Continuum mechanics and fluid-structure interaction problems: mathematical modelling and numerical approximation

deal.II LAB — FiniteElement, DoFHandler

Luca Heltai <luca.heltai@sissa.it>

International School for Advanced Studies (www.sissa.it)
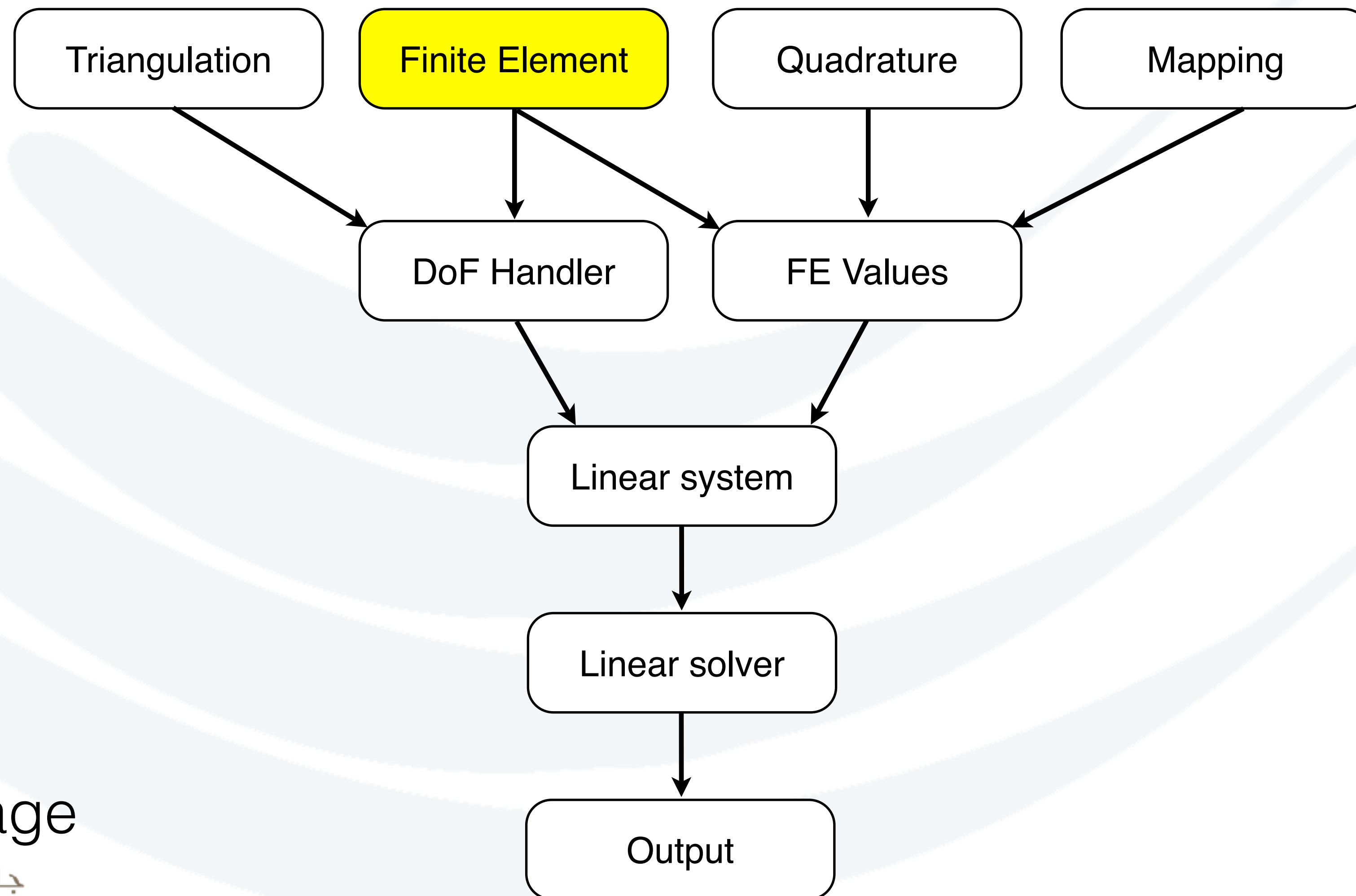Mathematical Analysis, Modeling, and Applications (math.sissa.it)
Master in High Performance Computing (www.mhpc.it)
SISSA mathLab (mathlab.sissa.it)
King Abdullah University of Science and Technology (kaust.edu.sa)

Structure of a prototypical FE problem

Main page

https://dealii.org/current/doxygen/deal.II/index.html
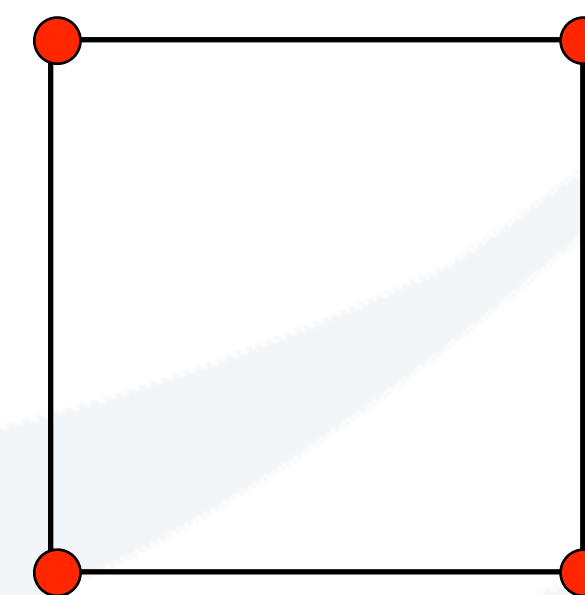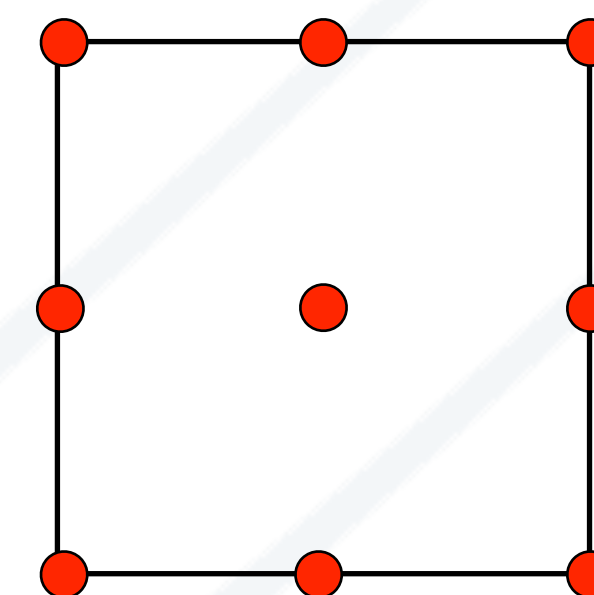
Assigning degrees-of-freedom: the FiniteElement classes

- Built in Finite Elements

  - Continuous

    - Piecewise Lagrange polynomials

  - Discontinuous

    - Monomials

    - Legendre polynomials

  - Vector-valued

    - Nedelec (H^{curl}, C/Dc)

    - Raviart-Thomas (H^{div}, C/Dc)

- A few more...

- Can develop finite elements from scratch

  - Specialisation for FE's derived by polynomial expansions
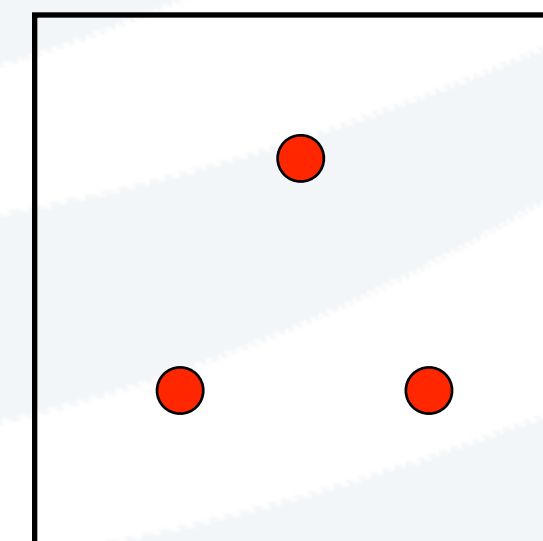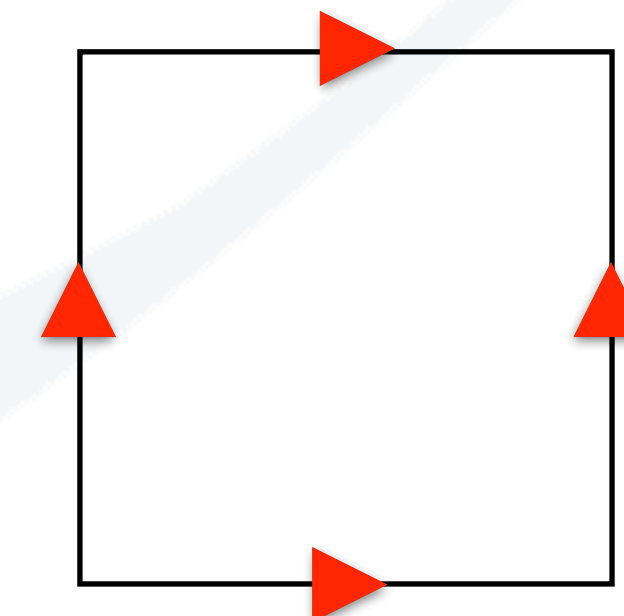
  - Enhanced/bubble elements

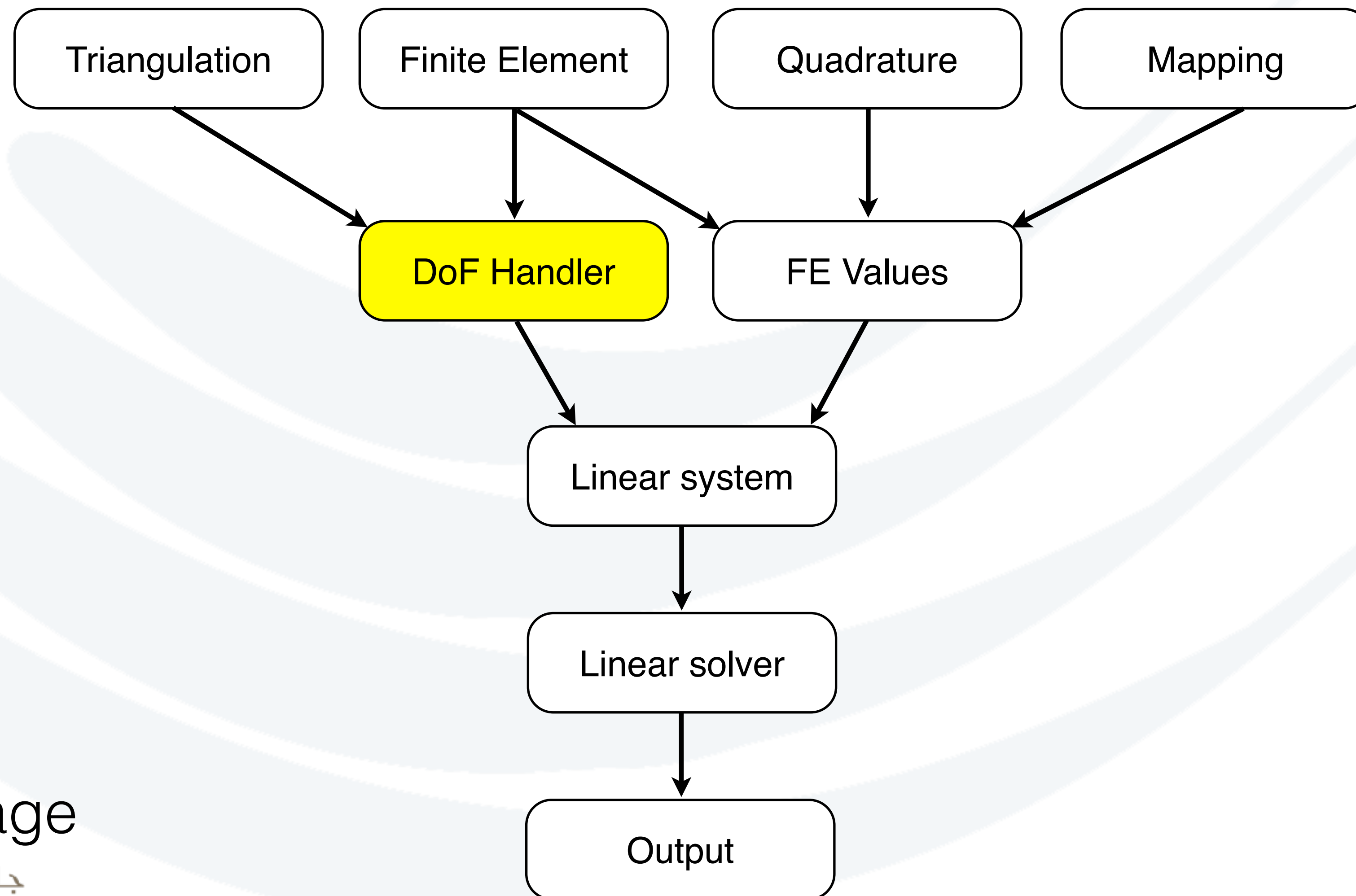`FE_Q<2>(1)`   `FE_Q<2>(2)`

`FE_DGPMonomial<2>(1)` `FE_Nedelec<2>(0)`

# Structure of a prototypical FE problem

Main page

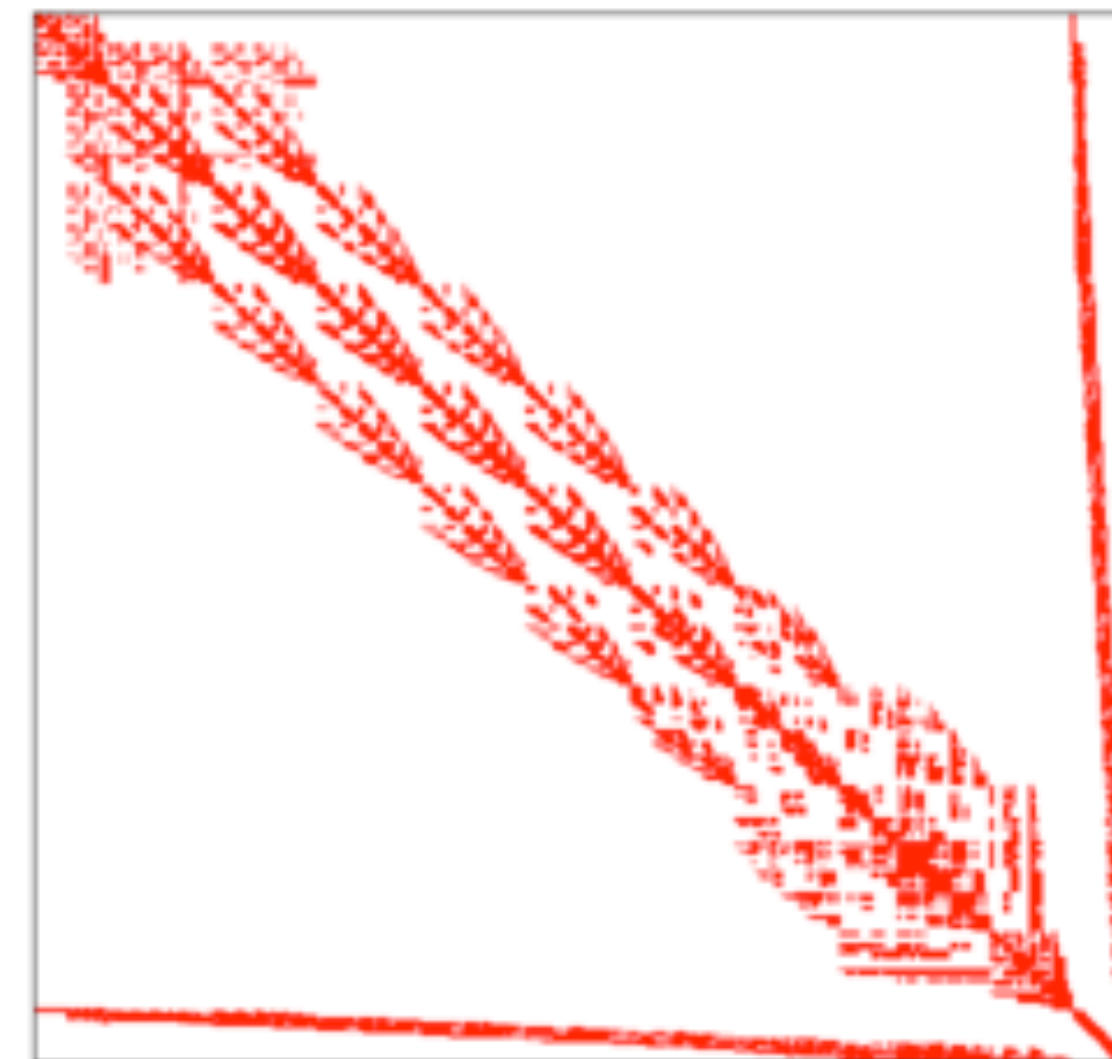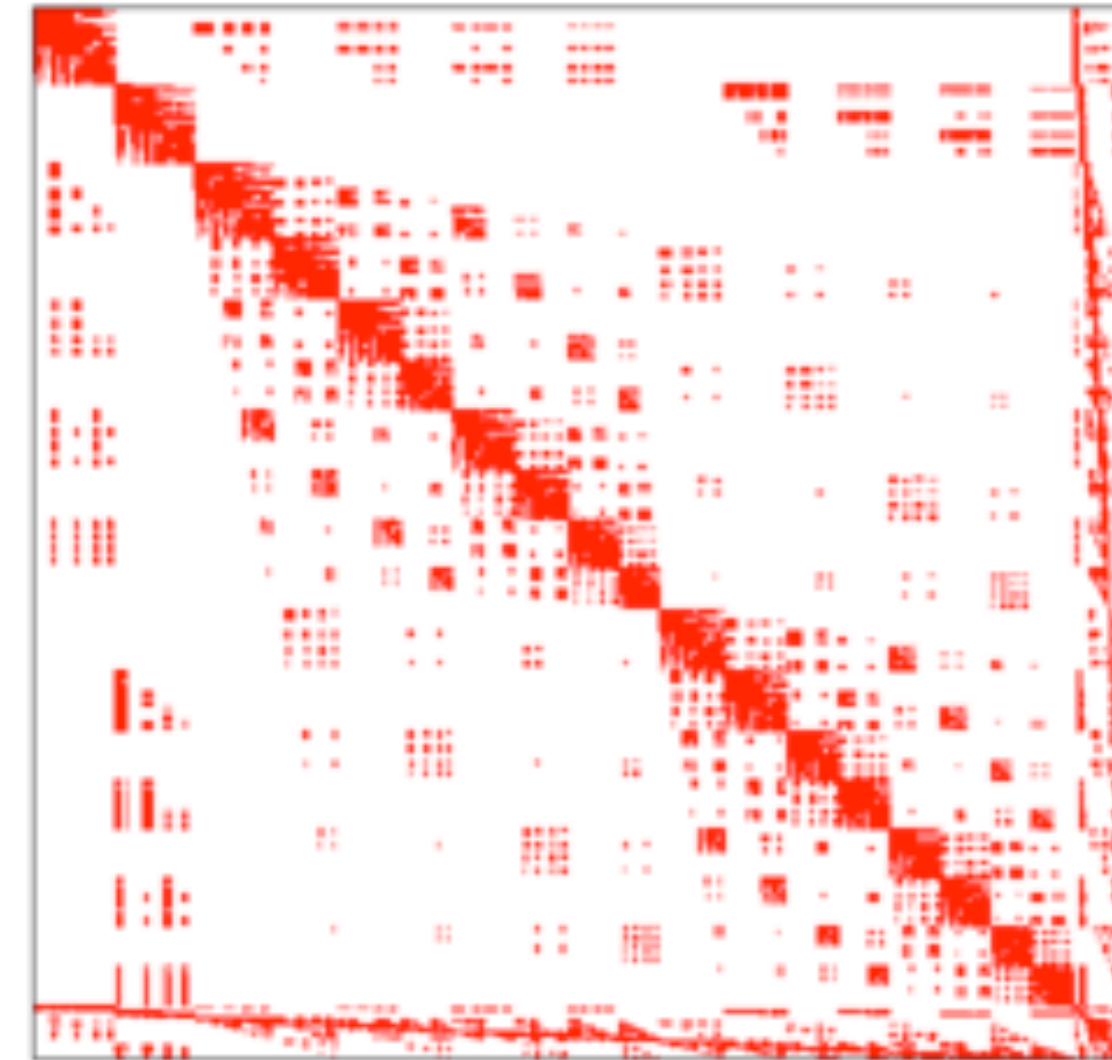# Assigning degrees-of-freedom: the DoFHandler class

- DoFHandler assigns DoF's to grid

  - Important: separate from Triangulation!

- Unified way to access DoF's, regardless of FE used

  - e.g. Discontinuous elements: support points not necessarily at vertices

- Fast access and grid traversal

  - STL-type cell iterators

  - Access to faces, edges through these
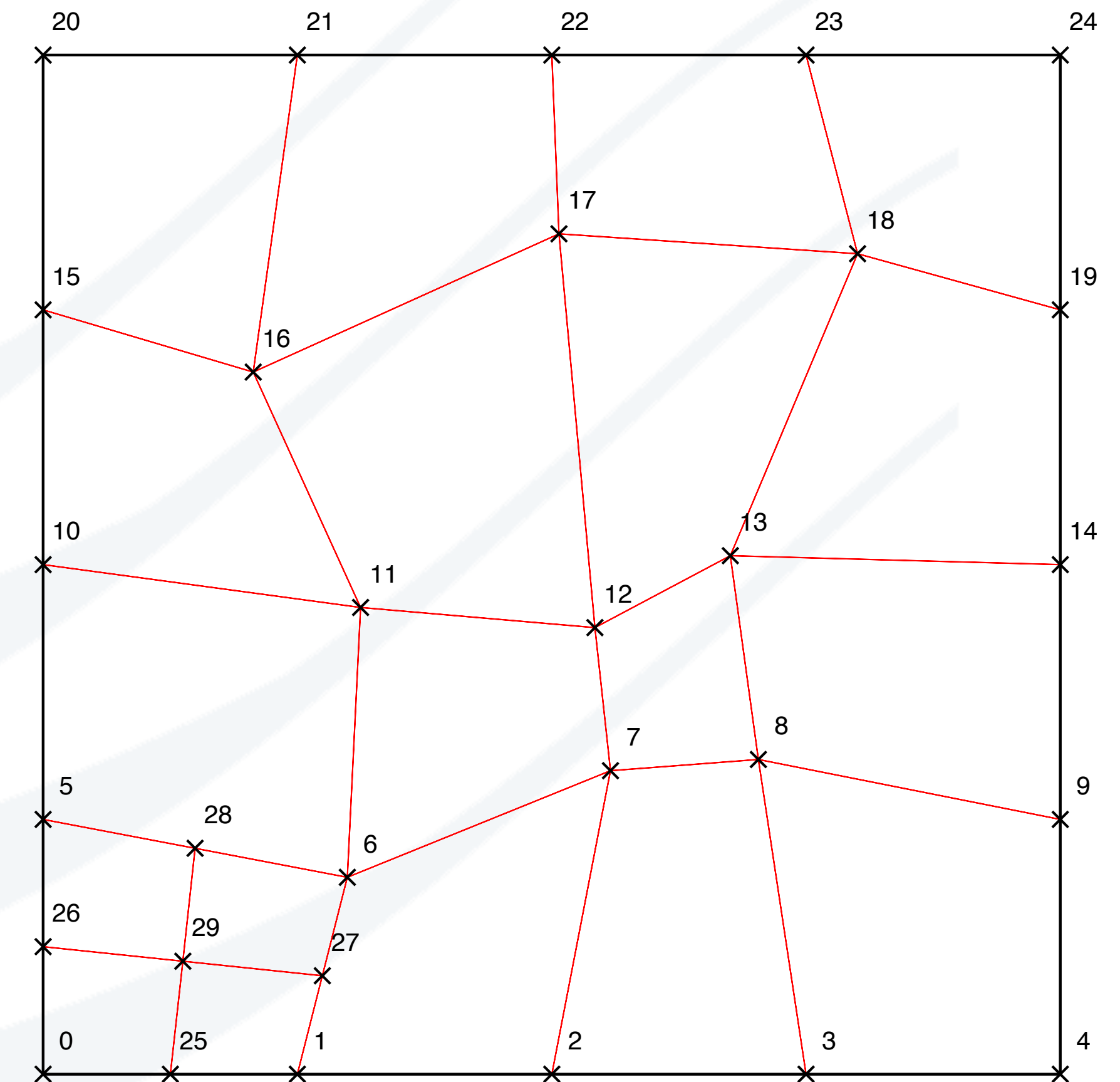
Assigning degrees-of-freedom: the DoFRenumbering namespace

- Renumbering schemes

  - Cuthill McKee

  - King

  - Downwind

- Reduce bandwidth

- Collect like-components

- Induce block-structure

- Directional (fluid flow)

- MPI subdomain

Assigning degrees-of-freedom:
the FiniteElement and DoFHandler classes

- Demonstration: Step-2
  https://www.dealii.org/current/doxygen/deal.II/step_2.html
  http://www.math.colostate.edu/~bangerth/videos.676.9.html

- Key points

  - Choosing a Finite Element

  - Distributing degrees-of-freedom on
    a mesh

  - Renumbering degrees-of-freedom

  - Visualizing sparsity patterns