ME-5554 Applied Linear System
Midterm Project

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The design engineer at Precision 3D Measuring Inc. have just developed a prototype 3D Coordinate Measuring Machine (CMM). In order to keep the cost low, the engineers have significantly reduced the amount of structural support in the frame, which unfortunately increase the compliance at the measurement probe. High compliance is generally not acceptable in a precision measurement system.

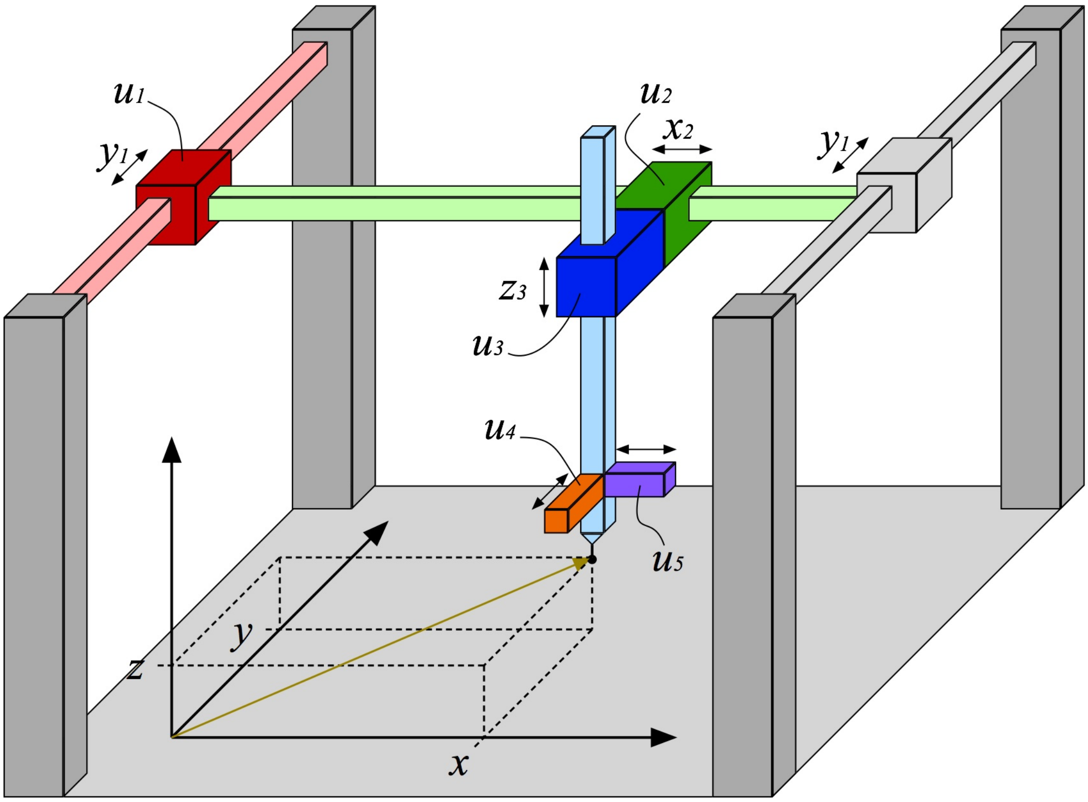

Figure 1: Prototype 3D Coordinate Measurement System

Equation of Motion

The equation of motion for prototype CMM have already been derived and are given by the following equations:

$$\dot{p}_1 = \alpha_1 u_1 - \left(\frac{b_y}{M_y}\right) p_1 - \dot{p}_4$$

$$\dot{q}_2 = \left(\frac{1}{M_y}\right) p_1 - \left(\frac{1}{m_4}\right) p_4 - \dot{q}_3$$

$$\dot{q}_3 = \left(\frac{1}{b_4}\right) (\alpha_4 u_4 + \dot{p}_4 - k_4 q_3)$$

$$\dot{p}_4 = k_y q_2$$

$$\dot{y}_1 = \left(\frac{1}{M_y}\right) p_1$$

$$y = y_1 - q_2$$

$$M_z \ddot{z} = -b_z \dot{z} + \alpha_3 u_3$$

$$\dot{p}_5 = \alpha_2 u_2 - \left(\frac{b_x}{M_x}\right) p_5 - \dot{p}_8$$

$$\dot{q}_6 = \left(\frac{1}{M_x}\right) p_5 - \left(\frac{1}{m_5}\right) p_8 - \dot{q}_7$$

$$\dot{q}_7 = \left(\frac{1}{b_5}\right) (\alpha_5 u_5 + \dot{p}_8 - k_5 q_7)$$

$$\dot{p}_8 = k_x q_6$$

$$\dot{x}_2 = \left(\frac{1}{M_x}\right) p_5$$

$$x = x_2 - q_6$$

Problem 1a. Using the differential and algebraic equation of motion contruct a complete mathematical State-Space model for the dynamic system:

State Space is defined:

$$\dot{X} = A.X + B.U$$

$$Y = C.X + D.U$$

We have:

Define: $\quad y_1 = p_9 \qquad x_2 = q_{10} \qquad z = q_{11} \qquad \dot{z} = p_{12}$

So:

$$M_z\ddot{z} = -b_z\dot{z} + \alpha_3 u_3 \Leftrightarrow M_z\dot{p}_{12} = -b_z q_{12} + \alpha_3 u_3 \Leftrightarrow \dot{p}_{12} = -\frac{b_z}{M_z}q_{12} + \frac{\alpha_3}{M_z}u_3$$

State variables: $X = \begin{bmatrix} p_1 & q_2 & q_3 & p_4 & p_5 & q_6 & q_7 & p_8 & p_9 & q_{10} & q_{11} & p_{12} \end{bmatrix}^T$

$$\dot{X} = \begin{bmatrix} \dot{p}_1 & \dot{q}_2 & \dot{q}_3 & \dot{p}_4 & \dot{p}_5 & \dot{q}_6 & \dot{q}_7 & \dot{p}_8 & \dot{p}_9 & \dot{q}_{10} & \dot{q}_{11} & \dot{p}_{12} \end{bmatrix}^T$$

1. $\dot{p}_1 = \alpha_1 u_1 - \left(\frac{b_y}{M_y}\right) p_1 - \dot{p}_4 = \alpha_1 u_1 - \left(\frac{b_y}{M_y}\right) p_1 - k_y q_2$
$$= \frac{b_y}{M_y}p_1 - k_y q_2 + \alpha_1 u_1$$
2. $\dot{q}_2 = \left(\frac{1}{M_y}\right) p_1 - \left(\frac{1}{m_4}\right) p_4 - \dot{q}_3 = \left(\frac{1}{M_y}\right) p_1 - \left(\frac{1}{m_4}\right) p_4 - \left(\frac{1}{b_4}\right)(\alpha_4 u_4 + \dot{p}_4 - k_4 q_3)$
$$= \frac{1}{M_y}p_1 - \frac{1}{m_4}p_4 + \frac{k_4}{b_4}q_3 - \frac{1}{b_4}k_y q_2 - \frac{\alpha_4}{b_4}u_4$$
$$= \frac{1}{M_y}p_1 - \frac{k_y}{b_4}q_2 + \frac{k_4}{b_4}q_3 - \frac{1}{m_4}p_4 - \frac{\alpha_4}{b_4}u_4$$
3. $\dot{q}_3 = \left(\frac{1}{b_4}\right)(\alpha_4 u_4 + \dot{p}_4 - k_4 q_3) = \left(\frac{1}{b_4}\right)(\alpha_4 u_4 + k_y q_2 - k_4 q_3)$
$$= \frac{k_y}{b_4}q_2 - \frac{k_4}{b_4}q_3 + \frac{\alpha_4}{b_4}u_4$$
4. $\dot{p}_4 = k_y q_2$
5. $\dot{p}_5 = \alpha_2 u_2 - \left(\frac{b_x}{M_x}\right) p_5 - \dot{p}_8 = \alpha_2 u_2 - \left(\frac{b_x}{M_x}\right) p_5 - k_x q_6$
$$= -\frac{b_x}{M_x}p_5 - k_x q_6 + \alpha_2 u_2$$
6. $\dot{q}_6 = \left(\frac{1}{M_x}\right) p_5 - \left(\frac{1}{m_5}\right) p_8 - \dot{q}_7 = \left(\frac{1}{M_x}\right) p_5 - \left(\frac{1}{m_5}\right) p_8 - \left(\frac{1}{b_5}\right)(\alpha_5 u_5 + \dot{p}_8 - k_5 q_7)$
$$= \frac{1}{M_x}p_5 - \frac{1}{m_5}p_8 + \frac{k_5}{b_5}q_7 - \frac{1}{b_5}k_x q_6 - \frac{\alpha_5}{b_5}u_5$$
$$= \frac{1}{M_x}p_5 - \frac{1}{b_5}k_x q_6 + \frac{k_5}{b_5}q_7 - \frac{1}{m_5}p_8 - \frac{\alpha_5}{b_5}u_5$$
7. $\dot{q}_7 = \left(\frac{1}{b_5}\right)(\alpha_5 u_5 + \dot{p}_8 - k_5 q_7) = \frac{1}{b_5}(\alpha_5 u_5 + k_x q_6 - k_5 q_7)$
$$= \frac{k_x}{b_5}q_6 - \frac{k_5}{b_5}q_7 + \frac{\alpha_5}{b_5}u_5$$

8. $\dot{p}_8 = k_x q_6$

9. $\dot{p}_9 = \dfrac{1}{M_y} p_1$

10. $\dot{q}_{10} = \dfrac{1}{M_x} p_5$

11. $\dot{q}_{11} = p_{12}$

12. $\dot{p}_{12} = -\dfrac{b_z}{M_z} p_{12} + \dfrac{\alpha_3}{M_z} u_3$

State equation: $X = A.X + B.U$

Base on States variables we have State matrix A is determined:

$$A = \begin{bmatrix}
-\frac{b_y}{M_y} & -k_y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{M_y} & -\frac{k_y}{b_4} & \frac{k_4}{b_4} & -\frac{1}{m_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{k_y}{b_4} & -\frac{k_4}{b_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & k_y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{b_x}{M_x} & -k_x & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{M_x} & -\frac{k_x}{b_5} & \frac{k_5}{b_5} & -\frac{1}{m_5} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{k_x}{b_5} & -\frac{k_5}{b_5} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & k_x & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{M_y} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{M_x} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{b_z}{M_z}
\end{bmatrix}$$

Input matrix B:

$$B = \begin{bmatrix} \alpha_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\dfrac{\alpha_4}{b_4} & 0 \\ 0 & 0 & 0 & \dfrac{\alpha_4}{b_4} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\dfrac{\alpha_5}{b_5} \\ 0 & 0 & 0 & 0 & \dfrac{\alpha_5}{b_5} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{\alpha_3}{M_z} & 0 & 0 \end{bmatrix}$$

Output equation: $Y = C.X + D.U$

Output states: $Y = \begin{bmatrix} x & y & z \end{bmatrix}^T$

Output matrix C:

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Direct Transmission matrix D: $D = [0]$

Problem 1b. Using the following numerical values to define an LTI object representation of above created state-space model in Matlab:

$M_y = 150kg, \quad M_x = 100kg, \quad M_z = 50kg, \quad b_y = 40Ns/m, \quad b_x = 50Ns/m,$

$b_z = 10Ns/m, \quad k_x = 0.2N/m, \quad m_4 = 20kg, \quad b_4 = 2.51Ns/m, \quad k_4 = 7.89N/m,$

$m_5 = 20kg, \quad b_5 = 3.77Ns/m, \quad k - 5 = 7.89N/m, \quad k_y = 0.1N/m,$

$\alpha_1 = 0.05N/V, \quad \alpha_2 = 0.1N/V, \quad \alpha_3 = 0.1N/V, \quad \alpha_4 = 0.3N/V, \quad \alpha_5 = 0.5N/V$

With above values, we compute elements in matrices:

$$\frac{b_y}{M_y} = \frac{40}{150} = 0.2667 \qquad -k_y = -0.1$$

$$\frac{1}{M_y} = \frac{1}{150} = 0.0067 \qquad -\frac{k_y}{b_4} = -\frac{0.1}{2.51} = -0.0398$$

$$\frac{k_4}{b_4} = \frac{7.89}{2.51} = 3.1434 \qquad -\frac{1}{m_4} = -\frac{1}{20} = -0.05$$

$$\frac{k_y}{b_4} = \frac{0.1}{2.51} = 0.0398 \qquad -\frac{k_4}{b_4} = -\frac{7.89}{2.51} = -3.1434$$

$$k_y = 0.1$$

$$\frac{b_x}{M_x} = \frac{50}{100} = 0.5 \qquad -k_x = -0.2$$

$$\frac{1}{M_x} = \frac{1}{100} = 0.01 \qquad -\frac{k_x}{b_5} = -\frac{0.2}{3.77} = -0.0531$$

$$\frac{k_5}{b_5} = \frac{7.89}{3.77} = 2.0928 \qquad -\frac{1}{m_5} = -\frac{1}{20} = -0.05$$

$$\frac{k_x}{b_5} = \frac{0.2}{3.77} = 0.0531 \qquad -\frac{k_5}{b_5} = -\frac{7.89}{3.77} = -2.0928$$

$$k_x = 0.2$$

$$\frac{b_z}{M_z} = \frac{10}{50} = 0.2$$

$$\frac{1}{M_x} = \frac{1}{100} = 0.01$$

$$\frac{1}{M_y} = \frac{1}{150} = 0.0067$$

$$\alpha_1 = 0.05 \qquad \frac{\alpha_4}{b_4} = \frac{0.3}{2.51} = 0.1195$$

$$\alpha_2 = 0.1 \qquad \frac{\alpha_5}{b_5} = \frac{0.5}{3.77} = 0.1326$$

$$\frac{\alpha_3}{M_z} = \frac{0.1}{50} = 0.002$$

We have:

State matrix:

$$A = \begin{bmatrix} -0.2667 & -0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0067 & -0.0398 & 3.1434 & -0.05 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0398 & -3.1434 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & -0.2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.01 & -0.0531 & 2.0928 & -0.05 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0531 & -2.0928 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0067 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.2 \end{bmatrix}$$

Input matrix B:

$$B = \begin{bmatrix} 0.05 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.1195 & 0 \\ 0 & 0 & 0 & 0.1195 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.1326 \\ 0 & 0 & 0 & 0 & 0.1326 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.002 & 0 & 0 \end{bmatrix}$$

```
% State input matrix
A = [-0.2667 -0.1 0 0 0 0 0 0 0 0 0 0; 0.0067 -0.0398 3.1434 -0.05 0 0 0 0 0 0 0 0;
0 0.0398 -3.1434 0 0 0 0 0 0 0 0 0; 0 0.1 0 0 0 0 0 0 0 0 0 0;
```

```
0 0 0 0 -0.5 -0.2 0 0 0 0 0 0; 0 0 0 0 0.01 -0.0531 2.0928 -0.05 0 0 0 0;
0 0 0 0 0 0.0531 -2.0928 0 0 0 0 0; 0 0 0 0 0 0.2 0 0 0 0 0 0;
0.0067 0 0 0 0 0 0 0 0 0 0 0; 0 0 0 0 0.01 0 0 0 0 0 0 0;
0 0 0 0 0 0 0 0 0 0 0 1; 0 0 0 0 0 0 0 0 0 0 0 -0.2];

% Input matrix
B = [0.05 0 0 0 0; 0 0 0 -0.12 0; 0 0 0 0.12 0; 0 0 0 0 0; 0 0.1 0 0 0; 0 0 0 0 -0.133;
0 0 0 0 0.133; 0 0 0 0 0; 0 0 0 0 0; 0 0 0 0 0; 0 0 0 0 0; 0 0 0.002 0 0];

% Output matrix
C = [0 0 0 0 0 -1 0 0 0 1 0 0; 0 -1 0 0 0 0 0 0 1 0 0 0; 0 0 0 0 0 0 0 0 0 0 1 0];

D = [];      % Transmisstion matrix

% Creating State-space model:
SYS = ss(A,B,C,D);

% Define statename, inputname and outputname
states = {'st1','st2','st3','st4','st5','st6','st7','st8','st9','st10','st11','st12'};
inputs = {'in1','in2','in3','in4','in5'};
outputs = {'out1','out2','out3'};
set(SYS,'statename',states);
set(SYS,'inputname',inputs);
set(SYS,'outputname',outputs);
```

Result for Problem 1b:

```
>> SYS

SYS =

a =
st1       st2       st3       st4       st5       st6       st7       st8       st9      st10
st11      st12
st1   -0.2667      -0.1         0         0         0         0         0         0         0
0         0         0
st2    0.0067   -0.0398     3.143     -0.05         0         0         0         0         0
0         0         0
st3         0    0.0398    -3.143         0         0         0         0         0         0
0         0         0
st4         0       0.1         0         0         0         0         0         0         0
0         0         0
st5         0         0         0         0      -0.5      -0.2         0         0         0
```

```
0         0         0
    st6           0         0         0         0      0.01   -0.0531     2.093     -0.05         0
0         0         0
    st7           0         0         0         0         0    0.0531    -2.093         0         0
0         0         0
    st8           0         0         0         0         0       0.2         0         0         0
0         0         0
    st9      0.0067         0         0         0         0         0         0         0         0
0         0         0
    st10          0         0         0         0      0.01         0         0         0         0
0         0         0
    st11          0         0         0         0         0         0         0         0         0
0         0         1
    st12          0         0         0         0         0         0         0         0         0
0         0      -0.2

    b =
    in1      in2      in3      in4      in5
    st1      0.05        0        0        0        0
    st2         0        0        0    -0.12        0
    st3         0        0        0     0.12        0
    st4         0        0        0        0        0
    st5         0      0.1        0        0        0
    st6         0        0        0        0   -0.133
    st7         0        0        0        0    0.133
    st8         0        0        0        0        0
    st9         0        0        0        0        0
    st10        0        0        0        0        0
    st11        0        0        0        0        0
    st12        0        0    0.002        0        0

    c =
    st1   st2   st3   st4   st5   st6   st7   st8   st9  st10  st11  st12
    out1      0     0     0     0     0    -1     0     0     0     1     0     0
    out2      0    -1     0     0     0     0     0     0     1     0     0     0
    out3      0     0     0     0     0     0     0     0     0     0     1     0

    d =
    in1  in2  in3  in4  in5
    out1    0    0    0    0    0
    out2    0    0    0    0    0
    out3    0    0    0    0    0

    Continuous-time state-space model.
```

Problem 1c. Use the MINREAL function in Matlab to demonstrate whether your LTI state-space model is a minimum realization or not.

```
sysr = minreal(SYS);
if (size(sysr) == size(SYS))
fprintf('LTI system is minimum realization.\n');
else
fprintf('LTI system is not minimum realization.\n');
end
```

Result of checking:

```
LTI system is minimum realization.
```

Problem 2: Demonstrate that this open-loop system is Completely Controllable.

```
Co = ctrb(SYS);      % calculate the control matrix
r = rank(Co);        % calculate rank of matrix
testC = size(A,1);
if r == testC
fprintf('System is Completely Controllable.\n');
else
fprintf('System is not completely controllable.\n');
end
```

Result of checking controllability:

```
System is Completely Controllable.
```

Subset of control inputs checking:

```
for i = 1:4
    if rank(ctrb(SYS(:,[1:i]))) == testC
        fprintf('Subset of control inputs could get %d inputs.\n',i);
        SYS(:,[1:i])
    end
end
```

Results:

```
Subset of control inputs could get 3 inputs.

ans =

a =

    st1       st2       st3       st4       st5       st6       st7       st8       st9      st10
st11      st12
    st1   -0.2667      -0.1         0         0         0         0         0         0         0
0         0         0
    st2    0.0067   -0.0398     3.143     -0.05         0         0         0         0         0
0         0         0
    st3         0    0.0398    -3.143         0         0         0         0         0         0
0         0         0
    st4         0       0.1         0         0         0         0         0         0         0
```

```
0         0         0
    st5          0         0         0         0      -0.5      -0.2         0         0         0
0         0         0
    st6          0         0         0         0      0.01   -0.0531     2.093     -0.05         0
0         0         0
    st7          0         0         0         0         0    0.0531    -2.093         0         0
0         0         0
    st8          0         0         0         0         0       0.2         0         0         0
0         0         0
    st9     0.0067         0         0         0         0         0         0         0         0
0         0         0
    st10         0         0         0         0      0.01         0         0         0         0
0         0         0
    st11         0         0         0         0         0         0         0         0         0
0         0         1
    st12         0         0         0         0         0         0         0         0         0
0         0      -0.2

    b =
    in1     in2     in3
    st1     0.05       0       0
    st2        0       0       0
    st3        0       0       0
    st4        0       0       0
    st5        0     0.1       0
    st6        0       0       0
    st7        0       0       0
    st8        0       0       0
    st9        0       0       0
    st10       0       0       0
    st11       0       0       0
    st12       0       0   0.002

    c =
    st1    st2    st3    st4    st5    st6    st7    st8    st9   st10   st11   st12
    out1      0      0      0      0      0     -1      0      0      0      1      0      0
    out2      0     -1      0      0      0      0      0      0      1      0      0      0
    out3      0      0      0      0      0      0      0      0      0      0      1      0

    d =
    in1   in2   in3
    out1     0     0     0
    out2     0     0     0
    out3     0     0     0
```

```
Continuous-time state-space model.

Subset of control inputs could get 4 inputs.

ans =

a =
    st1      st2      st3      st4      st5      st6      st7      st8      st9     st10
st11     st12
    st1   -0.2667     -0.1        0        0        0        0        0        0        0
0        0        0
    st2    0.0067  -0.0398    3.143    -0.05        0        0        0        0        0
0        0        0
    st3         0   0.0398   -3.143        0        0        0        0        0        0
0        0        0
    st4         0      0.1        0        0        0        0        0        0        0
0        0        0
    st5         0        0        0        0     -0.5     -0.2        0        0        0
0        0        0
    st6         0        0        0        0     0.01  -0.0531    2.093    -0.05        0
0        0        0
    st7         0        0        0        0        0   0.0531   -2.093        0        0
0        0        0
    st8         0        0        0        0        0      0.2        0        0        0
0        0        0
    st9    0.0067        0        0        0        0        0        0        0        0
0        0        0
    st10        0        0        0        0     0.01        0        0        0        0
0        0        0
    st11        0        0        0        0        0        0        0        0        0
0        0        1
    st12        0        0        0        0        0        0        0        0        0
0        0     -0.2

b =
    in1    in2    in3    in4
    st1    0.05      0      0      0
    st2       0      0      0  -0.12
    st3       0      0      0   0.12
    st4       0      0      0      0
    st5       0    0.1      0      0
    st6       0      0      0      0
    st7       0      0      0      0
```

```
st8       0       0       0       0
st9       0       0       0       0
st10      0       0       0       0
st11      0       0       0       0
st12      0       0   0.002       0

c =
st1   st2   st3   st4   st5   st6   st7   st8   st9  st10  st11  st12
out1     0     0     0     0     0    -1     0     0     0     1     0     0
out2     0    -1     0     0     0     0     0     0     1     0     0     0
out3     0     0     0     0     0     0     0     0     0     0     1     0

d =
in1  in2  in3  in4
out1    0    0    0    0
out2    0    0    0    0
out3    0    0    0    0

Continuous-time state-space model.
```

Problem 3: Compute the open loop poles of this system, the natural frequencies with unit of Hz, and the damping ratios for each eigenvalues.

```
Char = poly(A);          % define characteristic equation from state matrix
Poles = roots(Char);     % compute system poles
Eigs = eig(A);           % compute eigenvalues of state matrix
[wn,zeta] = damp(SYS);  % compute natural frequency and damping ratio of system

Wn = wn/(2*pi);          % convert natural frequency to unit of Hz
Zeta = abs(zeta);

column = [1:12];
Values = column';
table(Values, Wn, Zeta, Eigs, Poles)    % print out the table of values with Wn ascending
```

Result:

Values	Wn	Zeta	Eigs	Poles
1	0	1	0+0i	0+0i
2	0	1	0+0i	0+0i
3	0	1	-3.1832+0i	0+0i
4	0.011233	0.016702	-0.0011788+0.070567i	-3.1832+0i
5	0.011233	0.016702	-0.0011788-0.070567i	-2.1458+0i
6	0.015777	0.019661	-0.26436+0i	-0.49625+0i
7	0.015777	0.019661	-2.1458+0i	-0.26436+0i
8	0.031831	1	-0.49625+0i	-0.2+0i
9	0.042075	1	-0.001949+0.099112i	-0.001949+0.099112i
10	0.07898	1	-0.001949-0.099112i	-0.001949-0.099112i
11	0.34151	1	0+0i	-0.0011788+0.070567i
12	0.50662	1	-0.2+0i	-0.0011788-0.070567i

Problem 4: Simulate the open-loop response of the system for three seconds, assuming all initial states are zero except for non-zero IC's defined below.

$$y_1(0) = 0.5, \qquad p_1(0) = 300 \qquad z(0) = 0.7$$

$$x_2(0)0.6 \qquad p_5(0) = -150 \qquad \dot{z}(0) = -0.09348$$

```
X40 = [300; 0; 0; 0; -150; 0; 0; 0; 0.5; 0.6; 0.7; -0.09348];  % initial states for problem 4
figure;
[y,t,x] = initial(SYS,X40,3);           % apply initial function for 3 seconds
plot(t, y(:,1),'r',t,y(:,2),'b',t,y(:,3),'g');
title('Response for Initial Condition in 3 seconds');
xlabel('time');
ylabel('output');
legend('Output1 - X','Output2 - Y', 'Output3 - Z');
print('ResponseForInitialCondition','-dpng');    % save plot as an image
```

Result:

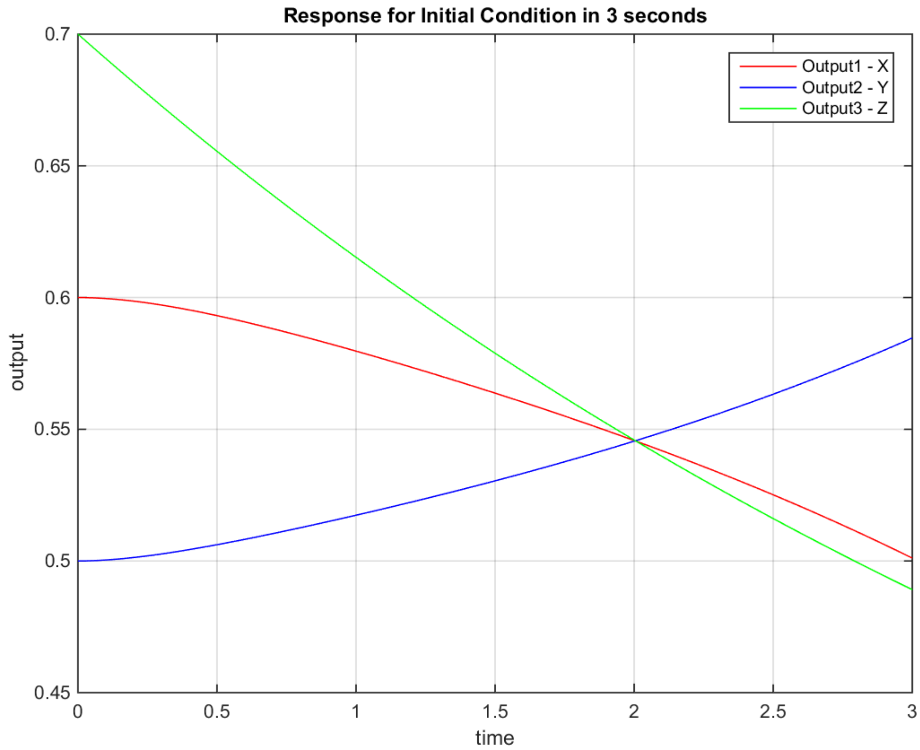

Figure 2: Response for Initial Condition in 3 seconds

Problem 5: Design a full-state feedback controller that meets the following performance requirements:

1. Each ouput must be within $\pm 1cm$, after 8 seconds
2. $|u_1| \leq 1000, \quad |u_2| \leq 1000, \quad |u_3| \leq 200, \quad |u_4| \leq 100, \quad |u_5| \leq 100$

Simulate the closed-loop response of the system for ten second, assuming all initial states are zero except for the non-zero IC's defined bellow:

$$y_1(0) = 1 \qquad x_2(0) = 1 \qquad z(0) = 1$$

```
% Poles 1
Poles1 = Poles;      % using poles from original state matrix A
G1 = place(A,B,Poles1);       % compute Gain matrix from Poles of open-loop

Ac1 = A - B*G1;            % compute state matrix for close-loop
B1 = [];              % new B matrix is 0
SYSC1 = ss(Ac1,B1,C,D);    % new close-loop system

%Checking step:
figure;
[yc1,tc1,xc1] = initial(SYSC1,X50,10);
plot(tc1,yc1);
grid on; legend('y1','y2','y3');

U1 = -G1*xc1';          % new control input space base on gain G
figure;
plot(tc1,U1);         % plot and check control inputs
grid on; legend('u1','u2','u3','u4','u5');
print('FullStateFeedback','-dpng');

% Poles 2
Poles2 = [-0.001 + 0.0001i; -0.001 - 0.0001i;  -0.002 + 0.015i; -0.002 - 0.015i; -2.5 + 0.0001i; -2
-0.5 + 0.001i; -0.65 - 0.001i; -0.2 + 0.0325i; -0.2 - 0.0325i; -0.0012 + 0.008i; -0.0012 - 0.008i];

G2 = place(A,B,Poles2);
Ac2 = A - B*G2;
B2 = [];
sysc2 = ss(Ac2, B2, C,D);

[yc2,tc2,xc2] = initial(sysc2,X50,10);
figure;
```

```
plot(tc2,yc2);
grid on; legend('y1','y2','y3');

U2 = -G2*xc2';
figure;
plot(tc2,U2);   grid on; legend('u1','u2','u3','u4','u5');

% Poles 3
Poles3 = [-0.001 + 0.0001i; -0.001 - 0.0001i;  -0.002 + 0.015i; -0.002 - 0.015i; -2.5 + 0.0001i; -2
-0.5 + 0.001i; -0.65 - 0.001i; -0.2 + 0.0325i; -0.2 - 0.0325i; -0.0015 + 0.006i; -0.0015 - 0.008i];
G3 = place(A,B,Poles3);
Ac3 = A - B*G3;
B3 = [];
sysc3 = ss(Ac3, B3, C,D);

[yc3,tc3,xc3] = initial(sysc3,X50,10);
figure;
plot(tc3,yc3);
grid on; legend('y1','y2','y3');
U3 = -G3*xc3';
figure;
plot(tc3,U3);
grid on; legend('u1','u2','u3','u4','u5');

% Poles 4
Poles4 = [-0.001 + 0.0001i; -0.001 - 0.0001i;  -0.002 + 0.015i; -0.002 - 0.015i; -2.5 + 0.0001i; -2
-0.5 + 0.001i; -0.65 - 0.001i; -0.2 + 0.0325i; -0.2 - 0.0325i; -0.0018 + 0.0065i; -0.0015 - 0.008i];
G4 = place(A,B,Poles4);
Ac4 = A - B*G4;
B4 = [];
sysc4 = ss(Ac4, B4, C,D);

[yc4,tc4,xc4] = initial(sysc4,X50,10);
figure;
plot(tc4,yc4);
grid on; legend('y1','y2','y3');
U4 = -G4*xc4';
figure;
plot(tc4,U4);
grid on; legend('u1','u2','u3','u4','u5');

% Poles 5
Poles5 = [-0.001 + 0.0001i; -0.001 - 0.0001i;  -0.002 + 0.015i; -0.002 - 0.015i; -2.5 + 0.0001i; -2
-0.5 + 0.001i; -0.65 - 0.001i; -0.2 + 0.0325i; -0.2 - 0.0325i; -0.0017 + 0.0055i; -0.0015 - 0.008i];
```

```
G5 = place(A,B,Poles5);
Ac5 = A - B*G5;
B5 = [];
sysc5 = ss(Ac5, B5, C,D);

[yc5,tc5,xc5] = initial(sysc5,X50,10);
figure;
plot(tc5,yc5);
grid on; legend('y1','y2','y3');
U5 = -G5*xc5';
figure;
plot(tc5,U5);
grid on; legend('u1','u2','u3','u4','u5');

%% Print out
figure;
subplot(15,2,7);
plot(tc5,yc5(:,1),'r');
xlabel('Time'); ylabel('Output');
grid on; title('First Output');
    subplot(15,2,17);
plot(tc5,yc5(:,2),'b');
xlabel('Time'); ylabel('Output');
grid on; title('Second Output');
    subplot(15,2,27);
plot(tc5,yc5(:,3),'g');
xlabel('Time'); ylabel('Output');
grid on; title('Third Output');
    subplot(15,2,6);
plot(tc5,U5(1,:),'r');
xlabel('Time'); ylabel('Control Input 1');
grid on; title('First Control Input');
    subplot(15,2,12);
plot(tc5,U5(2,:),'r');
xlabel('Time'); ylabel('Control Input 2');
grid on; title('Second Control Input');
    subplot(15,2,18);
plot(tc5,U5(3,:),'r');
xlabel('Time'); ylabel('Control Input 3');
grid on; title('Third Control Input');
    subplot(15,2,24);
plot(tc5,U5(4,:),'r');
xlabel('Time'); ylabel('Control Input 4');
grid on; title('Forth Control Input');
```

```
    subplot(15,2,30);
plot(tc5,U5(5,:),'r');
xlabel('Time'); ylabel('Control Input 5');
grid on; title('Fifth Control Input');
    print('FeedbackSystem','-dpng');
```

Full-State feedback system base on Poles 5:

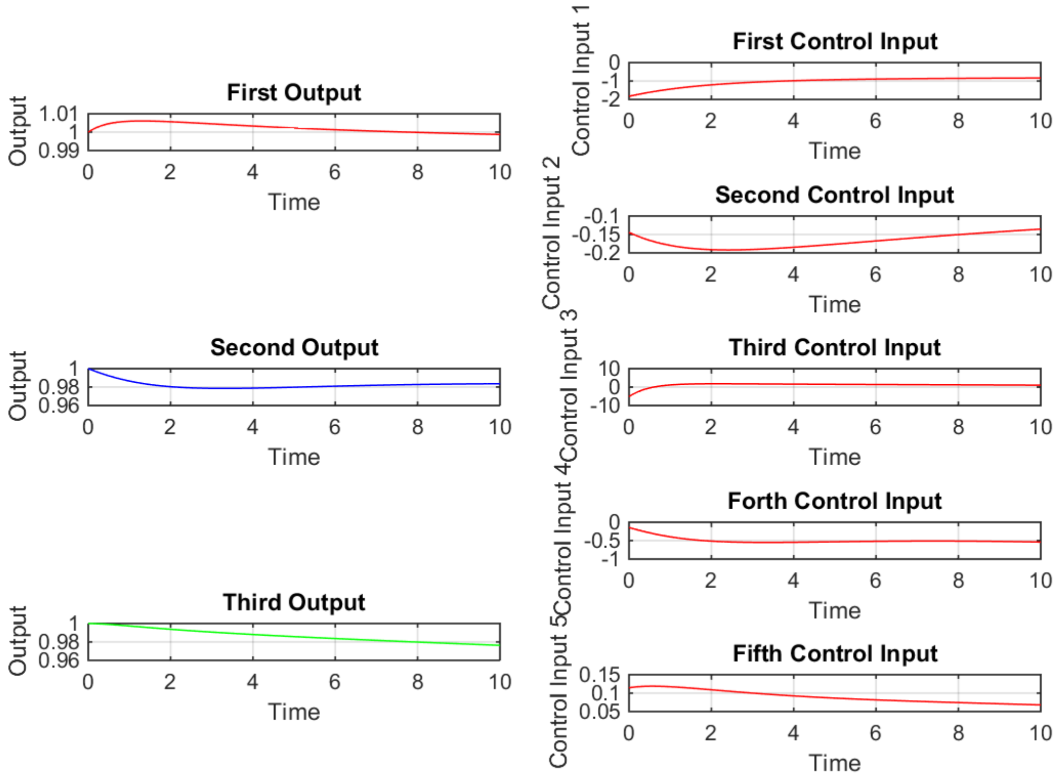

Figure 3: Full State Feedback System for 10 seconds