Abstract General Problem

V, W Banach spaces, W is reflexive

we want to solve, for $f \in W'$ find $u \in V$ s.t.

$A: V \longrightarrow W'$ (1) $Au = f$ in W'

(1) is well posed according to Hadamard:

$\exists \alpha > 0$, s.t. $\forall f \in W'$ 1) $\exists! u \in V$ 2) s.t.

$Au = f$ $\|u\|_V \leq \frac{1}{\alpha} \|f\|_{W'}$ 3)

1) A is surjective

2) A is injective

3) A is bounding $\|Au\|_{W'} = \|f\|_{W'} \geq \alpha \|u\|_V$

$\Longleftrightarrow$

Open Map Theorem and Closed Range Theorem