Math 5315 / CSE 7365, Fall 2017
Due October 5

Homework 3 – Interpolation Part I

1. Devise an iterative root-finding method for solving $f(x) = 0$ as follows:

- Given three previous iterates and their corresponding function values, $\{(x_n, f(x_n)),\ (x_{n-1}, f(x_{n-1})),\ (x_{n-2}, f(x_{n-2}))\}$, let $q_2(x)$ be the quadratic interpolating polynomial through these data.
- Construct the new iterate x_{n+1} as the root of q_2 closest to x_n (find this using the quadratic formula).
- Include a check for the possibility that q_2 has no real roots, and return an error message if that occurs.
- To initialize the algorithm (because you need three guesses), set the additional initial guesses as:

$$x_{-2} = \begin{cases} x_0(1 - 10^{-2}), & \text{if } x_0 \neq 0, \\ x_0 - 10^{-2}, & \text{if } x_0 = 0, \end{cases} \quad \text{and} \quad x_{-1} = \begin{cases} x_0(1 + 10^{-2}), & \text{if } x_0 \neq 0, \\ x_0 + 10^{-2}, & \text{if } x_0 = 0, \end{cases}$$

Write pseudocode for this algorithm so that it could be implemented without any missing steps.

2. (Matlab/Python) Write a function to implement your method from problem 1, having the Matlab form

```
function [x,its] = quadratic_sol(Ffun, x, maxit, Srtol, Satol, Rrtol, Ratol, output)
```

(or the equivalent in Python), in a file `quadratic_sol.m` or `quadratic_sol.py`. All arguments should have identical meaning to the corresponding arguments from homework 2.

In a script named `prob2.m` or `prob2.py`, compare the performance of this method on same root-finding problem as you used for Newton's and Steffensen's methods in homework 2. Use the same tolerances and initial guesses as you used there.

In code comments at the end of your `prob2.m` or `prob2.py` file, provide short answers answering the following questions:

(a) How does this perform in comparison with Newton and Steffensen?

(b) Why do you think that this method is not more widely used?

3. Prove that if $f \in \mathbb{P}_k$, then for any integer $n > k$, the divided difference $f[x_0, x_1, \ldots, x_n] = 0$.
Hint: re-read section 6.2; one of these theorems will be incredibly useful.

4. Prove that the functions $A_i(x)$ and $B_i(x)$ given on page 344 of the book satisfy the properties

$$A_i(x_j) = \delta_{ij}, \quad A_i'(x_j) = 0, \quad B_i(x_j) = 0, \quad B_i'(x_j) = \delta_{ij},$$

where δ_{ij} is the Kronecker delta, i.e.

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$