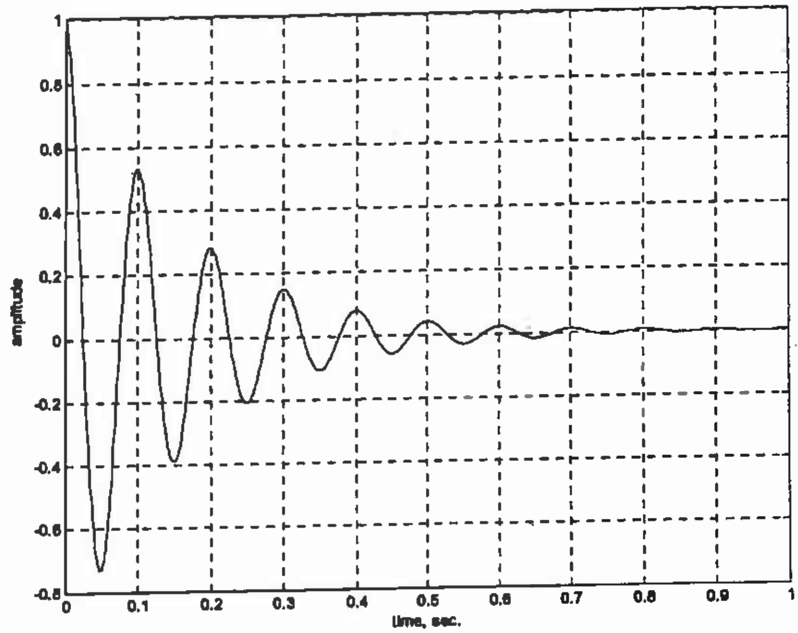
6. ~~8~~. 15 pts. For the system response below, determine:

a.) The damping ratio ζ

b.) The natural frequency ω_n



$$\delta = \frac{1}{N}\ln\left(\frac{x_n}{x_{n+1}}\right) = \frac{1}{4}\ln\left(\frac{1}{.08}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = .63$$

$$(2\pi\zeta)^2 = .63^2(1-\zeta^2)$$

$$4\pi^2\zeta^2 = .40(1-\zeta^2)$$

$$\zeta^2(4\pi^2+.4) = .4$$

$$\zeta = 0.10$$

$$w_d = \frac{2\pi}{T_d} = \frac{2\pi}{0.1} = 20\pi = w_n\sqrt{1-0.1^2}$$

$$w_n = 63.1 \text{ rad/sec}$$

2.28. Repeat Prob. 2.27 using the static deflection

$$y(x) = \frac{wl\ l^3}{24\ EI}\left[\left(\frac{x}{l}\right)^4 - 4\left(\frac{x}{l}\right) + 3\right]$$

for the uniformly loaded beam, and compare with previous result.

2.29. Determine the effective rotational stiffness of the shaft in Fig. P2.29 and calculate its natural period.

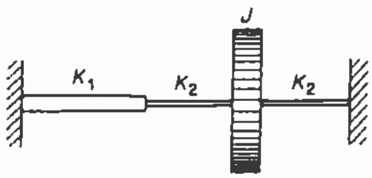


FIGURE P2.29.

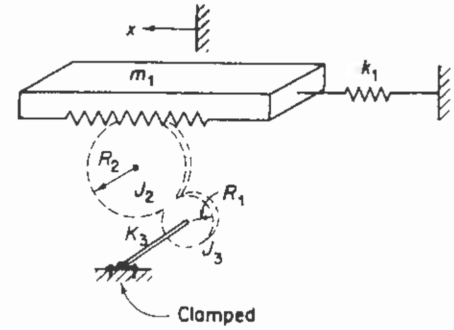


FIGURE P2.30.

2.30. For purposes of analysis, it is desired to reduce the system of Fig. P2.30 to a simple linear spring-mass system of effective mass m_{eff} and effective stiffness k_{eff}. Determine m_{eff} and k_{eff} in terms of the given quantities.

2.31. Determine the effective mass moment of inertia for shaft 1 in the system shown in Fig. P2.31.

2-30

$$T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} J_2 \left(\frac{\dot{x}}{R_2}\right)^2 + \frac{1}{2} J_3 \left(\frac{R_2}{R_1} \frac{\dot{x}}{R_2}\right)^2$$

$$= \frac{1}{2}\left[m_1 + J_2/R_2^2 + J_3/R_1^2\right]\dot{x}^2 = \frac{1}{2} m_{eff}\, \dot{x}^2$$

$$U = \frac{1}{2} k_1 x^2 + \frac{1}{2} K_3 \left(\frac{x}{R_1}\right)^2 = \frac{1}{2}\left[k_1 + K_3/R_1^2\right] x^2 = \frac{1}{2} k_{eff}\, x^2$$

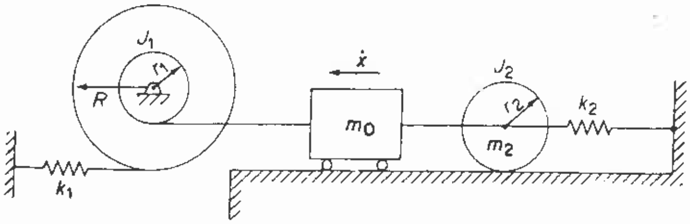
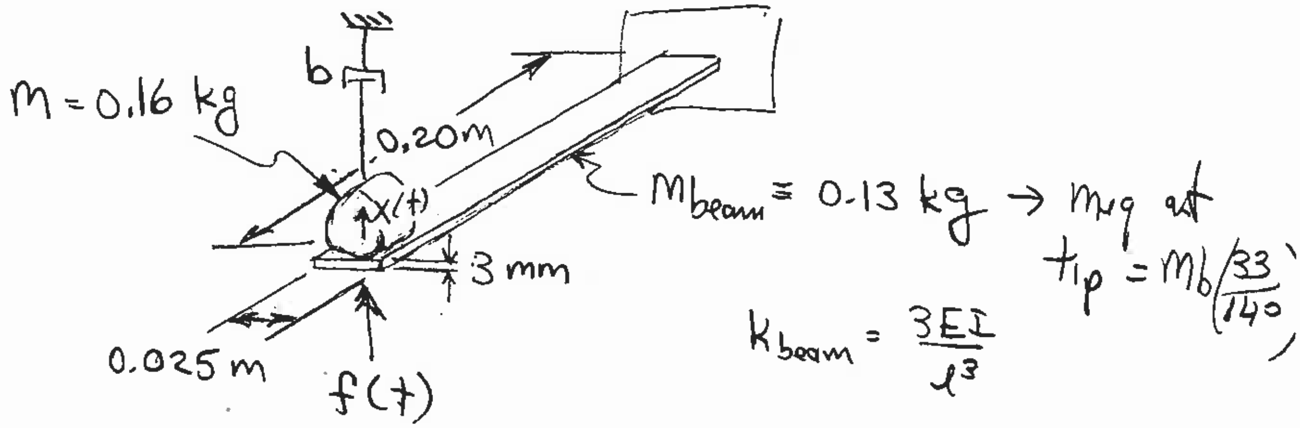


FIGURE P2.32.

4. 20 pts. The cantilever beam with an end mass is a model for a real system that is subjected to a harmonic forcing function. By observing the free vibration response, the damping ratio ζ was determined to be 0.02. Determine:

a.) The damping coefficient b

b.) The amplitude of vibration if the forcing function is defined by $f(t) = 80 \sin 300t$



$$m_{eff} = 0.16 + 0.13\left(\frac{33}{140}\right) = 0.19 \text{ kg}$$

note: assume steel $E = 200E9$ Pa

$$k = \frac{3EI}{\ell^3} = \frac{3(200E9)\left(\frac{1}{12}(.025).003^3\right)}{0.2^3} = 4219 \text{ N/m}$$

a) $$\zeta = \frac{b}{2\sqrt{m_{eff} k}}, \quad b = (.02)(2)\sqrt{.19(4219)}$$
$$= 1.13 \text{ N sec/m}$$

$$\omega = 300$$

$$\omega_n = \sqrt{\frac{4219}{.19}} = 149 \text{ rad/sec}$$

$$\frac{\omega}{\omega_n} = 2$$

$$\left|\frac{X}{F/k}\right| = \frac{1}{\sqrt{(1-2^2)^2 + (2(.02)2)^2}} = \frac{1}{\sqrt{(1-4)^2 + .08^2}} = .33$$

b) $$X = \frac{80}{4219}(.33) = .006 \text{ m} = \underline{6 \text{ mm}}$$

2. 25 pts. a) Determine the damping C if the system amplitude is observed to decrease by 20% for each consecutive peak when the system is given an initial displacement, θ_o, while $F(t) = 0$.

b) What is the steady state amplitude of motion given that $F(t) = 500\sin 10.47t$ and $\zeta = 0.1$?

$k_1 = k_2 = 5000$ N/m,
$a = 0.25$ m, $b = 0.5$ m, $l = 1$ m, $M = 50$ kg, $m = 10$ kg, $F_0 = 500$ N, $\omega =$ ~~[illegible] rpm.~~
100 rpm

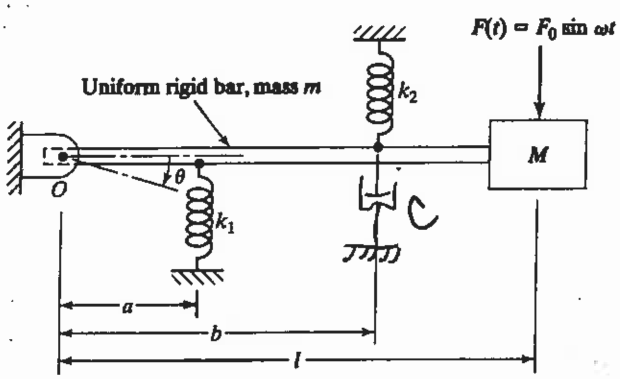


FIGURE 3.39

a) $\ln\left(\frac{x_n}{x_{n+1}}\right) = \ln\left(\frac{1}{.8}\right) = \ln(1.25) = .223 = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$

$\zeta = .035$

Eqn of motion:

$$J\ddot{\theta} + C_t\dot{\theta} + K_t\theta = F(t)\ell$$

$$J\ddot{\theta} + Cb^2\dot{\theta} + (K_1a^2 + K_2b^2)\theta = F(t)\ell$$

$$\zeta = \frac{C_T}{2\sqrt{JK_t}}, \quad C_T = \zeta 2\sqrt{JK_t}$$

$$K_t = 5000(.25^2 + .5^2) = 1563 \text{ Nm}$$

$$J = \frac{m\ell^2}{3} + M\ell^2 = \frac{10}{3} + 50 = 52.7 \text{ kg m}^2$$

$$C_t = .035(2)\sqrt{52.7(1563)} = 20 \text{ Nm sec}$$

$$C = \frac{C_t}{b^2} = 80 \, \frac{\text{N sec}}{\text{m}}$$

b) for $F(t) = 500 \sin 10.47t$

$$\omega_n = \sqrt{\frac{K_t}{J}} = \sqrt{\frac{1563}{52.7}} = 5.44 \text{ rad/sec}$$

$$\frac{\omega}{\omega_n} = \frac{10.47}{5.44} = 1.92$$

$T = 500(1) = 500$ Nm

$K_t = 1563$ Nm

$$\theta = |\Theta| \sin(\omega t + \phi), \quad |\Theta| = \frac{T/K_t}{\sqrt{(1-1.92^2)^2 + (2(.035)(1.92))^2}} = \underline{.12 \text{ rad}}$$