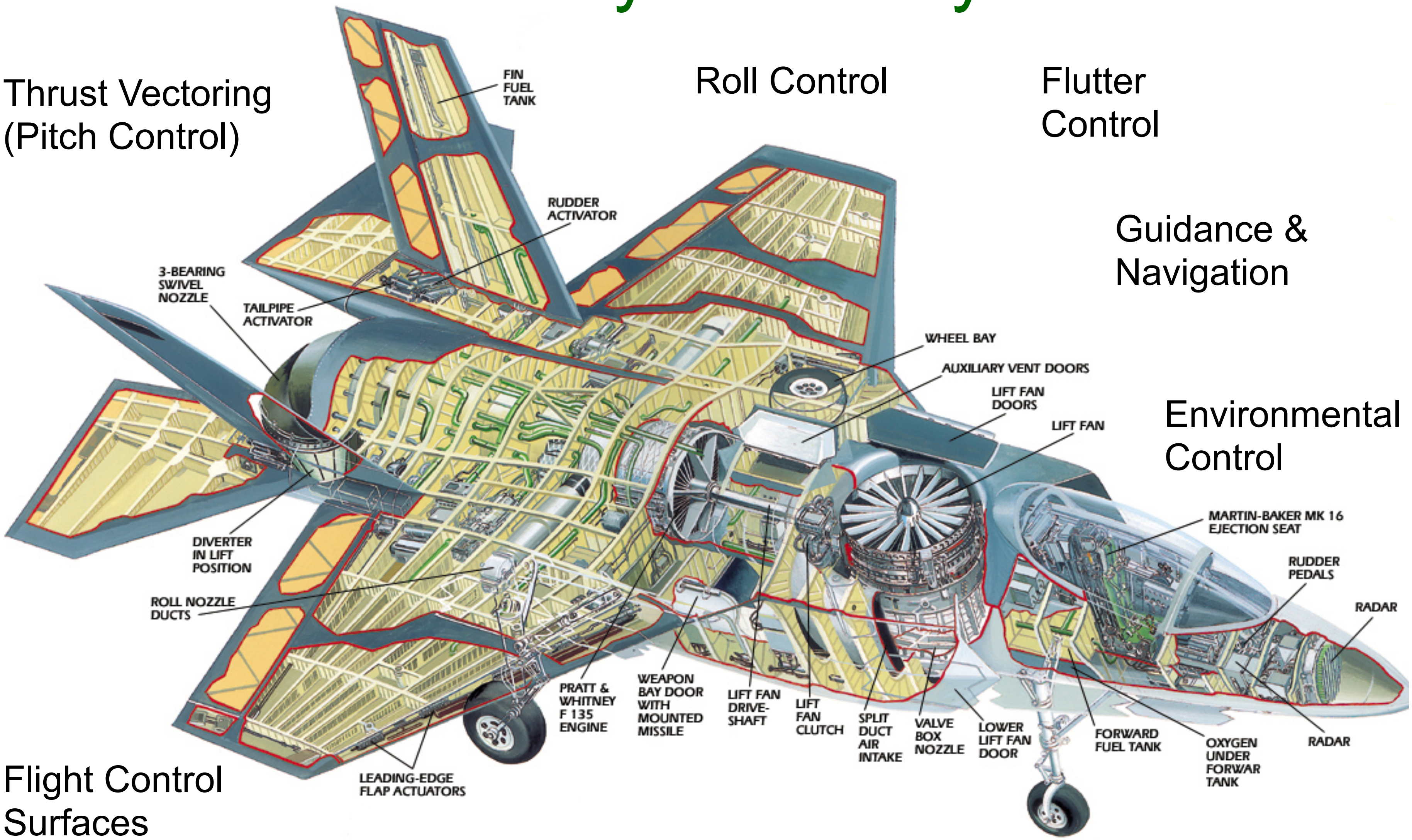
Roll Control

Flutter Control

Guidance & Navigation

Environmental Control

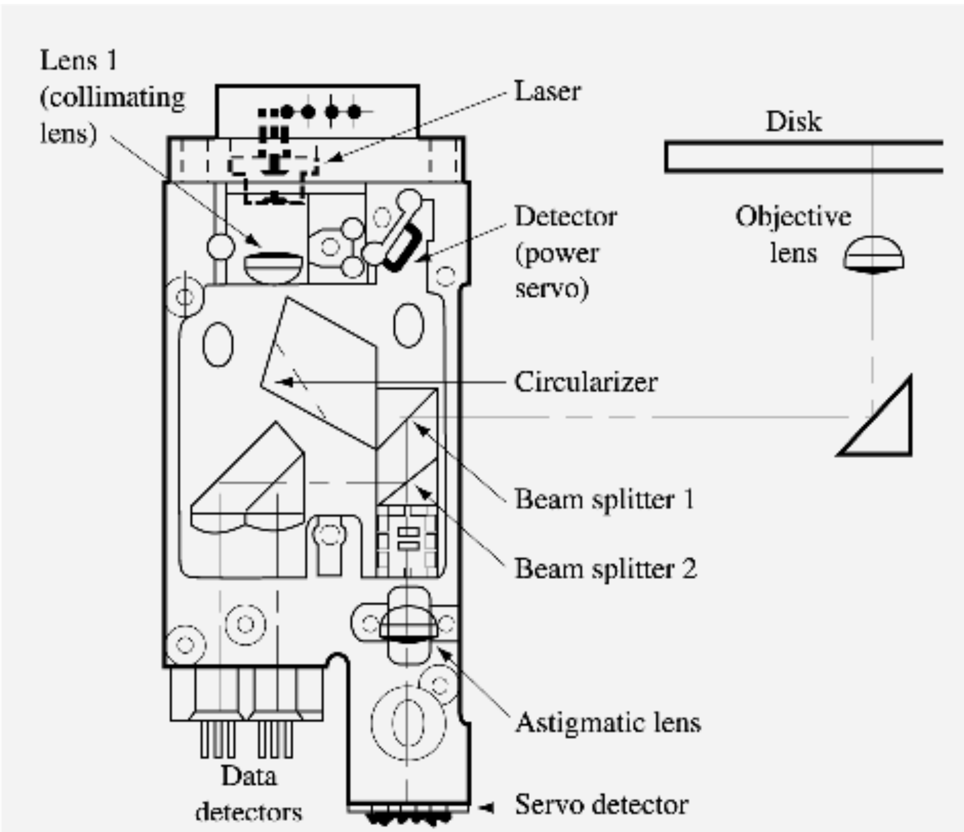
Flight Control Surfaces

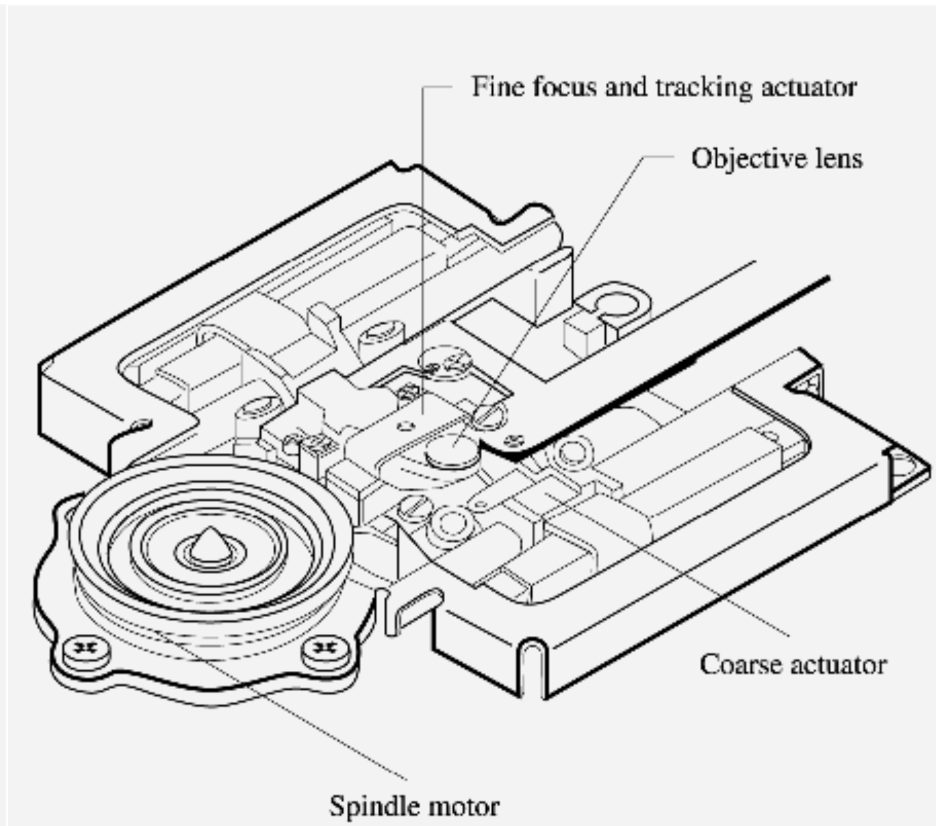


http://media.popularmechanics.com/images/lg_cutaway-lg.jpg

Example Control Systems

Control is an enabling element of disk drive technology:





http://www.research.ibm.com/journal/rd/405/asthana.html

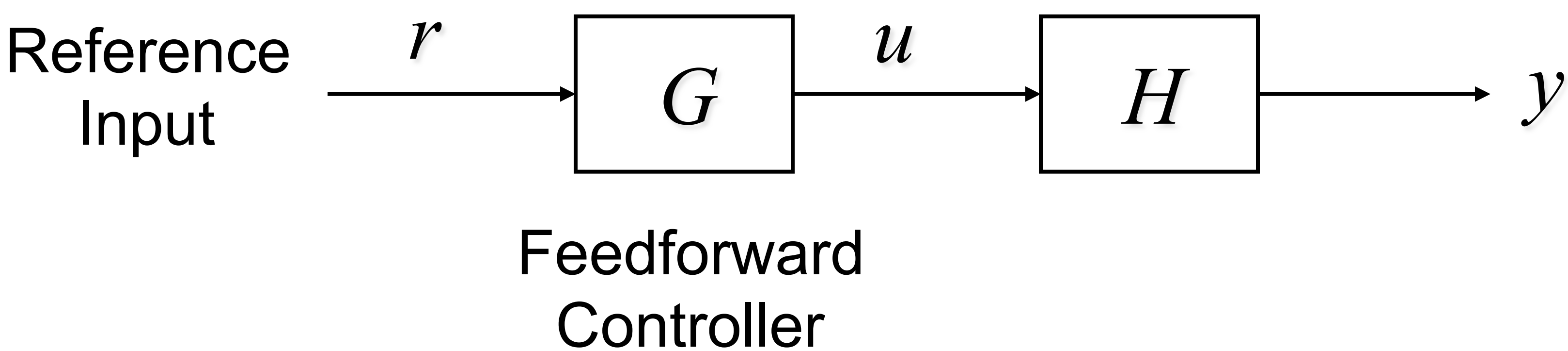
Example Control Systems

Control is also used in advanced biomedical devices:

Control must be used to position the artificial knee properly and change the 'feel' of the knee to match that of an actual knee.

Feedforward Control System

Feedforward control is a form of **Open-Loop** control (i.e. no explicit feedback loops)

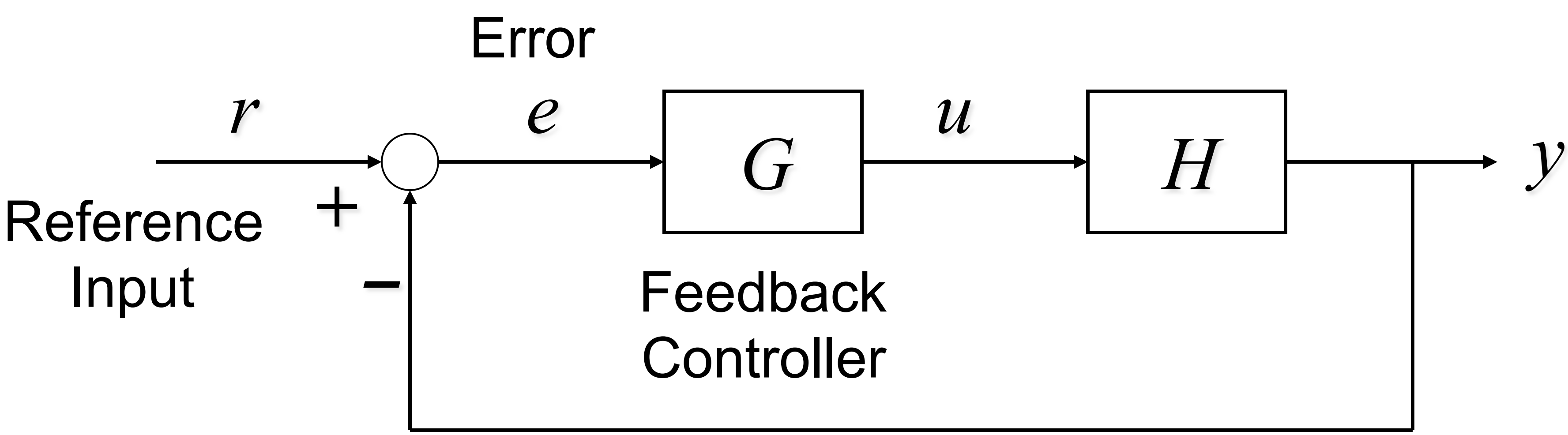


Feedforward control…

- is typically used for **command shaping** and/or **disturbance rejection**,
- does not alter the dynamics of the process

Feedback Control System

Feedback control is **Closed-Loop** control, where the control signal u is a function of the process output y



Feedback control…

is typically used for **regulation** or **tracking**,

changes the process dynamics

Question for Open Discussion

What are some relative strengths and weaknesses of feedback and feedforward control?

Course Focus

This course will focus on the analysis and design of continuous-time (analog) feedback control systems using state-space methods

– How do we model the process?
– How do we design a feedback controller?
– How do we analyze the performance?

Review of Continuous Control

Your previous study of control system design probably centered around ***classical control*** techniques for SISO systems.

Classical synthesis & analysis techniques are still valuable and widely used today

For the remainder of this session we will briefly review two main topics in classical control system design:

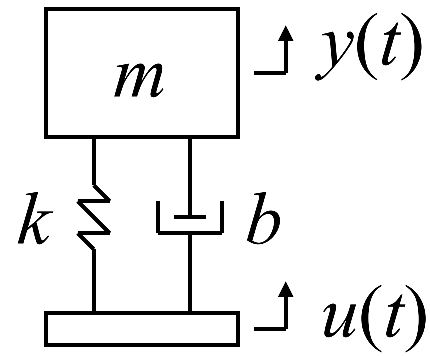
- Process modeling
- Compensator design techniques

Process Modeling - I

Example: base-excited mechanical oscillator

Free-body diagram force balance:

$$m\ddot{y} = \underbrace{b\left(\dot{u} - \dot{y}\right)}_{\text{damper force}} + \underbrace{k\left(u - y\right)}_{\text{spring force}}$$



Collect terms:

$$m\ddot{y} + b\dot{y} + ky = b\dot{u} + ku$$

In the Laplace domain:

$$\left(ms^2 + bs + k\right)Y\left(s\right) = \left(bs + k\right)U\left(s\right)$$

The resulting process <u>Transfer Function</u> is:

$$H\left(s\right) = \frac{Y\left(s\right)}{U\left(s\right)} = \left(\frac{bs + k}{ms^2 + bs + k}\right)$$

Process Modeling - II

Most continuous-time linear systems are represented in the Laplace domain as a ratio of polynomials:

$$H(s) = \left(\frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} \right) \qquad n \geq m$$

The variable $s \ (= \sigma + j\omega)$ is the Laplace variable from the Laplace Transform:

$$H(s) = \int_0^\infty e^{-st} h(t)\, dt$$

$h(t)$ is the process impulse response

Process Modeling - III

Any linear, causal input-output (I/O) relationship can be modeled using Laplace transforms

The controller (or compensator) is usually also represented by a transfer function $G(s)$ which is a ratio of polynomials

$$G(s) = \frac{U(s)}{E(s)}$$

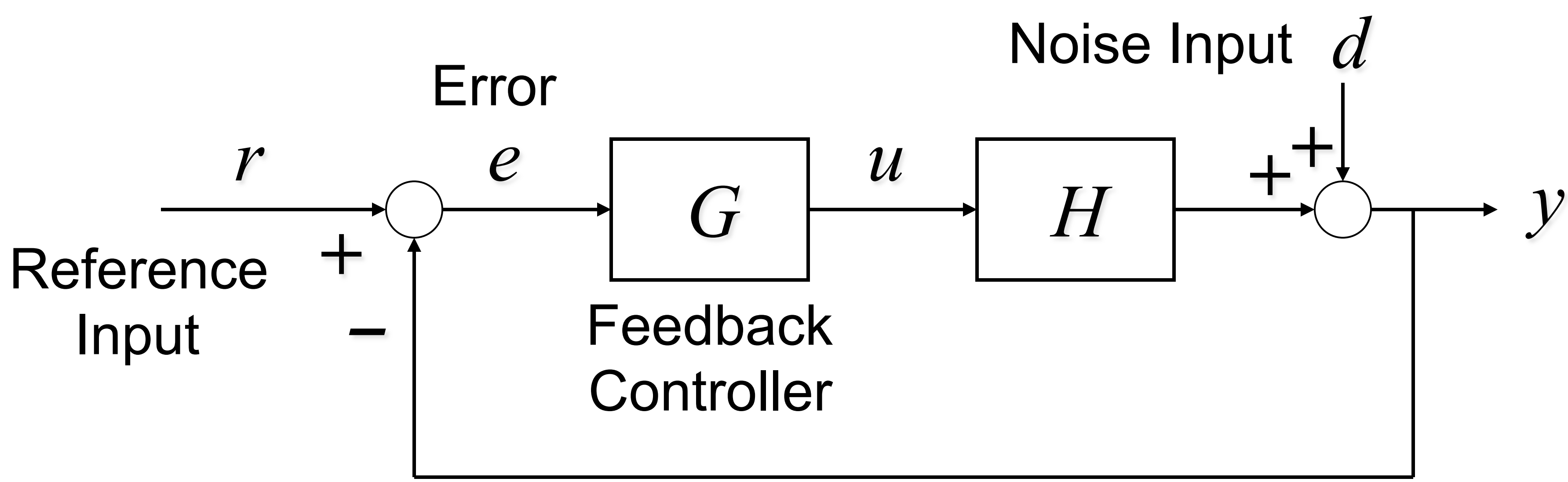
Classical Control Design - I

Typical design problem:

Design a controller $G(s)$ such that:

output $y(t)$ tracks the reference input $r(t)$

output $y(t)$ is insensitive to measurement noise $d(t)$



Classical Control Design - II

The output is given by:

$$Y(s) = H(s)U(s) + D(s) = H(s)\left[G(s)E(s)\right] + D(s)$$

And the error (for negative feedback) is:

$$E(s) = R(s) - Y(s)$$

Combining yields the following:

$$Y(s) = G(s)H(s)\left[R(s) - Y(s)\right] + D(s)$$

$$\left[1 + G(s)H(s)\right]Y(s) = \left[G(s)H(s)\right]R(s) + D(s)$$

Classical Control Design - III

The Closed-Loop I/O relationships are obtained by solving for $Y(s)$ in terms of the inputs $R(s)$ and $D(s)$

$$Y(s)=\underbrace{\left(\frac{G(s)H(s)}{1+G(s)H(s)}\right)}_{\substack{\text{Complementary}\\ \text{Sensitivity } T(s)}}R(s)+\underbrace{\left(\frac{1}{1+G(s)H(s)}\right)}_{\substack{\text{Sensitivity}\\ \text{Function } S(s)}}D(s)$$

Classical Control Design - IV

For Tracking, we want $Y(s)$ approximately equal to $R(s)$:

$$Y(s) \approx R(s) \quad \text{IF} \quad T(s) \approx 1 \quad \Rightarrow \quad G(s)H(s) >> 1$$

For Disturbance Rejection, we want:

$$S(s) \approx 0 \quad \Rightarrow \quad G(s)H(s) >> 1$$

Both goals appear to be satisfied with the same criterion.

Classical Control Design - V

Good news:

All we need to do is "turn up" the compensator gain to achieve both objectives.

Bad news:

Turning up the gain too much can cause instability. An unstable system is going to have worse performance than the uncontrolled process.

Classical Control Design - VI

Fundamental Design Tradeoff:

Designing a high-gain compensator $GH >> 1$ improves tracking performance and reduces the sensitivity to disturbances (good news) ….but….decreases the stability margin and increases the chance of closed-loop instability (bad news).

Classical Design Methods - I

Root Locus

The Characteristic Equation (CE) is given by:

$$1 + G(s)H(s) = 0$$

Objective: Insure that the roots of the CE remain in the left-half plane (LHP) as a selected control parameter is varied

For example, plot the roots as a function of the compensator gain k:

$$G(s) = k\left(\frac{B(s)}{A(s)}\right) \quad \Rightarrow \quad A(s) + kB(s)H(s) = 0$$

Classical Design Methods - II

Loop Shaping in the Frequency-Domain

The Open Loop transfer function is: $G(s)H(s)$

Objective: Manipulate the magnitude and phase of the loop to achieve the performance objectives while maintaining stability

Designs are analyzed in the frequency domain for Gain Margin and Phase Margin:

- Bode Diagram: Mag. & Phase vs. frequency
- Nyquist Diagram: Real vs. Imaginary
- Nichols Chart: Magnitude vs. Phase

Classical Design Methods - III

There are two common choices for classical compensator designs that you should be familiar with:

PID Compensation

$$G_{PID}(s) = \underbrace{k_P}_{\substack{\text{Proporational} \\ \text{Control}}} + \underbrace{k_I\left(\frac{1}{s}\right)}_{\substack{\text{Integral} \\ \text{Control}}} + \underbrace{k_D s}_{\substack{\text{Derivative} \\ \text{Control}}}$$

Lag/Lead Compensation

$$G(s) = k\left(\frac{s+b}{s+a}\right) \quad \begin{cases} a < b & \text{Lag Compensation} \\ b < a & \text{Lead Compensation} \end{cases}$$

Important Review Topics

Please review the following Classical Control topics:

- Block diagram manipulation
- Laplace transforms $(t \rightarrow s$ and $s \rightarrow t)$
- Frequency-domain design (Bode)
- Lead/Lag & PID compensation
- Root locus