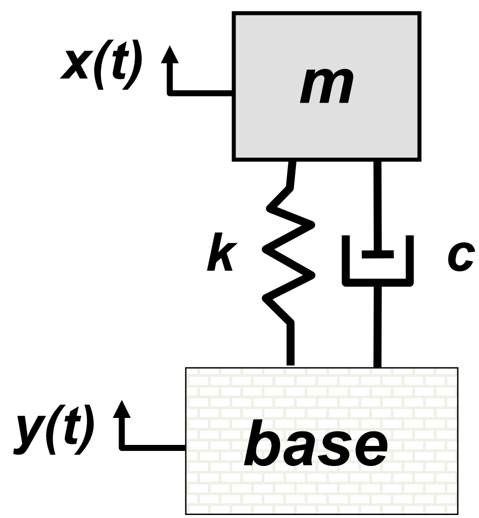
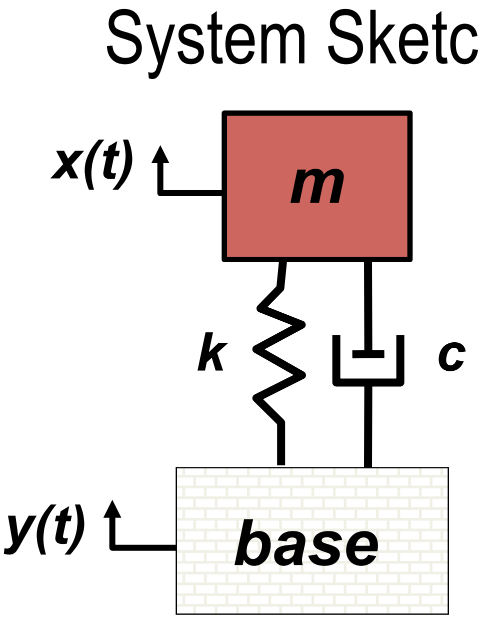
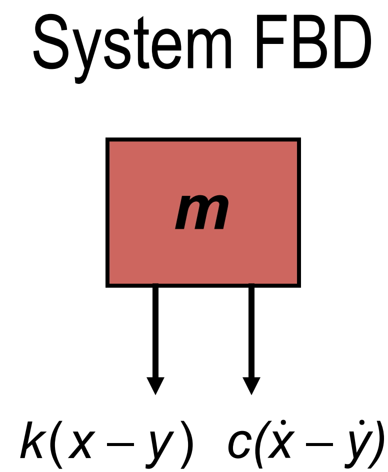
2.4 Base excitation

- Important class of vibration analysis
  - Preventing excitations from passing from a vibrating base through its mount into a structure
- Vibration isolation
  - Vibrations in your car
  - Satellite operation
  - Disk drives, etc.



FBD of SDOF Base Excitation





$$\sum F = -k(x-y) - c(\dot{x}-\dot{y}) = m\ddot{x}$$

$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky \qquad (2.61)$$

SDOF Base Excitation (cont)

Assume: $y(t) = Y\sin(\omega t)$ and plug into Equation(2.61)

$$m\ddot{x} + c\dot{x} + kx = \underbrace{c\omega Y\cos(\omega t) + kY\sin(\omega t)}_{\text{harmonic forcing functions}} \qquad (2.63)$$

For a car, $\omega = \dfrac{2\pi}{\tau} = \dfrac{2\pi V}{\lambda}$

The steady-state solution is just the superposition of the two individual particular solutions (system is linear).

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \underbrace{\overbrace{2\zeta\omega_n\omega Y}^{f_{0c}}\cos(\omega t)} + \underbrace{\overbrace{\omega_n^2 Y}^{f_{0s}}\sin(\omega t)} \qquad (2.64)$$

Particular Solution (sine term)

With a sine for the forcing function,

$$\ddot{x}+2\zeta\omega_n\dot{x} + \omega_n^2 x = f_{0s}\sin\omega t$$

$$x_{ps} = \underline{A_s\cos\omega t + B_s\sin\omega t} = X_s\sin(\omega t - \phi_s)$$

Use rectangular form to make it easier to add the cos term

where

$$A_s = \frac{-2\zeta\omega_n\omega f_{0s}}{(\omega_n^2-\omega^2)^2+\left(2\zeta\omega_n\omega\right)^2}$$

$$B_s = \frac{(\omega_n^2-\omega^2)f_{0s}}{(\omega_n^2-\omega^2)^2+\left(2\zeta\omega_n\omega\right)^2}$$

Particular Solution (cos term)

With a cosine for the forcing function, we showed

$$\ddot{x}+2\zeta\omega_n\dot{x} + \omega_n^2 x = f_{0c}\cos\omega t$$

$$x_{pc} = \underline{A_c\cos\omega t + B_c\sin\omega t} = X_c\cos(\omega t - \phi_c)$$

where

$$A_c = \frac{(\omega_n^2 - \omega^2) f_{0c}}{(\omega_n^2 - \omega^2)^2 + \left(2\zeta\omega_n\omega\right)^2}$$

$$B_c = \frac{2\zeta\omega_n\omega f_{0c}}{(\omega_n^2 - \omega^2)^2 + \left(2\zeta\omega_n\omega\right)^2}$$

Magnitude X/Y

Now add the *sin* and *cos* terms to get the magnitude of the full particular solution

$$X = \sqrt{\frac{f_{0c}^2 + f_{0s}^2}{(\omega_n^2 - \omega^2)^2 + \left(2\zeta\omega_n\omega\right)^2}} = \omega_n Y \sqrt{\frac{(2\zeta\omega)^2 + \omega_n^2}{(\omega_n^2 - \omega^2)^2 + \left(2\zeta\omega_n\omega\right)^2}}$$

where $f_{0c} = 2\zeta\omega_n\omega Y$ and $f_{0s} = \omega_n^2 Y$

if we define $r = \omega/\omega_n$ this becomes $$X = Y\sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+\left(2\zeta r\right)^2}} \quad (2.70)$$

$$\frac{X}{Y} = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+\left(2\zeta r\right)^2}} \quad (2.71)$$

What does this represent?

The relative magnitude plot of X/Y versus frequency ratio: Called the *Displacement Transmissibility*

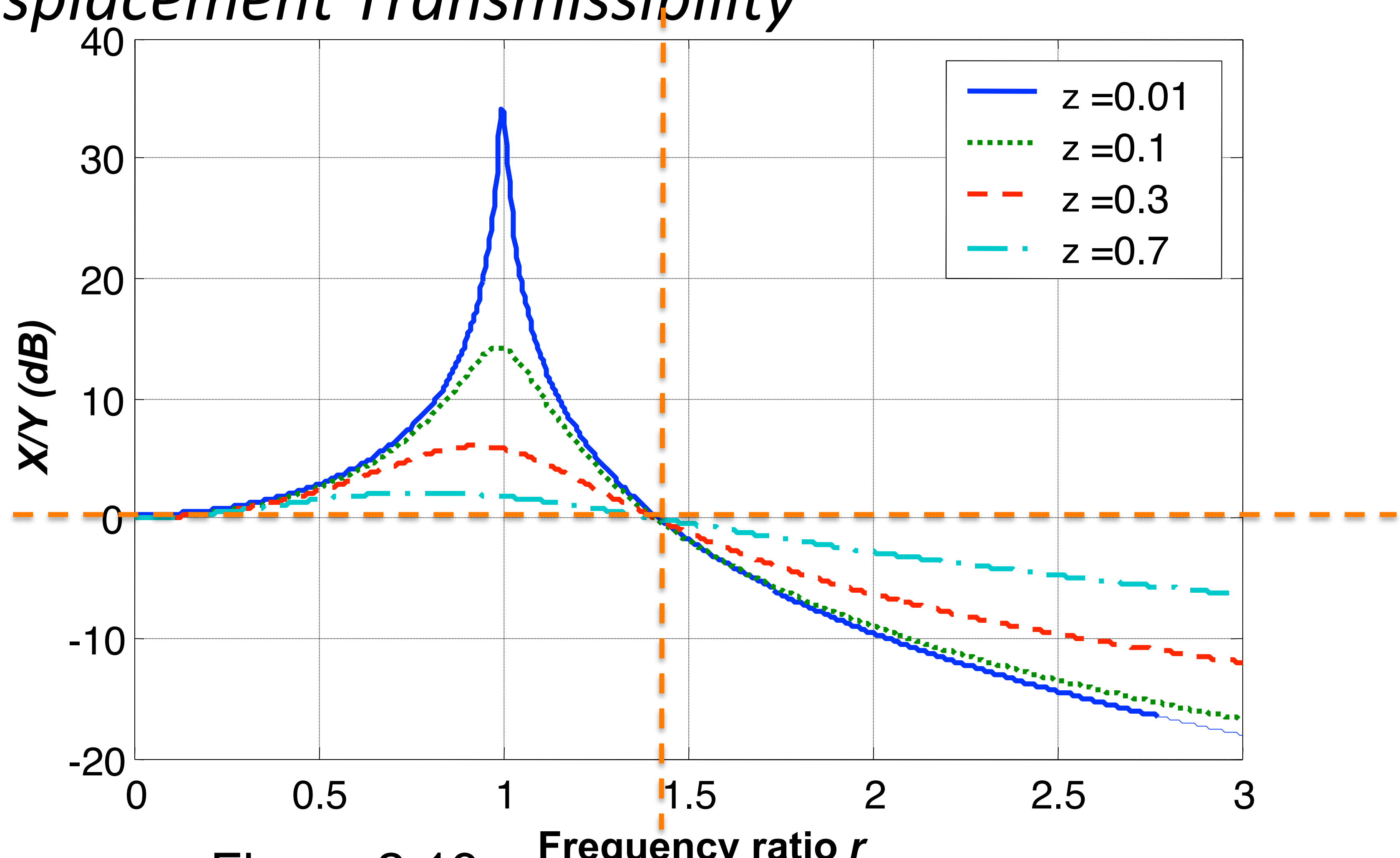


Figure 2.13

From the plot of relative Displacement Transmissibility observe that:

- X/Y is called Displacement Transmissibility Ratio
- Potentially severe amplification at resonance
- Attenuation for r > sqrt(2): Isolation Zone
- If r < sqrt(2) transmissibility decreases with damping ratio: Amplification Zone
- If r >> 1 then transmissibility increases with damping ratio $X_p \sim 2Y\zeta/r$

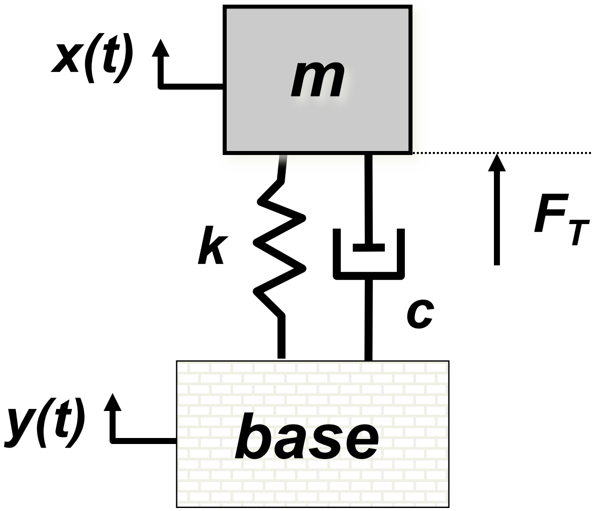
Next examine the *Force Transmitted to the mass* as a function of the frequency ratio

$$F_T = -k(x-y) - c(\dot{x}-\dot{y}) = m\ddot{x}$$ From FBD

At steady state, $x(t) = X\cos(\omega t - \phi)$,

so $\quad \ddot{x}$=-$\omega^2 X\cos(\omega t - \phi)$

$$|F_T| = m\omega^2 X = \underline{kr^2 X}$$



Plot of Force Transmissibility (in dB) versus frequency ratio

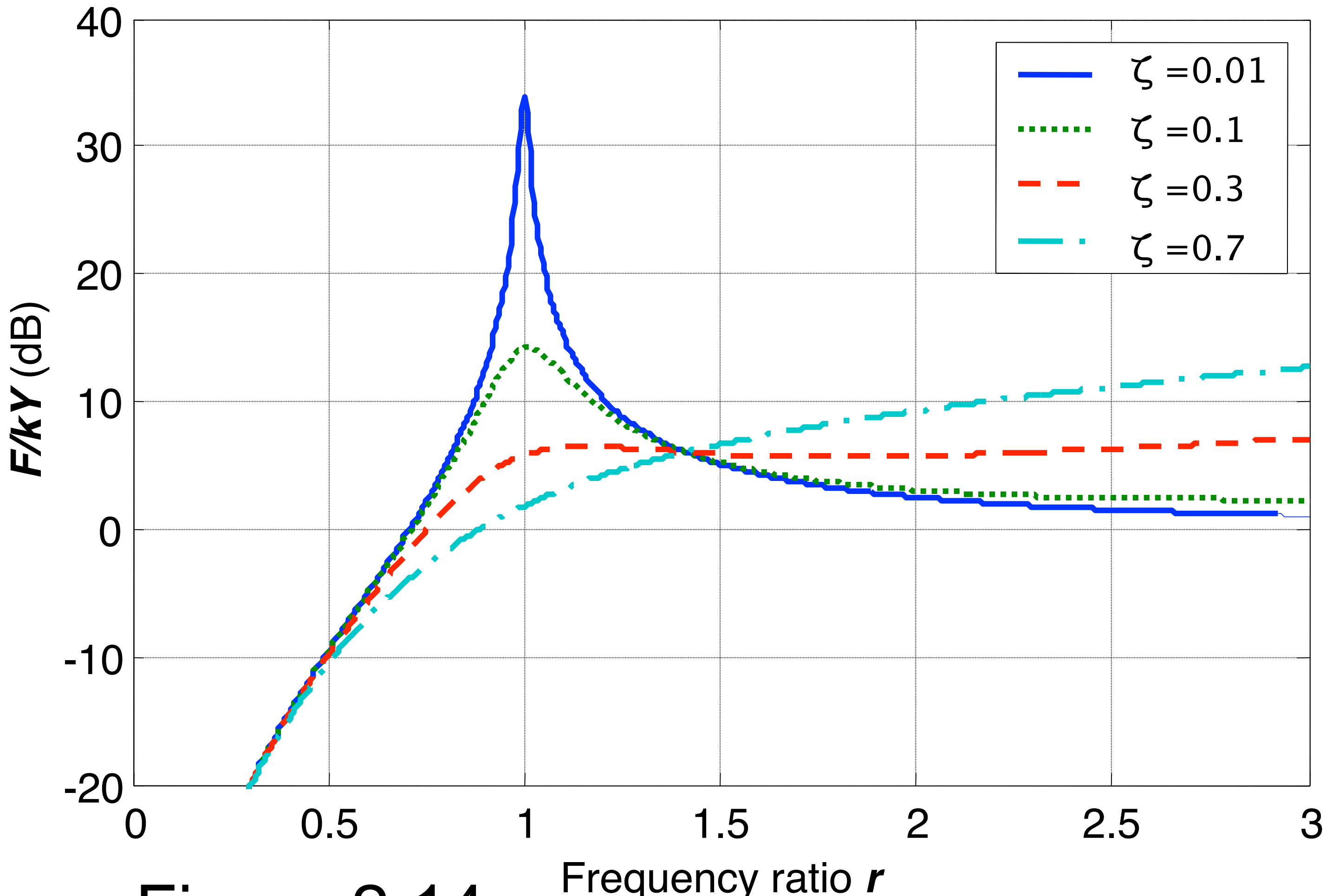
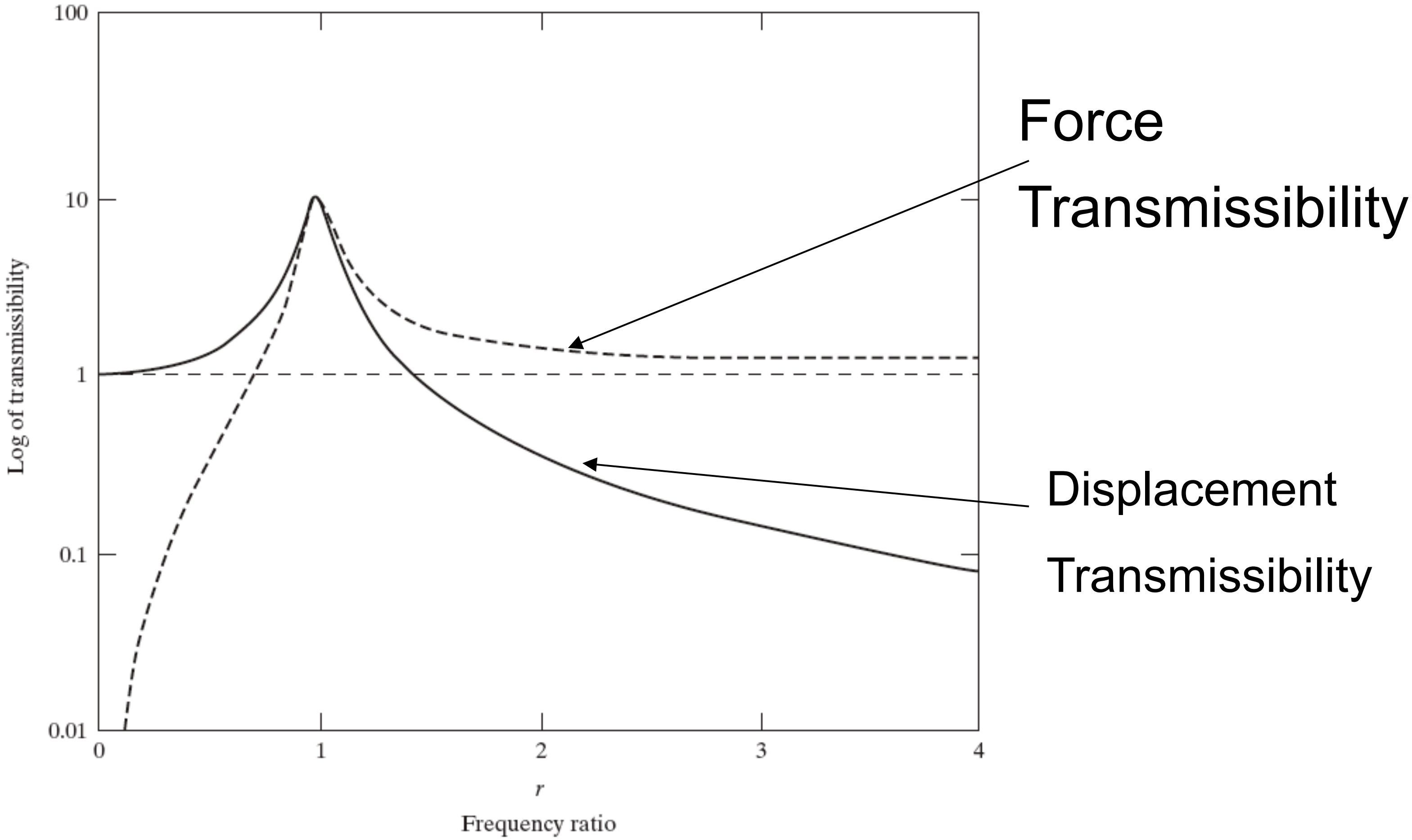
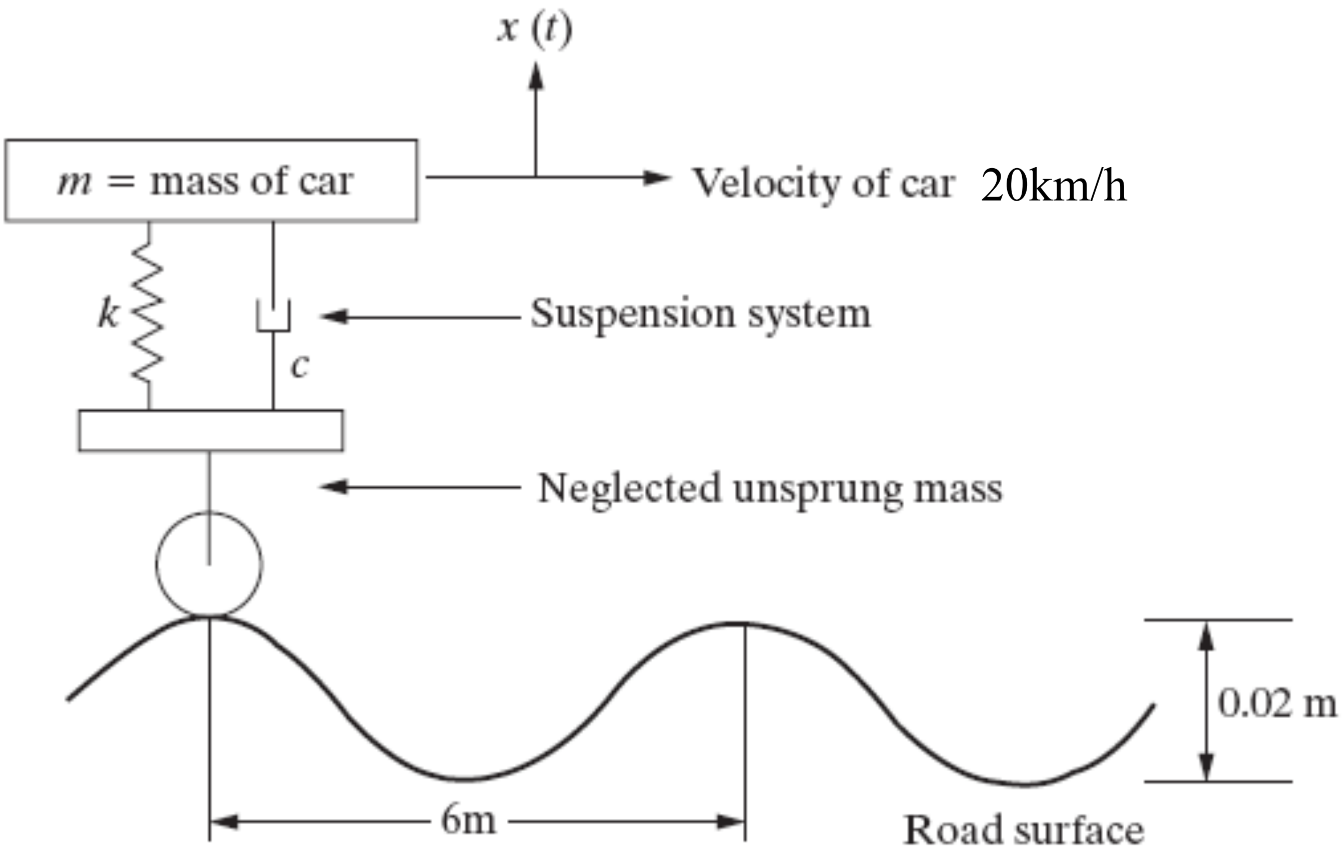


Figure 2.14

Figure 2.15 Comparison between force and displacement transmissibility



Example 2.4.1: Effect of speed on the amplitude of car vibration



Model the road as a sinusoidal input to base motion of the car model

Approximation of road surface:

$$y(t) = (0.01\text{ m})\sin\omega_b t$$

$$\omega_b = v(\text{km/hr})\left(\frac{1}{0.006\text{ km}}\right)\left(\frac{\text{hour}}{3600\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{\text{cycle}}\right) = 0.2909v\text{ rad/s}$$

$$\omega_b(20\text{km/hr}) = 5.818\text{ rad/s}$$

From the data give, determine the frequency and damping ratio of the car suspension:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{4\times10^4\text{ N/m}}{1007\text{ kg}}} = 6.303\text{ rad/s} \quad (\approx 1\text{ Hz})$$

$$\zeta = \frac{c}{2\sqrt{km}} = \frac{2000\text{ Ns/m}}{2\sqrt{\left(4\times10^4\text{ N/m}\right)\left(1007\text{ kg}\right)}} = 0.158$$

From the input frequency, input amplitude, natural frequency and damping ratio use equation (2.70) to compute the amplitude of the response:

$$r = \frac{\omega_b}{\omega} = \frac{5.818}{6.303}$$

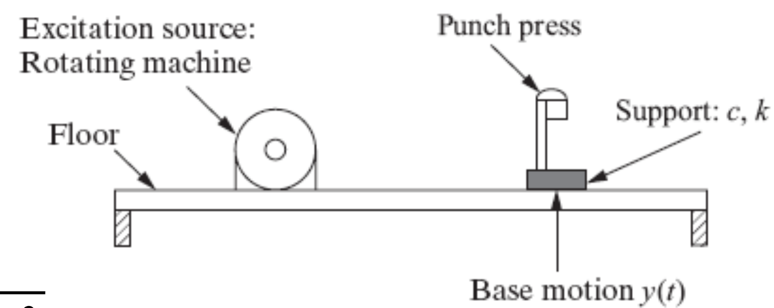
$$X = Y\sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$$

$$= (0.01\ \mathrm{m})\sqrt{\frac{1+\left[2(0.158)(0.923)\right]^2}{\left(1-(0.923)^2\right)^2+\left(2(0.158)(0.923)\right)^2}} = 0.0319\ \mathrm{m}$$

What happens as the car goes faster? See Table 2.1.

In what zone are we at this point?

Example 2.4.2: Compute the force transmitted to a machine through base motion at resonance



From (2.77) at r =1:

$$\frac{F_T}{kY} = \left[\frac{1+(2\zeta)^2}{(2\zeta)^2}\right]^{1/2} \Rightarrow F_T = \frac{kY}{2\zeta}\sqrt{1+4\zeta^2}$$

From given *m, c,* and *k*: $\zeta = \dfrac{c}{2\sqrt{km}} = \dfrac{900}{2\sqrt{40{,}000 \bullet 3000}} \cong 0.04$

From measured excitation Y = 0.001 m:

$$F_T = \frac{kY}{2\zeta}\sqrt{1+4\zeta^2} = \frac{(40{,}000\text{ N/m})(0.001\text{ m})}{2(0.04)}\sqrt{1+4(0.04)^2} = \underline{501.6\text{ N}}$$