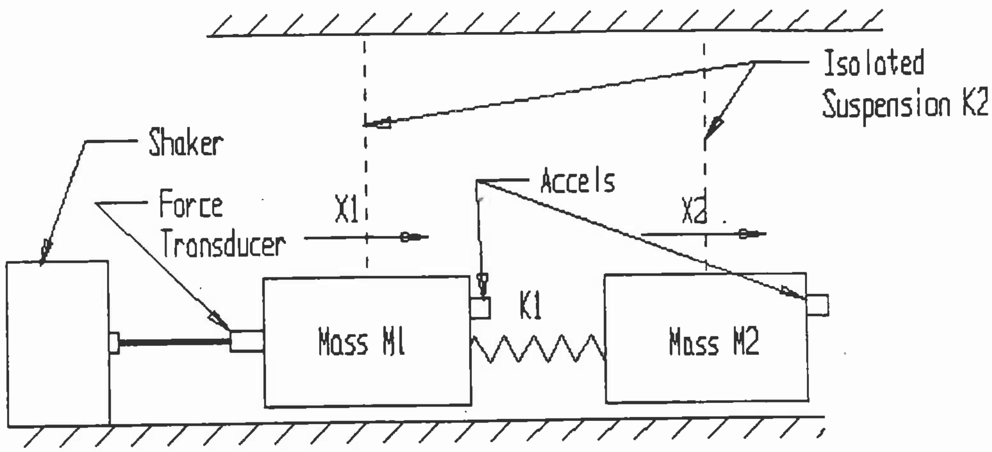
3.2 *Comparison of Measured and Actual System Parameters*

Consider the two-degree-of-freedom system below:



This is a model of an actual system in the lab. The actual damping is small, so for now it will not be included in the model.

Writing the differential equations of motion for this system results in:

$$\begin{bmatrix} M_1 & \\ & M_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} K_1+K_2 & -K_1 \\ -K_1 & K_1+K_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ 0 \end{Bmatrix} \tag{3.37}$$

Where K_2 is a *very small* stiffness compared to the stiffness between the masses. Neglecting this stiffness and writing the equation in the form of the solution gives:

$$\left(-\omega^2\begin{bmatrix} M_1 & \\ & M_2 \end{bmatrix} + \begin{bmatrix} K_1 & -K_1 \\ -K_1 & K_1 \end{bmatrix}\right)\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} e^{i\omega t} = \begin{Bmatrix} F_1 \\ 0 \end{Bmatrix} e^{i\omega t} \tag{3.38}$$

Using the same solution technique as presented before with $M_1 = M_2 = M$ results in:

$$\lambda M(\lambda M - 2K_1) = 0 \tag{3.39}$$

From this equation, it is seen that one of the eigenvalues is zero. This is a rigid body mode that is restrained only by the small effective stiffness of the suspension.

The approximate parameters of the actual system have been measured to be:

$$K_1 = 20{,}000 \text{ lb / in}$$

$$M_1 = M_2 = 0.0674 \frac{\text{lb sec}^2}{\text{in}}$$

This gives a non-zero natural frequency of 770.4 rad/sec or 122.6 Hz. The mode shapes are identical to those derived for the previous system:

$$\phi_1 = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$\phi_2 = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$

Solving for the FRF from Eqn. 3.33:

$$\frac{X_1}{F_1} = \sum_{r=1}^{n}\left(\frac{\phi_{1r}\phi_{1r}}{-\omega^2 m_r + k_r}\right) = \frac{1}{-\omega^2(2M)+0} + \frac{1}{-\omega^2(2M)+4K_1}$$
$$= \frac{1}{-0.135\omega^2} + \frac{1}{-0.135\omega^2 + 80{,}000} \qquad (3.40)$$

At zero frequency, the structure behaves as though it were a rigid body. Newton's 2nd law says:

$$\sum \vec{F} = M\vec{a} \tag{3.41}$$

To get the acceleration magnitude, we must differentiate Eqn. 3.32 twice:

$$\left|\frac{a_i}{F_j}\right| = \left|\frac{\ddot{x}_i}{F_j}\right| = \left|\frac{-\omega^2 X_i}{F_j}\right| = \left|\sum_{r=1}^{n}\left(\frac{-\omega^2 \phi_{ir}\phi_{jr}}{-\omega^2 m_r + k_r}\right)\right| \tag{3.42}$$

In the particular case:

$$\left.\left|\frac{a_1}{F_1}\right|\right|_{\omega=0} = \left.\left|\frac{1}{0.135} + \frac{-\omega^2}{-0.135\,\omega^2 + 80{,}000}\right|\right|_{\omega=0} = \frac{1}{0.135} \tag{3.43}$$

Notes:

- The second term in Eqn. 3.40 represents the oscillatory response, which will have zero acceleration at zero frequency. This is to be expected because the system will not oscillate with a constant force applied.
- The first term in Eqn. 3.40 will accelerate with an applied constant force, which is just a restatement of Newton's 2nd Law.

What does all this mean?

If we look at a plot of the acceleration vs force FRF, at zero frequency we will see a value attributed *only* to the rigid body mode. This value is exactly equal to $\frac{1}{2M}$, or $\frac{1}{\text{Total Mass}}$ of the structure.

- Analytically, the intercept will always occur in the same place however the form of the equation depends on the choice of the eigenvector. The zero-frequency amplitude would be $\frac{\tilde{A}}{\tilde{A}(\text{Total Mass})}$, where $\tilde{A}$ is the residue.

What about the response of the second mass?

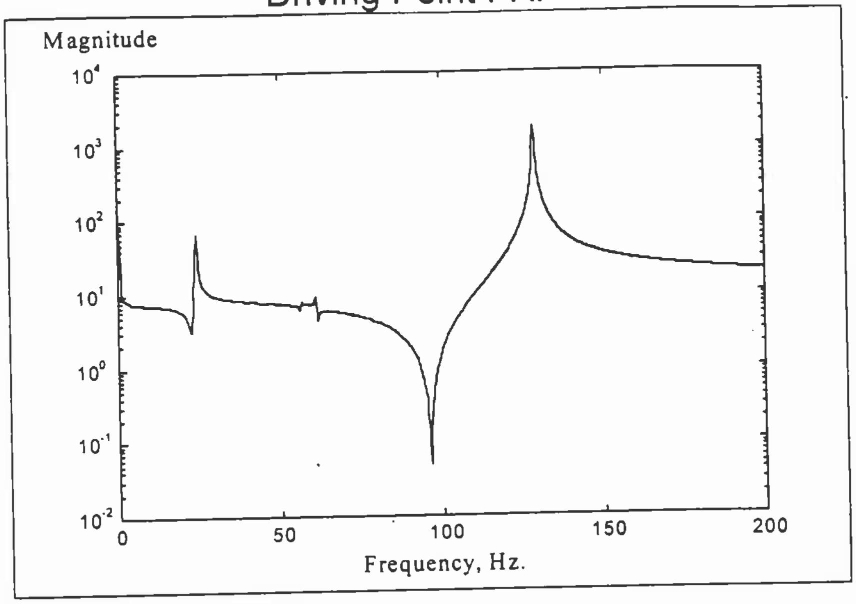
Rewriting Eqn. 3.42 for mass 2 and evaluating at $\omega = 0$ results in:

$$\left|\frac{a_2}{F_1}\right|_{\omega=0} = \left|\frac{\phi_{21}\phi_{11}}{0.135} + \frac{0}{80{,}000}\right| = \frac{1}{0.135} \qquad (3.44)$$

We would expect the zero-frequency intercept of both the driving point and response point FRF's to be the same.

The measured FRF's for the actual system are shown below.

Driving Point FRF



Response Point FRF