

**University of San Carlos
DCISM**

Problem Set 4

MATH 3109

PREPARED BY:
Woodrow A. Fajardo

SUBMITTED TO:
Prof. Elmer S. Poliquit

**MATH-3109
NOVEMBER 2025**

1 Raw Data

```
R // RAW DATA: income.csv
1 setwd("~/School/MATH-3109/PROBSET-4")
2
3 raw_data = read.csv("income.csv")
4 print(raw_data)
5 View(raw_data)
```

SYSTEM // EXPORT

	Age	Income	Education	Gender	Satisfaction	PurchaseAmount
1	56	71935.67	PhD	Male	8.60	155.18
2	NA	73080.72	Master	Male	7.73	NA
3	NA	13413.41		PhD Female	5.86	NA
4	25	59051.62	<NA>	Female	NA	143.81
5	38	46234.34		PhD <NA>	9.69	247.50
6	56	47541.99	Master	Female	6.46	NA
7	36	NA	Master	Female	3.48	222.89
8	40	72304.71	<NA>	Female	3.67	115.79
9	28	NA	<NA>	Female	2.49	216.34
10	28	55333.27		PhD Female	1.14	195.94
11	41	NA	Bachelor	Male	NA	223.39
12	53	NA		PhD Female	4.55	236.81
13	57	45599.01		PhD <NA>	3.64	161.01
14	41	49552.42	Bachelor	Female	1.13	157.81
15	NA	51426.89	Bachelor	Female	NA	192.47

2 Data Cleaning

- a. Identify the missing values in the dataset.
- b. What variables have missing values?
- c. What type of missingness (MCAR, MAR, MNAR) might be present?

3 Imputation

- a. Choose one imputation method (mean, regression, kNN, or multiple imputation) and justify your choice.

```
R // RAW DATA: income.csv
1 raw_data$Gender <- factor(raw_data$Gender, levels=c("Male", "Female"))
2 raw_data$Education <- factor(raw_data$Education, levels=c("PhD", "Master",
   ↪ "Bachelor", "Highschool"))
3
4 init <- mice(raw_data, maxit=0)
5
6 methods <- init$method
7 methods["Gender"] <- "logreg"
8 methods["Education"] <- "polyreg"
9 methods["Age"] <- "pmm"
10 methods["Income"] <- "pmm"
11 methods["Satisfaction"] <- "pmm"
12 methods["PurchaseAmount"] <- "pmm"
```

b. Impute the values using your own method.

R // RAW DATA: income.csv

```

1 imputed_data <- mice(raw_data, method = methods, m = 5, seed = 123,
  ↪ printFlag=FALSE)
2
3 complete_data <- complete(imputed_data, 1)
4 print(complete_data)

```

SYSTEM // EXPORT

	Age	Income	Education	Gender	Satisfaction	PurchaseAmount
1	56	71935.67	PhD	Male	8.60	155.18
2	36	73080.72	Master	Male	7.73	157.81
3	57	13413.41	PhD	Female	5.86	222.89
4	25	59051.62	Master	Female	1.14	143.81
5	38	46234.34	PhD	Male	9.69	247.50
6	56	47541.99	Master	Female	6.46	192.47
7	36	51426.89	Master	Female	3.48	222.89
8	40	72304.71	Master	Female	3.67	115.79
9	28	45599.01	Master	Female	2.49	216.34
10	28	55333.27	PhD	Female	1.14	195.94
11	41	51426.89	Bachelor	Male	9.69	223.39
12	53	47541.99	PhD	Female	4.55	236.81
13	57	45599.01	PhD	Female	3.64	161.01
14	41	49552.42	Bachelor	Female	1.13	157.81
15	41	51426.89	Bachelor	Female	2.49	192.47

4 Exploration

a. Generate summary statistics before and after imputation.

R // RAW DATA: income.csv

```
1 print(summary(raw_data))
```

SYSTEM // EXPORT

Age	Income	Education	Gender
Min. :25.00	Min. :13413	PhD :6	Male :3
1st Qu.:34.00	1st Qu.:46888	Master :3	Female:10
Median :40.50	Median :51427	Bachelor :3	NA's :2
Mean :41.58	Mean :53225	Highschool:0	
3rd Qu.:53.75	3rd Qu.:65494	NA's :3	
Max. :57.00	Max. :73081		
NA's :3	NA's :4		
 Satisfaction			
Min. :1.130	Min. :115.8		
1st Qu.:3.232	1st Qu.:157.2		
Median :4.110	Median :194.2		
Mean :4.870	Mean :189.1		
3rd Qu.:6.777	3rd Qu.:223.0		
Max. :9.690	Max. :247.5		
NA's :3	NA's :3		

R // RAW DATA: income.csv

```
1 print(summary(complete_data))
```

SYSTEM // EXPORT

Age	Income	Education	Gender
Min. :25.0	Min. :13413	PhD :6	Male : 4
1st Qu.:36.0	1st Qu.:46888	Master :6	Female:11
Median :41.0	Median :51427	Bachelor :3	
Mean :42.2	Mean :52098	Highschool:0	
3rd Qu.:54.5	3rd Qu.:57192		
Max. :57.0	Max. :73081		
Satisfaction	PurchaseAmount		
Min. :1.130	Min. :115.8		
1st Qu.:2.490	1st Qu.:157.8		
Median :3.670	Median :192.5		
Mean :4.784	Mean :189.5		
3rd Qu.:7.095	3rd Qu.:222.9		
Max. :9.690	Max. :247.5		

- b. Plot histograms or boxplots to compare distributions pre- and post- imputation.

R // Age

```
1 par(mfrow = c(1, 2))
2 hist(raw_data$Age,
3       main = "Before Imp: Age",
4       xlab = "Age",
5       col = "blue",
6       border = "black")
7 hist(complete_data$Age,
8       main = "After Imp: Age",
9       xlab = "Age",
10      col = "red",
11      border = "black")
```

4.1 Standard Image

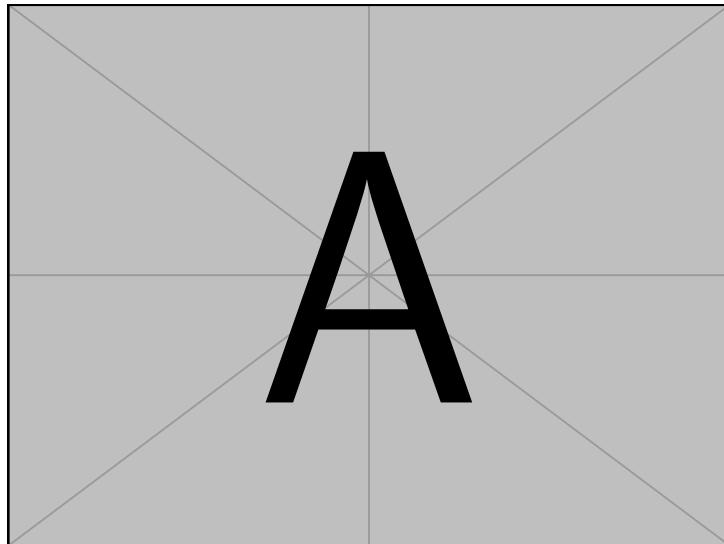


FIGURE 1

Standard single image.

Source: Generated by LaTeX

```
R // Income
1 par(mfrow = c(1, 2))
2 hist(raw_data$Income,
3       main = "Before Imp: Income",
4       xlab = "Income",
5       col = "blue",
6       border = "black")
7 hist(complete_data$Income,
8       main = "After Imp: Income",
9       xlab = "Income",
10      col = "red",
11      border = "black")
```

4.2 Standard Image

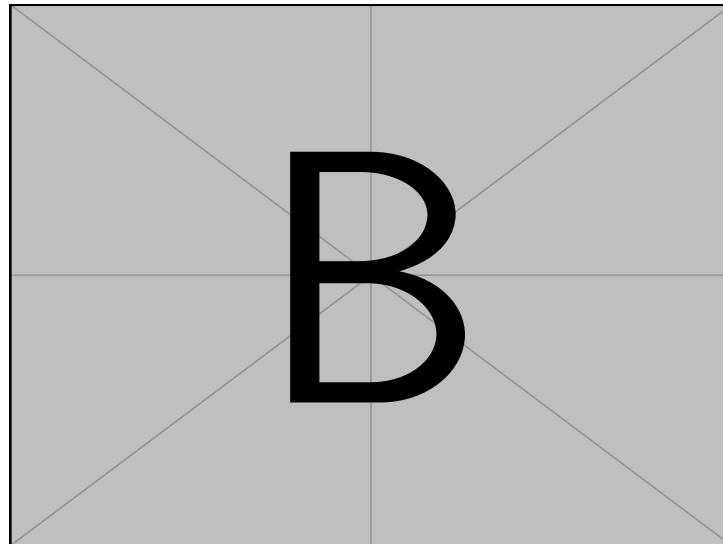


FIGURE 2

Standard single image.

Source: Generated by LaTeX

```
R // Purchase Amount
1 par(mfrow = c(1, 2))
2 hist(raw_data$PurchaseAmount,
3       main = "Before Imp: PurchaseAmount",
4       xlab = "Purchase Amt",
5       col = "blue",
6       border = "black")
7 hist(complete_data$PurchaseAmount,
8       main = "After Imp: Purchase Amt",
9       xlab = "Purchase Amt",
10      col = "red",
11      border = "black")
```

4.3 Standard Image

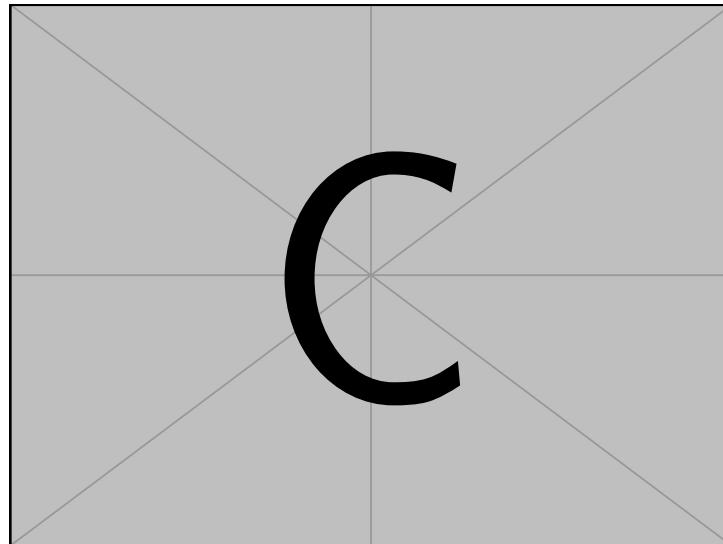


FIGURE 3

Standard single image.

Source: Generated by LaTeX

```
R // Satisfaction
1 par(mfrow = c(1, 2))
2 hist(raw_data$Satisfaction,
3       main = "Before Imp: Satisfaction",
4       xlab = "Satisfaction",
5       col = "blue",
6       border = "black"
7 )
8 hist(complete_data$Satisfaction,
9       main = "After Imp: Satisfaction",
10      xlab = "Satisfaction",
11      col = "red",
12      border = "black"
```

4.4 Standard Image

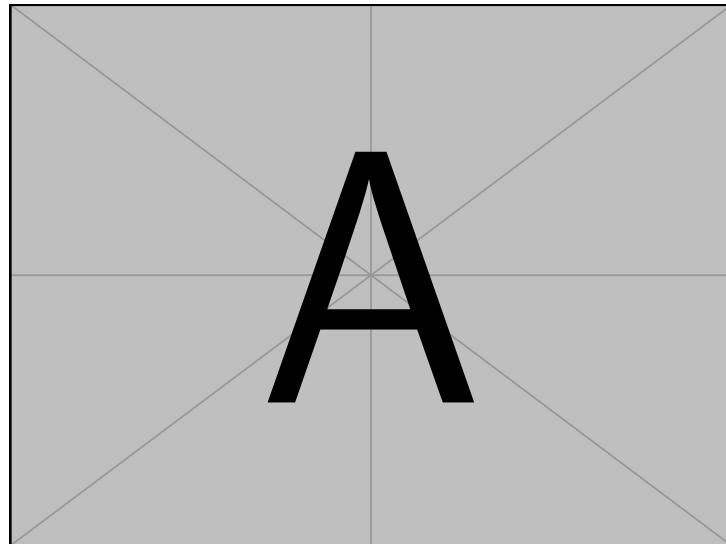


FIGURE 4

Standard single image.

Source: Generated by LaTeX

```
R // Gender
1 par(mfrow = c(1, 2))
2 barplot(table(raw_data$Gender),
3         main = "Before Imp: Gender",
4         col = "blue")
5 barplot(table(complete_data$Gender),
6         main = "After Imp: Gender",
7         col = "red")
```

4.5 Standard Image

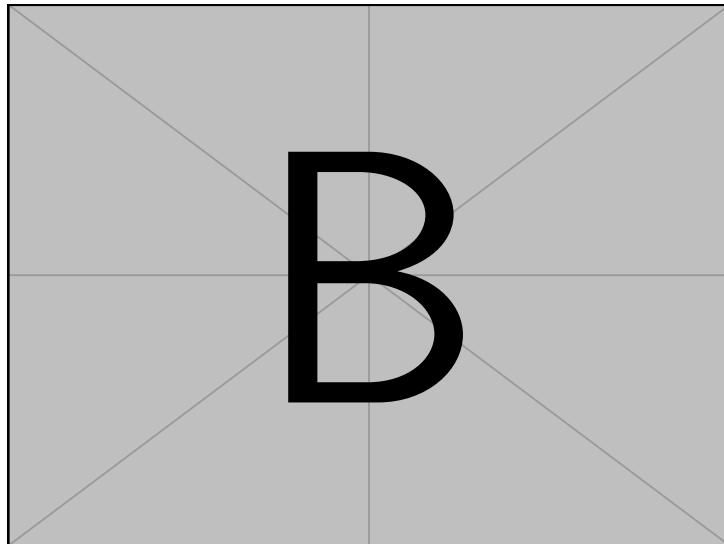


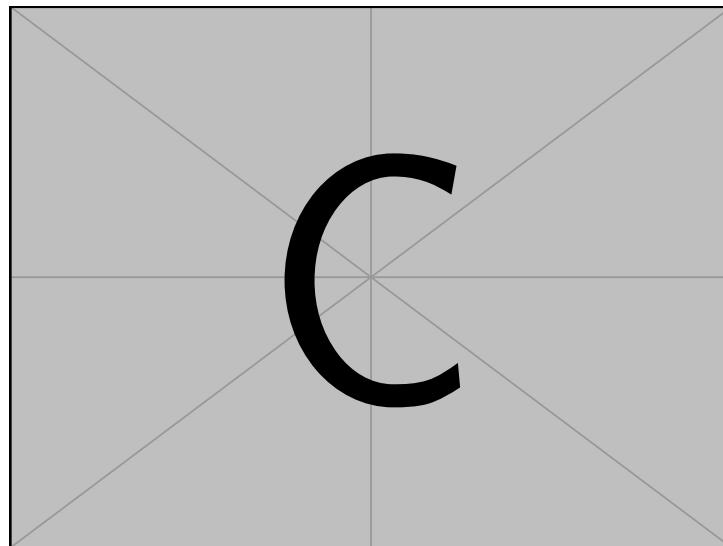
FIGURE 5

Standard single image.

Source: Generated by LaTeX

```
R // Education
1 par(mfrow = c(1, 2))
2 barplot(table(raw_data$Education),
3         main = "Before Imp: Education",
4         col = "blue")
5 barplot(table(complete_data$Education),
6         main = "After Imp: Education",
7         col = "red")
```

4.6 Standard Image

**FIGURE 6**

Standard single image.

Source: Generated by LaTeX

5 Simulate Synthetic Data

- a. Based on the imputed dataset, stimulate a synthetic dataset with $n = 100$.

```
R // RAW DATA: income.csv
1 make_syndata <- function(complete_data, method_vec, n_syn = 100){
2   na_data <- raw_data[rep(NA, n_syn), ]
3   combined <- rbind(complete_data, na_data)
4   imp_syn_data <- mice(combined, method = method_vec, m = 1, maxit = 1, seed =
  ↪ 123, printFlag = FALSE)
5   syn_data <- complete(imp_syn_data, 1)
6   syn_data[(nrow(complete_data) + 1):nrow(syn_data), ]
7 }
8
9 synthetic_data <- make_syndata(complete_data, methods, n_syn = 100)
10 print(tail(synthetic_data, 10))
```

SYSTEM // EXPORT

Age	Income	Education	Gender	Satisfaction	PurchaseAmount	
NA.90	41	51426.89	PhD	Female	2.49	216.34
NA.91	40	51426.89	Master	Male	6.46	236.81
NA.92	53	71935.67	Master	Male	9.69	157.81
NA.93	36	46234.34	Bachelor	Female	3.67	247.50
NA.94	57	46234.34	Master	Male	7.73	192.47
NA.95	28	49552.42	Bachelor	Male	6.46	236.81
NA.96	56	55333.27	Master	Male	9.69	195.94
NA.97	57	47541.99	Master	Male	8.60	223.39
NA.98	28	59051.62	Bachelor	Male	9.69	247.50
NA.99	25	59051.62	PhD	Female	3.67	222.89

- b. Ensure similar distribution and structure to the original imputed data.

```
R // RAW DATA: income.csv
1 print(summary(raw_data))
```

SYSTEM // EXPORT

Age	Income	Education	Gender
Min. :25.00	Min. :13413	PhD :6	Male : 3
1st Qu.:34.00	1st Qu.:46888	Master :3	Female:10
Median :40.50	Median :51427	Bachelor :3	NA's : 2
Mean :41.58	Mean :53225	Highschool:0	
3rd Qu.:53.75	3rd Qu.:65494	NA's : 3	
Max. :57.00	Max. :73081		
NA's :3	NA's :4		

Satisfaction	PurchaseAmount
Min. :1.130	Min. :115.8
1st Qu.:3.232	1st Qu.:157.2
Median :4.110	Median :194.2
Mean :4.870	Mean :189.1
3rd Qu.:6.777	3rd Qu.:223.0
Max. :9.690	Max. :247.5
NA's :3	NA's :3

```
R // RAW DATA: income.csv
```

```
1 print(summary(synthetic_data))
```

SYSTEM // EXPORT

Age	Income	Education	Gender
Min. :25.00	Min. :13413	PhD :29	Male :46
1st Qu.:36.00	1st Qu.:46234	Master :42	Female:54
Median :41.00	Median :51427	Bachelor :29	
Mean :42.18	Mean :52578	Highschool: 0	
3rd Qu.:56.00	3rd Qu.:55333		
Max. :57.00	Max. :73081		

Satisfaction	PurchaseAmount
Min. :1.130	Min. :115.8
1st Qu.:2.490	1st Qu.:192.5
Median :4.550	Median :216.3
Mean :5.199	Mean :203.3
3rd Qu.:7.730	3rd Qu.:223.4
Max. :9.690	Max. :247.5

6 Regression Analysis

- Use Purchase Amount as the outcome.
- Predictors: Age, Income, Satisfaction, Gender, Education
- Interpret Summary Results.

R // RAW DATA: income.csv

```

1 complete_regression <- lm(PurchaseAmount ~ Age + Income + Satisfaction + Gender
  ↵ + Education,
  ↵ data = complete_data)
2
3 print(summary(complete_regression))

```

SYSTEM // EXPORT

Call:
`lm(formula = PurchaseAmount ~ Age + Income + Satisfaction + Gender +
 Education, data = complete_data)`

Residuals:

Min	1Q	Median	3Q	Max
-40.471	-8.694	-1.920	7.119	37.842

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	2.267e+02	9.466e+01	2.395	0.0435 *
Age	-2.955e+00	1.304e+00	-2.267	0.0531 .
Income	-1.072e-03	8.618e-04	-1.244	0.2486
Satisfaction	1.924e+01	9.865e+00	1.950	0.0870 .
GenderFemale	9.231e+01	6.767e+01	1.364	0.2097
EducationMaster	-3.764e+01	2.518e+01	-1.495	0.1732
EducationBachelor	-6.761e+00	2.057e+01	-0.329	0.7508

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 27.89 on 8 degrees of freedom

Multiple R-squared: 0.7076, Adjusted R-squared: 0.4883

F-statistic: 3.226 on 6 and 8 DF, p-value: 0.06462

R // RAW DATA: income.csv

```

1 synthetic_regression <- lm(PurchaseAmount ~ Age + Income + Satisfaction + Gender
  ↵ + Education,
  ↵ data = synthetic_data)
2
3 print(summary(synthetic_regression))

```

SYSTEM // EXPORT

Call:

```
lm(formula = PurchaseAmount ~ Age + Income + Satisfaction + Gender +
   Education, data = synthetic_data)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-51.980	-16.896	1.313	17.227	44.868

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)		
(Intercept)	3.226e+02	1.993e+01	16.186	< 2e-16 ***		
Age	-6.077e-01	2.716e-01	-2.238	0.027622 *		
Income	-9.543e-04	2.239e-04	-4.262	4.86e-05 ***		
Satisfaction	-2.114e+00	1.748e+00	-1.210	0.229385		
GenderFemale	-3.860e+01	9.565e+00	-4.036	0.000112 ***		
EducationMaster	-2.751e+01	7.492e+00	-3.672	0.000401 ***		
EducationBachelor	-5.407e-01	7.205e+00	-0.075	0.940345		

Signif. codes:	0 '***'	0.001 '**'	0.01 '*'	0.05 '.'	0.1 ' '	1

Residual standard error: 23.98 on 93 degrees of freedom
 Multiple R-squared: 0.5131, Adjusted R-squared: 0.4817
 F-statistic: 16.34 on 6 and 93 DF, p-value: 9.215e-13

R // RAW DATA: income.csv

```
1 synthetic1000_data <- make_syndata(complete_data, methods, n_syn = 1000)
2 synthetic1000_regression <- lm(PurchaseAmount ~ Age + Income + Satisfaction +
  ↪ Gender + Education,
  ↪ data = synthetic1000_data)
3
4 print(summary(synthetic1000_regression))
```

SYSTEM // EXPORT

Call:

```
lm(formula = PurchaseAmount ~ Age + Income + Satisfaction + Gender +
   Education, data = synthetic1000_data)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-63.06	-14.83	0.91	16.08	60.76

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)		
(Intercept)	1.940e+02	6.274e+00	30.919	< 2e-16 ***		
Age	-2.415e+00	8.554e-02	-28.235	< 2e-16 ***		
Income	-3.455e-04	6.368e-05	-5.425	7.29e-08 ***		
Satisfaction	1.503e+01	5.390e-01	27.894	< 2e-16 ***		
GenderFemale	7.641e+01	2.773e+00	27.551	< 2e-16 ***		
EducationMaster	-3.862e+01	1.962e+00	-19.691	< 2e-16 ***		
EducationBachelor	-6.761e+00	2.324e+00	-2.910	0.0037 **		

Signif. codes:	0 '***'	0.001 '**'	0.01 '*'	0.05 '.'	0.1 ' '	1

Residual standard error: 24.13 on 993 degrees of freedom
 Multiple R-squared: 0.5944, Adjusted R-squared: 0.5919
 F-statistic: 242.5 on 6 and 993 DF, p-value: < 2.2e-16

```
R // RAW DATA: income.csv
1 pooled_model <- with(imputed_data, lm(PurchaseAmount ~ Age + Income +
  ↵ Satisfaction
  + Gender + Education))
2
3 pooled_results <- pool(pooled_model)
4 print(summary(pooled_results))
```

SYSTEM // EXPORT

	term	estimate	std.error	statistic	df	p.value
1	(Intercept)	240.780120493	1.158604e+02	2.0781912	6.128225	0.08196649
2	Age	-1.389056914	1.879782e+00	-0.7389457	2.194601	0.53089093
3	Income	-0.000924375	1.236205e-03	-0.7477523	4.092732	0.49528832
4	Satisfaction	9.459600865	1.231774e+01	0.7679656	3.269060	0.49417910
5	GenderFemale	28.879693995	7.892727e+01	0.3659026	3.114476	0.73788660
6	EducationMaster	-27.814550761	5.081571e+01	-0.5473613	2.229740	0.63400953
7	EducationBachelor	-3.169597038	3.072495e+01	-0.1031604	6.192437	0.92109579