Sets

Client Location Set $= \mathcal{J}$

Fixed Facilities Location Set $= \mathcal{I}_{\text{Fixed}}$

Possible Location Set $= \mathcal{I} = \mathcal{I}_{\text{Fixed}} \cup \mathcal{J}$

Variables

$$y[i] = \begin{cases} 1, & \text{if a facility is open at location } i \\ 0, & \text{otherwise} \end{cases}, \quad \forall i \in \mathcal{I}$$

$$x[i,j] = \begin{cases} 1, & \text{if the client demand at } j \text{ is served by facility at } i \\ 0, & \text{otherwise} \end{cases}, \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}$$

Parameters

$P =$ Total of located facilities

$\text{cost\_matrix}[i,j] =$ cost of client demand at j being served by the facility at i, $\quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}$

Objective function

Minimize $\quad \text{total\_costs} = \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{J}} \text{cost\_matrix}[i,j] \cdot x[i,j]$

Subject to

(1) $\quad \sum_{i\in\mathcal{I}} y[i] = P$

(2) $\quad \sum_{i\in\mathcal{I}} x[i,j] = 1, \quad \forall j \in \mathcal{J}$

(3) $\quad x[i,j] \leq y[i], \quad \forall i \in \mathcal{I}, \forall j \in \mathcal{J}$

(4) $\quad y[i] = 1, \quad \forall i \in \mathcal{I}_{\text{Fixed}}$

(5) $\quad y[j] = 0, \quad \forall j \in \mathcal{J},$ if $\exists i \in \mathcal{I}_{\text{Fixed}}$ such that $j \in \text{fixed\_facilities}[i].\text{exclusive\_region}$

(6) $\quad x[i,j] = 1, \quad \forall i \in \mathcal{I}_{\text{Fixed}}, \forall j \in \mathcal{J},$ if $j \in \text{fixed\_facilities}[i].\text{exclusive\_region}$

(7) $\quad \sum_{j\in\mathcal{J}} \text{clients}[j].\text{demand} \cdot x[i,j] \geq \text{fixed\_facilities}[i].\text{min\_demand}, \quad \forall i \in \mathcal{I}_{\text{Fixed}}$

(8) $\quad \sum_{j\in\mathcal{J}} \text{clients}[j].\text{demand} \cdot x[i,j] \leq \text{fixed\_facilities}[i].\text{max\_demand}, \quad \forall i \in \mathcal{I}_{\text{Fixed}}$