Likelihood of Significant Findings

Science is a cumulative process, and we should evaluate lines of research, not single studies. Researchers are advised to design studies that will provide informative data, but sometimes you need to look at sets of studies before patterns become clear. A commonly used lower limit for statistical power is 80%, which means you will find a non-significant result in one out of five studies, when there is a true effect. Because the true effect size is never known, you should think about your study not as having a specific power, but as having a power curve. Given the sample size you collect, the statistical power is high for a range of true effect sizes, but it will be low for another range of non-zero true effect sizes that are still interesting.

Whenever you perform studies where power is not 99.99%, you should expect to find mixed results when examining true effects. Indeed, some researchers have pointed out that *not* finding mixed results can be very unlikely (or 'too good to be true') in a set of studies (Francis, 2014; Schimmack, 2012). We don't have a very good feeling for what real patterns of studies look like, because we are continuously exposed to a scientific literature that does not reflect reality. Almost all multiple study papers in the scientific literature present only statistically significant results, even though this is unlikely.

The probability of observing a significant or non-significant result in a study depends on the Type 1 error rate (α), the statistical power of the test ($1-\beta$), and the probability that the null-hypothesis is true (cf. Ioannidis, 2005; Wacholder, Chanock, Garcia-Closas, El ghormli, & Rothman, 2004). A study might examine a *true effect*, which means the alternative hypothesis (H_1) is true (e.g., a correlation that differs from zero) or it might examine a *null effect*, which means the null-hypothesis (H_0) is true (e.g., a correlation that is zero). When performing a statistical test on data, the test result might be statistically significant at a specified alpha level ($p < \alpha$) or not. Thus, there are four possible outcomes of a study:

1) False positives or Type 1 errors (you observe a significant test result when H_0 is true)
2) False negatives or Type 2 errors (you observe a non-significant result when H_1 is true)
3) True negatives (a non-significant result when H_0 is true)
4) True positives (a significant test result when H_1 is true)

	H0 True	H1 True
Significant Finding	False Positive (α)	True Positive ($1-\beta$)
Non-Significant Finding	True Negative ($1-\alpha$)	False Negative (β)

To get a feeling for which results you can expect in the long run, we will use binomial probabilities. As an example of a binomial probability, consider the probability of observing *k* heads when flipping a coin *n* times. The observed data are generated by a statistical distribution determined by the unknown parameter θ, which ranges from 0 to 1, and is the true probability of getting heads (e.g., 0.5 for a fair coin). A logical question in a coin-flipping example would be to determine the probability of observing

k number of heads in n coin flips, assuming a fair coin (θ = 0.5). The probability of observing k significant results in n studies is:

$$\frac{n!}{k!\,(n-k)!} * \theta^k * (1-\theta)^{n-k} \tag{1}$$

The first term indicates the number of possible combinations of results (e.g., when two out of three studies are significant, either the first, the second, or the third study is non-significant, which gives three combinations), which is multiplied by the probability of observing significant results in each of the k significant studies, which is then multiplied by the probability of observing non-significant results in the remaining studies. This is known as the binomial likelihood function. When H_0 is true, the probability of significant results equals the alpha level, and thus when the alpha level is carefully controlled (e.g., in pre-registered studies) θ = 0.05. When H_1 is true, the probability of observing a significant result depends on the power, e.g., θ = 0.80 for studies with 80% power.

Open the online shiny app made in R which will perform these binomial likelihood calculations for you: http://shiny.ieis.tue.nl/mixed_results_likelihood/. The app displays binomial likelihoods for sets of studies.

By default, the settings in the app show the results for a situation when 2 out of 3 studies are significant, a Type 1 error rate of 5% is used, and the power for the studies is assumed to be 80%. The text below the figure shows that 2 out of 3 significant results should happen 38% of the time when there is a true effect, and power is 80%. Scroll down, and take a look at the table. In the left column, you can see the probabilities of 0, 1, 2, or 3 significant results when there is no effect (so you only observe Type 1 errors), and in the right columns you can see these probabilities for when there is a true effect.

Q1: Which statement is correct when you perform 3 studies?

A) When H1 is true, alpha = 0.05, and power = 0.80, it is almost as likely to observe one or more non-significant results (48.8%) as it is to observe only significant results (51.2%).
B) When alpha = 0.05 and power = 0.80, it is extremely rare that you will find 3 significant results (0.0125%), regardless of whether H0 is true or H1 is true.
C) When alpha = 0.05 and power = 0.80, 2 out of 3 statistically significant results is the most likely outcome overall (38.4%) when H1 is true.
D) When alpha = 0.05 and power = 0.80, the probability of finding at least one false positive (a significant result when H0 is true) in three studies is 5%.

Q2: Sometimes in lines of three studies, you'll find a significant effect in one study, but there is no effect in the other two related studies. Assume the two related studies were not exactly the same in every way (e.g., you have changed the manipulation, or the procedure, or some of the questions). It could be that the two other studies did not work because of minor differences that had some effect you do not fully understand yet. Or it could be that the single significant result was a Type 1 error, and H0 was true in all three studies. Which statement below is correct, assuming a 5% Type 1 error rate and 80% power?

A) All else being equal, the probability of a Type 1 error in one of three studies is 5% when there is no true effect in all three studies, and the probability of finding exactly 1 in three significant effects, assuming 80% power in all three studies, is 80%, which is substantially more likely.
B) All else being equal, the probability of a Type 1 error in one of three studies is 13.5% when there is no true effect in all three studies, and the probability of finding exactly 1 in three significant effects,

assuming 80% power in all three studies (and thus a true effect), is 9.6%, which is slightly, but not substantially less likely.
C) All else being equal, the probability of a Type 1 error in one of three studies is 85.7% when there is no true effect in all three studies, and the probability of finding exactly 1 in three significant effects, assuming 80% power in all three studies (and thus a true effect) (and thus a true effect), is 0.8%, which is substantially less likely.
D) It is not possible to know the probability you will observe a Type 1 error if you perform 3 studies.

The idea that most studies have 80% power is slightly optimistic. **Examine the correct answer to the previous question across a range of power values** (e.g., 50% power, and 30% power).

Q3: Several papers suggest it is a reasonable assumption that the power in the psychological literature might be around 50%. Set the number of studies to 4, the number of successes also to 4, and the assumed power slider to 50%, and look at the table at the bottom of the app. How likely is it to observe 4 significant results in 4 studies, assuming there is a true effect?

A) 6.25%
B) 12.5%
C) 25%
D) 37.5%

Imagine you perform 4 studies, and 3 show a significant result. **Change these numbers in the online app. Leave the power at 50%**. The output in the text tells you:

When the observed results are equally likely under H0 and H1, the likelihood ratio is 1. Benchmarks to interpret Likelihood Ratios suggest that when 1<LR<8 there is weak evidence, when 8<LR<32 there is moderate evidence, and when LR>32, there is strong evidence.
The data are more likely under the alternative hypothesis than the null hypothesis with a likelihood ratio of 526.32

These calculations show that, assuming you have observed three significant results out of four studies, and assuming each study had 50% power, it is 526 times more likely to have observed these data when the alternative hypothesis is true, than when the null hypothesis is true. In other words, it is 526 times more likely to find a significant effect in three studies when you have 50% power, than to find three Type 1 errors in a set of four studies.

Q4: Maybe you don't think 50% power is a reasonable assumption. How low can the power be (rounded to 2 digits), for the likelihood to remain higher than 32 in favor of H1 when observing 3 out of 4 significant results?

A) 5% power
B) 17% power
C) 34% power
D) 44% power

The main take home message of these calculations is to understand that 1) mixed results are supposed to happen, and 2) mixed results can contain strong evidence for a true effect, across a wide range of plausible power values. The app also tells you how much evidence, in a rough dichotomous way, you can expect. This is useful for our educational goal. But when you want to evaluate results from multiple studies, the formal way to do so is by performing a meta-analysis.

The above calculations make a very important assumption: The Type 1 error rate is controlled at 5%. If you try out many different tests in each study, and only report the result that yielded a $p < 0.05$, these calculations no longer hold.

Q5: Go back to the default settings of 2 out of 3 significant results, but now set the Type 1 error rate to 20%, to reflect a modest amount of p-hacking. Under these circumstances, what is the **highest** likelihood in favor of H1 you can get if you explore all possible values for the true power?

A) Approximately 1
B) Approximately 4.63
C) Approximately 6.70
D) Approximately 62.37

As the scenario above shows, *p*-hacking makes studies extremely uninformative. **If you inflate the error rate, you quickly destroy the evidence in the data.** You can no longer determine whether the data is more likely when there is no effect, than when there is an effect. Sometimes researchers complain that people who worry about *p*-hacking and try to promote better Type 1 error control are missing the point, and that other things (better measurement, better theory, etc.) are more important. I fully agree that these aspects of scientific research are at least as important as better error control. But better measures and theories will require decades of work. Better error control can be accomplished today, if researchers would stop inflating their error rates by flexibly analyzing their data. And as this assignment shows, inflated rates of false positives very quickly make it difficult to learn what is true from the data we collect. Because of the relative ease with which this part of scientific research can be improved, and because we can achieve this today (and not in a decade) I think it is worth stressing the importance of error control, and publish more realistic looking sets of studies.

Q6: Some 'prestigious' journals (which, when examined in terms of scientific quality such as reproducibility, reporting standards, and policies concerning data and material sharing, are quite low quality despite their prestige) only publish manuscripts with a large number of studies, which should all be statistically significant. If we assume an average power in psychology of 50%, only 3.125% of 5 study articles should contain exclusively significant results. If you pick up a random issue from such a prestigious journal, and see 10 articles, each reporting 5 studies, and all manuscripts have exclusively significant results, would you trust the reported findings more, or less, than when all these articles had reported mixed results? Why?

Q7: Unless you will power all your studies at 99.99% for the rest of your career (which would be slightly inefficient, but great if you don't like insecurity), you will observe mixed results in lines of research. How do you plan to deal with mixed results in lines of research?

The take-away message of this assignment is that you should expect mixed results under many circumstances, and that dichotomously ignoring non-significant results is not wise. A formal meta-analysis is the best way to evaluate sets of studies. But understanding what realistic patterns of results in research lines should look like is hopefully a first step in making you realize you should think meta-analytically.

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