Power Analysis for ANOVA Designs

Whereas power analysis for simple designs (e.g., a *t*-test or correlation) is relatively straightforward, performing an a-priori power analysis for ANOVA designs is more complex. Although the basic principles remain the same, to understand how to design well-powered ANOVA designs it is important to realize what impacts the effect size for an ANOVA. In my experience, it is often a good idea to think about the data you expect to observe when performing a power analysis (e.g., the means, standard deviations, and correlations between variables), but this is especially true for power analysis for ANOVA designs.

Another issue with power analyses for ANOVA designs is that software that is available is not that intuitive to use. Although G*Power is excellent software, it requires a lot of expertise to use, and is not that transparent about the input it requires. Together with Aaron Caldwell I have created an R package and Shiny App that lets you do simulation-based power analysis for ANOVA designs. We will use this package to get a better understanding of power analysis, and power analysis for ANOVA designs specifically. The basic approach is to first specify the design, and then to simulate datasets from these specifications, perform the planned statistical test on these datasets, and compute how many results are statistically significant.

You will notice that one of the greatest challenges in simulation based power analyses is coming up with values that you should expect. Despite this difficulty, I believe it is far more informative to perform power analyses based on means, standard deviations, and correlations, than just saying you will expect a 'medium' effect size. Indeed, one of the main lessons in this assignment should be that effect sizes in ANOVA designs are difficult to predict, and specifying the pattern you expect is a better way to do power analysis.

In completely novel lines of research where you have no good expectations, you might want to start with **sequential analyses**, or first perform a pilot study and estimate the standard deviation of your measure. Often, related studies have been done and you can use values from these studies to estimate parameters in the power analysis, perhaps being somewhat conservative. It is recommended to use validated measures, and for such measures there will often be some information available about standard deviations in similar populations.

You can use this Shiny app: http://shiny.ieis.tue.nl/anova_power/ for most parts of the assignment, and to perform power analyses in the future.

Simulation-based power analysis

There are two ways to calculate the statistical power of an ANOVA design based on simulations. The first is to **repeatedly simulate data** for each condition based on the means, sample size, standard deviation, and correlation, under specific statistical assumptions (i.e., normally distributed data). The ANOVA_power function allows you to perform power analyses based on repeatedly simulating normally distributed data. A second approach is to simulate a dataset that has *exactly* the desired properties - every cell of the design has n datapoints that have the desired mean and standard deviation, and correlation between groups (for a within design). By performing an ANOVA on this dataset, we can calculate the required statistics from the ANOVA result to calculate the statistical power. The ANOVA_exact function allows you to calculate power based on this approach. The ANOVA_power function is a bit more flexible (e.g., we can use different ways to correct for multiple comparisons, or in future versions of the package allow you to simulate more realistic datasets that are not normally distributed), but the ANOVA_exact function is much faster (and takes seconds, instead of minutes or hours for a large number of simulations).

An Example: 2 by 2 Mixed Design

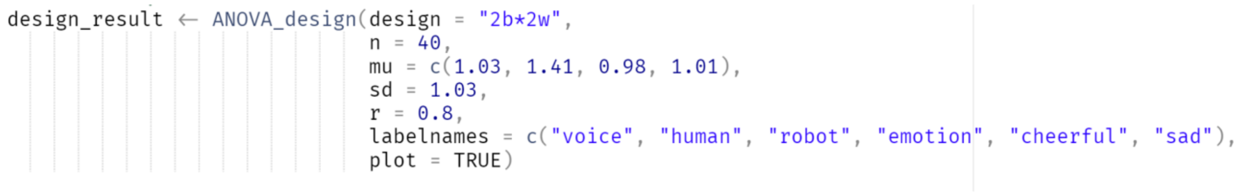
Imagine you plan to perform a study in which participants interact with an artificial voice assistant that sounds either like a human or like a robot, and who sounds either cheerful or sad. In the example below, 1000 simulations for a 2*2 mixed design (first factor, voice, is manipulated between participants, the second factor, emotion, is manipulated within participants) are performed. The sample size is 40 in each between subject condition (so 80 participants in total), the assumed population standard deviation is 1.03, the correlation for the within factors is 0.8, and the means are 1.03, 1.41, 0.98, 1.01. No correction for multiple comparisons is applied. The alpha level used as a significance threshold is set to 0.05 for this simulation.

A more detailed explanation of the functions in the Superpower package is provided at the end of the document (and in the Superpower help files in R). As a short summary, the **ANOVA_design** function requires specifying a design, the sample size (n), the pattern of means (mu), the standard deviations (sd), the correlation between dependent variables (r), and labelnames if you wish. The simulation functions (**ANOVA_power** which is slow but more flexible, and **ANOVA_exact** which is fast but cannot correct for multiple comparisons) then simulate data based on these designs and compute power. As we see more R packages and Shiny apps emerge that can perform simulation-based power analyses for users, without requiring users to program the simulation themselves, it is worth pointing out that **learning how these packages work will require time**. The more complex the designs that power is calculated for, the more time is needed to figure out how to enter the required input and perform the simulation. Nevertheless, this should be a lot more efficient than programming all these simulations from scratch yourself!

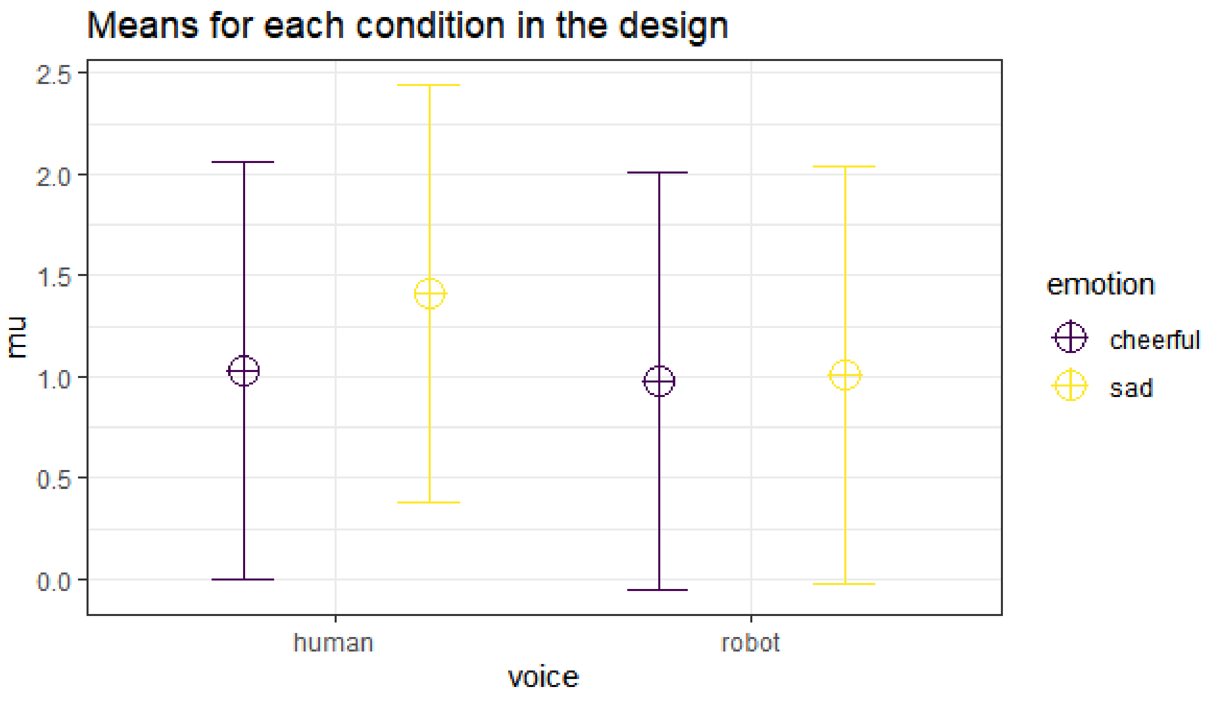
Open the ANOVA_power_analysis.R script, and **run the first 17 lines** to 1) install Superpower from GitHub, and 2) install required R packages and load them. **NOTE 1: You might get a pop-up that asks you "Would you like to use a personal library instead?"** If so, click 'YES'. If you run into issues, remember you can complete most of this exercise using the online shiny app.

NOTE 2: It makes sense to first update all packages by going to the 'packages' tab in RStudio (or in the menu go to Tools > Check for package updates), and clicking the 'Update' button. Updating all packages could take some time, depending on how many packages you have installed, and how long ago you updated them.

Run lines 21 to 27 to execute the ANOVA_design function and generate a plot of the means. Do not yet perform the power analysis with the code on line 30 – let's first examine our intuitions about the effect sizes we should expect based on the design we specified.



```
design_result ← ANOVA_design(design = "2b*2w",
                             n = 40,
                             mu = c(1.03, 1.41, 0.98, 1.01),
                             sd = 1.03,
                             r = 0.8,
                             labelnames = c("voice", "human", "robot", "emotion", "cheerful", "sad"),
                             plot = TRUE)
```



Q1: Look at the figure above. The means are 1.03, 1.41, 0.98, 1.01, and the emotion variable is manipulated within, and the voice variable between. The standard deviation for each condition is 1.08 and the correlation between dependent variables 0.8. **We will check if you**

are correct later, and it is ok if this is very difficult to get right – indeed, that is the main lesson! Which statement do you think is true?

A) The main effect of emotion is larger than the voice*emotion interaction, which is larger than the main effect of voice.

B) The main effect of voice is larger than the voice*emotion interaction, which is larger than the main effect of emotion.

C) The voice*emotion interaction is larger than the main effect of voice, which is larger than the main effect of emotion.

D) The voice*emotion interaction is larger than the main effect of emotion, which is larger than the main effect of voice.

Run line 30 to perform the power analysis and check your answer to the previous question. The result for the power simulation is printed and has two sections (which can be suppressed by setting verbose = FALSE). The **first table** provides power (from 0 to 100%) and effect sizes (partial eta-squared) for the **ANOVA** result. We see the power for the main effects of factor voice (17.43%), emotion (79.39%), and the voice*emotion interaction (66.03%). Was your intuition correct? It is very likely that is was very difficult to answer the previous question. Hopefully simulating different ANOVA designs in this assignment will boost your intuition!

```
# Simulate data using ANOVA_exact function and print results
ANOVA_exact(design_result)

Power and Effect sizes for ANOVA tests
              power  partial_eta_squared    cohen_f        non_centrality
voice         17.43               0.0134    0.1166         1.0604
emotion       79.39               0.0922    0.3187         7.9225
voice:emotion 66.03               0.0689    0.2721         5.7734

Power and Effect sizes for contrasts
                                                           power effect_size
p_voice_human_emotion_cheerful_voice_human_emotion_sad      94.92      0.5833
p_voice_human_emotion_cheerful_voice_robot_emotion_cheerful  5.53     -0.0485
p_voice_human_emotion_cheerful_voice_robot_emotion_sad       5.08     -0.0194
p_voice_human_emotion_sad_voice_robot_emotion_cheerful      45.39     -0.4175
p_voice_human_emotion_sad_voice_robot_emotion_sad           40.35     -0.3883
p_voice_robot_emotion_cheerful_voice_robot_emotion_sad       5.93      0.0461
```

The result for the power simulation reveals power is highest for the main effect of emotion, which is the within-subjects factor, and for which the means are highly correlated (0.8). Power is higher for strongly positively correlated (*r* > 0.5) within comparisons than uncorrelated between participants variables. Power is lower for the interaction, and very low for the between participant main effect of voice.

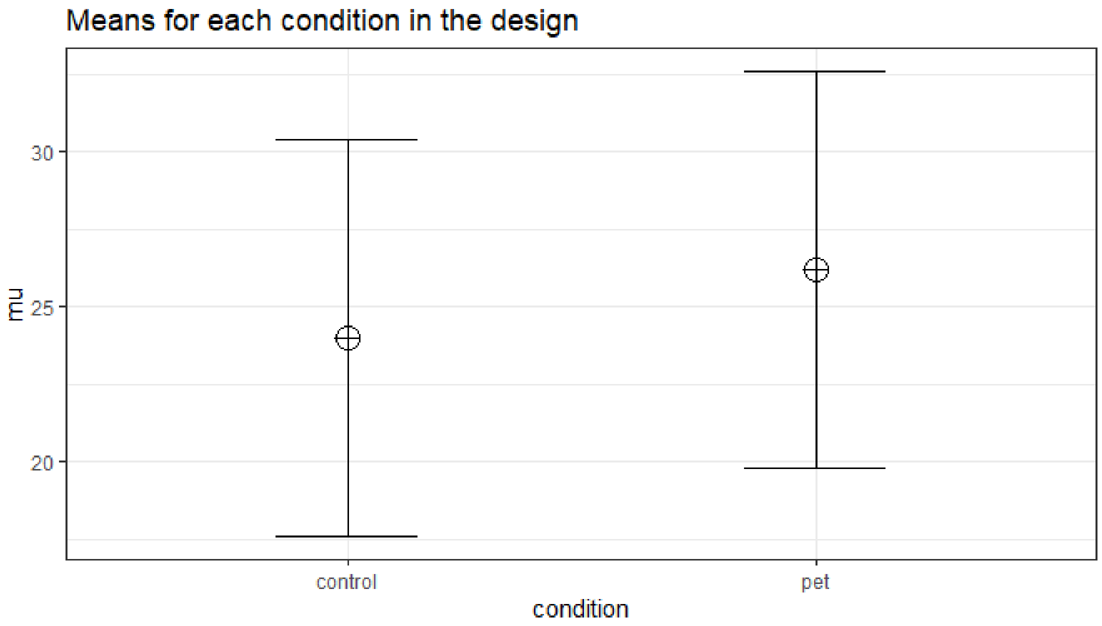
An ANOVA is typically followed up with simple comparisons. A statistical hypothesis often predicts not just an interaction, but also **the shape of an interaction**. For example, when looking at the plot of our design above, we might be specifically interested in comparing the independent effect for the cheerful vs. sad human voice assistant, and the difference between sad robot and sad human voice assistants. The **second table** *(Power and Effect sizes for contrasts)* provides the power for *t*-tests for all comparisons (94.92%, 5.53%, 5.08%, 45.39%, 40.35%, and 5.93%), and the effect sizes (Cohen's d for between-subject contrasts, and Cohen's d_z for within-subject contrasts, see Lakens, 2013). Power is relatively high for the within-participant comparison for human sad and human cheerful voices (94.92%). Even though the mean difference is almost the same as for the comparison between a sad-human voice assistant and a sad-robot voice assistant, since this latter comparison is a between-participants comparison, power is much lower (40.35%).

One take-home message is that **it is a good idea to manipulate variables within participants whenever possible**.

Power in a Two Group One-Way ANOVA

Imagine we aim to design a study to test the hypothesis that giving people a pet will increase their life satisfaction. We have an **experimental condition** where people who have no pet get one, and a **control condition** where people who have no pet do not get one, and randomly assign participants to either condition. We expect pets to increase life-satisfaction compared to the control condition. Based on work by Pavot and Diener (1993) we believe that we can expect responses on the life-satisfaction scale to have a mean of approximately 24 in our population, with a standard deviation of 6.4. We expect having a pet increases life satisfaction by approximately 2.2 scale points for participants who get a pet (so a mean score of 26.2). We plan to collect data from 200 participants in total, with 100 participants in each condition. We examine the statistical power our design would have to detect the differences we predict. **Run lines 34 to 41**.

```r
design_result <- ANOVA_design(design = "2b",
                   n = 100,
                   mu = c(24, 26.2),
                   sd = 6.4,
                   labelnames = c("condition", "control", "pet"),
                   plot = TRUE)
ANOVA_exact(design_result, alpha_level = 0.05)
plot_power(design_result, min_n = 10, max_n = 250, plot = TRUE)
```



```
Power and Effect sizes for ANOVA tests
          power    partial_eta_squared    cohen_f    non_centrality
condition 67.69    0.029                  0.1727     5.9082
```

```
Power and Effect sizes for contrasts
                                  Power     effect_size
p_condition_control_condition_pet 67.69          0.3437
```

The result of the simulation shows that we have exactly the same power for the ANOVA, as we have for the *t*-test for the simple comparison (67.69%). This **must be so**, because when there are only two groups, these tests are mathematically identical. In a study with 100 participants per condition, based on the parameters we have specified, we would have quite low power. For our example, Cohen's d (the standardized mean difference) is 2.2/6.4, or d = 0.3437 for the difference between the control condition and having pets. We see this power value in the section reporting the results for contrasts. The effect size for the ANOVA is a Cohen's f of 0.1727, which is exactly half as large as Cohen's d in a two-group between participant design. We can use this effect size to compute the expected power for the simple comparison using the pwr package.

```r
library(pwr)
pwr.t.test(d = 2.2/6.4,
           n = 100,
           sig.level = 0.05,
           type="two.sample",
           alternative="two.sided")$power
```

```
## [1] 0.6768572
```

The results match our simulation. We can also use the pwr.anova.test function, which yields the same result as the power.t.test.

```
pwr.anova.test(n = 100,
               k = 2,
               f = 0.171875,
               sig.level = 0.05)$power
```

```
## [1] 0.6768572
```

This analysis tells us that running the study with 100 participants in each condition is quite likely to *not* yield a significant test result, even if our expected pattern of differences is true (because the Type 2 error rate is 32.31%). This is clearly not optimal. If we perform a study, we would prefer the statistical power to be higher.

Q2: How many participants do we need for our design to have 90% power (rounded to the nearest digit)? Increase the number of participants in the ANOVA_design code until, based on the specified design, 90% power is reached.

A: 166 participants per condition.

B: 173 participants per condition.

C: 179 participants per condition.

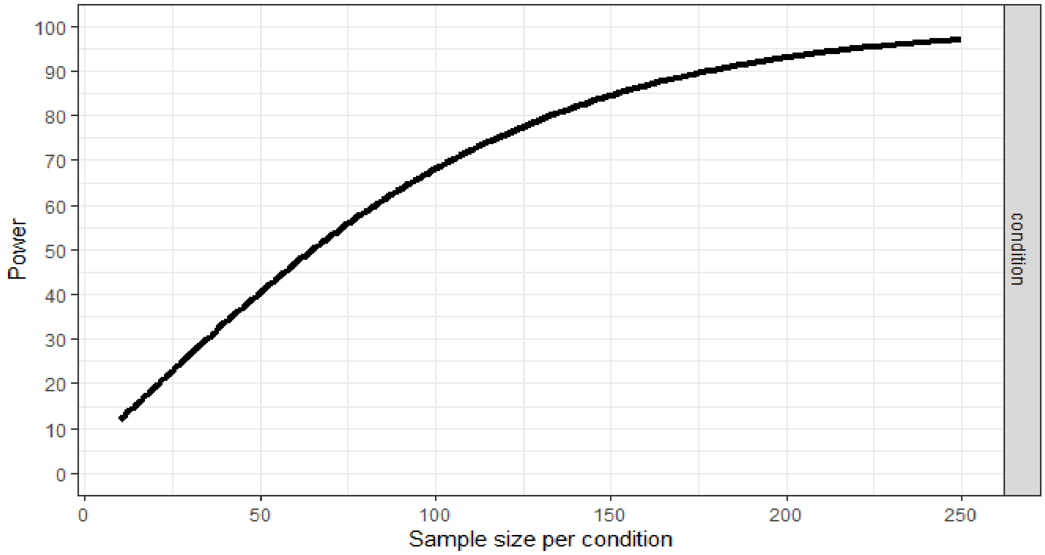
D: 187 participants per condition.

The plot_power function

Simulation based power analyses require you to increase the sample size until power is high enough to reach your desired Type 2 error rate. To facilitate this trial and error process you can use the plot_power function to plot the power across a range of sample sizes to produce a power curve.

The plot_power function simulates power for the specified ANOVA design up to a sample size of max_n using the ANOVA_exact function. Although it is relatively fast, with large sample size it can take some time to produce. **Run the code on line 44 to create the plot.** In the figure below we can easily see that, assuming the true pattern of means and standard deviations represents our expected pattern of means and standard deviations, we have 80% power around 135 participants per condition, 90% power around 180 participants per condition, and 95% power around 225 participants per condition.

```
plot_power(design_result, min_n = 10, max_n = 250, plot = TRUE)
```



Because the true pattern of means is always unknown, it is sensible to examine the power across a range of scenarios. For example, if the difference in means is somewhat smaller, because the means are 24 and 26 instead of 24 and 26.2, we can compare the power curve above with the power curve for a slightly less optimistic scenario (where the true effect size is slightly smaller). **Adjust the mu (means) from 24, 26.2 to 24, 26**. Run the code.

```
design_result <- ANOVA_design(design = "2b",
                   n = 100,
                   mu = c(24, 26),
                   sd = 6.4,
                   labelnames = c("condition", "control", "pet"),
                   plot = TRUE)
ANOVA_exact(design_result, alpha_level = 0.05)
plot_power(design_result, max_n = 250, plot = TRUE)
```

Q3: If the expected mean difference is 2 instead of 2.2, and looking at the power plot generated by the code, at which sample size per condition do we achieve 90% power?

A: around approximately n = 150.

B: around approximately n = 190.

C: around approximately n = 220.

D: around approximately n = 250.

It could be that in addition to a slightly smaller effect size, the standard deviation is slightly larger than we expected as well. This will reduce the power even further, and is thus an even less optimistic scenario. Let's assume the true standard deviation is 6.8 instead of 6.4, in addition to the smaller mean difference examined in question 3 above. Run the code:

```r
design_result <- ANOVA_design(design = "2b",
                   n = 180,
                   mu = c(24, 26),
                   sd = 6.8,
                   labelnames = c("condition", "control", "pet"),
                   plot = TRUE)
ANOVA_exact(design_result, alpha_level = 0.05)
plot_power(design_result, min_n = 10, max_n = 250, plot = TRUE)
```

Q4: If the expected mean difference is 2 and in addition the standard deviation is 6.8 instead of 6.4, and looking at the power plot generated by the code, at which sample size per condition do we achieve 90% power?

A: around approximately n = 190

B: around approximately n = 220.

C: around approximately n = 245.

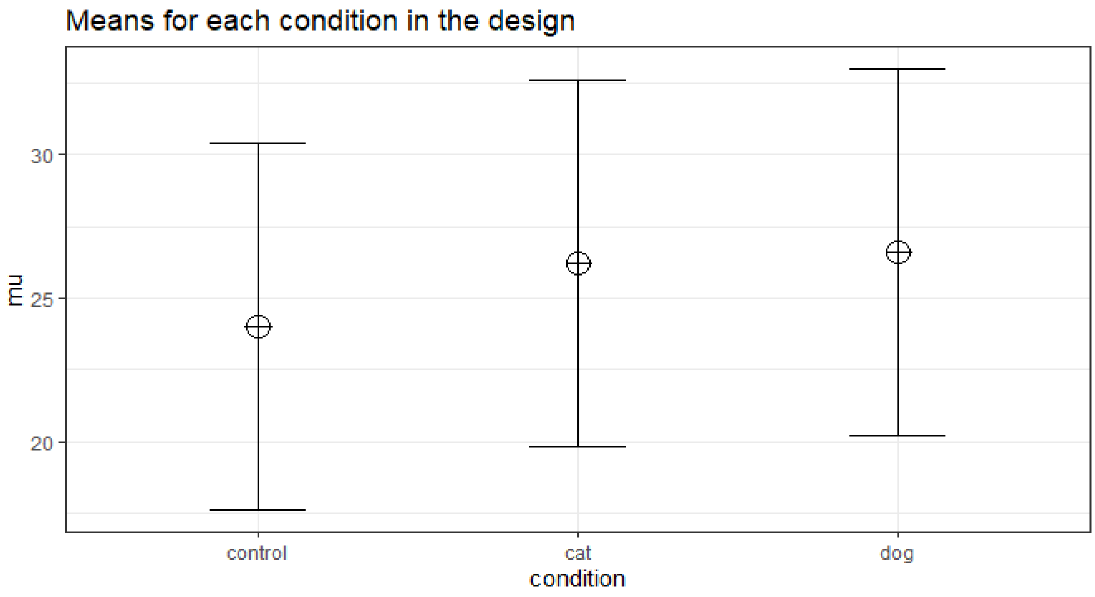
D: the graph does not reach 90% power with n = 250, so > 250 per group.

As these different plots make clear, your study never really has a **known** statistical power. Because the true effect size (i.e., the pattern of means and standard deviations) is unknown, the true power of your study is unknown. A study has 90% power *assuming a specific effect size*, but if the effect size is different than what you expected, the true power can be either higher or lower. We should therefore always talk about the 'expected' power when we do an a-priori power analysis, and provide a good justification for our expectations (i.e., for the expected pattern of means, standard deviations, and correlations). Performing **sensitivity analyses** (like the plots above) is recommended.

Three between subject conditions

Let's move from two conditions to three conditions – since an ANOVA was developed as an extension of a *t*-test to situations when there are more than two groups. Imagine we aim to design a study to test the hypothesis that giving people a pet to take care of will increase their life satisfaction. We have a control condition, a 'cat' pet condition, and a 'dog' pet condition. We can simulate a One-Way ANOVA with a specified alpha level, sample size, and effect size, and see the statistical power we would have for the ANOVA and the follow-up comparisons. We expect all pets to increase life-satisfaction compared to the control condition. Obviously, we also expect the people who are in the 'dog' pet condition to have even greater life-satisfaction than people in the 'cat' pet condition. Based on work by Pavot and Diener (1993) we again assume that we can expect responses on the life-satisfaction scale to have a mean of approximately 24 in our population, with a standard deviation of 6.4. We expect having a pet increases life satisfaction with approximately 2.2 scale points for participants who get a cat, and 2.6 scale points for participants who get a dog. We initially consider collecting data from 150 participants in total, with 50 participants in each condition. But before we proceed with the data collection, we examine the statistical power our design would have to detect the differences we predict. You can use the code in the assignment to look at this new problem. Run all the code that is part of **# Question 5 and 6**.

```
design_result <- ANOVA_design(design = "3b",
                   n = 180,
                   mu = c(24, 26.2, 26.6),
                   sd = 6.4,
                   labelnames = c("condition", "control", "cat", "dog"),
                   plot = TRUE)
ANOVA_exact(design_result, alpha_level = 0.05)
plot_power(design_result, min_n = 10, max_n = 250, plot = TRUE)
```



```
ANOVA_exact(design_result)

## Power and Effect sizes for ANOVA tests
##           power partial_eta_squared cohen_f non_centrality
## condition 47.69              0.0315  0.1804         4.7852
##
## Power and Effect sizes for contrasts
##                                   power effect_size
## p_condition_control_condition_cat 39.83      0.3437
## p_condition_control_condition_dog 52.05      0.4063
## p_condition_cat_condition_dog      6.10      0.0625
```

The result shows that you would have quite low power with 50 participants, both for the overall ANOVA (47.69%), as for the follow up comparisons (approximately 40% power for the control vs cat condition, just above 50% for the control vs dogs condition), and a really low power (around 6%, just above the Type 1 error rate of 5%) for the expected difference between cats and dogs. For our example, Cohen's d (the standardized mean difference, or the mean difference divided by the standard deviation) is 2.2/6.4, or d = 0.34375 for the difference between the control condition and cats, 2.6/6.4 of d = 0.40625 for the difference between the control condition and dogs, and 0.4/6.4 or d = 0.0625 for the difference between cats and dogs as pets. Especially this last effect size is incredibly small, and would require a very large sample size to detect.

It is clear we would need to collect more participants in each group to increase our power. Let's plot a power curve across a range of sample sizes to see how much we need to increase the sample size to have sufficient power for the ANOVA.

```
plot_power(design_result, min_n = 10, max_n = 250, plot = TRUE)
```

Q5: For the specified design above, and looking at the power plot generated by the code, at which sample size per condition do we achieve 90% power?

A: around approximately n = 120.

B: around approximately n = 135.

C: around approximately n = 175.

D: around approximately n = 200.

We should also check the power for the comparison of the control condition and the cat condition.

Q6: Enter the correct answer for question 5 as the sample size (replacing the n = 50 in line 49 – so we are simulating power for a study with a different sample size per group). What statement is true if we use an alpha level of 0.05 for each of the three simple comparisons?

A) We have more than 80% power for all three comparisons.

B) We have more than 80% power when comparing the dog condition against the control condition but for the other two simple comparisons, power is too low.

C) We have more than 80% power when comparing the dog condition against the control condition and when comparing the cat condition against the control condition, but for comparison between dogs and cats power is too low.

D) We have less than 80% power for all three comparisons.

Note that if you want to correct for three multiple comparisons, the code below (which is based on a slower simulation, but allows you to correct for multiple comparisons using the holm procedure) will show power is even lower after adjusting the alpha level for multiple comparisons.

```
ANOVA_power(design_result,
            alpha = 0.05,
            nsims = 1000,
            p_adjust = "holm"))
```

Power for the ANOVA might be 90%, but power for the simple comparison can be much lower. In case you are interested: We would need 5380 participants to have 90% power for the comparison between people who received a dog versus a cat as a pet! This is possible if you can easily collect data, but might make this specific question impossible to reliably answer if data collection is costly. Realizing you can't reliably study a question is an important aspect of a-priori power analysis.

Q7: What is the relation between the power for the main effect in a One-Way ANOVA, and the power for planned comparisons between all possible combinations of the 2 groups?

A: The effect size for the planned contrasts is at least half as big as the effect size for the main effects of the ANOVA.

B: The effect size for the ANOVA is at least half as big as the effect size for the planned contrasts.

C: The effect size for the planned contrasts is always smaller than the effect size for any of the effects in an ANOVA when there are more than 2 groups.

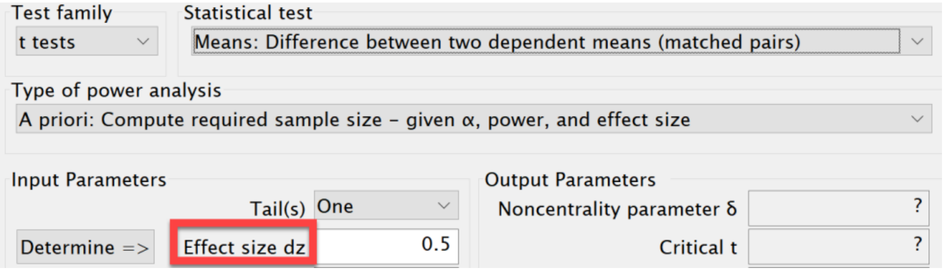
D: The effect size for the ANOVA can be either larger, the same, or smaller than the effect size of any of the planned contrasts.

Power in Repeated Measures ANOVA

In a repeated measures design, multiple observations are collected from the **same participants**. In the simplest case, where there are two repeated observations, a repeated measures ANOVA equals a dependent or paired *t*-test.

Two within conditions

To illustrate the effect of correlated observations, we start by simulating data for a medium effect size for a dependent (or paired, or within-subject) *t*-test. Let's first look at G*power. If we want to perform an a-priori power analysis, we are asked to fill in the effect size d_z. As Cohen (1988) writes, "The Z subscript is used to emphasize the fact that our raw score unit is no longer X or Y, but Z", where Z are the difference scores of X-Y. You can see the d_z in G*power when selecting the option to calculate power for a dependent *t*-test:



Test family: t tests

Statistical test: Means: Difference between two dependent means (matched pairs)

Type of power analysis: A priori: Compute required sample size – given α, power, and effect size

Input Parameters

Tail(s): One

Determine =>

Effect size dz: 0.5

Output Parameters

Noncentrality parameter δ: ?

Critical t: ?

Within designs can have greater power to detect differences than between designs because **the repeated observations are correlated**, and a within design requires less participants because **each participant provides multiple observations**. One difference between an independent *t*-test and a dependent *t*-test is that an independent *t*-test has 2(n-1) degrees of freedom (where n is the sample size per group), while a dependent *t*-test has (n-1) degrees of freedom. The total sample size needed in a two-group within-design (NW) relative to the sample needed in two-group between-designs (NB), assuming normal distributions, and ignoring the difference in degrees of freedom between the two types of tests, is (from Maxwell & Delaney, 2004, p. 561, formula 45):

$$N_W = \frac{N_B(1-\rho)}{2}$$

The division by 2 in the equation is due to the fact that in a two-condition within design every participant provides two data-points. The extent to which this reduces the sample size compared to a between-subject design depends on the correlation (ρ) between the two dependent variables, as indicated by the 1- ρ part of the equation. If the correlation is 0, a within-subject design needs half as many participants as a between-subject design (e.g., 64 instead of 128 participants), simply because every participant provides 2 datapoints. **The higher the correlation, the larger the relative benefit of within designs**, and whenever the correlation is negative (up to -1) the relative benefit disappears. **A positive correlation reduces the standard deviation of the difference scores**, which increases the standardized mean difference, and thus increases power, compared to a between participant design, but **a negative correlation increases the standard deviation of the difference scores** compared to a between design, lowering the standardized effect size, and reducing the power. For a longer explanation, see my blog.

Whereas in an independent *t*-test pairs of observations are uncorrelated, in a within design pairs of observations are correlated. This has an effect on the standard deviation of the difference scores. In turn, because the standardized effect size is the mean difference divided by the standard deviation of the difference scores, the correlation has an effect on the standardized mean difference in a within design, Cohen's d_z. The relation, as Cohen (1988, formula 2.3.7) explains, is:

$$\sigma_z = \sigma\sqrt{2(1-\rho)}$$

Therefore, the relation between dz and d is $\sqrt{2(1-\rho)}$. As Cohen (1988) writes: "In other words, a given difference between population means for matched (dependent) samples is standardized by a value which is $\sqrt{2(1-\rho)}$ as large as would be the case were they independent. If we enter a correlation of 0.5 in the formula, we get $\sqrt{2(0.5)} = 1$. When the correlation is 0.5, d = d_z. When there is a strong correlation between dependent variables, for example r = 0.9, we get $d = d_z\sqrt{2(1-0.9)}$, and a d_z of 1 would be a d = 0.45. Reversely, $d_z = \frac{d}{\sqrt{2(1-r)}}$, so with a r = 0.9, a d of 1 would be a d_z = 2.24. Some consider this increase in d_z compared to d when observations are strongly correlated an 'inflation' when estimating effect sizes, but since the reduction in the standard deviation of the difference scores due to the correlation makes it easier to distinguish signal from noise in a hypothesis test, it leads to a clear power benefit.

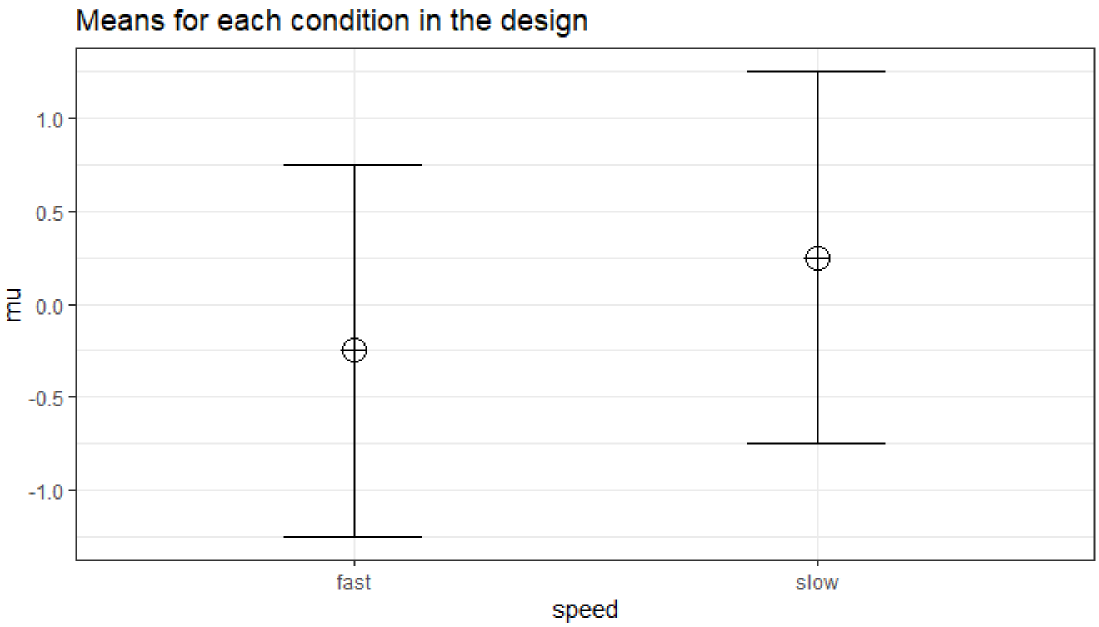
Q8: Although we often say that within designs have more power than between designs, and practically speaking this is often true in psychology, it is not necessarily true. For which values of the correlation will the power in a within subject design actually decrease compared to a between subject design?

A: Only when the correlation between dependent variables is -1.
B: Only when the correlation between dependent variables is smaller than -0.5.
C: Only when the correlation between dependent variables is smaller than 0.0.
D: When the correlation between dependent variables is smaller than 0.5.

There is no equivalent f_z for Cohen's f for a within subject ANOVA with more than 2 groups, but **the correlation between dependent variables always factors into a power analysis, both for a dependent *t*-test, and for a repeated measures ANOVA**.

Run lines 63 to 71. We perform a power analysis for a design with 2 groups, with 34 participants, a correlation among repeated measures of 0.5, and an alpha of 0.05. In this case, let's assume we simulate responses when instructing people to make a decision quickly or more slowly. We simulate data with means -0.25 and 0.25, and set the sd to 1. This means we have a mean difference of 0.5, and a Cohen's d of 0.5/1 = 0.5. For 2 groups we can calculate Cohen's f as d/2, so f = 0.25. We also calculate partial eta-squared for the ANOVA, which equals $\frac{f^2}{f^2+1}$, or $\frac{0.0625}{0.0625+1}$, or 0.05882353. Note that these calculations ignore the correlation between dependent variables – and in the output below, we will see a calculated Cohen’s f and partial eta-squared that are much larger than the values we calculated here, because they do incorporate the correlation. Some software calculates effect sizes while ignoring the correlation (for example G*Power) while other software incorporates the correlation (e.g., SPSS) – this is important to keep in mind!

```
design_result <- ANOVA_design(design = "2w",
                              n = 34,
                              mu = c(-0.25, 0.25),
                              sd = 1,
                              r = 0.5,
                              labelnames = c("speed", "fast", "slow"),
                              plot = TRUE)
```



```
ANOVA_exact(design_result)

## Power and Effect sizes for ANOVA tests
##       power partial_eta_squared cohen_f non_centrality
## speed 80.78              0.2048  0.5075            8.5
##
## Power and Effect sizes for contrasts
##                          power effect_size
## p_speed_fast_speed_slow 80.78         0.5
```

The results of the exact simulation approach reveal we have 80.777% power. Note that the simulation calculates Cohen’s d_z effect sizes for simple comparisons - which here given the correlation of 0.5 is also $d_z = 0.5$ for a medium effect size.

Q9: We repeat the simulation, and the only difference is a correlation between dependent variables of 0.7. **Change the value for the correlation in line 67 from r = 0.5 to r = 0.7**. Which statement is true?

A) Power has **decreased** to 74.5%, Cohen's dz has **decreased** to 0.4655

B) Power has **increased** to 95.47%, Cohen's dz has **increased** to 0.6455

C) Power has **decreased** to 74.5%, Cohen's dz has **increased** to 0.6455

D) Power has **increased** to 95.47%, Cohen's dz has **decreased** to 0.4655

Q10: What is Cohen's d_z?

A) The effect size for independent *t*-tests, that ignores the correlation between variables.

B) The effect size used in dependent *t*-tests, that ignores the correlation between variables.

C) The effect size used in independent *t*-tests, that takes into account the correlation between variables.

D) The effect size used in dependent *t*-tests, that takes into account the correlation between variables.

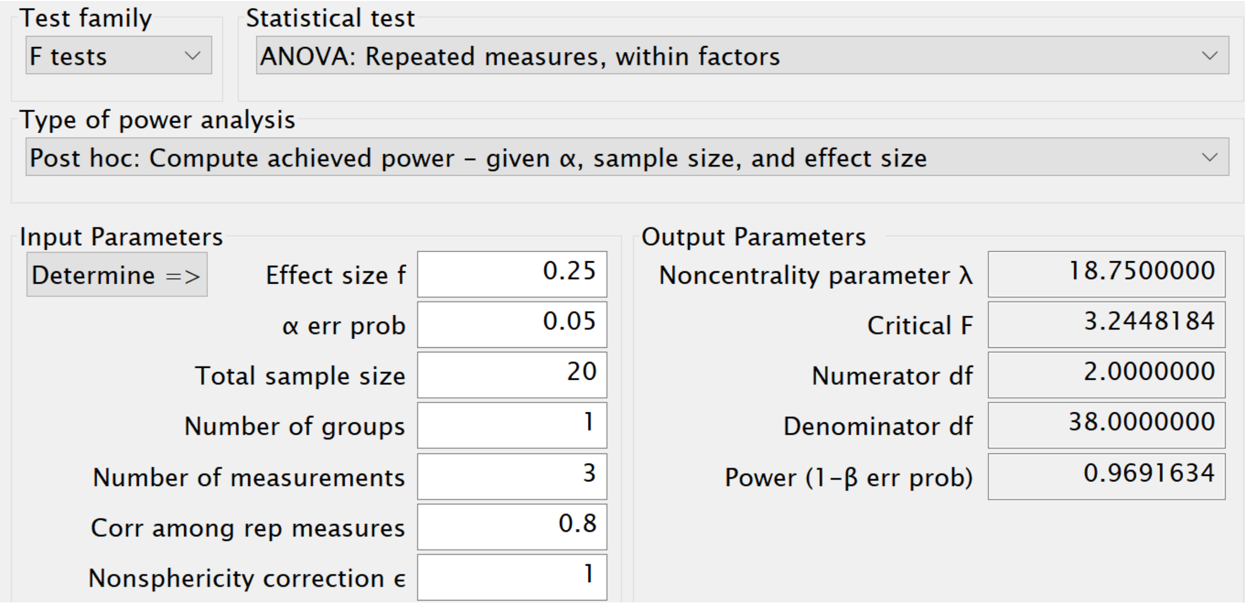
Power in Repeated Measures ANOVA with More than 2 Groups

We will examine a repeated measures experiment with 3 within-participant conditions, to illustrate how a repeated measures ANOVA extends a dependent *t*-test. In the example for a two-group within design we provided a specific formula for the sample size benefit for two groups. The sample size needed in within-designs (NW) with more than 2 conditions, relative to the sample needed in between-designs (NB), assuming normal distributions and compound symmetry, and ignoring the difference in degrees of freedom between the two types of tests, is (from Maxwell & Delaney, 2004, p. 562, formula 47):

$$N_W = \frac{N_B(1-\rho)}{a}$$

Where a is the number of within-subject levels.

We can perform power analyses for within designs using simulations. We set n = 20 for the simulation, and the correlation between dependent variables to 0.8. In this case, we simulate data with means -0.3061862, 0.0000000, and 0.3061862, the alpha level is 0.05, and set the sd to 1. This means the true effect size f = 0.25. The effect size is thus identical to that used in the previous example of a dependent *t*-test or two group ANOVA, but the pattern of means is different. **Run lines 75 to 83**. The results of the simulation show power is 96.9%. We can see this is in line with the power estimate from G*power:



Test family: F tests

Statistical test: ANOVA: Repeated measures, within factors

Type of power analysis: Post hoc: Compute achieved power – given α, sample size, and effect size

Input Parameters

Effect size f	0.25
α err prob	0.05
Total sample size	20
Number of groups	1
Number of measurements	3
Corr among rep measures	0.8
Nonsphericity correction ϵ	1

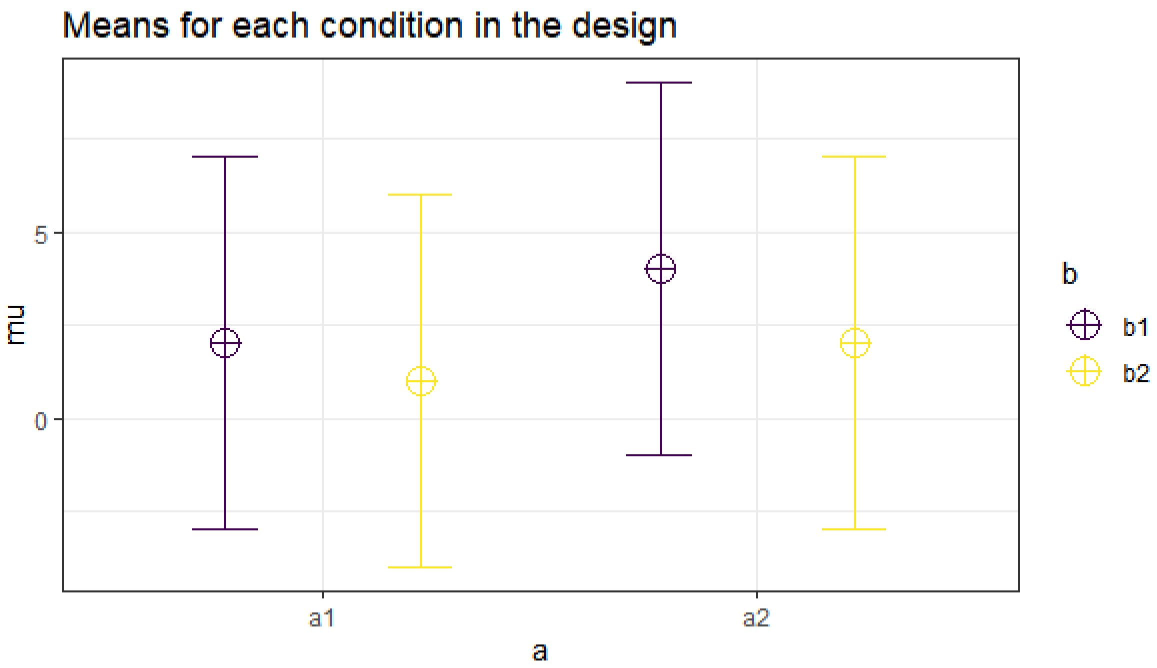
Output Parameters

Noncentrality parameter λ	18.7500000
Critical F	3.2448184
Numerator df	2.0000000
Denominator df	38.0000000
Power (1−β err prob)	0.9691634

Simulating More Complex Designs

One of the benefits of simulations is that we can perform power analyses for more complex designs, for which no analytic solutions exist. For the following power analysis, we will use the ANOVA_power function, because it allows us to control for multiple comparisons. Although researchers often do not control for multiple comparisons in exploratory ANOVA designs, they should, because errors inflate quite rapidly due to multiple comparisons (although as with all statistics, different statisticians value different things, and not everyone thinks error control is equally important – see the comments).

Run lines 88 to 107. The simulation will take a while, so read on as it runs. First, we simulate a 2x2 within design with means as specified below. We also specify the correlation between each comparison (as you can see in the correlation matrix that is printed). We did not label the factors, so they are indicated by default labels, but the correlation between the condition a1_b1 and a2_b2 is r = 0.4, but the correlation is higher for most other between participant variables. Note that power analysis software such as G*power does not allow you to perform power analyses for such designs. Let's examine the results. **Please note that your results will differ slightly, because we are not using the exact calculation, but we are simulating datasets 1000 times.**



```
Power and Effect sizes for ANOVA tests
          power effect_size
anova_a    52.7      0.1034
anova_b    95.5      0.2451
anova_a:b  57.0      0.1116

Power and Effect sizes for contrasts
                        power effect_size
p_a_a1_b_b1_a_a1_b_b2  55.6   -0.323521
p_a_a1_b_b1_a_a2_b_b1  75.7    0.402522
p_a_a1_b_b1_a_a2_b_b2   5.1   -0.002677
p_a_a1_b_b2_a_a2_b_b1  96.0    0.552530
p_a_a1_b_b2_a_a2_b_b2  23.8    0.196067
p_a_a2_b_b1_a_a2_b_b2  98.8   -0.652523
```

Re-run line 107, but change “none” into “holm”. Now we can compare the power we have when correcting for multiple comparisons using the Holm procedure for both the ANOVA (2 main effects and one interaction) and the 6 simple comparisons that we could perform. The results are printed below:

```
Power and Effect sizes for ANOVA tests
          power effect_size
anova_a    43.6      0.1034
anova_b    91.0      0.2451
anova_a:b  47.7      0.1116
```

```
Power and Effect sizes for contrasts
                      power effect_size
p_a_a1_b_b1_a_a1_b_b2  35.0   -0.323521
p_a_a1_b_b1_a_a2_b_b1  53.1    0.402522
p_a_a1_b_b1_a_a2_b_b2   1.4   -0.006677
p_a_a1_b_b2_a_a2_b_b1  83.1    0.552530
p_a_a1_b_b2_a_a2_b_b2  13.2    0.196067
p_a_a2_b_b1_a_a2_b_b2  95.0   -0.652523
```

We see that power is reduced, and so is the Type 1 error rate. The simple comparison indicated with "`p_a_a1_b_b1_a_a2_b_b2`" compares the two cells with means of 2 – so the true difference is 0. The Type 1 error rate varies as you simulate data. When we did not control for Type 1 errors, 5.1% of results of this single comparison were statistically significant, which is very close to the Type 1 error rate (your results can vary a bit – increase the number of simulations for more accurate results). After using the Holm correction for multiple comparisons, the Type 1 error rate is 1.4. The overall error rate for the 6 tests is controlled at the 5% level with the Holm correction, but this also means a lower alpha is used. This subsequently has an effect on the power of the tests.

This shows how it is important to incorporate corrections for multiple comparisons in your power analyses, and simulations can do so in a very flexible manner. Simulating designs by specifying the pattern of means, standard deviations, and correlations is a good way to build intuitions about the effect sizes you can expect in ANOVA designs.

For more information on how to perform power analysis for ANOVA designs using Superpower, see: https://arcaldwell49.github.io/SuperpowerBook/

References

Cohen, J. (1988). Statistical power analysis for the behavioral sciences (2nd ed). Hillsdale, N.J: L. Erlbaum Associates.

Lakens, D. (2013). Calculating and reporting effect sizes to facilitate cumulative science: A practical primer for t-tests and ANOVAs. Frontiers in Psychology, 4. https://doi.org/10.3389/fpsyg.2013.00863

Lakens, D., & Caldwell, A. R. (2019). Simulation-Based Power-Analysis for Factorial ANOVA Designs. https://doi.org/10.31234/osf.io/baxsf

Maxwell, S. E., & Delaney, H. D. (2004). Designing experiments and analyzing data: A model comparison perspective (2nd ed). Mahwah, N.J: Lawrence Erlbaum Associates.