BITTIGER

DS 501 Data scientist express bootcamp

Week 2 [Ella]

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Open ended question

- FB launched new UI for news feed, but user interaction decreases, what would you do?
  - How should we proceed?
  - What questions are expected from us?
- General interview tips
  - Ask clarifying questions
  - Show positive attitude, brainstorm
  - Read interviewers' feedback, understand what interviewer is looking for, build chemistry

How to deal with open ended question?

- Understand the context
  - Which metric decreases? What about other metrics?
  - How much is decreased? Significant?
  - How many users? Same users or different users?
- Understand the goal
  - 1. Do we want to **verify the new UI is the cause**?
  - 2. Do we want to understand why new UI has such impact?
- How to achieve the goal
  - 1. Randomized experiments
  - 2. Check other metrics, slice dice users into groups, and etc

Causal inference

- Causality has always been studied
  - Does a decision to smoke cigarettes increase the likelihood of a person getting lung cancer?
  - Does number of homeless increase crime rate?
- Definition of causal inference
  - Process of drawing a conclusion about a causal connection
- Causal inference v.s. (statistical) inference
  - (Statistical) inference is drawn from observation
  - Causal inference cannot drawn from observation, but from experiment.

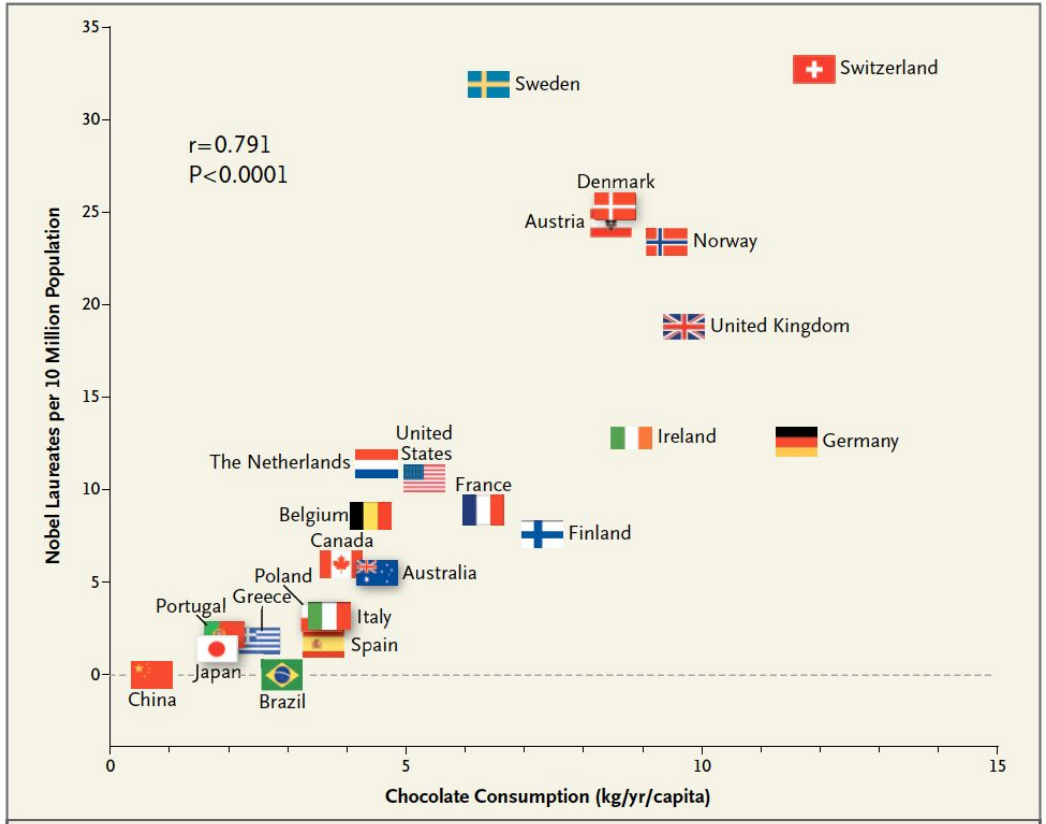
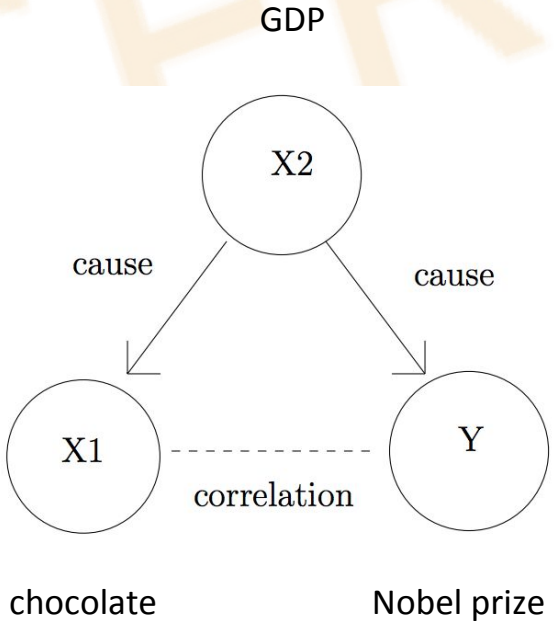


Figure 1. **Correlation between Countries' Annual Per Capita Chocolate Consumption and the Number of Nobel Laureates per 10 Million Population.**

Correlation is not causation

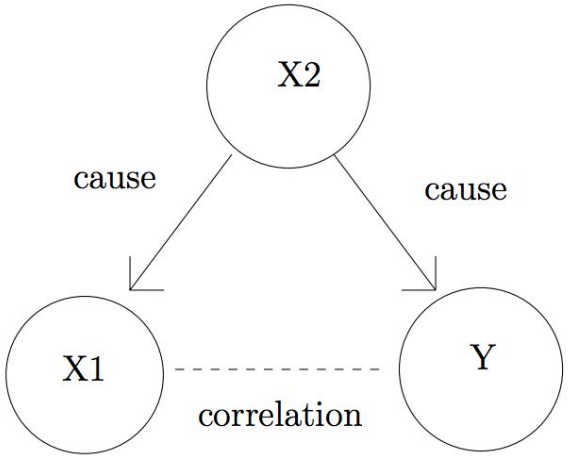
- Quote from chairman of Nobel chemistry committee
  - “Chocolate is a luxury. Wealthy individuals are more likely to be able to afford it.

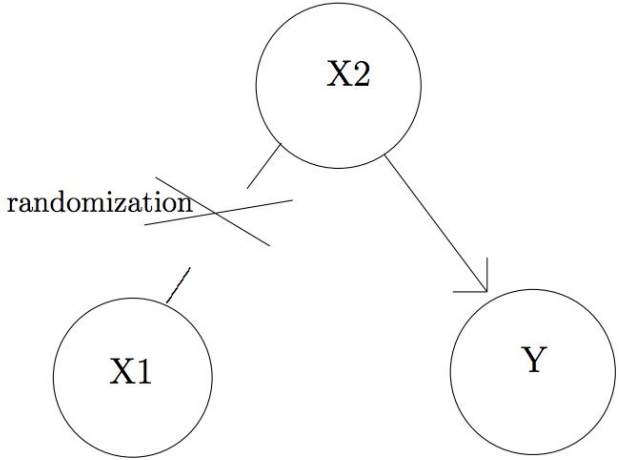
    Education is also a luxury. Poor people can’t afford to go to college for 10 years to get a PhD in chemistry. But you can’t win the Nobel prize in chemistry unless you’re a chemist. “
- Common factor
  - “GDP or wealth of a country will be correlated both with chocolate eating and with Nobel prizes. ”



Observational study v.s. Randomized experiment

- Observational studies can suggest good experiments to run, but can't definitively show causation.
- Randomization can eliminate correlation between X_1 and Y due to a different cause X_2(confounder).
  - FB example





A/B Testing

- Define the causal relationship to be explored
  - New UI decreases user interaction
- Define metric
  - Number of posts per user
- Design randomized experiments (A/B test)
  - Two groups of users, comparable
  - control group: old UI, experiment group: new UI
- Collect data and conduct **hypothesis testing**
  - Compare the metrics using two sample t test
- Draw conclusion

Hypothesis testing

- Definition
  - Use sample of data to test an assumption regarding a population parameter, which could be
    - A population mean μ
    - The difference in two population means, $\mu 1 - \mu 2$
    - A population variance
    - The ratio of two population variances
    - A population proportion p
    - The difference in two population proportions, $p1 - p2$
    - Three (or more!) means, $\mu 1$, $\mu 2$, and $\mu 3$

Hypothesis testing (cont'd)

- Definition
  - Two opposing hypotheses about a population
    - Null hypothesis, H_0 , is usually the hypothesis that sample observations result purely from chance.
    - Alternative hypothesis, H_1 or H_a, is the hypothesis that sample observations are influenced by some non-random cause.

Bet over flip coin, win $1 if head, what do you feel after playing 10 times and losing $10?

Flip coin example

1

$$X_1, \ldots, X_n \sim \text{Bernoulli}(p)$$

Given sample data

p is population statistic of our interest

2

$$H_0 : p = \frac{1}{2} \qquad H_1 : p \neq \frac{1}{2}.$$

Null hypothesis

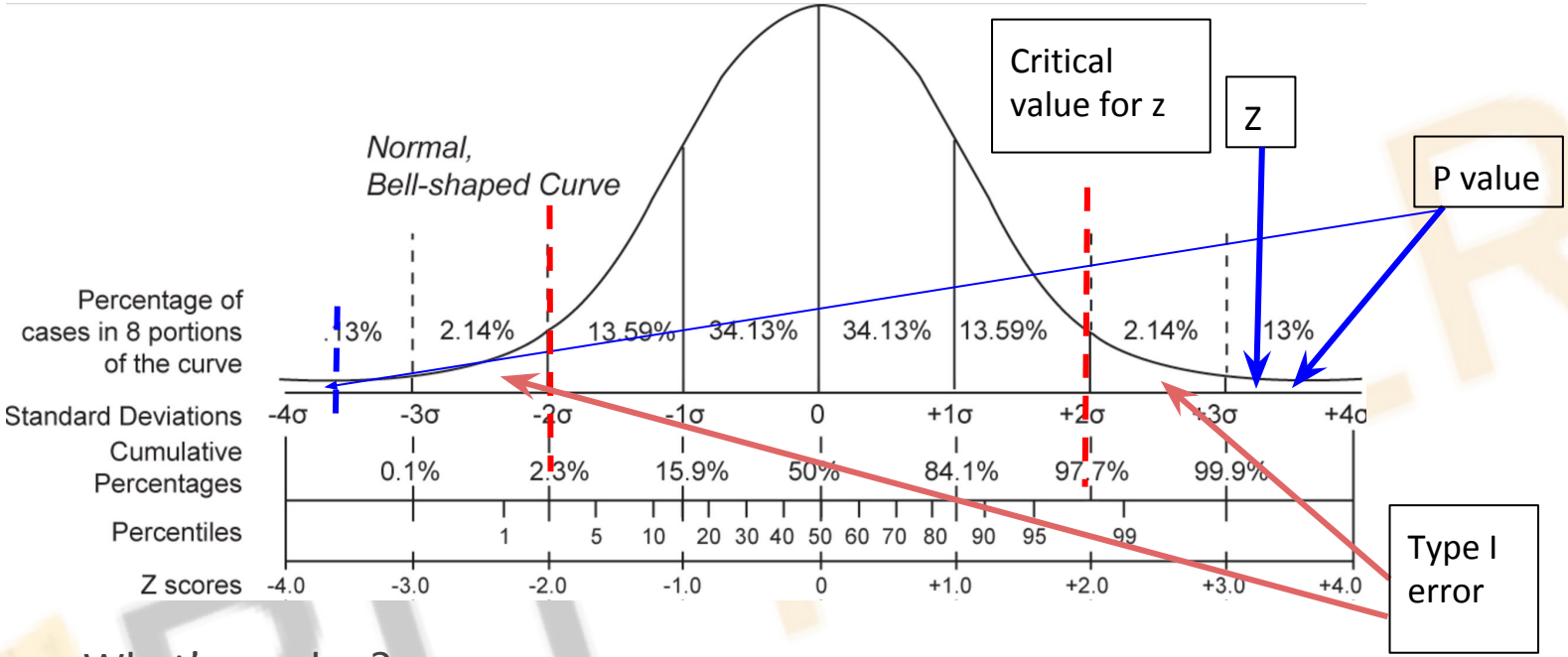
Alternative hypothesis

3 How to decide if we should reject or not reject null hypothesis?

- We say not reject (or retain) null hypothesis, but not say accept null hypothesis. Why?

How to decide reject or not reject?

- According to CLT, sample mean p = y/n = 1 following $N(\mu, \sigma^2/n)$
  - Population mean $p_0 = 0.5$, $\sigma^2/n = p_0(1 - p_0)/n$
  - So $z = (p - p_0)/\sqrt{(p_0(1 - p_0)/n)}$ following N(0, 1)
  - Is p so far from p_0 (or z too large) that true population mean p_0 is not 0.5 ? (when null hypothesis should be rejected.)
  - Two ways to decide
    - "Critical value" approach, compare z with critical value
    - "P value" approach, compare p value with threshold



- What's p value?
  - Assuming H_0 is true, what's the probability of observing a more extreme test stats in the direction of H_a
- No matter how large threshold is, there is still error of rejecting H_0 when we shouldn't (i.e. H_0 is true), type I error, α.
  - Type I error should be small, like 0.1, 0.05, 0.01.
  - Type I error is threshold for p value to compare with.
  - Type I error determines critical value for z score to compare with.

How to construct hypothesis testing

- Define H_0, H_a
- Decide which test statistics to be calculated, e.g., z score
  - Option 1

$$z = \frac{(\bar{x} - \mu)}{\sigma / \sqrt{n}}$$

    - Calculate z score and compare with critical value ($z^* = z_{1-\alpha/2}$)
    - If $|z| > z^*$, reject H_0; else do not reject H_0
  - Option 2
    - Calculate p value and compare with type I error (α)
    - If p is smaller, reject H_0; else do not reject H_0

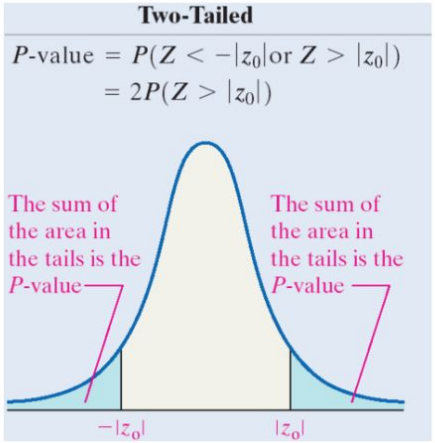
p value of z^* is α

- Other ways to define alternative hypothesis?

Different types of alternative hypothesis

- Two tailed v.s. one tailed

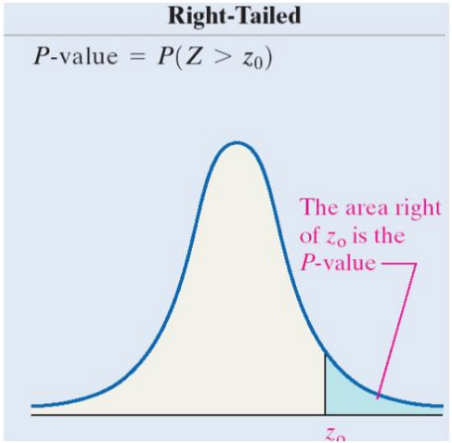
Two-tail



$H_0 : \mu = \mu_0$

$H_1 : \mu \neq \mu_0$

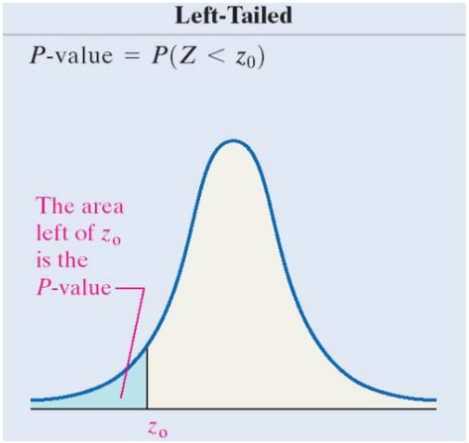
Right Tail



$H_0 : \mu = \mu_0$

$H_1 : \mu > \mu_0$

Left Tail



$H_0 : \mu = \mu_0$

$H_1 : \mu < \mu_0$

- When to use one sided test? Reading

Type I, II error, power

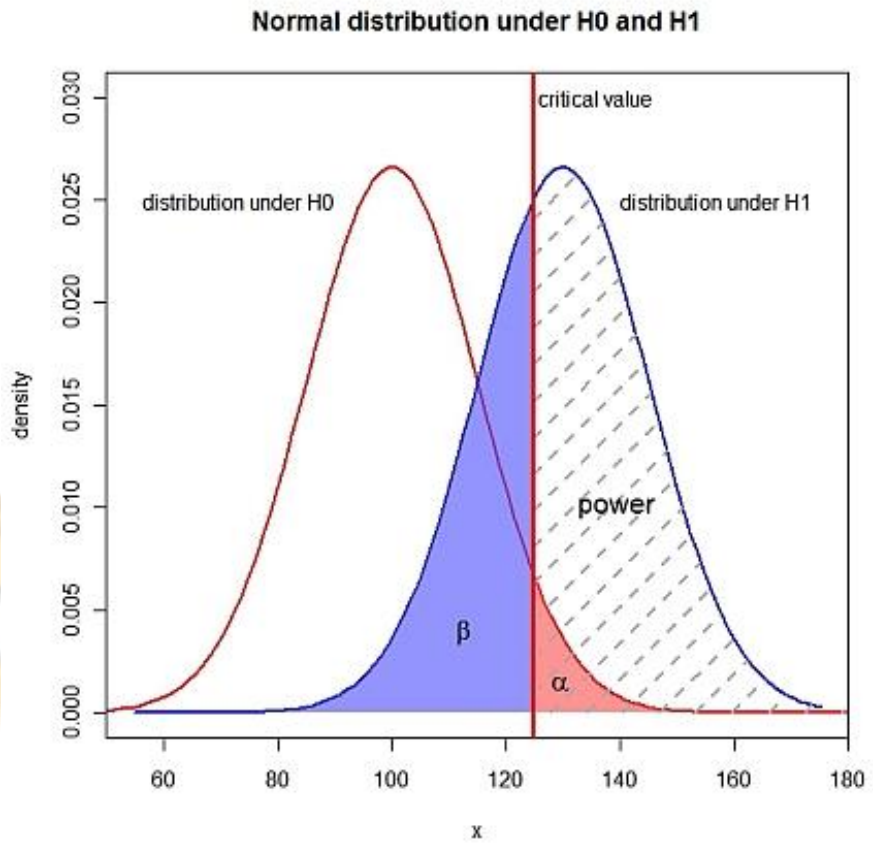
	Ground Truth	
Decision	H_0	H_a
Not reject H_0	Correctly (not reject null)	Type II error, β
Reject H_0	Type I error, α	Correctly (reject null), power

$$\alpha = P(reject\ H_0 \mid H_0)$$

$$\beta = P(not\ reject\ H_0 \mid H_a)$$

$$power = P(reject\ H_0 \mid H_a)$$

Type I, II error, power



- Power and power function
- Trade off between Type I error and Type II error
- How can we get small type I error and type II error?
- Sample size calculation

Difference between flip coin and FB example?

One sample v.s. Two sample tests

- One sample test
  - One population, compare test statistic, e.g, sample mean, with a known number
- Two sample test
  - Two populations, compare two population means
  - Paired test
    - Two dependent groups, for example, same group been measured at two different times.
    - Essentially one sample test
  - Unpaired test
    - Two independent groups, may have different sample sizes.

Z test v.s. T test

- Hypothesis test for the population mean
  - If population variance σ^2 is known, z test
  - If the population variance σ^2 is unknown (most of the time), or n is small, t test
    - Consider CLT
    - T score calculation, same as z score
    - Only that critical value $t^* = t_{1 - \alpha;\, n - 1}$ from t distribution, not normal distribution.
- One sample t test
- Two sample t test
  - Paired
  - Unpaired

Two sample t-test

- Compares the means of the two groups of data
  - X_1 random sample from $N(\mu_1, \sigma_1^2)$

    X_2 random sample from $N(\mu_2, \sigma_2^2)$
  - $H_0 : \mu_2 = \mu_1 \quad H_a : \mu_2 != \mu_1$ (other H_a?)

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{Var(\bar{x}_1 - \bar{x}_2)}} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{Var(\bar{x}_1) + Var(\bar{x}_2)}} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

  - When $\sigma_1^2 = \sigma_2^2$

$$S^2_{\text{pooled}} = \frac{\sum_{k=1}^{n_1}(X_{1,k} - \bar{X}_1)^2 + \sum_{k=1}^{n_2}(X_{2,k} - \bar{X}_2)^2}{n_1 + n_2 - 2} = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

Student t test v.s. Welch t test

- If population variance from two samples are equal, use pooled variance (student t test)

$$t^* = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \qquad s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} \qquad df = n_1 + n_2 - 2$$

- If population variance from two samples are not equal, use unpooled variance (Welch t test)

$$t^* = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \qquad df = \frac{(n_1-1)\cdot(n_2-1)}{(n_2-1)C^2+(1-C)^2(n_1-1)}$$

$$C = \frac{s_1^2/n_1}{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$

Assumptions of t test

- Student t test
  - Normality
  - Independence
  - Equal variance (two sample test)
- What if normality is violated
  - Just do it (CLT)
  - Transformation
  - Other nonparametric methods
- What if equal variance is violated
  - Welch t test, preferred over student t test reading

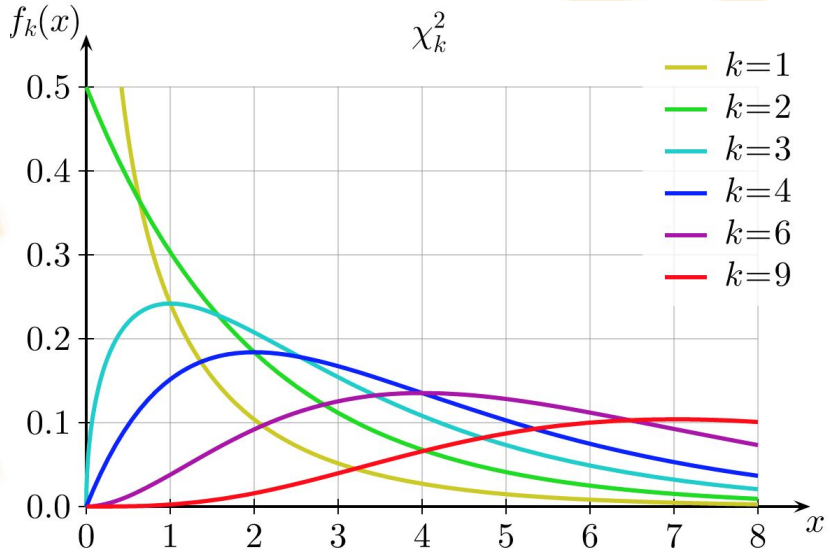
Test about variance

Chi square distribution

- Definition
  - Sum of square of k standard normal random variables, N (0, 1)

$$\chi^2(k) = \sum_{i=1}^{k} Z_i^2$$

  - k-1 degree of freedom



- Chi square test for one variance

$H_0 : \sigma^2 = \sigma_0^2$ $\quad H_A : \sigma^2 \neq \sigma_0^2, \quad H_A : \sigma^2 < \sigma_0^2,$ or $H_A : \sigma^2 > \sigma_0^2$

$$\chi^2 = \frac{(n-1)S^2}{\sigma_0^2}$$

What can Chi-square test be used for?

- One sample test
  - A population variance
  - Compare categorical variable with known distribution (goodness of fit)
  - Null and alternative hypothesis?
- Two sample test
  - Compare two population variance
  - Compare two categorical variables to test if they have different distribution.
  - Chi-square independence test
  - Null and alternative hypothesis?

Chi square test for goodness of fit

- Test if X follows certain distribution F, X is categorical var
  - H_0: X follows F
    H_a: X does not follow F
  - Calculate (chi-square) statistics
    - Recall t/z score, which measures the (normalized) distance between observed stats and expected stats given H_0 is true
    - Similarly, $chi\ square = \sum_{all\ cells} \frac{(observed - expected)^2}{expected}$
    - Flip coin example

	head	tail
observed	10	0
expected	5	5

$$\chi^2 = \frac{(10-5)^2}{5} + \frac{(5-0)^2}{5} = 10$$

Critical value chart

Connection between binomial, normal, chi square

- N trial, m success, p is success rate, q = 1 - p

	success	fail
observed	m	N-m
expected	Np	Nq

$$\chi^2 = \frac{(m - Np)^2}{(Np)} + \frac{(N - m - Nq)^2}{(Nq)}$$

$$\chi^2 = \frac{(m - Np)^2}{(Npq)}$$

$$N = Np + N(1 - p)$$
$$N = m + (N - m)$$
$$q = 1 - p$$

$$\chi = \frac{m - Np}{\sqrt{(Npq)}}$$

CLT

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

O_i = the number of observations of type *i*.
$E_i = N * p_i$, the expected frequency of type *i*,
n = the number of cells in the table.

Chi square test for independence

- Test if X, Y are independent
  - H_0: X and Y are independent.
    H_a: X and Y are dependent.
  - Example
    - Two types of bidder, human and computer. Different categories of bidding product. Test if bidder type is independent with product category.

$$chi\ square = \sum_{all\ cells} \frac{(observed - expected)^2}{expected}$$

How to calculate the expected stats?

Chi square test (cont'd)

	Auto parts	Books music	cloth	computer	furniture	home goods	jewelry	mobile	Office equip	Sporting goods	**Row sum**
Human	9757	13733	476	9733	87807	389249	555634	492350	160671	939398	2658808
Robot	0	1509	0	11667	0	18708	37101	105138	7967	230326	412416
column sum	9757	15242	476	21400	87807	407957	592735	597488	168638	1169724	

X_1 → Human

Y_1 → Robot

Col sum_1 → column sum

Row sum_1

Row sum_2

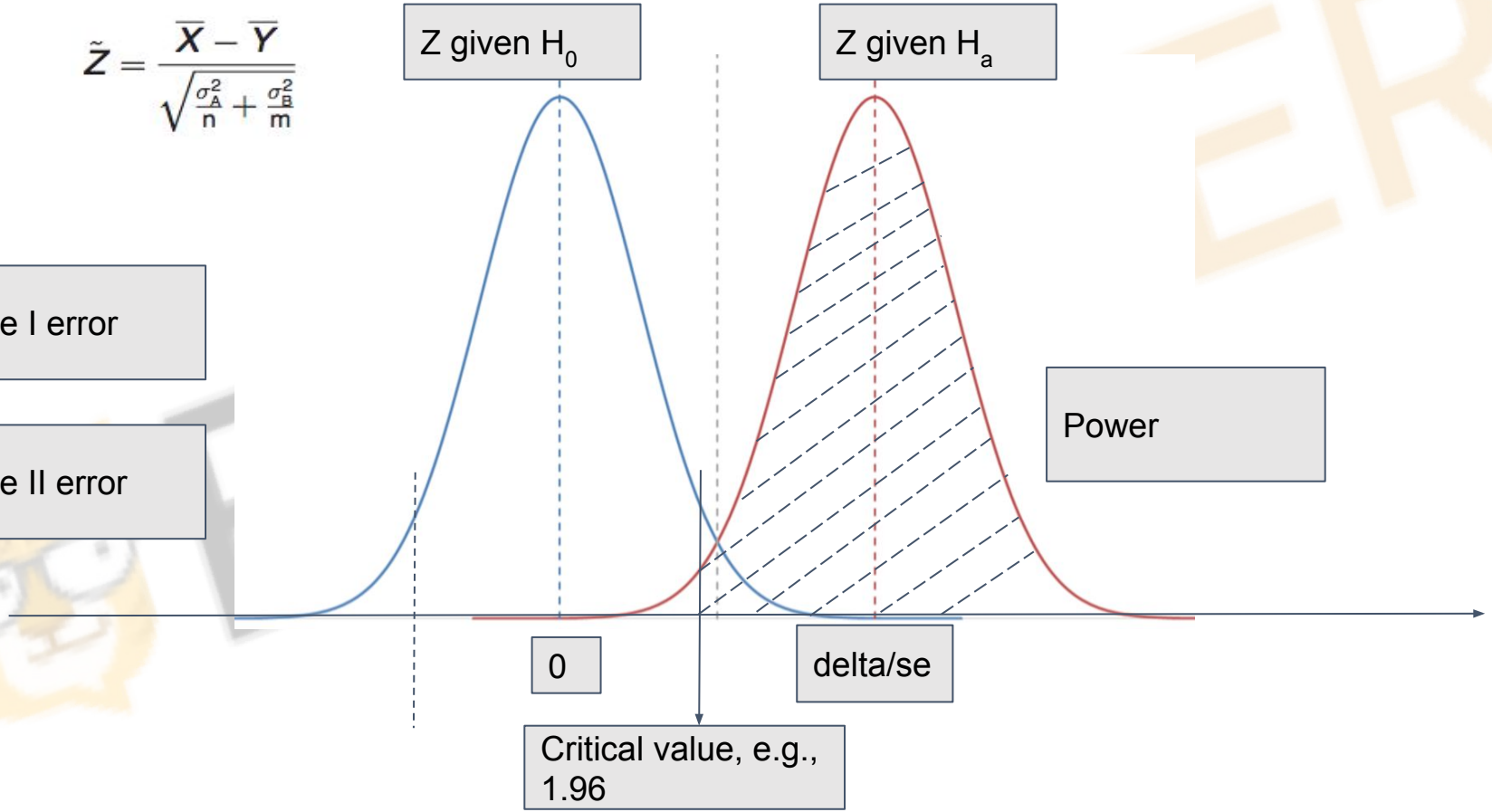
- If independent
  - $X_1 : Y_1$ = row sum_1 : row sum_2
  - $X_1 + Y_1$ = col sum_1
  - Expected value X_1 = col sum_1 * row sum_1 / (row sum_1 + row sum_2)

$$\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$

df: (row num - 1) (column num - 1)

One more word about power

$$\tilde{Z} = \frac{\overline{X} - \overline{Y}}{\sqrt{\frac{\sigma_A^2}{n} + \frac{\sigma_B^2}{m}}}$$



Power analysis (cont'd)

- Larger type I error (alpha), greater power
- Larger delta/se, greater power
  - Large delta
  - Smaller standard error
- Larger sample size (n, m), greater power
  - Mostly influence n, m to control power
- Check the relationships in R

- Suppose we want power = 0.8 to detect a particular value of $\mu_A - \mu_B = \Delta$ with known σ_A^2, σ_B^2, calculate n
  - $\Delta = 1$
  - $\sigma_A^2 = \sigma_B^2 = 1$
  - n = m

HW1

- Calculate n
- Check how n change along with Δ ?
- Check how n change along with σ_A^2 ?

HW3 Feature engineering

- How would you transform existing features if needed? Any new features could be generated?

- A new casino game involves rolling 3 dice. The winnings are directly proportional to the total number of sixes rolled. Suppose a gambler plays the game 101 times, with the following observed counts:

Number of Sixes	Number of Rolls
0	48
1	35
2	15
3	3

HW2: test if this is fair dice.

- What test to use?
- Calculate stats and p value