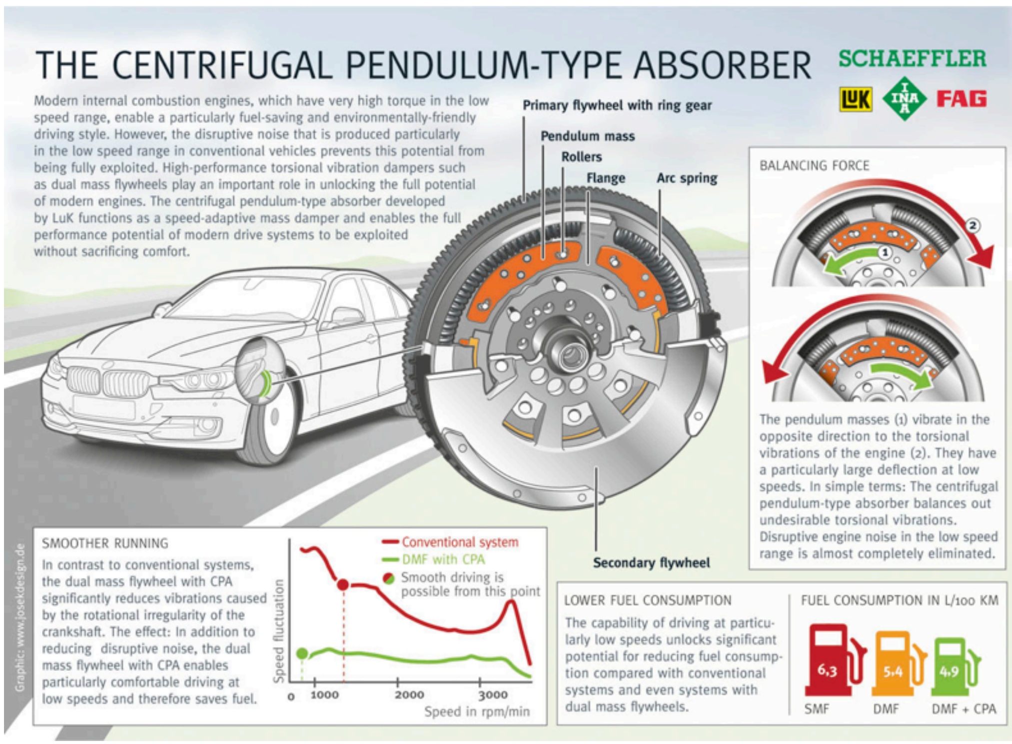
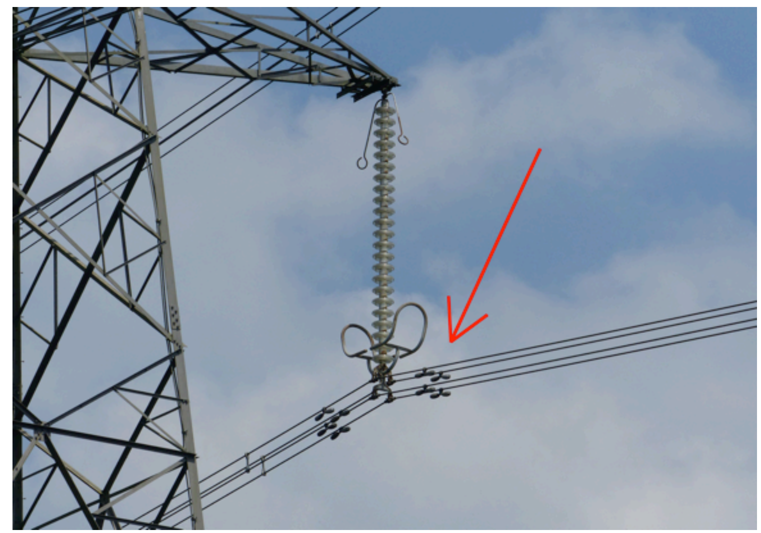
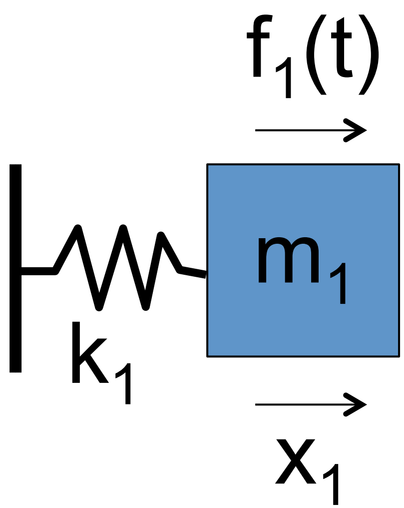
The Vibration Absorber

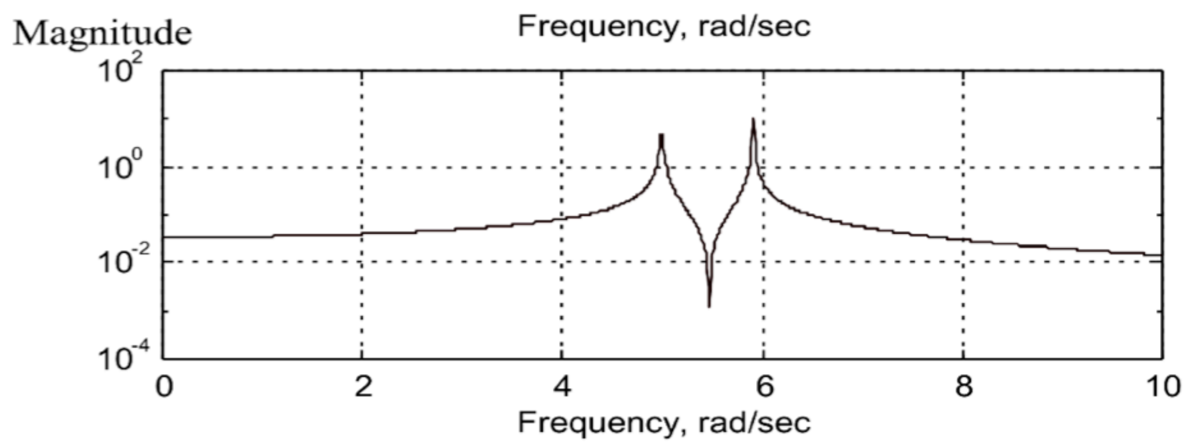




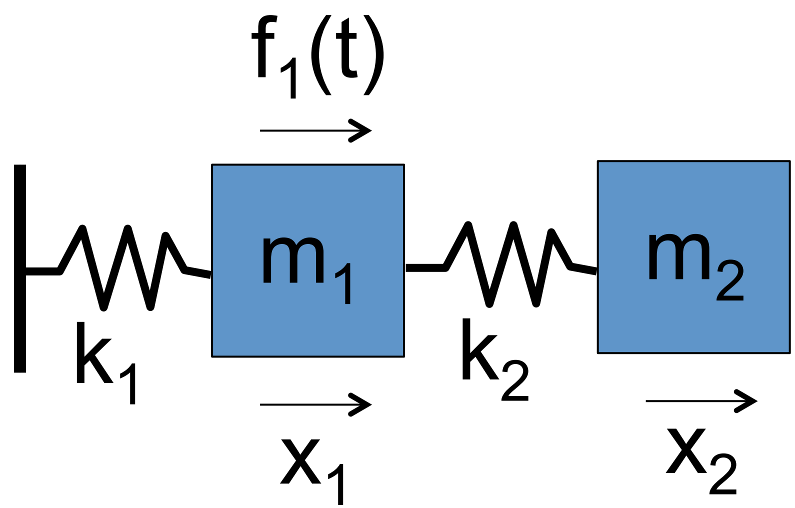
Starting with a single degree of freedom system…

- It is desired to cancel the oscillatory motion of an SDOF system subject to a harmonic forcing function, $f_1(t) = \sin \omega t$
- Recall that the driving point response of a 2-DOF system shows an anti-resonance between the resonant points





So we will add a second mass to the system



- The second mass dynamics will be tuned to the input excitation frequency:

$$\frac{k_2}{m_2} = \omega = \omega_{22}$$

Now to equations of motion give:

$$\begin{bmatrix} m_1 & \\ & m_2 \end{bmatrix}\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix}+\begin{bmatrix} (k_1+k_2) & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} F_1 \sin\omega t \\ 0 \end{pmatrix}$$

Assuming harmonic motion results in:

$$\begin{bmatrix} (k_1+k_2-m_1\omega^2) & -k_2 \\ -k_2 & k_2-m_2\omega^2 \end{bmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix}=\begin{pmatrix} F_1 \\ 0 \end{pmatrix}$$

We can use Cramer's Rule to find the solution to X_1:

- Recall Cramer's Rule:

$$[A](X) = (F)$$

$$[\alpha_1] = [(F)(A_2)(A_3)\cdots]$$
$$[\alpha_2] = [(A_1)(F)(A_3)\cdots]$$

Then: $$X_1 = \frac{|\alpha_1|}{|A|}; \quad X_2 = \frac{|\alpha_2|}{|A|}$$

For our particular case:

$$X_1 = \frac{\begin{vmatrix} F_1 & -k_2 \\ 0 & -m_2\omega^2 + k_2 \end{vmatrix}}{\begin{vmatrix} \left(-m_1\omega^2 + k_1 + k_2\right) & -k_2 \\ -k_2 & \left(-m_2\omega^2 + k_2\right) \end{vmatrix}}$$

$$= \frac{F_1\left(k_2 - m_2\omega^2\right)}{\left(-m_1\omega^2 + k_1 + k_2\right)\left(-m_2\omega^2 + k_2\right) - k_2^2}$$

This is simplified to:

$$X_1 = \frac{F_1\left(1-\dfrac{\omega^2}{\omega_{22}^2}\right)}{\left(k_1+k_2-m_1\omega^2\right)\left(1-\dfrac{\omega^2}{\omega_{22}^2}\right)-k_2}$$

In dimensionless form this becomes:

$$\frac{X_1 k_1}{F_1} = \frac{F_1\left(1-\dfrac{\omega^2}{\omega_{22}^2}\right)}{\left(1+\dfrac{k_1}{k_2}-\dfrac{\omega^2}{\omega_{11}^2}\right)\left(1-\dfrac{\omega^2}{\omega_{22}^2}\right)-\dfrac{k_2}{k_1}}$$

Now, when $\omega = \omega_{22}$ we see that $X_1 = 0$

- Solving for $X_{2:}$

$$\frac{X_2 k_1}{F_1} = \frac{1}{\left(1 + \frac{k_1}{k_2} - \frac{\omega^2}{\omega_{11}^2}\right)\left(1 - \frac{\omega^2}{\omega_{22}^2}\right) - \frac{k_2}{k_1}}$$

Or: $$X_2 = -\frac{F_1}{k_2}$$

We see that the motion of m_1 is zero, and the motion of m_2 is 180 degrees out of phase with the force, and the force transmitted through spring k_2 is $k_2 X_2 = -F_1$

Some things to consider in the design of the vibration absorber

- The size of k_2 is dependent on the allowable value of X_2. We will have two resonant peaks on either side of the operating frequency. The spread in the frequency is controlled by both k_2 and m_2.

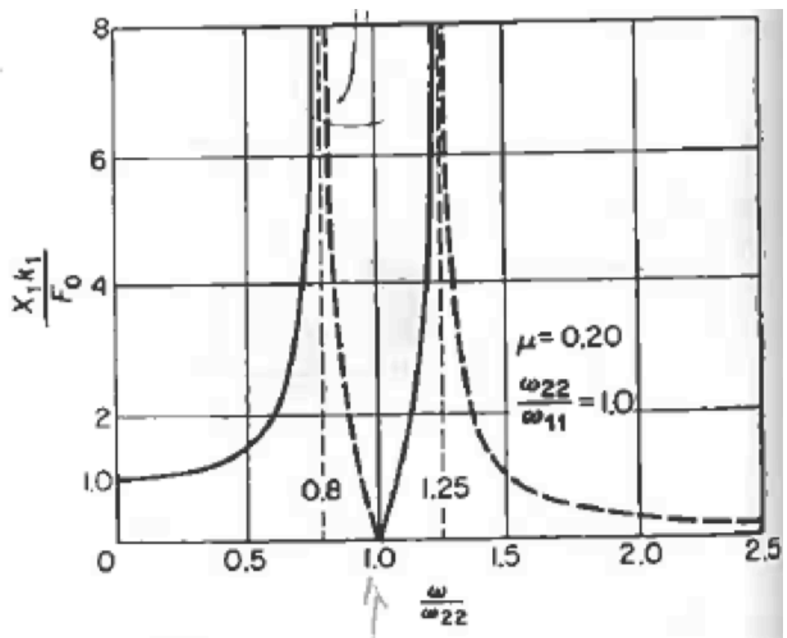


FIGURE 5.6.2. Response vs. frequency.

A nice metric to consider in designing a vibration absorber is the mass ratio m_2 / m_1 when determining the resonant frequency spread:

From this plot, the resonant peaks are determined relative to the center frequency where $X_2 = 0$.

A larger spread of resonant frequencies results in a greater range of operating frequencies that will not cause damaging resonant response

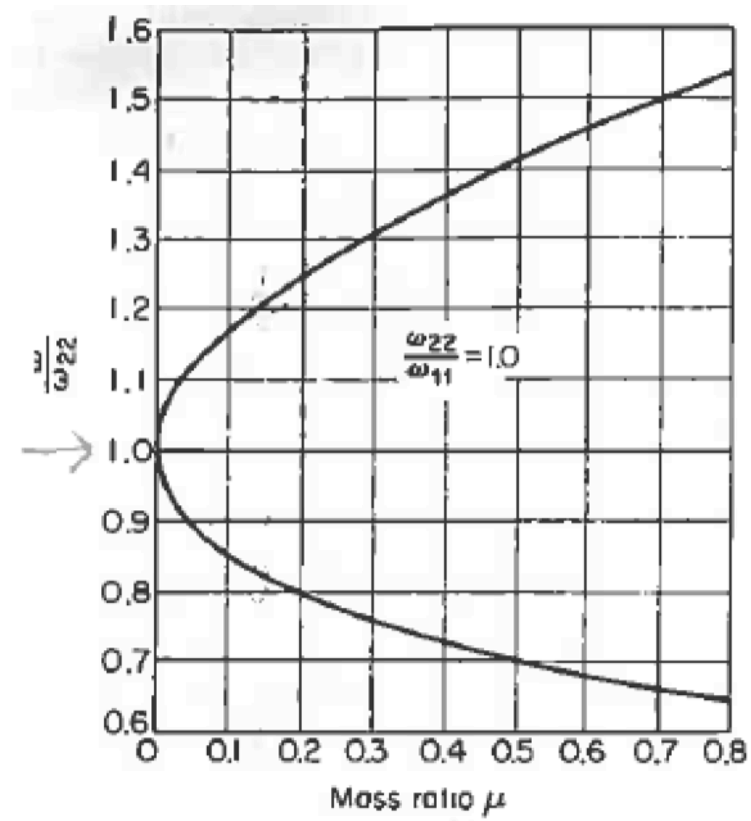
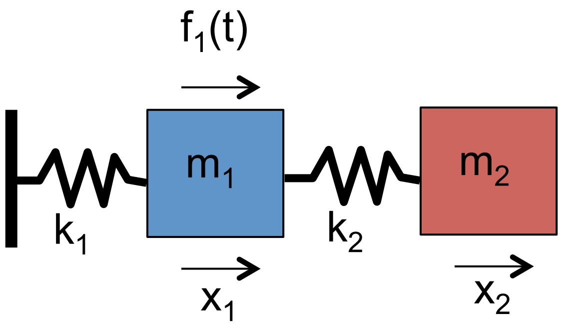


FIGURE 5.6.3 Natural frequencies vs. μ ~~$\times$~~ m_2/m_1.

Example

- Design a vibration absorber for the system shown where we desire the system natural frequencies to be at least 30% away from the operating frequency of 400 rad/s



$$m_1 = .01295\, lbs^2 / in$$

$$k_1 = 2075\, lb / in$$

$$\omega_n = 400\, rad / s = \omega_{operating}$$

From the figure on the previous slide, $\dfrac{m_2}{m_1} \geq 0.5$

$$m_2 \geq .006475\, lbs^2 / in$$

Example cont.

$$k_2 = \omega_{22}^2 m_2$$
$$= 400^2 \left(.006475\right) = 1036\, lb / in$$

Checking this work shows that the two resonant frequencies are:

$$\omega_1 = 282 rad / s$$
$$\omega_2 = 566 rad / s$$

Which are greater than 30% away from the operating frequency

Now if the forcing function is determined to be $f\left(t\right) = \sin \omega t$

Then: $$X_2 = -\frac{F_1}{k_2} = -\frac{1}{1036} \approx -0.001\, in$$