

A Survey of Discretization Techniques: Taxonomy and Empirical Analysis in Supervised Learning

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Abstract—Discretization is an essential preprocessing technique used in many knowledge discovery and data mining tasks. Its main goal is to transform a set of continuous attributes into discrete ones, by associating categorical values to intervals and thus transforming quantitative data into qualitative data. In this manner, symbolic data mining algorithms can be applied over continuous data and the representation of information is simplified, making it more concise and specific. The literature provides numerous proposals of discretization and some attempts to categorize them into a taxonomy can be found. However, in previous papers, there is a lack of consensus in the definition of the properties and no formal categorization has been established yet, which may be confusing for practitioners. Furthermore, only a small set of discretizers have been widely considered, while many other methods have gone unnoticed. With the intention of alleviating these problems, this paper provides a survey of discretization methods proposed in the literature from a theoretical and empirical perspective. From the theoretical perspective, we develop a taxonomy based on the main properties pointed out in previous research, unifying the notation and including all the known methods up to date. Empirically, we conduct an experimental study in supervised classification involving the most representative and newest discretizers, different types of classifiers, and a large number of data sets. The results of their performances measured in terms of accuracy, number of intervals, and inconsistency have been verified by means of nonparametric statistical tests. Additionally, a set of discretizers are highlighted as the best performing ones.

Index Terms—Discretization, continuous attributes, decision trees, taxonomy, data preprocessing, data mining, classification

1 INTRODUCTION

KNOWLEDGE extraction and data mining (DM) are important methodologies to be performed over different databases which contain data relevant to a real application [1], [2]. Both processes often require some previous tasks such as problem comprehension, data comprehension or data preprocessing in order to guarantee the successful application of a DM algorithm to real data [3], [4]. Data preprocessing [5] is a crucial research topic in the DM field and it includes several processes of data transformation, cleaning and data reduction. Discretization, as one of the basic data reduction techniques, has received increasing research attention in recent years [6] and has become one of the preprocessing techniques most broadly used in DM.

The discretization process transforms quantitative data into qualitative data, that is, numerical attributes into

discrete or nominal attributes with a finite number of intervals, obtaining a nonoverlapping partition of a continuous domain. An association between each interval with a numerical discrete value is then established. In practice, discretization can be viewed as a data reduction method since it maps data from a huge spectrum of numeric values to a greatly reduced subset of discrete values. Once the discretization is performed, the data can be treated as nominal data during any induction or deduction DM process. Many existing DM algorithms are designed to only learn in categorical data, using nominal attributes, while real-world applications usually involve continuous features. Those numerical features have to be discretized before using such algorithms.

In supervised learning, and specifically classification, the topic of this survey, we can define the discretization as follows: Assuming a data set consisting of N examples and C target classes, a discretization algorithm would discretize the continuous attribute A in this data set into m discrete intervals $D = \{[d_0, d_1], (d_1, d_2], \dots, (d_{m-1}, d_m]\}$, where d_0 is the minimal value, d_m is the maximal value and $d_i < d_{i+1}$, for $i = 0, 1, \dots, m - 1$. Such a discrete result D is called a discretization scheme on attribute A and $P = \{d_1, d_2, \dots, d_{m-1}\}$ is the set of cut points of attribute A .

The necessity of using discretization on data can be caused by several factors. Many DM algorithms are primarily oriented to handle nominal attributes [7], [6], [8], or may even only deal with discrete attributes. For instance, three of the 10 methods considered as the top 10 in DM [9] require an embedded or an external discretization

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Manuscript received 28 May 2011; revised 17 Oct. 2011; accepted 14 Dec. 2011; published online 13 Feb. 2012.

Recommended for acceptance by J. Dix.

For information on obtaining reprints of this article, please send e-mail to: tkde@computer.org, and reference IEEECS Log Number TKDE-2011-05-0301. Digital Object Identifier no. 10.1109/TKDE.2012.35.

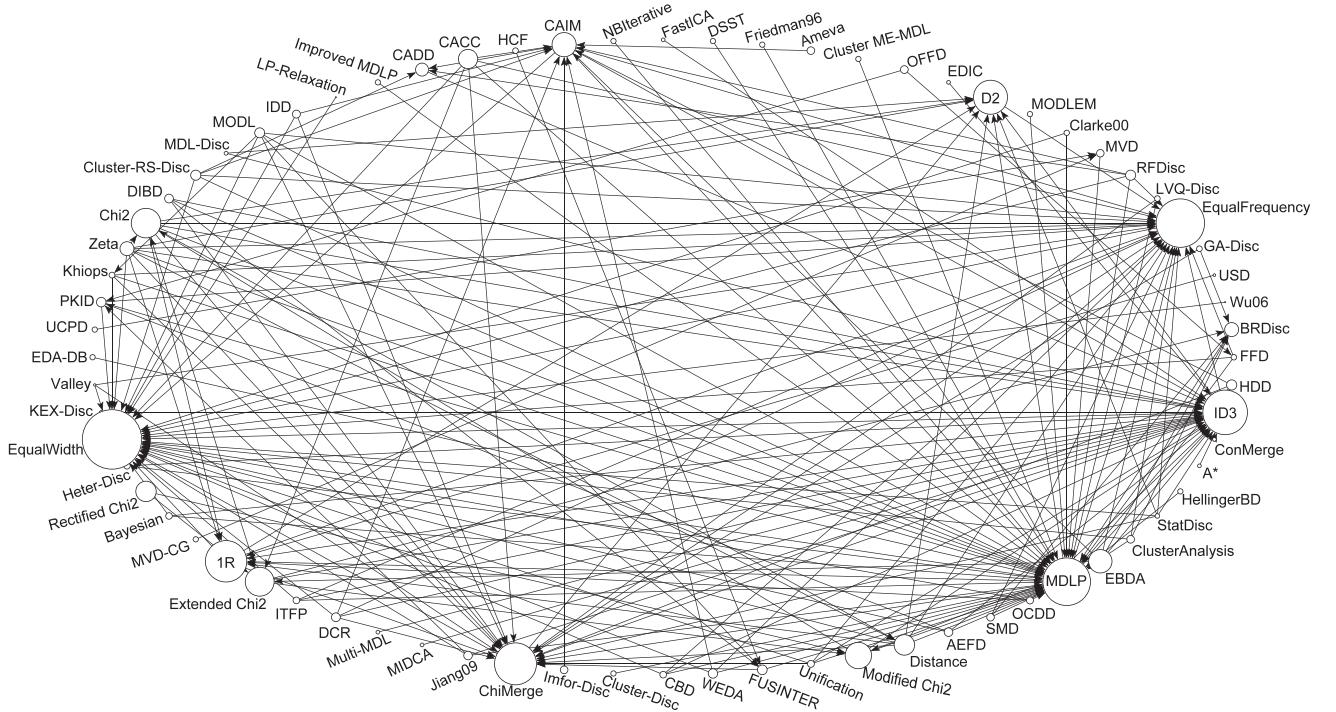


Fig. 1. Comparison network of discretizers. Later, the methods will be defined in Table 1.

of data: C4.5 [10], Apriori [11] and Naive Bayes [12], [13]. Even with algorithms that are able to deal with continuous data, learning is less efficient and effective [14], [15], [4]. Other advantages derived from discretization are the reduction and the simplification of data, making the learning faster and yielding more accurate, compact and shorter results; and noise possibly present in the data is reduced. For both researchers and practitioners, discrete attributes are easier to understand, use, and explain [6]. Nevertheless, any discretization process generally leads to a loss of information, making the minimization of such information loss the main goal of a discretizer.

Obtaining the optimal discretization is NP-complete [15]. A vast number of discretization techniques can be found in the literature. It is obvious that when dealing with a concrete problem or data set, the choice of a discretizer will condition the success of the posterior learning task in accuracy, simplicity of the model, etc. Different heuristic approaches have been proposed for discretization, for example, approaches based on information entropy [16], [7], statistical χ^2 test [17], [18], likelihood [19], [20], rough sets [21], [22], etc. Other criteria have been used in order to provide a classification of discretizers, such as univariate/multivariate, supervised/unsupervised, top-down/bottoum-up, global/local, static/dynamic, and more. All these criteria are the basis of the taxonomies already proposed and they will be deeply elaborated upon in this paper. The identification of the best discretizer for each situation is a very difficult task to carry out, but performing exhaustive experiments considering a representative set of learners and discretizers could help to decide the best choice.

Some reviews of discretization techniques can be found in the literature [7], [6], [23], [8]. However, the characteristics of the methods are not studied completely, many

discretizers, even classic ones, are not mentioned, and the notation used for categorization is not unified. For example, in [7], the static/dynamic distinction is different from that used in [6] and the global/local property is usually confused with the univariate/multivariate property [24], [25], [26]. Subsequent papers include one notation or other, depending on the initial discretization study referenced by them: [7], [24] or [6].

In spite of the wealth of literature, and apart from the absence of a complete categorization of discretizers using a unified notation, it can be observed that, there are few attempts to empirically compare them. In this way, the algorithms proposed are usually compared with a subset of the complete family of discretizers and, in most of the studies, no rigorous empirical analysis has been carried out. Furthermore, many new methods have been proposed in recent years and they are going unnoticed with respect to the discretizers reviewed in well-known surveys [7], [6]. Fig. 1 illustrates a comparison network where each node corresponds to a discretization algorithm and a directed vertex between two nodes indicates that the algorithm of the start node has been compared with the algorithm of the end node. The direction of the arrows is always from the newest method to the oldest, but it does not influence the results. The size of the node is correlated with the number of input and output vertices. We can see that most of the discretizers are represented by small nodes and that the graph is far from being complete, which has prompted the present paper. The most compared techniques are EqualWidth, EqualFrequency, MDLP [16], ID3 [10], ChiMerge [17], 1R [27], D2 [28], and Chi2 [18].

These reasons motivate the global purpose of this paper, which can be divided into three main objectives:

- To propose a complete taxonomy based on the main properties observed in the discretization methods. The taxonomy will allow us to characterize their advantages and drawbacks in order to choose a discretizer from a theoretical point of view.
- To make an empirical study analyzing the most representative and newest discretizers in terms of the number of intervals obtained and inconsistency level of the data.
- Finally, to relate the best discretizers for a set of representative DM models using two metrics to measure the predictive classification success.

The experimental study will include a statistical analysis based on nonparametric tests. We will conduct experiments involving a total of 30 discretizers; six classification methods belonging to lazy, rules, decision trees, and Bayesian learning families; and 40 data sets. The experimental evaluation does not correspond to an exhaustive search for the best parameters for each discretizer, given the data at hand. Then, its main focus is to properly relate a subset of best performing discretizers to each classic classifier using a general configuration for them.

This paper is organized as follows: The related and advanced work on discretization is provided in Section 2. Section 3 presents the discretizers reviewed, their properties, and the taxonomy proposed. Section 4 describes the experimental framework, examines the results obtained in the empirical study and presents a discussion of them. Section 5 concludes the paper. Finally, we must point out that the paper has an associated web site <http://sci2s.ugr.es/discretization> which collects additional information regarding discretizers involved in the experiments such as implementations and detailed experimental results.

2 RELATED AND ADVANCED WORK

Research in improving and analyzing discretization is common and in high demand currently. Discretization is a promising technique to obtain the hoped results, depending on the DM task, which justifies its relationship to other methods and problems. This section provides a brief summary of topics closely related to discretization from a theoretical and practical point of view and describes other works and future trends which have been studied in the last few years.

- *Discretization specific analysis:* Susmaga proposed an analysis method for discretizers based on binarization of continuous attributes and rough sets measures [29]. He emphasized that his analysis method is useful for detecting redundancy in discretization and the set of cut points which can be removed without decreasing the performance. Also, it can be applied to improve existing discretization approaches.
- *Optimal multisplitting:* Elomaa and Rousu characterized some fundamental properties for using some classic evaluation functions in supervised univariate discretization. They analyzed entropy, information gain, gain ratio, training set error, gini index, and normalized distance measure, concluding that they are suitable for use in the optimal multisplitting of

- an attribute [30]. They also developed an optimal algorithm for performing this multisplitting process and devised two techniques [31], [32] to speed it up.
- *Discretization of continuous labels:* Two possible approaches have been used in the conversion of a continuous supervised learning (regression problem) into a nominal supervised learning (classification problem). The first one is simply to use regression tree algorithms, such as CART [33]. The second consists of applying discretization to the output attribute, either statically [34] or in a dynamic fashion [35].
- *Fuzzy discretization:* Extensive research has been carried out around the definition of linguistic terms that divide the domain attribute into fuzzy regions [36]. Fuzzy discretization is characterized by membership value, group or interval number, and affinity corresponding to an attribute value, unlike crisp discretization which only considers the interval number [37].
- *Cost-sensitive discretization:* The objective of cost-based discretization is to take into account the cost of making errors instead of just minimizing the total sum of errors [38]. It is related to problems of imbalanced or cost-sensitive classification [39], [40].
- *Semi-supervised discretization:* A first attempt to discretize data in semi-supervised classification problems has been devised in [41], showing that it is asymptotically equivalent to the supervised approach.

The research mentioned in this section is out of the scope of this survey. We point out that the main objective of this paper is to give a wide overview of the discretization methods found in the literature and to conduct an exhaustive experimental comparison of the most relevant discretizers without considering external and advanced factors such as those mentioned above or derived problems from classic supervised classification.

3 DISCRETIZATION: BACKGROUND AND TECHNIQUES

This section presents a taxonomy of discretization methods and the criteria used for building it. First, in Section 3.1, the main characteristics which will define the categories of the taxonomy will be outlined. Then, in Section 3.2, we enumerate the discretization methods proposed in the literature we will consider by using their complete and abbreviated name together with the associated reference. Finally, we present the taxonomy.

3.1 Common Properties of Discretization Methods

This section provides a framework for the discussion of the discretizers presented in the next section. The issues discussed include several properties involved in the structure of the taxonomy, since they are exclusive to the operation of the discretizer. Other, less critical issues such as parametric properties or stopping conditions will be presented although they are not involved in the taxonomy. Finally, some criteria will also be pointed out in order to compare discretization methods.

3.1.1 Main Characteristics of a Discretizer

In [6], [7], [8], various axes have been described in order to make a categorization of discretization methods. We review and explain them in this section, emphasizing the main aspects and relations found among them and unifying the notation. The taxonomy proposed will be based on these characteristics:

- *Static versus Dynamic:* This characteristic refers to the moment and independence which the discretizer operates in relation with the learner. A dynamic discretizer acts when the learner is building the model, thus they can only access partial information (local property, see later) embedded in the learner itself, yielding compact and accurate results in conjunction with the associated learner. Otherwise, a static discretizer proceeds prior to the learning task and it is independent from the learning algorithm [6]. Almost all known discretizers are static, due to the fact that most of the dynamic discretizers are really subparts or stages of DM algorithms when dealing with numerical data [42]. Some examples of well-known dynamic techniques are ID3 discretizer [10] and ITFP [43].
- *Univariate versus Multivariate:* Multivariate techniques, also known as 2D discretization [44], simultaneously consider all attributes to define the initial set of cut points or to decide the best cut point altogether. They can also discretize one attribute at a time when studying the interactions with other attributes, exploiting high order relationships. By contrast, univariate discretizers only work with a single attribute at a time, once an order among attributes has been established, and the resulting discretization scheme in each attribute remains unchanged in later stages. Interest has recently arisen in developing multivariate discretizers since they are very influential in deductive learning [45], [46] and in complex classification problems where high interactions among multiple attributes exist, which univariate discretizers might obviate [47], [48].
- *Supervised versus Unsupervised:* Unsupervised discretizers do not consider the class label whereas supervised ones do. The manner in which the latter consider the class attribute depends on the interaction between input attributes and class labels, and the heuristic measures used to determine the best cut points (entropy, interdependence, etc.). Most discretizers proposed in the literature are supervised and theoretically, using class information, should automatically determine the best number of intervals for each attribute. If a discretizer is unsupervised, it does not mean that it cannot be applied over supervised tasks. However, a supervised discretizer can only be applied over supervised DM problems. Representative unsupervised discretizers are Equal-Width and EqualFrequency [49], PKID and FFD [12], and MVD [45].
- *Splitting versus Merging:* This refers to the procedure used to create or define new intervals. Splitting methods establish a cut point among all the possible boundary points and divide the domain into two intervals. By contrast, merging methods start with a predefined partition and remove a candidate cut point to mix both adjacent intervals. These properties are highly related to *Top-Down* and *Bottom-up*, respectively, (explained in the next section). The idea behind them is very similar, except that top-down or bottom-up discretizers assume that the process is incremental (described later), according to a hierarchical discretization construction. In fact, there can be discretizers whose operation is based on splitting or merging more than one interval at a time [50], [51]. Also, some discretizers can be considered *hybrid* due to the fact that they can alternate splits with merges in running time [52], [53].
- *Global versus Local:* To make a decision, a discretizer can either require all available data in the attribute or use only partial information.. A discretizer is said to be local when it only makes the partition decision based on local information. Examples of widely used local techniques are MDLP [16] and ID3 [10]. Few discretizers are local, except some based on top-down partition and all the dynamic techniques. In a top-down process, some algorithms follow the divide-and-conquer scheme and when a split is found, the data are recursively divided, restricting access to partial data. Regarding dynamic discretizers, they find the cut points in internal operations of a DM algorithm, so they never gain access to the full data set.
- *Direct versus Incremental:* Direct discretizers divide the range into k intervals simultaneously, requiring an additional criterion to determine the value of k . They do not only include one-step discretization methods, but also discretizers which perform several stages in their operation, selecting more than a single cut point at every step. By contrast, incremental methods begin with a simple discretization and pass through an improvement process, requiring an additional criterion to know when to stop it. At each step, they find the best candidate boundary to be used as a cut point and afterwards the rest of the decisions are made accordingly. Incremental discretizers are also known as hierarchical discretizers [23]. Both types of discretizers are widespread in the literature, although there is usually a more defined relationship between incremental and supervised ones.
- *Evaluation measure:* This is the metric used by the discretizer to compare two candidate schemes and decide which is more suitable to be used. We consider five main families of evaluation measures:
 - *Information:* This family includes *entropy* as the most used evaluation measure in discretization (MDLP [16], ID3 [10], FUSINTER [54]) and other derived information theory measures such as the *Gini index* [55].
 - *Statistical:* Statistical evaluation involves the measurement of dependency/correlation among attributes (Zeta [56], ChiMerge [17], Chi2 [18]), probability and Bayesian properties

- [19] (MODL [20]), interdependency [57], contingency coefficient [58], etc.
- *Rough sets*: This group is composed of methods that evaluate the discretization schemes by using rough set measures and properties [21], such as lower and upper approximations, class separability, etc.
- *Wrapper*: This collection comprises methods that rely on the error provided by a classifier that is run for each evaluation. The classifier can be a very simple one, such as a majority class voting classifier (Valley [59]) or general classifiers such as Naive Bayes (NBIterative [60]).
- *Binning*: This category refers to the absence of an evaluation measure. It is the simplest method to discretize an attribute by creating a specified number of bins. Each bin is defined a priori and allocates a specified number of values per attribute. Widely used binning methods are EqualWidth and EqualFrequency.

3.1.2 Other Properties

We can remark other properties related to discretization. They also influence the operation and results obtained by a discretizer, but to a lower degree than the characteristics explained above. Furthermore, some of them present a large variety of categorizations and may harm the interpretability of the taxonomy.

- *Parametric versus Nonparametric*: This property refers to the automatic determination of the number of intervals for each attribute by the discretizer. A nonparametric discretizer computes the appropriate number of intervals for each attribute considering a tradeoff between the loss of information or consistency and obtaining the lowest number of them. A parametric discretizer requires a maximum number of intervals desired to be fixed by the user. Examples of nonparametric discretizers are MDLP [16] and CAIM [57]. Examples of parametric ones are ChiMerge [17] and CADD [52].
- *Top-Down versus Bottom Up*: This property is only observed in incremental discretizers. Top-Down methods begin with an empty discretization. Its improvement process is simply to add a new cutpoint to the discretization. On the other hand, Bottom-Up methods begin with a discretization that contains all the possible cutpoints. Its improvement process consists of iteratively merging two intervals, removing a cut point. A classic Top-Down method is MDLP [16] and a well-known Bottom-Up method is ChiMerge [17].
- *Stopping condition*: This is related to the mechanism used to stop the discretization process and must be specified in nonparametric approaches. Well-known stopping criteria are the Minimum Description Length measure [16], confidence thresholds [17], or inconsistency ratios [24].
- *Disjoint versus Nondisjoint*: Disjoint methods discretize the value range of the attribute into disassociated intervals, without overlapping, whereas nondisjoint

methods dicsreteze the value range into intervals that can overlap. The methods reviewed in this paper are disjoint, while fuzzy discretization is usually non-disjoint [36].

- *Ordinal versus Nominal*: Ordinal discretization transforms quantitative data intro ordinal qualitative data whereas nominal discretization transforms it into nominal qualitative data, discarding the information about order. Ordinal discretizers are less common, not usually considered classic discretizers [113].

3.1.3 Criteria to Compare Discretization Methods

When comparing discretization methods, there are a number of criteria that can be used to evaluate the relative strengths and weaknesses of each algorithm. These include the number of intervals, inconsistency, predictive classification rate, and time requirements

- *Number of intervals*: A desirable feature for practical discretization is that discretized attributes have as few values as possible, since a large number of intervals may make the learning slow and ineffective. [28].
- *Inconsistency*: A supervision-based measure used to compute the number of unavoidable errors produced in the data set. An unavoidable error is one associated with two examples with the same values for input attributes and different class labels. In general, data sets with continuous attributes are consistent, but when a discretization scheme is applied over the data, an inconsistent data set may be obtained. The desired inconsistency level that a discretizer should obtain is 0.0.
- *Predictive classification rate*: A successful algorithm will often be able to discretize the training set without significantly reducing the prediction capability of learners in test data which are prepared to treat numerical data.
- *Time requirements*: A static discretization process is carried out just once on a training set, so it does not seem to be a very important evaluation method. However, if the discretization phase takes too long it can become impractical for real applications. In dynamic discretization, the operation is repeated many times as the learner requires, so it should be performed efficiently.

3.2 Discretization Methods and Taxonomy

At the time of writing, more than 80 discretization methods have been proposed in the literature. This section is devoted to enumerating and designating them according to a standard followed in this paper. We have used 30 discretizers in the experimental study, those that we have identified as the most relevant ones. For more details on their descriptions, the reader can visit the URL associated to the KEEL project.¹ Additionally, implementations of these algorithms in Java can be found in KEEL software [114], [115].

Table 1 presents an enumeration of discretizers reviewed in this paper. The complete name, abbreviation, and reference are provided for each one. This paper does not collect the descriptions of the discretizers due to space

1. <http://www.keel.es>.

TABLE 1
Discretizers

Complete name	Abbr. name	Reference	Complete name	Abbr. name	Reference
Equal Width Discretizer	EqualWidth	[61]	Self Organizing Map Discretizer	SOM-Disc	[62]
Equal Frequency Discretizer	EqualFrequency	[61]	Optimal Class-Dependent Discretizer	OCDD	[26]
No name specified	Chou91	[63]	No name specified	Butterworth04	[64]
Adaptive Quantizer	AQ	[65]	No name specified	Zhang04	[22]
Discretizer 2	D2	[28]	Khiops	Khiops	[66]
ChiMerge	ChiMerge	[17]	Class-Attribute Interdependence Maximization	CAIM	[57]
One-Rule Discretizer	1R	[27]	Extended Chi2	Extended Chi2	[67]
Iterative Dichotomizer 3 Discretizer	ID3	[10]	Heterogeneity Discretizer	Heter-Disc	[68]
Minimum Description Length Principle	MDLP	[16]	Unsupervised Correlation Preserving Discretizer	UCPD	[44]
Valley	Valley	[59], [69]	No name specified	Multi-MDL	[47]
Class-Attribute Dependent Discretizer	CADD	[52]	Difference Similitude Set Theory Discretizer	DSST	[70]
Relieff Discretizer	Relieff	[71]	Multivariate Interdependent Discretizer	MIDCA	[72]
Class-driven Statistical Discretizer	StatDisc	[14]	MODL	MODL	[20]
No name specified	NBIterative	[60]	Information Theoretic Fuzzy Partitioning	ITFP	[43]
Boolean Reasoning Discretizer	BRDisc	[21]	No name specified	Wu06	[73]
Minimum Description Length Discretizer	MDL-Disc	[74]	Fast Independent Component Analysis	FastICA	[75]
Bayesian Discretizer	Bayesian	[19]	Linear Program Relaxation	LP-Relaxation	[76]
No name specified	Friedman96	[77]	Hellinger-Based Discretizer	HellingerBD	[50]
Cluster Analysis Discretizer	ClusterAnalysis	[24]	Distribution Index-Based Discretizer	DIBD	[78]
Zeta	Zeta	[56]	Wrapper Estimation of Distribution Algorithm	WEDA	[53]
Distance-based Discretizer	Distance	[79]	Clustering + Rought Sets Discretizer	Cluster-RS-Disc	[25]
Finite Mixture Model Discretizer	FMM	[80]	Interval Distance Discretizer	IDD	[51]
Chi2	Chi2	[18]	Class-Attribute Contingency Coefficient	CACC	[58]
No name specified	FischerExt	[81]	Rectified Chi2	Rectified Chi2	[82]
Contextual Merit Numerical Feature Discretizer	CM-NFD	[83]	Ameva	Ameva	[84]
Concurrent Merger	ConMerge	[85]	Unification	Unification	[55]
Knowledge EXplorer Discretizer	KEX-Disc	[86]	Multiple Scanning Discretizer	MultipleScan	[87]
LVQ-based Discretization	LVQ-Disc	[88]	Optimal Flexible Frequency Discretizer	OFFD	[89]
No name specified	Multi-Bayesian	[90]	Proportional Discretizer	PKID	[12]
No name specified	A*	[91]	Fixed Frequency Discretizer	FFD	[12]
FUSINTER	FUSINTER	[54]	Discretization Class intervals Reduce	DCR	[92]
Cluster-based Discretizer	Cluster-Disc	[93]	MVD-CG	MVD-CG	[94]
Entropy-based Discretization According to Distribution of Boundary points	EDA-DB	[95]	Approximate Equal Frequency Discretizer	AEFD	[96]
No name specified	Clarke00	[97]	No name specified	Jiang09	[96]
Relative Unsupervised Discretizer	RUDE	[98]	Random Forest Discretizer	RFDisc	[99]
Multivariate Discretization	MVD	[45]	Supervised Multivariate Discretizer	SMD	[100]
Modified Learning from Examples Module	MODLEM	[101]	Clustering Based Discretization	CBD	[46]
Modified Chi2	Modified Chi2	[102]	Improved MDLP	Improved MDLP	[103]
HyperCluster Finder	HCF	[104]	Imfor-Disc	Imfor-Disc	[105]
Entropy-based Discretization with Inconsistency Checking	EDIC	[49]	Clustering ME-MDL	Cluster ME-MDL	[106]
Unparametrized Supervised Discretizer	USD	[107]	Effective Bottom-up Discretizer	EBDA	[108]
Rough Set Discretizer	RS-Disc	[109]	Contextual Discretizer	Contextual-Disc	[110]
Rough Set Genetic Algorithm Discretizer	RS-GA-Disc	[111]	Hypercube Division Discretizer	HDD	[48]
Genetic Algorithm Discretizer	GA-Disc	[112]			

restrictions. Instead, we recommend that readers consult the original references to understand the complete operation of the discretizers of interest. Discretizers used in the experimental study are depicted in bold. The ID3 discretizer used in the study is a static version of the well-known discretizer embedded in C4.5.

The properties studied above can be used to categorize the discretizers proposed in the literature. The seven characteristics studied allows us to present the taxonomy of discretization methods following an established order. All techniques enumerated in Table 1 are collected in the taxonomy drawn in Fig. 2. It illustrates the categorization following a hierarchy based on this order: static/dynamic, univariate/multivariate, supervised/unsupervised, splitting/merging/hybrid, global/local, direct/incremental, and evaluation measure. The rationale behind the choice of this order is to achieve a clear representation of the taxonomy.

The proposed taxonomy assists us in the organization of many discretization methods so that we can classify them into categories and analyze their behavior. Also, we can highlight other aspects in which the taxonomy can be useful. For example, it provides a snapshot of existing methods and relations or similarities among them. It also depicts the size of the families, the work done in each one, and what currently is missing. Finally, it provides a general overview on the state-of-the art in discretization for researchers/practitioners who are starting in this topic or need to discretize data in real applications.

4 EXPERIMENTAL FRAMEWORK, EMPIRICAL STUDY, AND ANALYSIS OF RESULTS

This section presents the experimental framework followed in this paper, together with the results collected and discussions on them. Section 4.1 will describe the complete

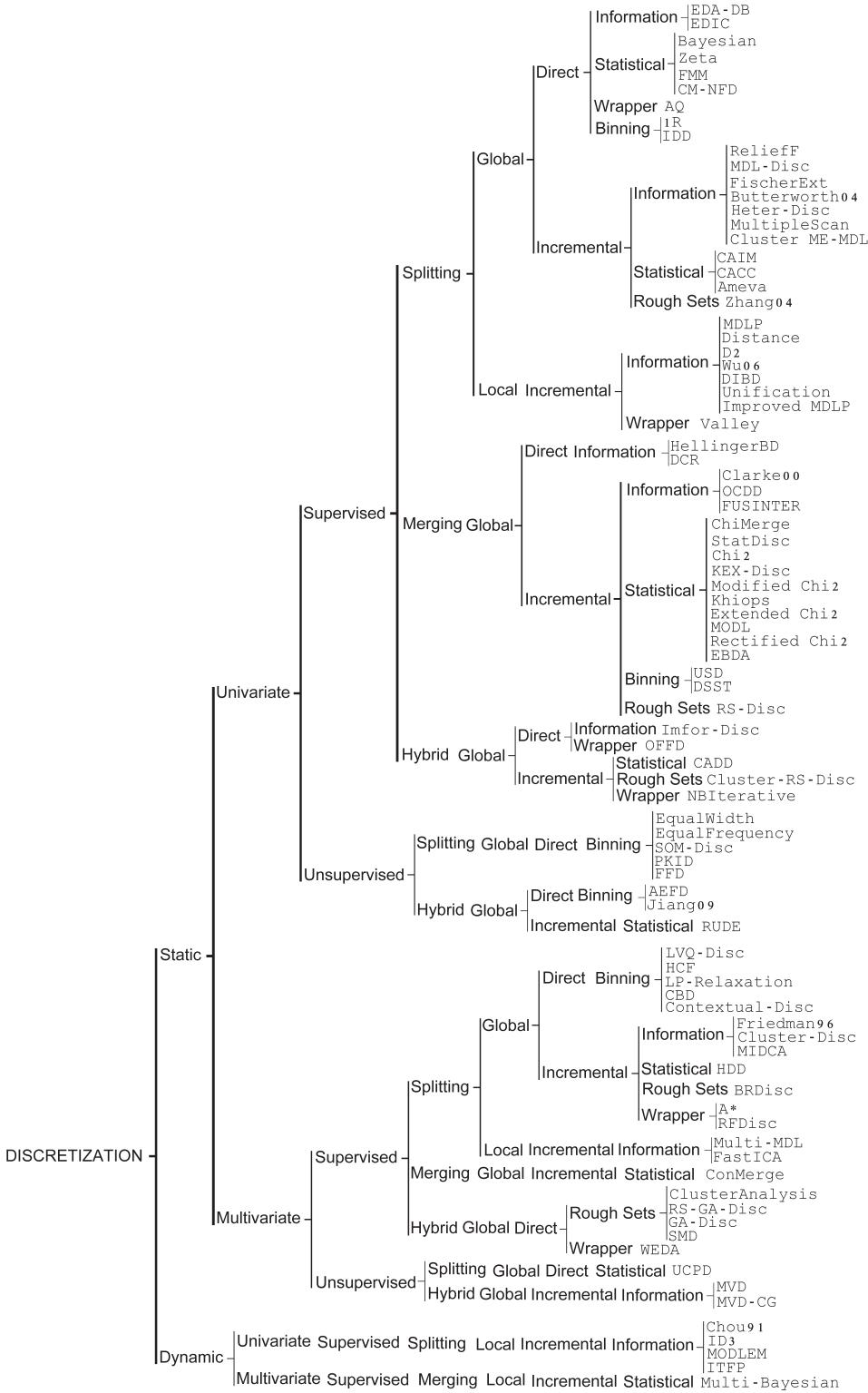


Fig. 2. Discretization taxonomy.

experimental set up. Then, we offer the study and analysis of the results obtained over the data sets used in Section 4.2.

4.1 Experimental Set Up

The goal of this section is to show all the properties and issues related to the experimental study. We specify the data sets, validation procedure, classifiers used, parameters of the classifiers and discretizers, and performance metrics.

The statistical tests used to contrast the results are also briefly commented at the end of this section.

The performance of discretization algorithms is analyzed by using 40 data sets taken from the UCI Machine Learning Database Repository [116] and KEEL data set repository [115].² The main characteristics of these data sets are

2. <http://www.keel.es/datasets.php>.

TABLE 2
Summary Description for Classification Data Sets

Data Set	#Ex.	#Atts.	#Num.	#Nom.	#Cl.
abalone	4,174	8	7	1	28
appendicitis	106	7	7	0	2
australian	690	14	8	6	2
autos	205	25	15	10	6
balance	625	4	4	0	3
banana	5,300	2	2	0	2
bands	539	19	19	0	2
bupa	345	6	6	0	2
cleveland	303	13	13	0	5
contraceptive	1,473	9	9	0	3
crx	690	15	6	9	2
dermatology	366	34	34	0	6
ecoli	336	7	7	0	8
flare-solar	1066	9	9	0	2
glass	214	9	9	0	7
haberman	306	3	3	0	2
hayes	160	4	4	0	3
heart	270	13	13	0	2
hepatitis	155	19	19	0	2
iris	150	4	4	0	3
mammographic	961	5	5	0	2
movement	360	90	90	0	15
newthyroid	215	5	5	0	3
pageblocks	5,472	10	10	0	5
penbased	10,992	16	16	0	10
phoneme	5,404	5	5	0	2
pima	768	8	8	0	2
saheart	462	9	8	1	2
satimage	6,435	36	36	0	7
segment	2,310	19	19	0	7
sonar	208	60	60	0	2
spambase	4,597	57	57	0	2
specfheart	267	44	44	0	2
tae	151	5	5	0	3
titanic	2,201	3	3	0	2
vehicle	846	18	18	0	4
vowel	990	13	13	0	11
wine	178	13	13	0	3
wisconsin	699	9	9	0	2
yeast	1484	8	8	0	10

summarized in Table 2. For each data set, the name, number of examples, number of attributes (numeric and nominal), and number of classes are defined.

In this study, six classifiers have been used in order to find differences in performance among the discretizers. The classifiers are:

- C4.5 [10]: A well-known decision tree, considered one of the top 10 DM algorithms [9].
- *DataSqueezer* [117]: This learner belongs to the family of inductive rule extraction. In spite of its relative simplicity, *DataSqueezer* is a very effective learner. The rules generated by the algorithm are compact and comprehensible, but accuracy is to some extent degraded in order to achieve this goal.
- KNN: One of the simplest and most effective methods based on similarities among a set of objects. It is also considered one of the top 10 DM algorithms [9] and it can handle nominal attributes using proper

TABLE 3
Parameters of the Discretizers and Classifiers

Method	Parameters
C4.5	pruned tree, confidence = 0.25, 2 examples per leaf
DataSqueezer	pruning and generalization threshold = 0.05
KNN	$K=3$, HVDM distance
PUBLIC	25 nodes between prune
Ripper	$k=2$, grow set = 0.66
1R	6 examples of the same class per interval
CADD	confidence threshold = 0.01
Chi2	inconsistency threshold = 0.02
ChiMerge	confidence threshold = 0.05
FDD	frequency size = 30
FUSINTER	$\alpha = 0.975$, $\lambda = 1$
HDD	coefficient = 0.8
IDD	neighborhood = 3, windows size = 3, nominal distance
MODL	optimized process type
UCPD	intervals = [3, 6], KNN map type, neighborhood = 6, minimum support = 25, merged threshold = 0.5, scaling factor = 0.5, use discrete

distance functions such as HVDM [118]. It belongs to the lazy learning family [119], [120].

- *Naive Bayes*: This is another of the top 10 DM algorithms [9]. Its aim is to construct a rule which will allow us to assign future objects to a class, assuming independence of attributes when probabilities are established.
- *PUBLIC* [121]: It is an advanced decision tree that integrates the pruning phase with the building stage of the tree in order to avoid the expansion of branches that would be pruned afterwards.
- *Ripper* [122]: This is a widely used rule induction method based on a *separate and conquer* strategy. It incorporates diverse mechanisms to avoid overfitting and to handle numeric and nominal attributes simultaneously. The models obtained are in the form of decision lists.

The data sets considered are partitioned using the 10-fold cross-validation (10-fcv) procedure. The parameters of the discretizers and classifiers are those recommended by their respective authors. They are specified in Table 3 for those methods which require them. We assume that the choice of the values of parameters is optimally chosen by their own authors. Nevertheless, in discretizers that require the input of the number of intervals as a parameter, we use a rule of thumb which is dependent on the number of instances in the data set. It consists in dividing the number of instances by 100 and taking the maximum value between this result and the number of classes. All discretizers and classifiers are run one time in each partition because they are nonstochastic.

Two performance measures are widely used because of their simplicity and successful application when multiclass classification problems are dealt. We refer to accuracy and Cohen's kappa [123] measures, which will be adopted to measure the efficacy of discretizers in terms of the generalization classification rate.

- *Accuracy*: is the number of successful hits relative to the total number of classifications. It has been by far

the most commonly used metric for assessing the performance of classifiers for years [2], [124].

- *Cohen's kappa*: is an alternative to *accuracy*, a method, known for decades, which compensates for random hits [123]. Its original purpose was to measure the degree of agreement or disagreement between two people observing the same phenomenon. Cohen's kappa can be adapted to classification tasks and its use is recommended because it takes random successes into consideration as a standard, in the same way as the AUC measure [125].

An easy way of computing Cohen's kappa is to make use of the resulting confusion matrix in a classification task. Specifically, the Cohen's kappa measure can be obtained using the following expression:

$$\kappa = \frac{N \sum_{i=1}^C y_{ii} - \sum_{i=1}^C y_{ii}y_{ii}}{N^2 - \sum_{i=1}^C y_{ii}y_{ii}},$$

where y_{ii} is the cell count in the main diagonal of the resulting confusion matrix, N is the number of examples, C is the number of class values, and y_{ii}, y_{ii} are the columns' and rows' total counts of the confusion matrix, respectively. Cohen's kappa ranges from -1 (total disagreement) through 0 (random classification) to 1 (perfect agreement). Being a scalar, it is less expressive than ROC curves when applied to binary classification. However, for multiclass problems, kappa is a very useful, yet simple, meter for measuring the accuracy of the classifier while compensating for random successes.

The empirical study involves 30 discretization methods from those listed in Table 1. We want to outline that the implementations are only based on the descriptions and specifications given by the respective authors in their papers.

Statistical analysis will be carried out by means of nonparametric statistical tests. In [126], [127], [128], authors recommend a set of simple, safe, and robust nonparametric tests for statistical comparisons of classifiers. The Wilcoxon test [129] will be used in order to conduct pairwise comparisons among all discretizers considered in the study. More information about these statistical procedures specifically designed for use in the field of Machine Learning can be found at the SCI2S thematic public website on *Statistical Inference in Computational Intelligence and Data Mining*.³

4.2 Analysis and Empirical Results

Table 4 presents the average results corresponding to the number of intervals and inconsistency rate in training and test data by all the discretizers over the 40 data sets. Similarly, Tables 5 and 6 collect the average results associated with accuracy and kappa measures for each classifier considered. For each metric, the discretizers are ordered from the best to the worst. In Tables 5 and 6, we highlight those discretizers whose performance is within 5 percent of the range between the best and the worst method in each measure, that is, $\text{value}_{\text{best}} - (0.05 \cdot (\text{value}_{\text{best}} - \text{value}_{\text{worst}}))$. They should be

TABLE 4
Average Results Collected from Intrinsic Properties of the Discretizers: Number of Intervals Obtained and Inconsistency Rate in Training and Test Data

Number Int.		Incons. Train		Incons. Tst	
Heter-Disc	8.3125	ID3	0.0504	ID3	0.0349
MVD	18.4575	PKID	0.0581	PKID	0.0358
Distance	23.2125	Modified Chi2	0.0693	FFD	0.0377
UCPD	35.0225	FFD	0.0693	HDD	0.0405
MDLP	36.6600	HDD	0.0755	Modified Chi2	0.0409
Chi2	46.6350	USD	0.0874	USD	0.0512
FUSINTER	59.9850	ClusterAnalysis	0.0958	Khiops	0.0599
DIBD	64.4025	Khiops	0.1157	ClusterAnalysis	0.0623
CADD	67.7100	EqualWidth	0.1222	EqualWidth	0.0627
ChiMerge	69.5625	EqualFrequency	0.1355	EqualFrequency	0.0652
CAIM	72.5125	Chi2	0.1360	Chi2	0.0653
Zeta	75.9325	Bayesian	0.1642	FUSINTER	0.0854
Ameva	78.8425	MODL	0.1716	MODL	0.0970
Khiops	130.3000	FUSINTER	0.1735	HellingerBD	0.1054
IR	162.1925	HellingerBD	0.1975	Bayesian	0.1139
EqualWidth	171.7200	IDD	0.2061	UCPD	0.1383
Extended Chi2	205.2650	ChiMerge	0.2504	ChiMerge	0.1432
HellingerBD	244.6925	UCPD	0.2605	IDD	0.1570
EqualFrequency	267.7250	CAIM	0.2810	CAIM	0.1589
PKID	295.9550	Extended Chi2	0.3048	Extended Chi2	0.1762
MODL	335.8700	Ameva	0.3050	Ameva	0.1932
FFD	342.6050	IR	0.3112	CACC	0.2047
IDD	349.1250	CACC	0.3118	IR	0.2441
Modified Chi2	353.6000	MDLP	0.3783	Zeta	0.2454
CACC	505.5775	Zeta	0.3913	MDLP	0.2501
ClusterAnalysis	1116.1800	MVD	0.4237	DIBD	0.2757
USD	1276.1775	Distance	0.4274	Distance	0.2987
Bayesian	1336.0175	DIBD	0.4367	MVD	0.3171
ID3	1858.3000	CADD	0.6532	CADD	0.5688
HDD	2202.5275	Heter-Disc	0.6749	Heter-Disc	0.5708

considered as outstanding methods in each category, regardless of their specific position in the table.

All detailed results for each data set, discretizer and classifier (including average and standard deviations), can be found at the URL <http://sci2s.ugr.es/discretization>. In the interest of compactness, we will include and analyze summarized results in the paper.

The Wilcoxon test [129], [126], [127] is adopted in this study considering a level of significance equal to $\alpha = 0.05$. Tables 7, 8, and 9 show a summary of all possible comparisons involved in the Wilcoxon test among all discretizers and measures, for number of intervals and inconsistency rate, accuracy and kappa, respectively. Again, the individual comparisons between all possible discretizers are exhibited in the aforementioned URL mentioned above, where a detailed report of statistical results can be found for each measure and classifier. The tables in this paper (Tables 7, 8, and 9) summarize, for each method in the rows, the number of discretizers outperformed by using the Wilcoxon test under the column represented by the "+" symbol. The column with the " \pm " symbol indicates the number of wins and ties obtained by the method in the row. The maximum value for each column is highlighted by a shaded cell.

Finally, to illustrate the magnitude of the differences in average results and the relationship between the number of intervals yielded by each discretizer and the accuracy obtained for each classifier, Fig. 3 depicts a confrontation between the average number of intervals and accuracy reflected by an X-Y axis graphic, for each classifier. It also helps us to see the differences in the behavior of discretization when it is used over distinct classifiers.

3. <http://sci2s.ugr.es/sicidm/>.

TABLE 5
Average Results of Accuracy Considering the Six Classifiers

C4.5		DataSqueezer	KNN	Naive Bayes	PUBLIC	Ripper
FUSINTER	0.7588	Distance	0.5666	PKID	0.7587	FUSINTER
ChiMerge	0.7494	CAIM	0.5547	Modified Chi2	0.7578	CAIM
Zeta	0.7488	Ameva	0.5518	FUSINTER	0.7576	ChiMerge
CAIM	0.7484	MDLP	0.5475	ChiMerge	0.7543	MDLP
UCPD	0.7447	Zeta	0.5475	FFD	0.7535	Distance
Distance	0.7446	ChiMerge	0.5472	EqualWidth	0.7535	Zeta
MDLP	0.7444	CACC	0.5430	FUSINTER	0.7535	Khiops
Chi2	0.7442	Heter-Disc	0.5374	ChiMerge	0.7517	Chi2
Modified Chi2	0.7396	DIBD	0.5322	CAIM	0.7507	UCPD
Ameva	0.7351	UCPD	0.5172	MODL	0.7507	EqualWidth
Khiops	0.7312	MVD	0.5147	EqualFrequency	0.7491	EqualFrequency
MODL	0.7310	FUSINTER	0.5126	MODL	0.7479	Modified Chi2
EqualFrequency	0.7304	Bayesian	0.4915	MODL	0.7479	MODL
EqualWidth	0.7252	Extended Chi2	0.4913	Ameva	0.7220	ID3
HellingerBD	0.7240	Chi2	0.4874	ID3	0.7220	Khiops
CACC	0.7203	HellingerBD	0.4868	ClusterAnalysis	0.7132	Chi2
Extended Chi2	0.7172	MODL	0.4812	Zeta	0.7126	MODL
DIBD	0.7141	CADD	0.4780	HDD	0.7104	EqualWidth
FFD	0.7091	EqualFrequency	0.4711	UCPD	0.7090	EqualFrequency
PKID	0.7079	1R	0.4702	MDLP	0.7002	ClusterAnalysis
HDD	0.6941	EqualWidth	0.4680	Distance	0.6888	Extended Chi2
USD	0.6835	IDD	0.4679	IDD	0.6860	PKID
ClusterAnalysis	0.6813	USD	0.4651	Bayesian	0.6844	PKID
ID3	0.6720	Khiops	0.4567	CACC	0.6813	USD
1R	0.6695	Modified Chi2	0.4526	DIBD	0.6731	Bayesian
Bayesian	0.6675	HDD	0.4308	1R	0.6721	CACC
IDD	0.6606	ClusterAnalysis	0.4282	Extended Chi2	0.6695	DIBD
MVD	0.6499	PKID	0.3942	MVD	0.6062	1R
Heter-Disc	0.6443	ID3	0.3896	Heter-Disc	0.5524	Heter-Disc
CADD	0.5689	FFD	0.3848	CADD	0.5064	CADD

Once the results are presented in the mentioned tables and graphics, we can stress some interesting properties observed from them, and we can point out the best performing discretizers:

- Regarding the number of intervals, the discretizers which divide the numerical attributes in fewer intervals are *Heter-Disc*, *MVD*, and *Distance*, whereas discretizers which require a large number of cut points are *HDD*, *ID3*, and *Bayesian*. The Wilcoxon test confirms that *Heter-Disc* is the discretizer that obtains the least intervals outperforming the rest.
- The inconsistency rate both in training data and test data follows a similar trend for all discretizers, considering that the inconsistency obtained in test data is always lower than in training data. *ID3* is the discretizer that obtains the lowest average inconsistency rate in training and test data, albeit the Wilcoxon test cannot find significant differences between it and the other two discretizers: *FFD* and *PKID*. We can observe a close relationship between the number of intervals produced and the inconsistency rate, where discretizers that compute fewer cut points are usually those which have a high inconsistency rate. They risk the consistency of the data in order to simplify the result, although the consistency is not usually correlated with the accuracy, as we will see below.
- In decision trees (C4.5 and PUBLIC), a subset of discretizers can be stressed as the best performing ones. Considering average accuracy, *FUSINTER*, *ChiMerge*, and *CAIM* stand out from the rest. Considering average kappa, *Zeta* and *MDLP* are also added to this subset. The Wilcoxon test confirms this

result and adds another discretizer, *Distance*, which outperforms 16 of the 29 methods. All methods emphasized are supervised, incremental (except *Zeta*) and use statistical and information measures as evaluators. Splitting/Merging and Local/Global properties have no effect on decision trees.

- Considering rule induction (*DataSqueezer* and *Ripper*), the best performing discretizers are *Distance*, *Modified Chi2*, *Chi2*, *PKID*, and *MODL* in average accuracy and *CACC*, *Ameva*, *CAIM*, and *FUSINTER* in average kappa. In this case, the results are very irregular due to the fact that the Wilcoxon test emphasizes the *ChiMerge* as the best performing discretizer for *DataSqueezer* instead of *Distance* and incorporates *Zeta* in the subset. With *Ripper*, the Wilcoxon test confirms the results obtained by averaging accuracy and kappa. It is difficult to discern a common set of properties that define the best performing discretizers due to the fact that rule induction methods differ in their operation to a greater extent than decision trees. However, we can remark that, in the subset of best methods, incremental and supervised discretizers predominate in the statistical evaluation.
- Lazy and Bayesian learning can be analyzed together, due to the fact that the HVDM distance used in KNN is highly related to the computation of Bayesian probabilities considering attribute independence [118]. With respect to lazy and Bayesian learning, *KNN* and *Naive Bayes*, the subset of remarkable discretizers is formed by *PKID*, *FFD*, *Modified Chi2*, *FUSINTER*, *ChiMerge*, *CAIM*, *EqualWidth*, and *Zeta*, when average accuracy is used; and *Chi2*, *Khiops*, *EqualFrequency* and *MODL* must be added when average kappa is

TABLE 6
Average Results of Kappa Considering the Six Classifiers

C4.5		DataSqueez		KNN		Naive Bayes		PUBLIC		Ripper	
FUSINTER	0.5550	CACC	0.2719	PKID	0.5784	PKID	0.5762	CAIM	0.5279	Modified Chi2	0.5180
ChiMerge	0.5433	Ameva	0.2712	FFD	0.5617	Modified Chi2	0.5742	FUSINTER	0.5204	Chi2	0.5163
CAIM	0.5427	CAIM	0.2618	Modified Chi2	0.5492	FUSINTER	0.5737	ChiMerge	0.5158	MODL	0.5123
Zeta	0.5379	ChiMerge	0.2501	Khiops	0.5457	FFD	0.5710	MDLP	0.5118	FUSINTER	0.5073
MDLP	0.5305	FUSINTER	0.2421	EqualFrequency	0.5438	ChiMerge	0.5650	Distance	0.5074	Khiops	0.4939
UCPD	0.5299	UCPD	0.2324	EqualWidth	0.5338	Chi2	0.5620	Zeta	0.5010	PKID	0.4915
Ameva	0.5297	Zeta	0.2189	CAIM	0.5260	CAIM	0.5616	Ameva	0.4986	EqualFrequency	0.4892
Chi2	0.5290	USD	0.2174	FUSINTER	0.5242	EqualWidth	0.5593	Chi2	0.4899	ChiMerge	0.4878
Distance	0.5288	Distance	0.2099	ChiMerge	0.5232	Khiops	0.5570	UCPD	0.4888	EqualWidth	0.4875
Modified Chi2	0.5163	Khiops	0.2038	MODL	0.5205	EqualFrequency	0.5564	Khiops	0.4846	CAIM	0.4870
MODL	0.5131	HDD	0.2030	HellingerBD	0.5111	MODL	0.5564	CACC	0.4746	Ameva	0.4810
EqualFrequency	0.5108	EqualFrequency	0.2016	Chi2	0.5100	USD	0.5458	HellingerBD	0.4736	FFD	0.4809
Khiops	0.5078	HellingerBD	0.1965	Ameva	0.5041	Zeta	0.5457	Modified Chi2	0.4697	Zeta	0.4769
HellingerBD	0.4984	Bayesian	0.1941	USD	0.4943	USD	0.5456	MODL	0.4620	HellingerBD	0.4729
CACC	0.4961	MODL	0.1918	HDD	0.4878	ID3	0.5403	EqualFrequency	0.4535	USD	0.4560
EqualWidth	0.4909	MDLP	0.1875	ClusterAnalysis	0.4863	HDD	0.5394	DIBD	0.4431	UCPD	0.4552
Extended Chi2	0.4766	PKID	0.1846	Zeta	0.4831	MDLP	0.5389	EqualWidth	0.4386	CACC	0.4504
DIBD	0.4759	ID3	0.1818	ID3	0.4769	Distance	0.5368	Extended Chi2	0.4358	MDLP	0.4449
FFD	0.4605	EqualWidth	0.1801	UCPD	0.4763	HellingerBD	0.5353	HDD	0.4048	Distance	0.4429
PKID	0.4526	Modified Chi2	0.1788	MDLP	0.4656	ClusterAnalysis	0.5252	FFD	0.3969	HDD	0.4403
HDD	0.4287	DIBD	0.1778	Distance	0.4470	UCPD	0.5194	PKID	0.3883	ID3	0.4359
USD	0.4282	Chi2	0.1743	CACC	0.4367	CACC	0.5128	USD	0.3845	Extended Chi2	0.4290
ClusterAnalysis	0.4044	IDD	0.1648	IDD	0.4329	Extended Chi2	0.4910	MVD	0.3461	ClusterAnalysis	0.4252
ID3	0.3803	FFD	0.1635	Extended Chi2	0.4226	Bayesian	0.4757	ClusterAnalysis	0.3453	Bayesian	0.3987
IDD	0.3803	ClusterAnalysis	0.1613	Bayesian	0.4201	DIBD	0.4731	Bayesian	0.3419	DIBD	0.3759
MVD	0.3759	Extended Chi2	0.1465	DIBD	0.4167	IDD	0.4618	ID3	0.3241	IDD	0.3650
Bayesian	0.3716	MVD	0.1312	IR	0.3940	IR	0.3980	IDD	0.3066	MVD	0.3446
IR	0.3574	IR	0.1147	MVD	0.3429	MVD	0.3977	IR	0.3004	IR	0.3371
Heter-Disc	0.2709	Heter-Disc	0.1024	Heter-Disc	0.2172	Heter-Disc	0.2583	Heter-Disc	0.2570	Heter-Disc	0.2402
CADD	0.1524	CADD	0.0260	CADD	0.1669	CADD	0.1729	CADD	0.1489	CADD	0.1602

considered. The statistical report by Wilcoxon informs us of the existence of two outstanding methods: *PKID* for *KNN*, which outperforms 27/29 and *FUSINTER* for *Naive Bayes*. Here, supervised and unsupervised, direct and incremental, binning, and statistical/information evaluation are characteristics present in the best performing methods. However, we can see that all of them are global, thus identifying a trend toward binning methods.

- In general, accuracy and kappa performance registered by discretizers do not differ too much. The behavior in both evaluation metrics are quite similar, taking into account that the differences in kappa are usually lower due to the compensation of random success offered by it. Surprisingly, in *DataSqueez*, accuracy and kappa offer the greatest differences in behavior, but they are motivated by the fact that this method focuses on obtaining simple rule sets, leaving precision in the background.
- It is obvious that there is a direct dependence between discretization and the classifier used. We have pointed out that a similar behavior in decision trees and lazy/bayesian learning can be detected, whereas in rule induction learning, the operation of the algorithm conditions the effectiveness of the discretizer. Knowing a subset of suitable discretizers for each type of discretizer is a good starting point to understand and propose improvements in the area.
- Another interesting remark can be made about the relationship between accuracy and the number of intervals yielded by a discretizer. Fig. 3 supports the hypothesis that there is no direct correlation between them. A discretizer that computes few cut points

does not have to obtain poor results in accuracy and vice versa. Figs. 3a, 3c, 3d, and 3e point out that there is a minimum limit in the number of intervals to guarantee accurate models, given by the cut points computed by *Distance*. Fig. 3b shows how *DataSqueez* is worse as the number of intervals increases, but this is an inherent behavior of the classifier.

- Finally, we can stress a subset of global best discretizers considering a tradeoff between the number of intervals and accuracy obtained. In this subset, we can include *FUSINTER*, *Distance*, *Chi2*, *MDLP*, and *UCPD*.

On the other hand, an analysis centered on the 30 discretizers studied is given as follows:

- Many classic discretizers are usually the best performing ones. This is the case of *ChiMerge*, *MDLP*, *Zeta*, *Distance*, and *Chi2*.
- Other classic discretizers are not as good as they should be, considering that they have been improved over the years: *EqualWidth*, *EqualFrequency*, *IR*, *ID3* (the static version is much worse than the dynamic inserted in C4.5 operation), *CADD*, *Bayesian*, and *ClusterAnalysis*.
- Slight modifications of classic methods have greatly enhanced their results, such as, for example, *FUSINTER*, *Modified Chi2*, *PKID*, and *FFD*; but in other cases, the extensions have diminished their performance: *USD*, *Extended Chi2*.
- Promising techniques that have been evaluated under unfavorable circumstances are *MVD* and *UCP*, which are unsupervised methods useful for application to other DM problems apart from classification.

TABLE 7
Wilcoxon Test Results in Number of Intervals
and Inconsistencies

	N. Intervals		Incons. Tra		Incons. Tst	
	+	±	+	±	+	±
1R	10	21	3	17	2	20
Ameva	13	21	6	16	4	21
Bayesian	2	4	10	29	7	29
CACC	7	22	4	17	4	21
CADD	21	28	0	1	0	1
CAIM	14	23	6	19	6	20
Chi2	15	26	9	20	9	20
ChiMerge	15	23	6	20	6	23
ClusterAnalysis	1	4	15	29	9	29
DIBD	21	27	2	7	2	8
Distance	26	28	2	6	2	6
EqualFrequency	7	12	12	26	11	29
EqualWidth	11	18	16	26	13	29
Extended Chi2	14	27	2	14	2	18
FFD	5	8	21	29	16	29
FUSINTER	14	22	11	23	8	29
HDD	0	2	18	29	14	29
HellingerBD	9	15	8	21	7	26
Heter-Disc	29	29	0	1	0	1
ID3	0	1	23	29	16	29
IDD	5	11	8	28	6	29
Khiops	9	15	15	27	12	29
MDLP	22	27	3	9	3	11
Modified Chi2	7	13	17	26	15	29
MODL	5	14	12	24	7	29
MVD	23	28	2	13	2	13
PKID	5	8	22	29	16	29
UCPD	17	25	6	17	5	20
USD	2	4	18	29	15	29
Zeta	12	23	3	9	3	13

- Recent proposed methods that have been demonstrated to be competitive compared with classic methods and even outperforming them in some scenarios are *Khiops*, *CAIM*, *MODL*, *Ameva*, and *CACC*. However, recent proposals that have reported bad results in general are *Heter-Disc*, *HellingerBD*, *DIBD*, *IDD*, and *HDD*.
- Finally, this study involves a higher number of data sets than the quantity considered in previous works and the conclusions achieved are impartial toward an specific discretizer. However, we have to stress some coincidences with the conclusions of these previous works. For example in [102], the authors propose an improved version of *Chi2* in terms of accuracy, removing the user parameter choice. We check and measure the actual improvement. In [12], the authors develop an intense theoretical and analytical study concerning *Naive Bayes* and propose *PKID* and *FFD* according to their conclusions. In this paper, we corroborate that *PKID* is the best suitable method for *Naive Bayes* and even for *KNN*. Finally, we may note

TABLE 8
Wilcoxon Test Results in Accuracy

	C4.5		Data Squeez		KNN		Naive Bayes		PUBLIC		Ripper	
	+	±	+	±	+	±	+	±	+	±	+	
1R	1	12	3	23	2	19	1	9	1	12	1	11
Ameva	14	29	17	29	8	26	9	29	13	29	9	29
Bayesian	1	9	5	26	2	12	2	11	0	11	2	17
CACC	9	28	16	29	2	18	5	28	9	29	4	26
CADD	0	1	1	22	0	1	0	1	0	6	0	0
CAIM	16	29	16	29	11	28	10	29	16	29	11	28
Chi2	13	29	4	26	6	27	9	29	11	29	19	29
ChiMerge	17	29	18	29	13	28	10	29	17	29	9	28
ClusterAnalysis	1	10	0	12	5	24	6	27	1	11	2	20
DIBD	6	21	8	29	2	9	2	8	9	23	1	5
Distance	13	29	16	29	2	17	7	26	13	28	2	13
EqualFrequency	10	27	3	21	18	29	9	29	10	26	11	27
EqualWidth	7	20	2	18	11	28	8	29	6	20	9	27
Extended Chi2	9	27	4	26	3	19	3	17	6	25	2	25
FFD	5	15	0	5	20	28	8	29	1	13	10	27
FUSINTER	21	29	9	29	12	28	15	29	20	29	11	29
HDD	1	18	0	14	4	23	5	28	0	24	7	26
HellingerBD	10	27	4	22	7	26	7	28	10	26	6	26
Heter-Disc	0	9	9	29	0	2	0	3	0	11	1	10
ID3	1	10	0	5	5	22	4	28	0	11	5	26
IDD	1	10	3	23	4	21	2	14	0	12	1	16
Khiops	12	27	3	18	18	29	9	29	9	27	11	29
MDLP	14	29	14	29	3	22	8	29	15	29	2	16
Modified Chi2	11	27	3	21	17	28	10	29	9	29	23	29
MODL	12	28	5	23	14	28	9	29	10	28	17	29
MVD	1	15	5	29	1	8	1	7	0	19	1	13
PKID	5	15	0	6	27	29	9	29	1	13	15	29
UCPD	14	29	7	26	4	17	2	15	15	28	3	19
USD	1	13	3	19	6	23	6	29	1	19	7	25
Zeta	14	29	17	29	4	20	9	29	14	29	7	27

TABLE 9
Wilcoxon Test Results in Kappa

	C4.5		Data Squeez		KNN		Naive Bayes		PUBLIC		Ripper	
	+	±	+	±	+	±	+	±	+	±	+	
1R	1	11	0	15	2	16	2	8	1	13	1	11
Ameva	15	29	24	29	11	26	11	29	16	29	9	29
Bayesian	1	8	1	24	2	10	2	8	1	11	2	17
CACC	11	28	25	29	3	16	7	25	13	29	4	26
CADD	0	1	0	3	0	1	0	1	0	2	0	0
CAIM	17	29	22	29	13	28	11	29	21	29	11	28
Chi2	14	29	2	24	11	27	10	29	13	29	19	29
ChiMerge	19	29	22	29	13	28	11	29	18	29	9	28
ClusterAnalysis	2	10	2	21	5	23	6	22	1	11	2	20
DIBD	8	20	1	24	2	10	2	7	7	18	1	5
Distance	16	29	1	26	2	16	7	28	16	29	2	13
EqualFrequency	11	25	3	25	18	29	10	29	10	23	11	27
EqualWidth	7	20	2	23	14	27	8	28	6	18	9	27
Extended Chi2	10	27	1	20	2	17	3	16	6	23	2	25
FFD	6	14	1	19	23	28	12	29	2	14	10	27
FUSINTER	21	29	16	29	14	28	18	29	19	29	11	29
HDD	2	17	5	25	5	22	6	25	1	22	7	26
HellingerBD	11	23	4	23	9	26	7	21	11	24	6	26
Heter-Disc	0	6	0	12	0	2	0	2	0	8	1	10
ID3	1	8	2	22	4	20	6	26	0	10	5	26
IDD	1	9	1	23	2	18	2	15	1	11	1	16
Khiops	11	24	5	24	18	29	10	29	13	25	11	29
MDLP	16	29	1	24	6	22	8	29	19	29	2	16
Modified Chi2	12	27	1	21	17	27	14	29	9	28	23	29
MODL	12	28	4	24	14	27	12	29	11	28	17	29
MVD	1	12	0	19	1	10	1	6	1	16	1	13
PKID	5	14	2	23	27	29	14	29	2	14	15	29
UCPD	14	29	15	28	4	16	4	16	13	25	3	19
USD	4	13	9	25	6	23	6	25	3	15	7	25
Zeta	15	29	9	27	3	18	6	27	16	29	7	27

that *CAIM* is one of the simplest discretizers and its effectiveness has also been shown in this study.

5 CONCLUDING REMARKS AND GLOBAL GUIDELINES

The present paper offers an exhaustive survey of the discretization methods proposed in the literature. Basic and advanced properties, existing work, and related fields have been studied. Based on the main characteristics

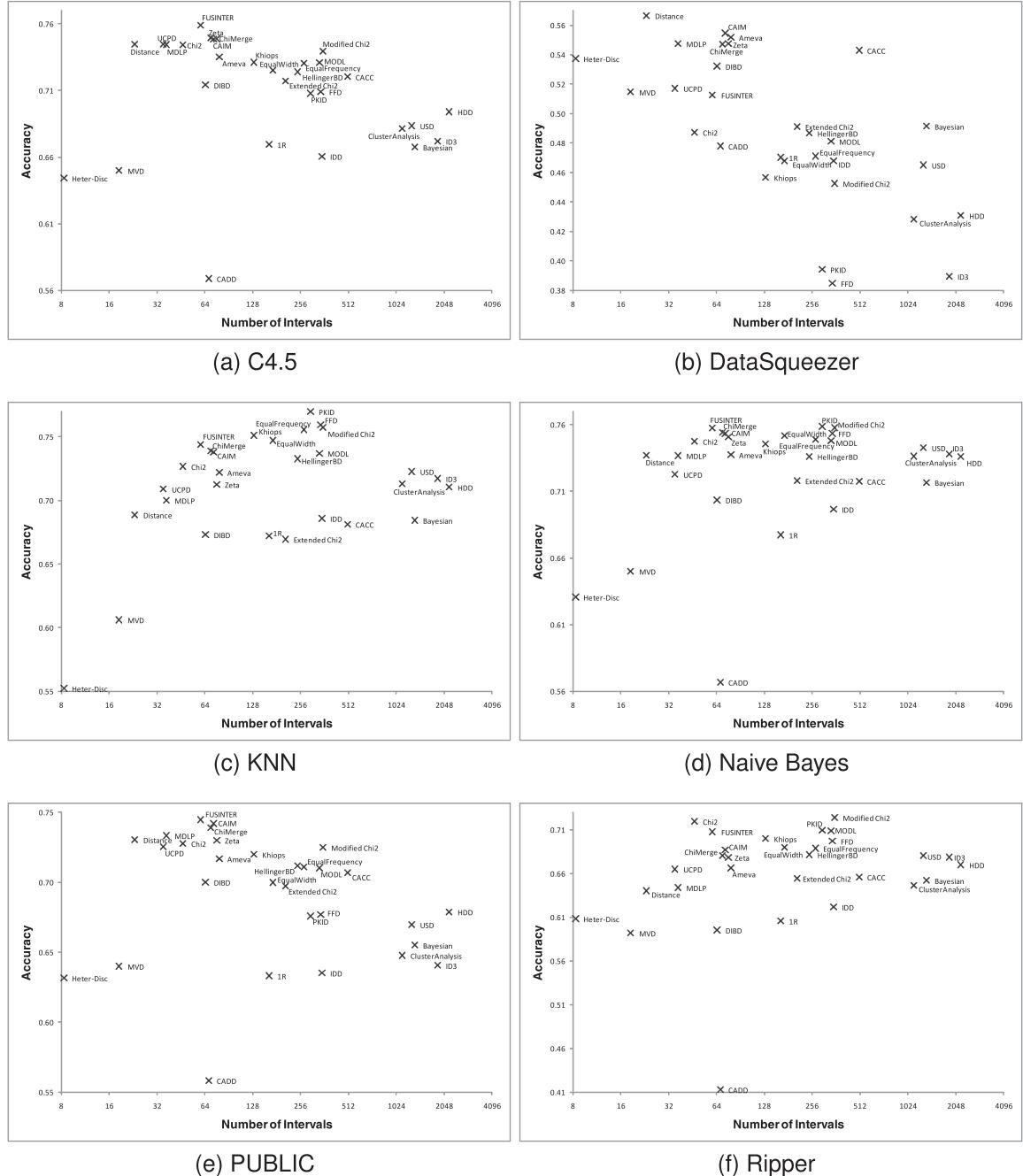


Fig. 3. Accuracy versus number of intervals.

studied, we have designed a taxonomy of discretization methods. Furthermore, the most important discretizers (classic and recent) have been empirically analyzed over a vast number of classification data sets. In order to strengthen the study, statistical analysis based on nonparametric tests has been added supporting the conclusions drawn. Several remarks and guidelines can be suggested:

- A researcher/practitioner interested in applying a discretization method should be aware of the properties that define them in order to choose the most appropriate in each case. The taxonomy developed and the empirical study can help to make this decision.

- In the proposal of a new discretizer, the best approaches and those which fit with the basic properties of the new proposal should be used in the comparison study. In order to do this, the taxonomy and the analysis of results can guide a future proposal in the correct way.
- This paper assists nonexperts in discretization to differentiate among methods, making an appropriate decision about their application and understanding their behavior.
- It is important to know the main advantages of each discretizer. In this paper, many discretizers have been empirically analyzed but we cannot give a single conclusion about which is the best

performing one. This depends upon the problem tackled and the data mining method used, but the results offered here could help to limit the set of candidates.

- The empirical study allows us to stress several methods among the whole set:

- *FUSINTER*, *ChiMerge*, *CAIM*, and *Modified Chi2* offer excellent performances considering all types of classifiers.
- *PKID*, *FFD* are suitable methods for lazy and Bayesian learning and *CACC*, *Distance*, and *MODL* are good choices in rule induction learning.
- *FUSINTER*, *Distance*, *Chi2*, *MDLP*, and *UCPD* obtain a satisfactory tradeoff between the number of intervals produced and accuracy.

It would be desirable that a researcher/practitioner who wants to decide which discretization scheme to apply to his/her data needs to know how the experiments of this paper or data will benefit and guide him/her. As future work, we propose the analysis of each property studied in the taxonomy with respect to some data characteristics, such as number of labels, dimensions or dynamic range of original attributes. Following this trend, we expect to find the most suitable discretizer taking into consideration some basic characteristic of the data sets.

ACKNOWLEDGMENTS

This work was supported by the Research Projects TIN2011-28488 and TIC-6858. J.A. Sáez and V. Lopez hold an FPU scholarship from the Spanish Ministry of Education. The authors would like to thank the reviewers for their suggestions.

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