

Description of PWC Model

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January 12, 2024

The class *PWCModel* can be used to infer the D -dimensional means of Piece-Wise Constant (PWC) data. This means, for our given N observations, we assume that they were generated by an Additional White Gaussian Noise (AWGN) channel with known noise (i.e., normally generated Gaussian noise with known covariance matrix) and unknown piecewise constant means. The goal is now to estimate these means, optionally with some measure of the certainty about our estimation.

To emphasize the desired *piecewise constant* solutions, a sparsifying prior is applied to each state jump (state jumps are denoted by U_i , $i \in \{1, \dots, N - 1\}$ in Figure 1). The idea is that solutions with U_i very close to zero should be preferred, while occasional jumps can (and should) happen and, therefore, should not be punished too severely. Having this desired behaviour in mind, a log-cost NUV prior is used. Its behaviour is extensively studied in [1], along with a derivation of its generated outgoing messages used in the algorithm.

A factor graph of this model is shown in Figure 1. It specifies the used nomenclature of the algorithm, including the (somewhat arbitrary) directions of the messages. Furthermore, it shows how prior knowledge about the initial state of K shall be incorporated into the model by applying it as a prior on K'_1 (denoted by $\rho(\cdot)$). Finally, it shall be noted that for given incoming messages (specified either in terms of their means and covariances or dual-means and precision matrices) the final estimates are computed by Iteratively Reweighed Least-Squared (IRLS) as discussed in [1]. The forward-backward message passing is either be handled by the Modified Bryson-Frazies (MBF) or BIFM algorithm (both described in [2]), depending on the chosen representation of the given incoming messages (chooses the appropriate algorithm such that the number of required matrix inversions is minimized).

References

- [1] H.-A. Loeliger, “Lecture notes for model-based estimation and signal analysis,” 2023.
- [2] H. M. F. W. N. Z. Hans-Andreas Loeliger, Lukas Bruderer, “On sparsity by nuv-em, gaussian message passing, and kalman smoothing,” 2016.

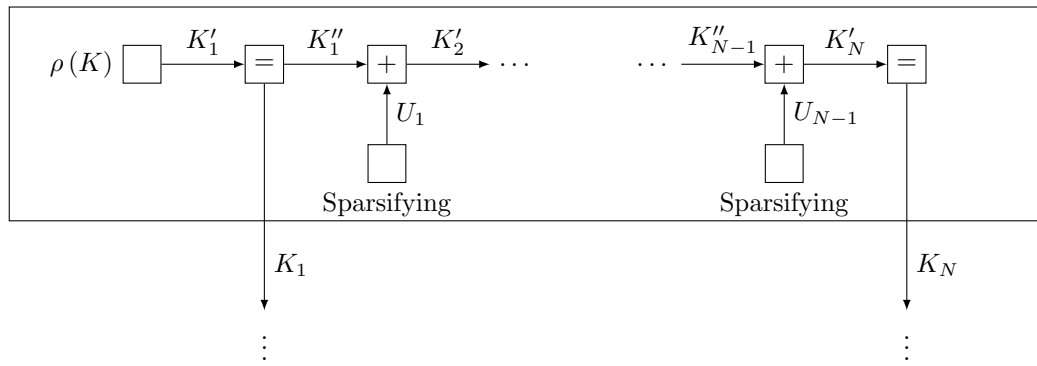


Figure 1: Factor graph of PWC model.