ME 5514 VIBRATION MECHANICS SYSTEMS
Homework 1

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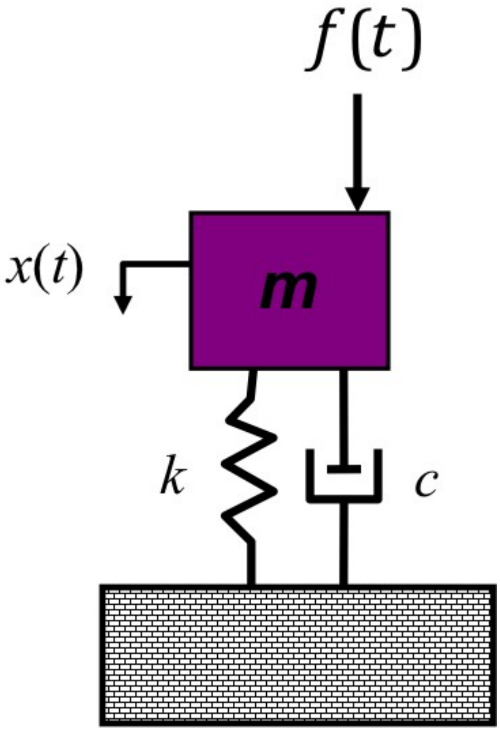
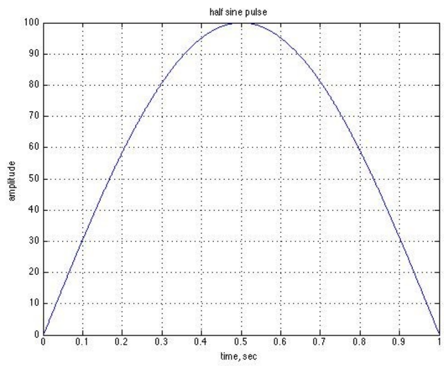
The system shown is subject to a half-sine pulse input and all initial conditions = 0. Determine the time domain response for 5 seconds ($0 < t < 5$) using the following methods:





with: $m = 100$ lbs; $k = 7$ lb/in; $\zeta = 0.1$

We have the motion differential equation of system:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F_0 \sin \omega t$$

$$\Leftrightarrow \ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = f_0 \sin \omega t$$

with: ω_n is natural frequency: $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{7}{100}} = 0.2646$ rad/s

ζ is damping ratio: $\zeta = \frac{c}{2m\omega_n} = 0.1$

and $f_0 = \frac{F_0}{m}$

1. Compute numerically using the Euler form with time step $\Delta t = 0.1$:

   For all initial conditions are 0: $x(0) = x_1(0) = 0; \dot{x}(0) = x_2(0) = 0$.

   From half-sine pulse input we could point out that:

   - Amplitude of force is: 100 lbs.

   - Circle time of force is: $T = 2.1 = 2$ seconds

   $\Rightarrow$ force's frequency is: $f = \frac{1}{T} = \frac{1}{2} = 0.5$ Hz

   Equivalently we have: $\omega = 2\pi f = 2 * \pi * 0.5 = \pi$ rad/s

   We could temporary form the force input model: $f(t) = 100 \sin(\omega t + \phi)$

   - For $t_0 = 0, t_1 = 1$ we both have $f(t) = 0 \Rightarrow \phi = 0$

   $\Rightarrow$ The half-sine input equation: $f(t) = 100 \sin(\pi t)$

Applied half-sine pulse input we have the system's motion equation:

$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = \sin\pi t \qquad \text{(because } f_0 = \frac{F_0}{m} = \frac{100}{100} = 1\text{)}$$

Define $\begin{cases} x_1 = x(t) \\ x_2 = \dot{x}(t) = \dot{x_1} \end{cases}$ we have: $\begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = -2\zeta\omega_n x_2 - \omega_n^2 x_1 + \sin\pi t \end{cases}$

We have the state-space form of motion equation: $\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + \mathbf{f}(t)$

with: $A = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix}$; $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$; $\mathbf{f}(t) = \begin{bmatrix} 0 \\ f_0\cos\omega t \end{bmatrix}$

Applying Euler form to motion equation we solution:

$$x(t_{i+1}) = x(t_i) + Ax(t_i)\Delta t + \mathbf{f}(t_i)\Delta t$$

Solving problem in `MATLAB` using the `ODE45` function we have result:

```
TSPAN = 0:0.1:5;
Y0 = [0;0]; % for initial condition
[t,y1b] = ode45('num_for_hw1a',TSPAN,Y0);
figure; plot(t,y1b(:,1));
xlabel('Time(sec)'); ylabel('Displacement (m)');
title('Time response for 5 second with f(t) = 100sin\pi t');
grid on;
print('vibration_hw1a','-dpng');

function Xdot = num_for_hw1a(t,X)
m = 100; k = 7; ze = 0.1;
wn = sqrt(k/m);
w = pi; F = 100; f = F/m;
f = [0; f*cos(w*t)];
A = [0 1; -wn*wn -2*ze*wn];
Xdot = A*X + f;
end
```

Result is presented in following figure:

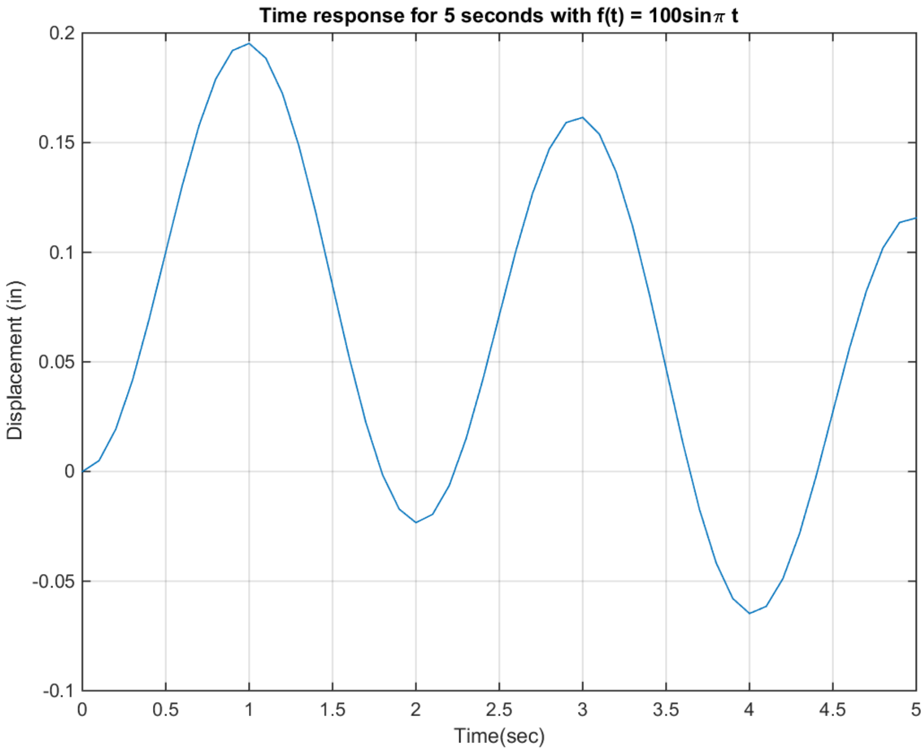


Figure 1: Result by applying Euler form for 5 seconds

2. Compare with harmonic excitation solution using $f(t) = 100\sin\pi t$ as the input at $t = 0$:

System's motion equation: $m\ddot{x}(t) + c\dot{x}(t) + kx(t) = 100\sin\pi t$

$$\Leftrightarrow \ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = \sin\pi t \qquad (*)$$

we have the particular solution: $x_p(t) = A_s\cos\omega t + B_s\sin\omega t$

with: $A_s = \dfrac{(\omega_n^2 - \omega^2)f_0}{(\omega_n^2-\omega^2)^2 + (2\zeta\omega_n\omega)^2}$ and $B_s = \dfrac{2\zeta\omega_n\omega f_0}{(\omega_n^2-\omega^2)^2 + (2\zeta\omega_n\omega)^2}$

Overall, we have particular solution:

$$x_p(t) = \frac{f_0}{\sqrt{(\omega_n^2-\omega^2)^2 + (2\zeta\omega_n\omega)^2}}\cos\left(\omega t - \tan^{-1}\frac{2\zeta\omega_n\omega}{\omega_n^2-\omega^2}\right)$$

Ignoring the transient response we have solution for (*):

$$x(t) = x_p(t) = \frac{f_0}{\sqrt{(\omega_n^2-\omega^2)^2 + (2\zeta\omega_n\omega)^2}}\cos\left(\omega t - \tan^{-1}\frac{2\zeta\omega_n\omega}{\omega_n^2-\omega^2}\right)$$

Solving problem in `MATLAB` we have solution:

```
m = 100; k = 7; ze = 0.1;
wn = sqrt(k/m);
w = pi; F = 100; f = F/m;
X = f/sqrt((wn*wn - w*w)^2 + (2*ze*wn*w)^2);
phi = atan(2*ze*wn*w/(wn*wn-w*w));
t = 0:0.1:5;
```

```
xt = X*cos(w*t - phi);
figure; plot(t,xt); grid on;
xlabel('Time (sec)'); ylabel('Displacement (in)');
title('Time response for 5 second with Harmonic motion');
print('vibration_hw1b','-dpng');
```

Result is presented in following figure:

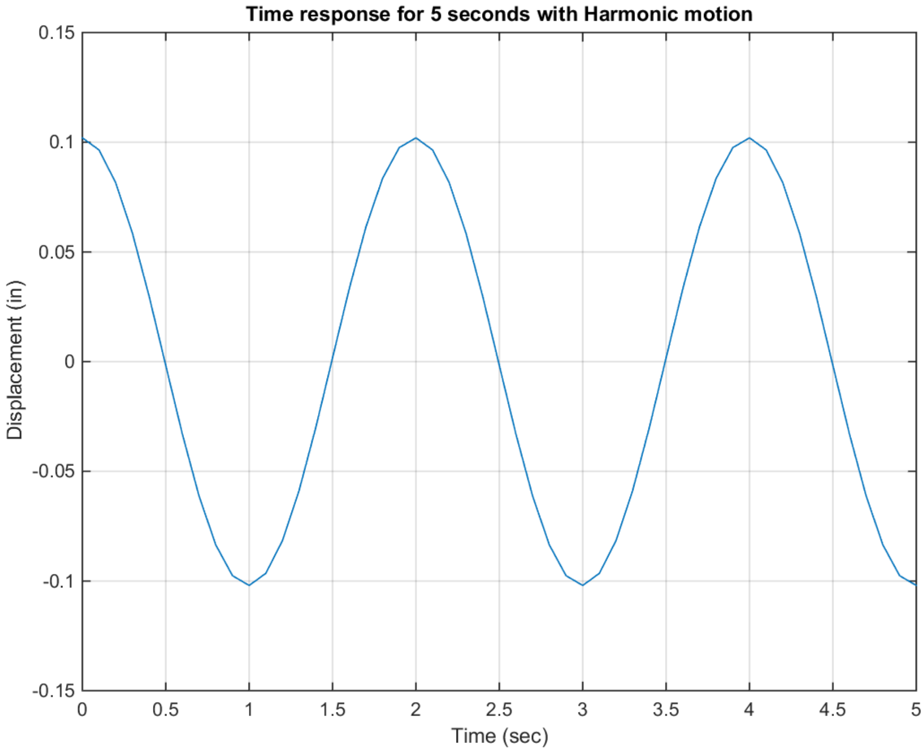


Figure 2: Result by applying Harmonic model for 5 seconds

Subtract out a time delayed and negative harmonic input starting at $t = 1$ sec to provide the response beyond 1 sec:

```
delta_x = y1a(:,1)' - xt;
t1 = 0:0.1:1;
t2 = 0:0.1:4;
disp1 = zeros(size(t1));
disp2 = zeros(size(t2));
for i = 1:size(t1,2)
    disp1(1,i) =  y1a(i,1);
end
```

```
for j = 1:size(t2,2)
    disp2(1,j) = delta_x(1,size(t1,2)+j-1);
end
tt = [t1,1+t2];
disp = [disp1,disp2];
figure; plot(tt,disp); grid on;
xlabel('Time (sec)'); ylabel('Displacement (in)');
title('Time response for 5 seconds');
print('vibration_hw1b2','-dpng');
```

Result is presented in following figure:

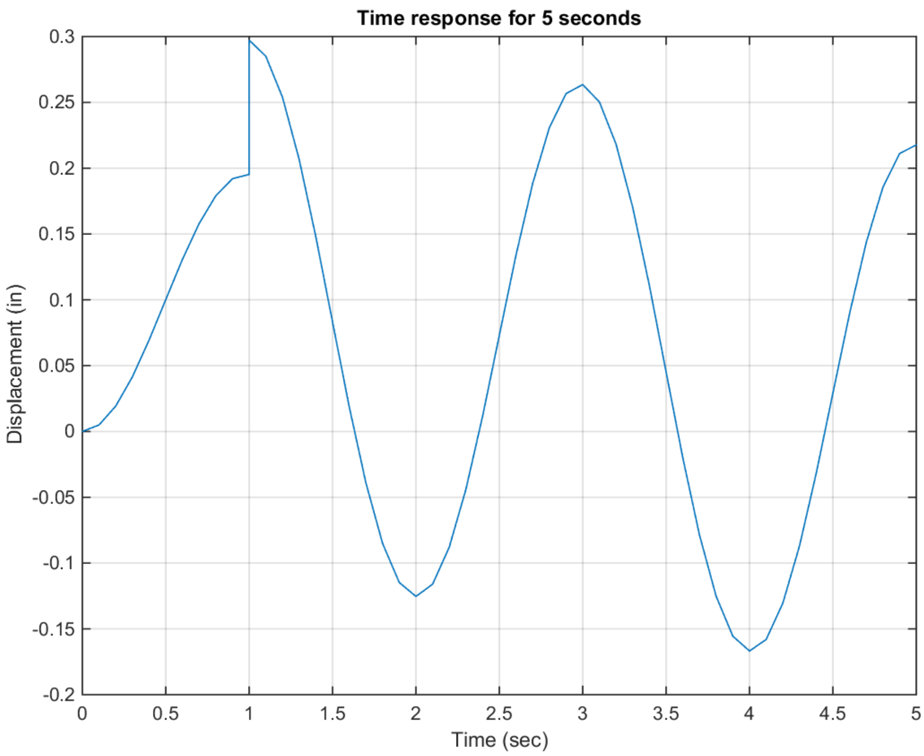


Figure 3: Time response for 5 seconds

3. Compare the maximum amplitude response with the shock response spectrum (*on slide* #40):

   System is undamped ($\zeta = 0.1 < 1$) so based on slide #40 we have the maximum magnitude response in 5 second is the highest peak on plot of figure 3.

   Based on `MATLAB` result, we have ***maximum magnitude response*** is `0.2973` at time step $\boldsymbol{t = 1}$ second.