

 $\mathrm{ODE} \hspace{3cm} 2023/2024$ 

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The results of the project "Collocation Methods, Time Step Adaptivity and Nonlinear System Solvers" are reported in this document. Notice all the eps and png files can be found inside the Grading directory.

## 1. Collocation Methods

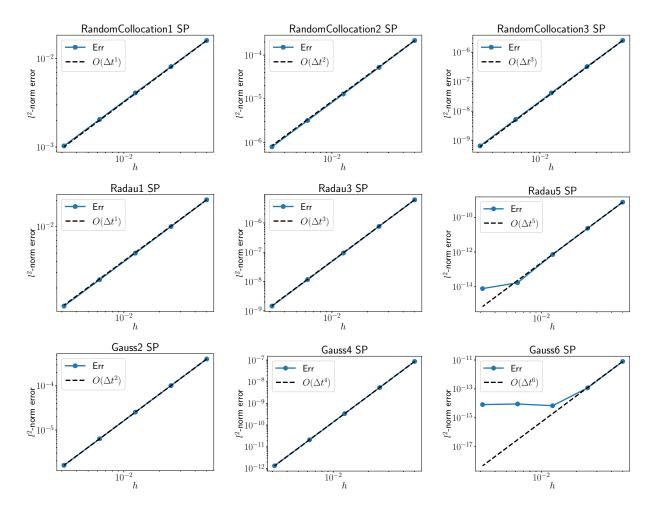
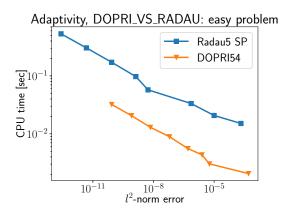
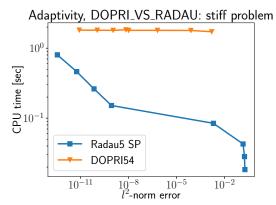


Figure 1: Collocation Methods

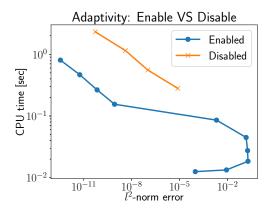
In Figure 1 the order of different collocation methods has been visualized through the use of convergence experiments.

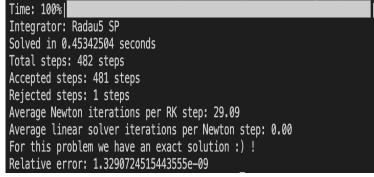
## 2. Time Step Adaptivity





- (A) Adaptivity\_DOPRI\_VS\_RADAU\_easy.py
- (B) Adaptivity\_DOPRI\_VS\_RADAU\_stiff.py



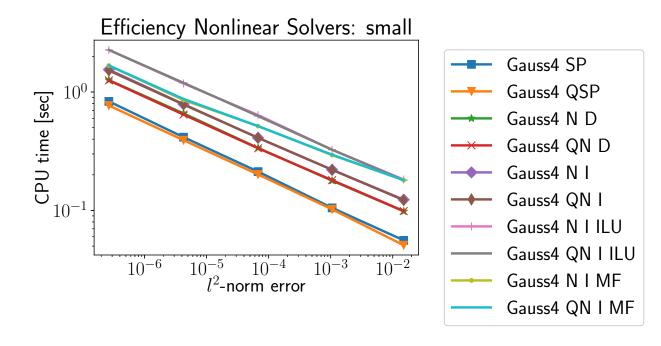


- (C) Adaptivity\_ENABLE\_VS\_DISABLE\_stiff.py
- (D) Adaptivity\_number\_of\_steps.py

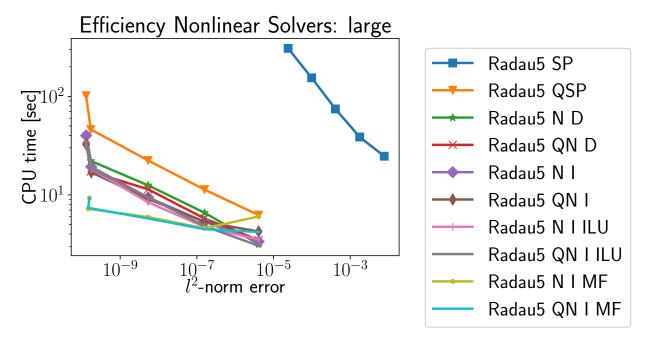
Figure 2: Time step adaptivity efficiency results.

The main insight from Figure 2 is that enabling time step adaptivity results in better performance in terms of error per computational time. On easy problems we see that DOPRI achieves a lower error than Radau5 if we give them the same time. Nevertheless on stiff problems Radau5 exhibits higher performance than DOPRI. This makes sense since DOPRI belongs to the set of explicit methods; Indeed, these methods are known for requiring ridiculously small step sizes in order to handle stiff problems.

## 3. Nonlinear Solvers



(A) NonlinearSolvers\_small.py



(B) NonlinearSolvers\_large.py

Figure 3: Nonlinear solvers efficiency.

Finally, in Figure 3 various non linear solvers are compared; stepping methods considered are Gauss4 for small problems and Radau5 for large problems.

For large problems, we can observe that the quasi-Newton approach results in computational gains. Furthermore, we can also observe that iterative solvers perform better than direct ones.