1 Introduction and Data Acquisition

The transportation network (also called traffic road network) is a common structure of pathways for the mobilized vehicle and pedestrian. The transportation network essentially consists of two components: the intersection with multiple entries and exits, and the road either directional or bi-directional. These specific real-world structures can be interpreted and converted with Network Flow problems. The roads will be viewed as edges with specific capacities and the intersection for roads can be simplified into the nodes in Network Flow problems. Figure 1(a) shows maps of traffic maps in Isthmus, Madison, and corresponding network flow. The objective of this study is to use the Minimum Cost Network Flow Model to simulate the traffic flow around the Isthmus area-Madison City. The traffic data will be collected from the Department of Transportation-Wisconsin, and prepossessed to construct a simplified but valid network model. The MNFP will be the principal approach to solve the optimization model. In advance, to simulate the pattern of movement of vehicles, which will include the vehicle speed, vehicle parking, traffic jamming, we will introduce the multi-period linear programming for interests of exploring.
The source of transports data was collected between 2013 to 2014 by the Department of Transportation-Wisconsin. The raw data collection vary from hourly to weekly and the time series. To simplify the problem, we will use the weekly average data in the solution to represent a generic view of the traffic flow. Furthermore, to simulate the vary pattern caused by car parking or loading, the Parking Garages & Lots Web Map from DOT will also be our data source. The details of constructing the traffic network will be illustrated in the next section.

2 Mathematical model

2.1 Network constructing

The traffic network will be implemented in direct graph with cycles. The nodes are set up at the location where there is an intersection or traffic signal. We adapt the weekly average traffic (Figure 1(b)) as the capacity for each edges. Furthermore, the demands for each nodes will be setup as 0 since no car is allowed to stay in the intersection of road. To simplify the model, there are multiple dummy source nodes and dummy sink nodes which are manually inserted based on local traffic condition. Specifically, E Washington Ave and University Ave will be the dummy sources or sinks. To modify the network with part lots, we add couples dummy edges and nodes with infinite capacities and demands to simulate the retentive flows left in parking lots.

2.2 Model variables, constraints, and objective

The objective of optimization is to maximize the total traffic flow or minimize the network cost in the network. The variables are the traffic flow in each edges, which have unit as cars/week. Each edges is bounded by upper limitation provided in previous session. Meanwhile, the dummy lot nodes and edges for parking lot will be setup either as infinity or a specific storage based on data feasibility. The general forms of MCNF problem is presented as following:

- N: the set of nodes
- E: the set of (directed) edges
- b_i: the supply / demand of node i
- u_{ij}: the capacity of the road.
- c_{ij}: the cost can be a metric correlated to we want to minimize for the traffic flow. For now, we can take $c_{ij} = 1/u_{ij}$ such that roads with larger capacity is naturally preferred over small roads with less capacity.
- x_{ij}: total flow on edge $(i,j) \in E$.

Then the mathematical model of the problem can be expressed as a standard MCNF problem:

$$
\begin{aligned}
\min_{x} \quad & \sum_{(i,j)\in E} c_{ij}x_{ij} \\
\text{s.t.} \quad & \sum_{j\in N} x_{kj} - \sum_{i\in N} x_{ik} = b_k \; \forall k \in N \\
& 0 \leq x_{ij} \leq u_{ij} \; \forall (i,j) \in E
\end{aligned}
$$

3 Project TimeLine

1. Jul 10 to Jul 17: Data wrangling and data prepossessing
2. Jul 18 to Jul 25: Model implementing and debugging
3. Jul 26 to Aug 01: Statistic analysing and sensitivity analysing
4. Aug 02 to deadline: Manuscript revision and enhancement of visualization.

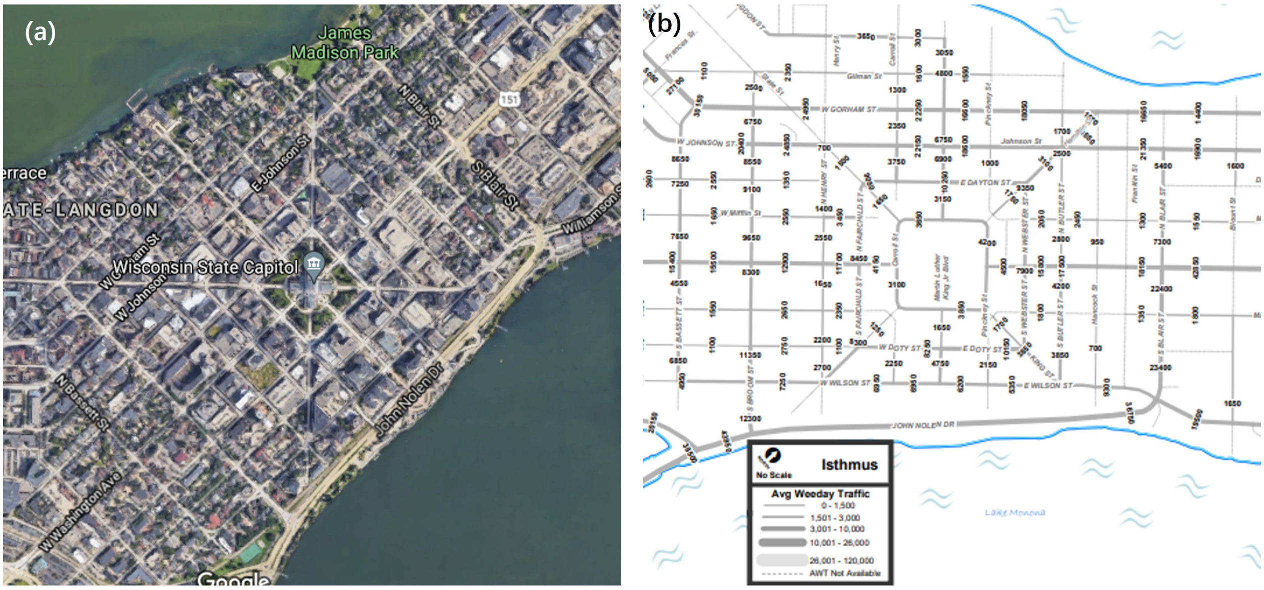

Figure 1: (a) The aerial image of Isthmus; (b) The traffic monitored flow by DOT. The number on the edge represents the road average weekday traffic.

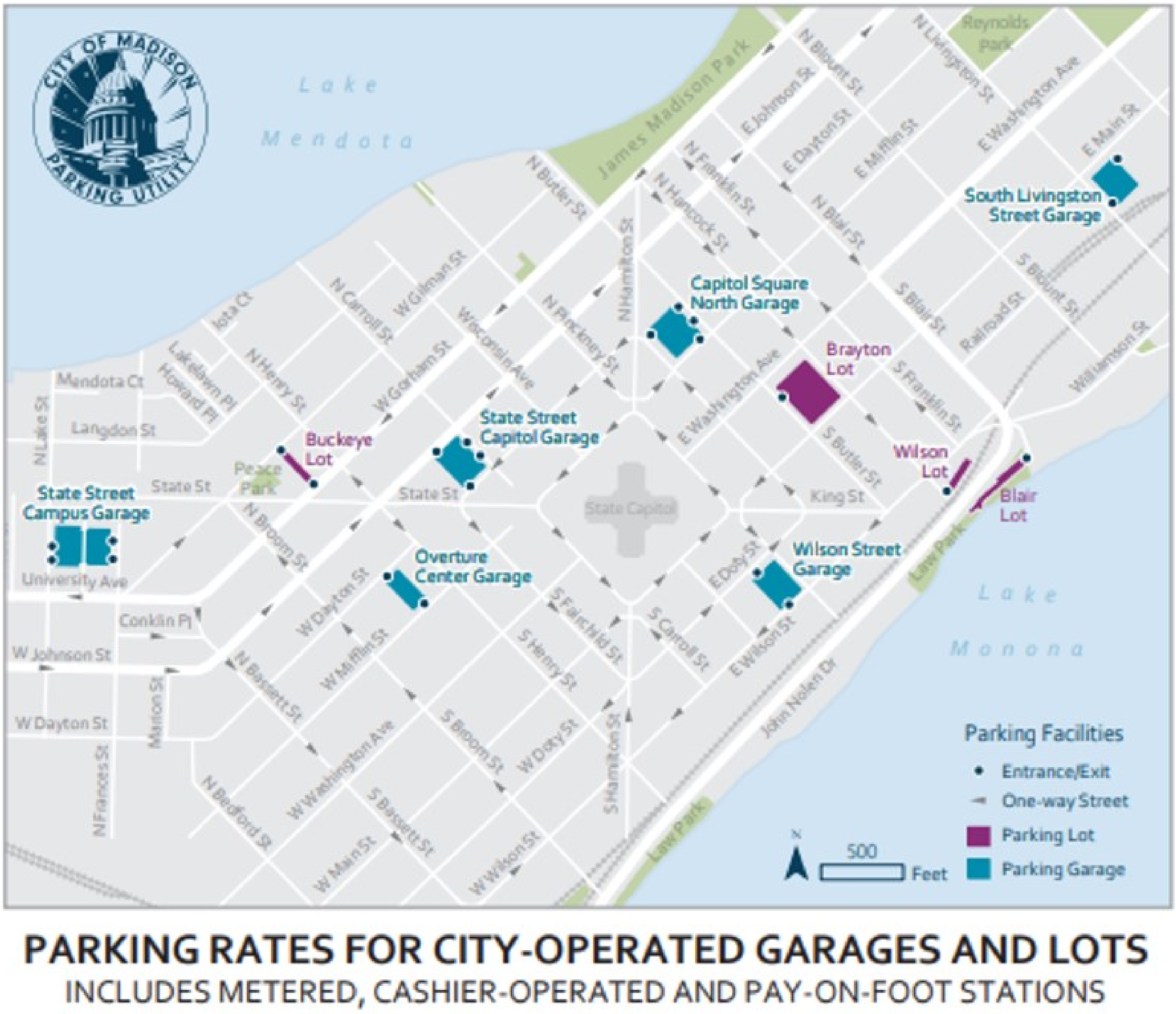

Figure 2: The parking map at Isthmus provided by City of Madison

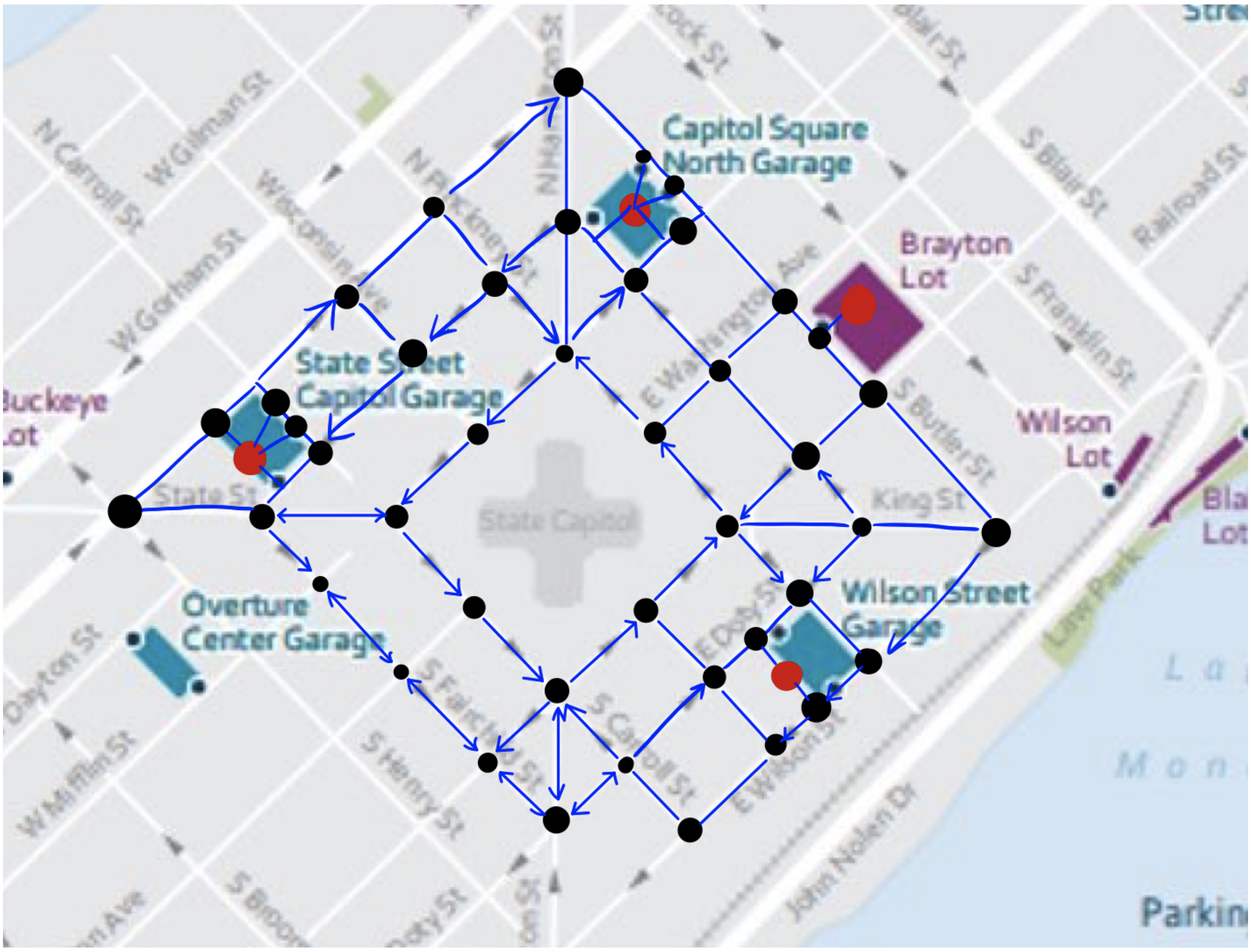

Figure 3: Modeling of the network (drawn by hand) overlaid to the original Isthmus transportation map. Black dot represents an intersection; red dot represents a parking lot; directed blue edge represent a one-way road; undirected or bidirectional edge represents a two-way road.