

# 999786

June 12, 2023

## 1 Investigating periods of increasing interest rates for the S&P 1500

### 1.0.1 by Luca Reichelt, 999786

#### 1.1 1. Outline

This project aims to investigate periods of increasing interest rates by the federal reserve based on the constituents of the S&P 1500. As an end result we attempt to implement our findings into a portfolio strategy.

The first month of observation is defined as the month of the first hike. The last date of observation is defined as the last month before a decreasing or stagnant interest rate (no change). Company data is based on the latest data available which have been published before the first day. We do not adjust for changes in the SP1500, but will include all the data available for its constituents as of the first month of the respective period.

All of the data that isn't retrieved while executing the code has been exported from Bloomberg Terminal or investing.com and is available within the folder "data\_raw" in xlsx format. The data which is retrieved during the code execution is retrieved from Yahoo Finance or the FRED API, which is a API provided by the Federal Reserve of St.Lewis (<https://fred.stlouisfed.org/docs/api/fred/>).

You should be able to run this notebook after creating a local environment using anconda/miniconda and executing following command while referring to the provided yaml file:

```
conda env create --file=env.yaml
```

#### 1.2 2. Data

```
[ ]: # importing the main packages
      from datetime import datetime
      # import dateutil.parser
      import os
      import numpy as np
      import pandas as pd

      pd.options.mode.chained_assignment = None
      import yfinance as yf
      import pandas_datareader.data as web
      import pickle as pkl
```

```
import dataframe_image as dfi
```

### 1.2.1 2.1 Data Retrieval

```
[ ]: # getting the last periods of increasing interest rates (Federal Funds  
    ↪Effective MONTHLY Rate) from 1965 onwards  
fed_rates = web.DataReader("FEDFUNDS", "fred", 1965)  
fed_rates.set_index(pd.to_datetime(fed_rates.index.date), inplace=True)  
  
# test for empty values  
print(fed_rates.index.isna().sum())
```

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```
[ ]: def period_df(start, duration, fed_rates=fed_rates):  
    fed_rates = fed_rates[fed_rates.index >= start]  
  
    df = pd.DataFrame(columns=["Name", "Start", "Last"])  
    df.loc[df.shape[0]] = [None, None, None]  
  
    df.is_copy = False  
  
    period = 0  
    j = 0  
  
    for i in range(0, len(fed_rates) - 1):  
        if (fed_rates.iloc[i + 1]["FEDFUNDS"] <= fed_rates.iloc[i]["FEDFUNDS"])  
        ↪and (  
            i - j >= duration  
        ):  
            df.loc[period, "Last"] = datetime.strftime(fed_rates.index[i],  
                ↪"%Y-%m-%d")  
            period += 1  
            if (fed_rates.index[-1] - fed_rates.index[i]).days >= 365:  
                df.loc[len(df)] = [None, None, None]  
                j = i  
            elif (fed_rates.iloc[i + 1]["FEDFUNDS"] <= fed_rates.  
                ↪iloc[i]["FEDFUNDS"]) and (  
                i - j < duration  
            ):  
                df.loc[period, "Name"] = "Period " + str(period + 1)  
                df.loc[period, "Start"] = datetime.strftime(fed_rates.index[i],  
                    ↪"%Y-%m-%d")  
                j = i  
  
    # add last date  
    df.loc[period, "Last"] = "2023-04-01"
```

```

# add duration column
df["Duration"] = (
    round(
        (pd.to_datetime(df["Last"]) - pd.to_datetime(df["Start"]))
        / np.timedelta64(1, "M")
    )
).astype(int)

# export df as image
dfi.export(
    df.style.set_properties(
        **{"background-color": "white", "color": "black", "border-color": "#948b8b"}
    ),
    "periods" + start + ".png",
)
# df["Start"] = pd.to_datetime(df["Start"])
# df["Last"] = pd.to_datetime(df["Last"])
return df

period_df(start="1965", duration=12)
# unfortunately lack of data regarding constituents for this period from bloomberg

periods = period_df(start="1995", duration=9)

```

```
[ ]: def web_import(rate, start, end):
    df = web.DataReader(rate, "fred", start, end)
    df.index = pd.to_datetime(df.index)
    df.sort_index(inplace=True)
    return df
```

```
[ ]: # def yf_import(ticker, start, end):
#     data = yf.download(ticker, start, end)
#     monthly = data.groupby(pd.PeriodIndex(data.index, freq="M"))["Close"].mean()
#     print(monthly)
#     if data.size > 0:
#         if (
#             not data.empty == True
#             and len(monthly.values)
#             == round((data.index[-1] - data.index[0]) / np.timedelta64(1, "M"))
#             and len(monthly.values) >= 9
#         ):

```

```

#             df = pd.DataFrame(
#                 index=pd.date_range(start, end, freq="MS"),
#                 data=np.append(monthly.values, data.iloc[-1]["Close"]),
#             )
#             df.rename(columns={df.columns[0]: ticker}, inplace=True)
#             df.drop(columns=df.columns.difference([ticker]), inplace=True)
#             df.index = pd.to_datetime(df.index)
#             return df
#         elif len(monthly.values) < 9:
#             empty = np.empty((1, 11 - (len(monthly))))
#             empty[:] = np.nan
#             df_data = np.append(empty, monthly.values)
#             df = pd.DataFrame(index=pd.date_range(start, end, freq="MS"), ▾
#             ↵data=df_data)
#             df.rename(columns={df.columns[0]: ticker}, inplace=True)
#             df.drop(columns=df.columns.difference([ticker]), inplace=True)
#             df.index = pd.to_datetime(df.index)
#             return df
#         else:
#             df = pd.DataFrame(index=pd.date_range(start, end, freq="MS"), ▾
#             ↵columns=[ticker])
#             df.index = pd.to_datetime(df.index)
#             return df

# above is overkill

def yf_import(ticker, start, end):
    data = yf.download(ticker, start, end)
    monthly = data.groupby(pd.PeriodIndex(data.index, freq="M"))["Close"].mean()
    monthly.index = monthly.index.to_timestamp()
    df = pd.DataFrame(index=pd.date_range(start, end, freq="MS"))
    df.index = pd.to_datetime(df.index)
    df.insert(0, ticker, monthly)
    return df

```

```

[ ]: raw = "data_raw/"

gold = pd.read_csv(raw + "Gold_Futures.csv").set_index("Date")
gold["Gold"] = gold["Price"].str.replace(", ", "").astype(float)
gold.drop(columns=gold.columns.difference(["Gold"]), inplace=True)
gold.index = pd.to_datetime(gold.index)
gold.sort_index(inplace=True)

```

## 1.2.2 2.2 Introductory Visualizations

```
[ ]: import plotly.express as px
import plotly.graph_objects as go
from plotly.subplots import make_subplots

tickers = ["^SP1500", "^IXIC"]
rates = ["FEDFUNDS", "CORESTICKM159SFRBATL", "UNRATE"]

def comparison(tickers, start, end):
    df = gold.loc[start:end]
    for ticker in tickers:
        df = pd.concat([df, yf_import(ticker, start, end)], axis=1)
    df = pd.concat([df, web.DataReader("WTISPLC", "fred", start, end)], axis=1)
    df_chg = df.pct_change() * 100

    legend = {
        "Gold": "Gold",
        "^SP1500": "S&P 1500",
        "^IXIC": "Nasdaq",
        "WTISPLC": "Spot Crude Oil Price WTI",
        "FEDFUNDS": "FED Rate",
        "CORESTICKM159SFRBATL": "CPI",
        "UNRATE": "Rate of Unemployment",
    }

    fig = px.line(
        df_chg,
        # ["Gold", "^SP1500", "^IXIC", "WTISPLC"],
        labels=legend,
        title="Indices/Assets monthly change in %",
    )
    fig.update_yaxes(title="Change in % compared to month before")
    fig.update_xaxes(title="Date")
    fig.for_each_trace(
        lambda t: t.update(
            name=legend[t.name],
            legendgroup=legend[t.name],
            hovertemplate=t.hovertemplate.replace(t.name, legend[t.name]),
        )
    )

    for index, row in periods.iterrows():
        fig.add_vline(
            x=row["Start"], line_width=2, line_dash="dash", line_color="green"
        )
```

```

    fig.add_vline(x=row["Last"], line_width=2, line_dash="dash",  

    ↪line_color="red")

    fig.update_layout(legend_title="Legend", autosize=False, width=1200,  

    ↪height=600)
    fig.show()

df_perf = df.apply(lambda x: x.div(x.iloc[0]).subtract(1).mul(100))

for rate in rates:
    df_perf = pd.concat([df_perf, web.DataReader(rate, "fred", start,  

    ↪end)], axis=1)

fig = make_subplots(specs=[[{"secondary_y": True}]])

x = df_perf.index

for ticker in tickers:
    fig.add_trace(
        go.Scatter(x=x, y=df_perf[ticker], name=legend[ticker]),
        secondary_y=False,
    )

fig.add_trace(
    go.Scatter(x=x, y=df_perf["Gold"], name="Gold"),
    secondary_y=False,
)

fig.add_trace(
    go.Scatter(x=x, y=df_perf["WTISPLC"], name="Spot Crude Oil Price WTI"),
    secondary_y=False,
)

for rate in rates:
    fig.add_trace(
        go.Scatter(x=x, y=df_perf[rate], name=legend[rate]),
        secondary_y=True,
    )

for index, row in periods.iterrows():
    fig.add_vline(
        x=row["Start"], line_width=2, line_dash="dash", line_color="green"
    )
    fig.add_vline(x=row["Last"], line_width=2, line_dash="dash",  

    ↪line_color="red")

```

```

fig.update_yaxes(title_text="Total Indices/Asset change in %",  

    ↪secondary_y=False)  

fig.update_yaxes(title_text="Rates for FED/CPI/Unemployment",  

    ↪secondary_y=True)  
  

fig.update_xaxes(title="Date")  
  

fig.update_layout(  

    title="Total performance to FED/CPI/Unemployment Rate",  

    legend_title="Legend",  

    autosize=False,  

    width=1200,  

    height=600,  

)  

fig.show()

```

```
[ ]: comparison(  

    tickers,  

    periods.iloc[0]["Start"],  

    periods.iloc[-1]["Last"],  

)

```

[\*\*\*\*\*100%\*\*\*\*\*] 1 of 1 completed  
[\*\*\*\*\*100%\*\*\*\*\*] 1 of 1 completed

### 1.2.3 2.3 Pre-Processing the features data

This data has been retrieved beforehand via the Bloomberg Excel Add-ins and is imported from the files in data-raw

```
[ ]: # the periods refer to the four timeframes from the outline
```

```
period_data = [  

    "SPR_Period_1.xlsx",  

    "SPR_Period_2.xlsx",  

    "SPR_Period_3.xlsx",  

    "SPR_Period_4.xlsx",  

]
```

```
[ ]: def scatterplots(df):  

    fig = px.scatter(  

        df,  

        x="Beta:M-1",  

        y="Market Cap_perf",  

        size="Market Cap",  

        title="Risk/Compared Volatility to SP1500 compared to Return/  

    ↪Performance for single companies",  

        color="GICS Sector",  

        hover_name="Name",
```

```

        log_x=True,
        size_max=100,
        width=1200,
        height=600,
    )

fig.show()

grouped = df.drop(columns=["Name"]).groupby("GICS Sector").mean()

fig = px.scatter(
    grouped,
    x="Beta:M-1",
    y="Market Cap_perf",
    size="Market Cap",
    title="Risk/Compared Volatility to SP1500 compared to Return/  
Performance by GICS Sector",
    color="Market Cap_perf",
    hover_name=grouped.index,
    log_x=True,
    size_max=100,
    width=1200,
    height=600,
)

```

)

```

fig.show()

```

```

[ ]: import matplotlib.pyplot as plt
import seaborn as sns

def heatmap(df):
    df = df.corr(numeric_only=True)

    f, ax = plt.subplots(figsize=(18, 18))
    sns.heatmap(df, annot=True, linewidths=0.5, fmt=".1f", ax=ax)
    sns.set(font_scale=2)
    plt.xticks(rotation=45)
    plt.yticks(rotation=0)
    plt.show()

```

```

[ ]: # this function cleans the datasets and provides first tabular and visual  
↳ descriptions of the data
# its also calls the heatmap visualization function above to give us an idea on  
↳ how to structure the model

```

```

def df(filename):

```

```

df = pd.read_excel(open(raw + filename, "rb"))

string_cols = ["Ticker", "Name", "GICS Sector"]

for col in set(df.columns) - set(string_cols):
    df.loc[:, col] = pd.to_numeric(df.loc[:, col], errors="coerce")

df.dropna(inplace=True)

sector_dummies = pd.get_dummies(df[["GICS Sector"]])
df = pd.concat([df, sector_dummies], axis=1)

df["Revenue per Employee"] = (
    df["Revenue T12M"] / df["Number of Employees:Y"]
).astype(float)

df["Market Cap_perf"] = (
    (df["Market Cap_last"] - df["Market Cap"]) / df["Market Cap"]
).astype(float)

df.drop(["Market Cap_last", "Price"], axis=1, inplace=True)

df.set_index("Ticker", inplace=True)
df.sort_values("GICS Sector", inplace=True)

print("\nData description for cleaned " + filename)
perf = df.pop("Market Cap_perf")
df.insert(0, "Market Cap_perf", perf)

rpe = df.pop("Revenue per Employee")
df.insert(1, "Revenue per Employee", rpe)

print(df["GICS Sector"].value_counts(ascending=False))
print(df.iloc[:, :12].describe())

scatterplots(df)

heatmap(df)

return df

```

```

[ ]: P1 = df("SPR_Period_1.xlsx")
P1.to_pickle("data/P1.pkl")
P2 = df("SPR_Period_2.xlsx")
P2.to_pickle("data/P2.pkl")
P3 = df("SPR_Period_3.xlsx")
P3.to_pickle("data/P3.pkl")

```

```

CrP = df("SPR_Period_4.xlsx")
CrP.to_pickle("data/CrP.pkl")

```

Data decription for cleaned SPR\_Period\_1.xlsx

GICS Sector

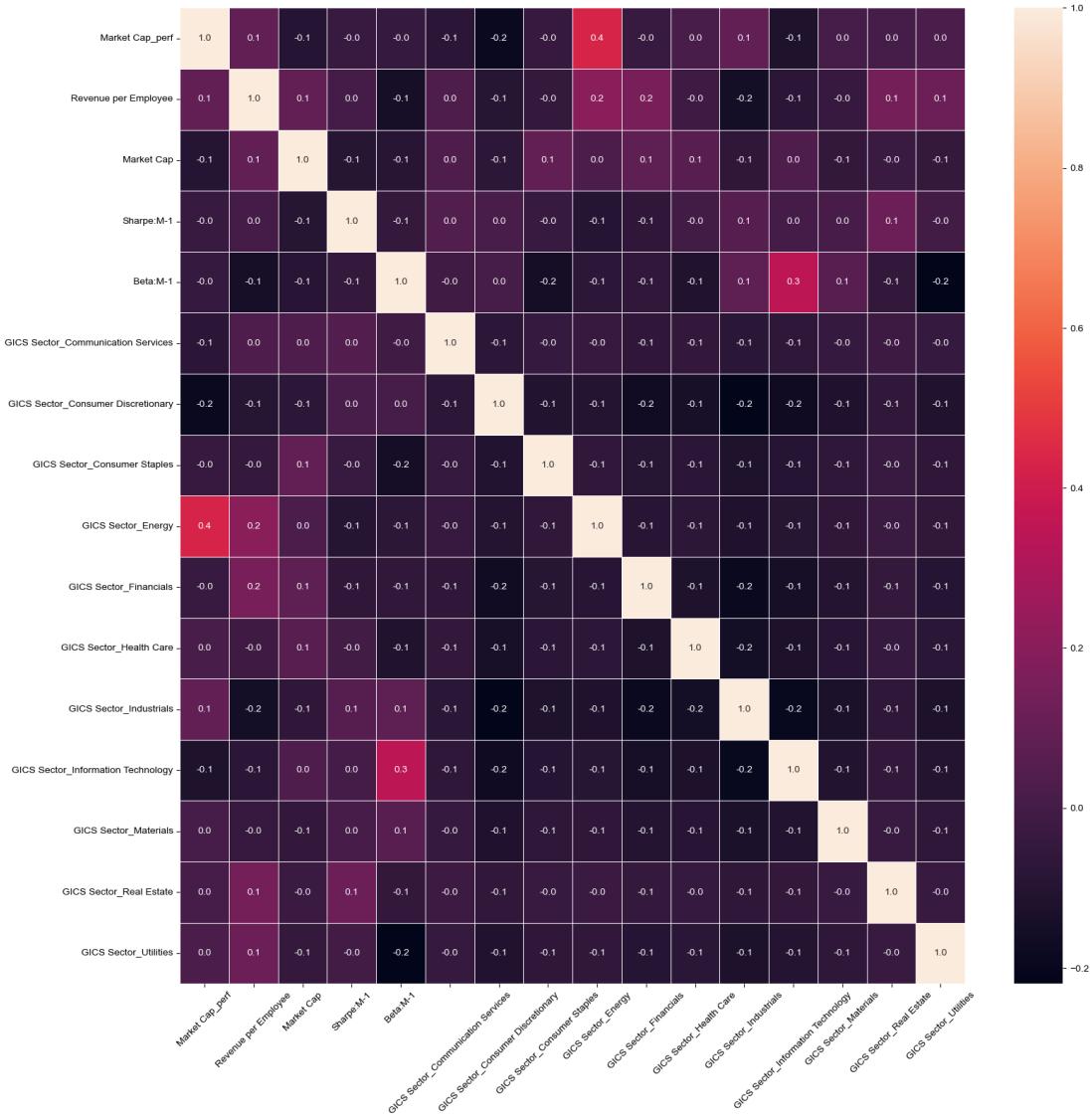
|                        |     |
|------------------------|-----|
| Industrials            | 145 |
| Consumer Discretionary | 120 |
| Information Technology | 98  |
| Financials             | 94  |
| Health Care            | 69  |
| Materials              | 51  |
| Utilities              | 47  |
| Consumer Staples       | 44  |
| Energy                 | 40  |
| Communication Services | 22  |
| Real Estate            | 14  |

Name: count, dtype: int64

|       | Market Cap_perf | Revenue per Employee | Market Cap    | Sharpe:M-1 | \ |
|-------|-----------------|----------------------|---------------|------------|---|
| count | 744.000000      | 7.440000e+02         | 744.000000    | 744.000000 |   |
| mean  | 0.294032        | 4.468768e+05         | 12065.701248  | 0.715024   |   |
| std   | 0.658582        | 6.624057e+05         | 32176.136903  | 3.805139   |   |
| min   | -0.916199       | 2.557610e+04         | 146.250700    | -4.577547  |   |
| 25%   | -0.113207       | 1.789160e+05         | 1152.156775   | -1.530829  |   |
| 50%   | 0.179333        | 2.688665e+05         | 2761.541750   | -0.361485  |   |
| 75%   | 0.503834        | 4.660680e+05         | 9059.699000   | 1.725719   |   |
| max   | 5.033335        | 1.063660e+07         | 308608.843100 | 36.750693  |   |

Beta:M-1

|       |            |
|-------|------------|
| count | 744.000000 |
| mean  | 1.079571   |
| std   | 0.353574   |
| min   | 0.274396   |
| 25%   | 0.829210   |
| 50%   | 1.030910   |
| 75%   | 1.278337   |
| max   | 2.533833   |



## Data description for cleaned SPR\_Period\_2.xlsx

### GICS Sector

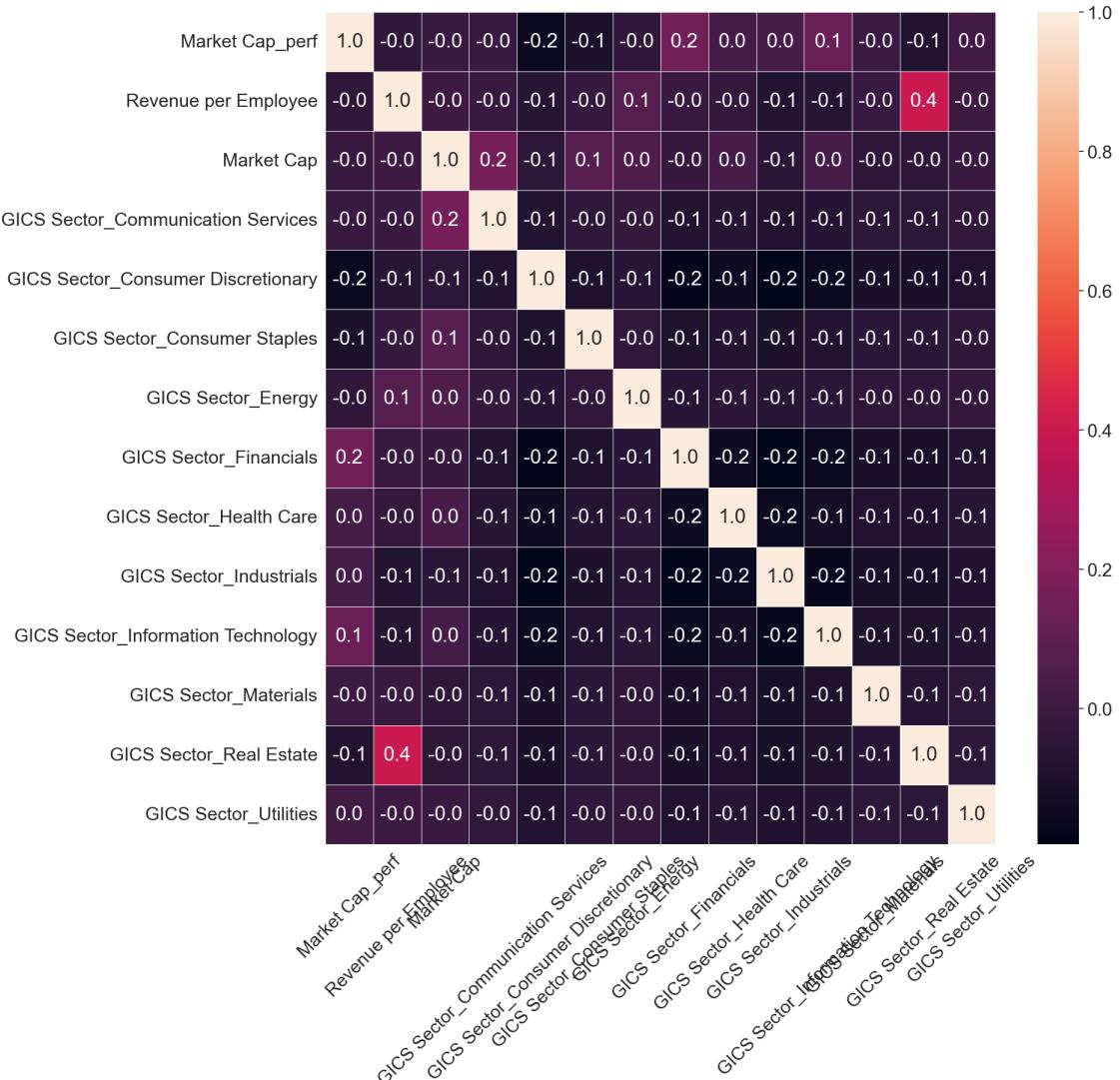
|                        |     |
|------------------------|-----|
| Industrials            | 216 |
| Financials             | 198 |
| Consumer Discretionary | 196 |
| Information Technology | 162 |
| Health Care            | 139 |
| Real Estate            | 82  |
| Materials              | 81  |
| Consumer Staples       | 65  |
| Utilities              | 52  |
| Communication Services | 47  |

Energy

31

Name: count, dtype: int64

|       | Market Cap_perf | Revenue per Employee | Market Cap    |
|-------|-----------------|----------------------|---------------|
| count | 1269.000000     | 1.269000e+03         | 1269.000000   |
| mean  | 0.145752        | 9.019770e+05         | 16865.722069  |
| std   | 0.310371        | 2.643064e+06         | 47038.230613  |
| min   | -0.839416       | 2.457104e+04         | 144.281000    |
| 25%   | -0.031057       | 2.482764e+05         | 1402.683200   |
| 50%   | 0.138096        | 3.864179e+05         | 3528.513100   |
| 75%   | 0.286780        | 7.353425e+05         | 12265.221500  |
| max   | 2.616234        | 5.176925e+07         | 575108.507600 |

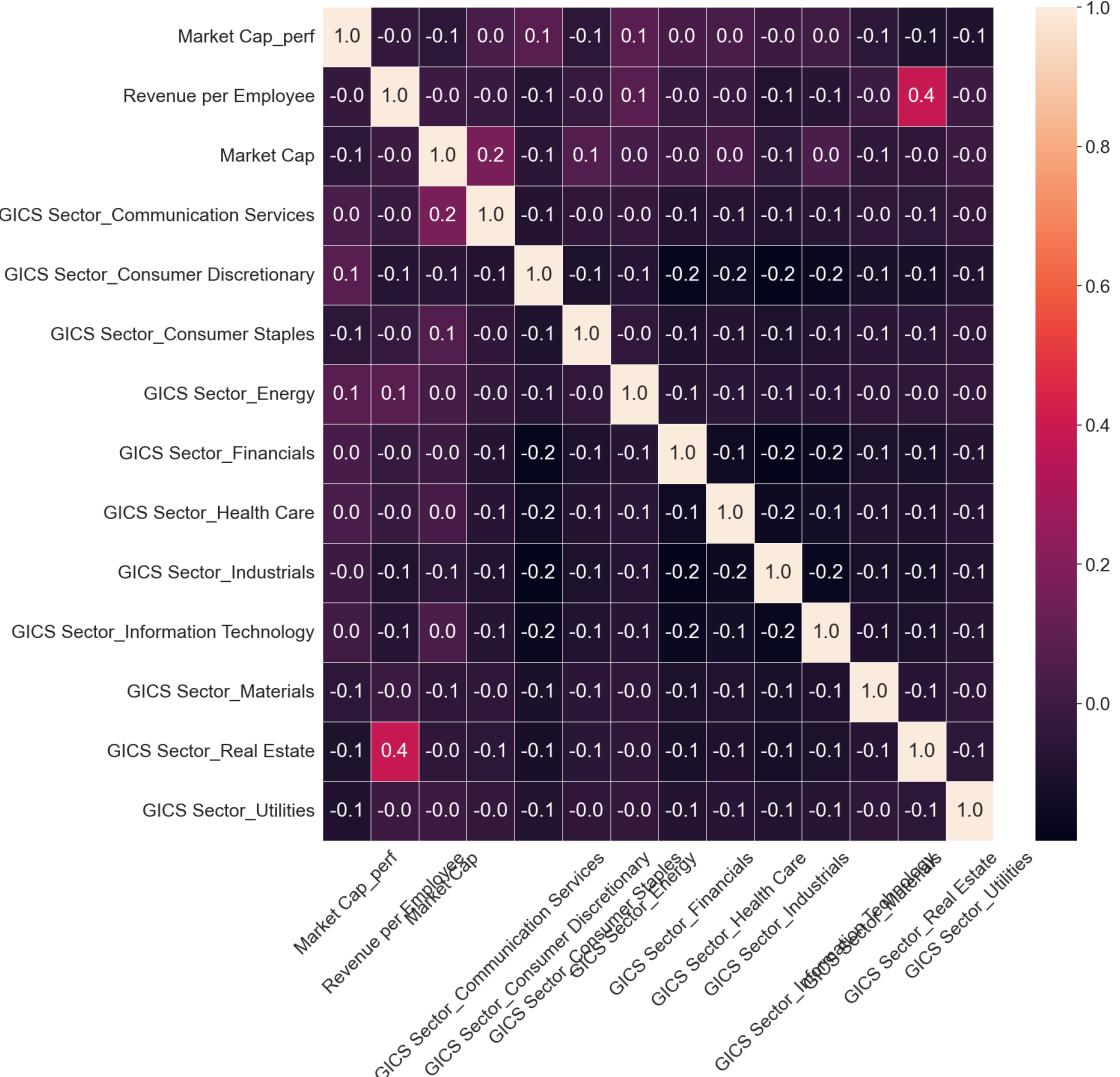


Data description for cleaned SPR\_Period\_3.xlsx

```

GICS Sector
Industrials           221
Consumer Discretionary 207
Financials            196
Information Technology 164
Health Care            143
Real Estate             91
Materials                77
Consumer Staples          65
Utilities                  48
Communication Services      46
Energy                      39
Name: count, dtype: int64
   Market Cap_perf  Revenue per Employee      Market Cap
count      1297.000000    1.297000e+03    1297.000000
mean        0.150803    9.279740e+05   18790.798789
std         0.314666    2.660655e+06   55517.775553
min        -0.882032    2.241154e+04    105.860000
25%        -0.042641    2.495037e+05   1526.499100
50%         0.102934    4.043673e+05   3917.466100
75%         0.284772    7.870889e+05  13333.739800
max        2.378570    4.657489e+07  847355.653400

```

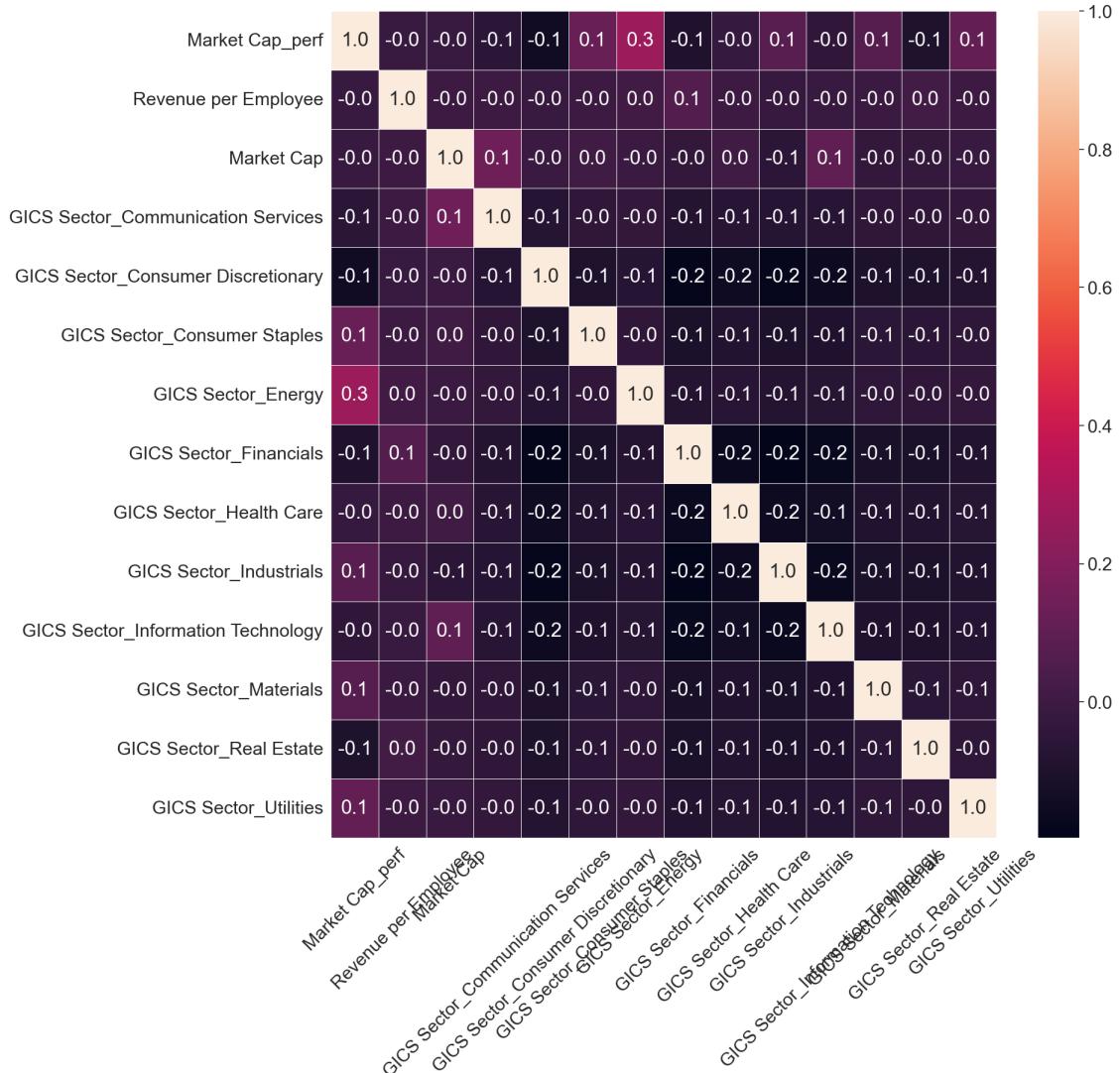


#### Data description for cleaned SPR\_Period\_4.xlsx

##### GICS Sector

|                        |     |
|------------------------|-----|
| Financials             | 214 |
| Industrials            | 204 |
| Consumer Discretionary | 187 |
| Information Technology | 163 |
| Health Care            | 147 |
| Materials              | 77  |
| Consumer Staples       | 73  |
| Real Estate            | 73  |
| Utilities              | 48  |
| Energy                 | 43  |
| Communication Services | 41  |

```
Name: count, dtype: int64
      Market Cap_perf  Revenue per Employee  Market Cap
count      1270.000000          1.270000e+03  1.270000e+03
mean       -0.148952          1.681369e+06  3.716714e+04
std        0.285485          2.446662e+07  1.588635e+05
min        -0.925602          0.000000e+00  2.625325e+02
25%       -0.325974          2.873606e+05  2.475682e+03
50%       -0.156679          4.597162e+05  6.106569e+03
75%        0.023768          9.459195e+05  2.262599e+04
max        1.598834          8.679870e+08  2.986128e+06
```



```
[ ]: # saving the pre-processed dfs for easy access
```

```
P1 = pd.read_pickle("data/P1.pkl")
```

```

P2 = pd.read_pickle("data/P2.pkl")
P3 = pd.read_pickle("data/P3.pkl")
CrP = pd.read_pickle("data/CrP.pkl")

[ ]: # creating a dataframe consisting of all data across periods
all_dfs = [P1, P2, P3, CrP]
all_df = pd.concat(all_dfs)

print("\nData description for all periods (including current period)")
perf = all_df.pop("Market Cap_perf")
all_df.insert(0, "Market Cap_perf", perf)

rpe = all_df.pop("Revenue per Employee")
all_df.insert(1, "Revenue per Employee", rpe)

print(all_df["GICS Sector"].value_counts(ascending=False))
print(all_df.iloc[:, :12].describe())

scatterplots(all_df)

heatmap(all_df)

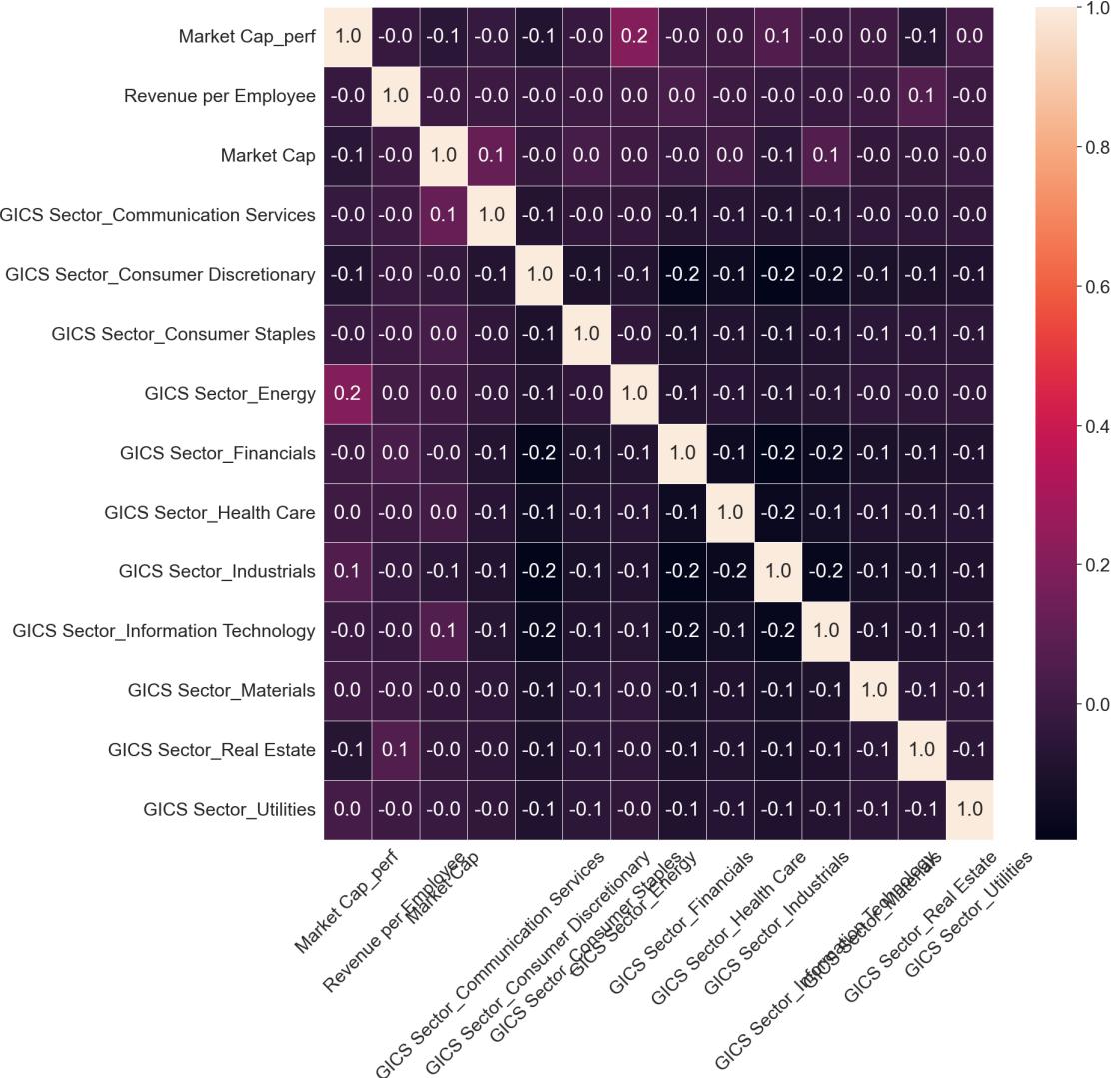
```

Data description for all periods (including current period)

| GICS Sector            | count |
|------------------------|-------|
| Industrials            | 786   |
| Consumer Discretionary | 710   |
| Financials             | 702   |
| Information Technology | 587   |
| Health Care            | 498   |
| Materials              | 286   |
| Real Estate            | 260   |
| Consumer Staples       | 247   |
| Utilities              | 195   |
| Communication Services | 156   |
| Energy                 | 153   |

Name: count, dtype: int64

|       | Market Cap_perf | Revenue per Employee | Market Cap   |
|-------|-----------------|----------------------|--------------|
| count | 4580.000000     | 4.580000e+03         | 4.580000e+03 |
| mean  | 0.089550        | 1.051530e+06         | 2.226057e+04 |
| std   | 0.414938        | 1.304172e+07         | 9.347337e+04 |
| min   | -0.925602       | 0.000000e+00         | 1.058600e+02 |
| 25%   | -0.143823       | 2.424445e+05         | 1.559422e+03 |
| 50%   | 0.049928        | 3.851413e+05         | 4.083165e+03 |
| 75%   | 0.251214        | 7.485898e+05         | 1.406112e+04 |
| max   | 5.033335        | 8.679870e+08         | 2.986128e+06 |



```
[ ]: # creating a df consisting only of concluded periods
```

```
concluded_dfs = [P1, P2, P3]
concluded_df = pd.concat(concluded_dfs)

print("\nData description for only concluded periods")
perf = concluded_df.pop("Market Cap_perf")
concluded_df.insert(0, "Market Cap_perf", perf)

rpe = concluded_df.pop("Revenue per Employee")
concluded_df.insert(1, "Revenue per Employee", rpe)

print(concluded_df["GICS Sector"].value_counts(ascending=False))
```

```

print(concluded_df.iloc[:, :12].describe())

scatterplots(concluded_df)

heatmap(concluded_df)

```

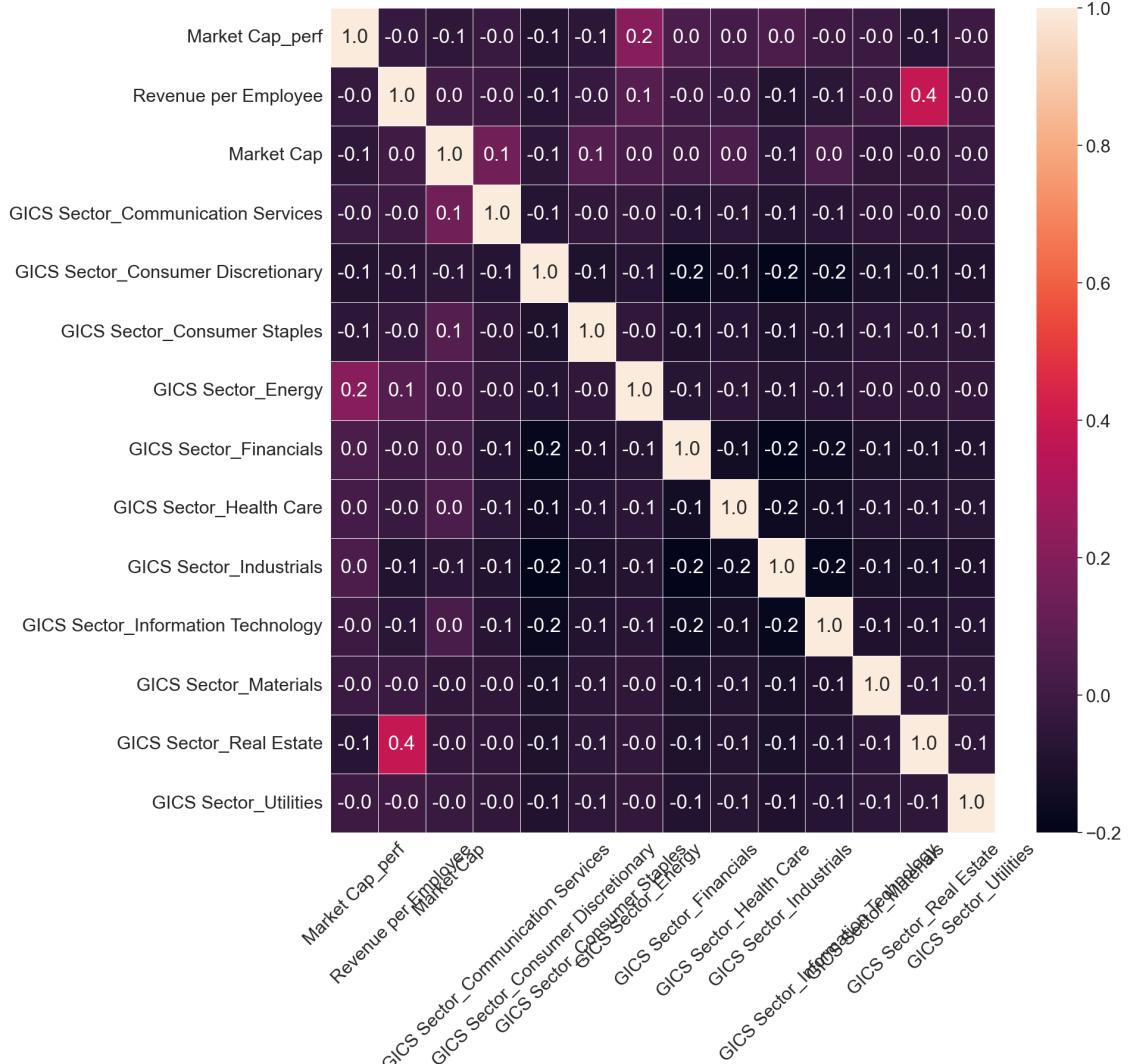
Data description for only concluded periods

GICS Sector

|                        |     |
|------------------------|-----|
| Industrials            | 582 |
| Consumer Discretionary | 523 |
| Financials             | 488 |
| Information Technology | 424 |
| Health Care            | 351 |
| Materials              | 209 |
| Real Estate            | 187 |
| Consumer Staples       | 174 |
| Utilities              | 147 |
| Communication Services | 115 |
| Energy                 | 110 |

Name: count, dtype: int64

|       | Market Cap_perf | Revenue per Employee | Market Cap    |
|-------|-----------------|----------------------|---------------|
| count | 3310.000000     | 3.310000e+03         | 3310.000000   |
| mean  | 0.181060        | 8.098693e+05         | 16541.132647  |
| std   | 0.420463        | 2.363555e+06         | 47896.088008  |
| min   | -0.916199       | 2.241154e+04         | 105.860000    |
| 25%   | -0.045230       | 2.300026e+05         | 1401.645550   |
| 50%   | 0.130913        | 3.599069e+05         | 3471.415900   |
| 75%   | 0.315884        | 6.920711e+05         | 11867.860650  |
| max   | 5.033335        | 5.176925e+07         | 847355.653400 |



```
[ ]: # insight into the characteristics of the best performing observations

top10 = pd.DataFrame()

for df in concluded_dfs:
    top10 = pd.concat(
        [top10, df.sort_values("Market Cap_perf", ascending=False).head(10)])
)

perf = top10.pop("Market Cap_perf")
top10.insert(0, "Market Cap_perf", perf)

rpe = top10.pop("Revenue per Employee")
top10.insert(1, "Revenue per Employee", rpe)
```

```

print(top10["GICS Sector"].value_counts(ascending=False))
print(top10.iloc[:, :12].describe())

top10.sort_values("Market Cap_perf", ascending=False)

heatmap(top10)

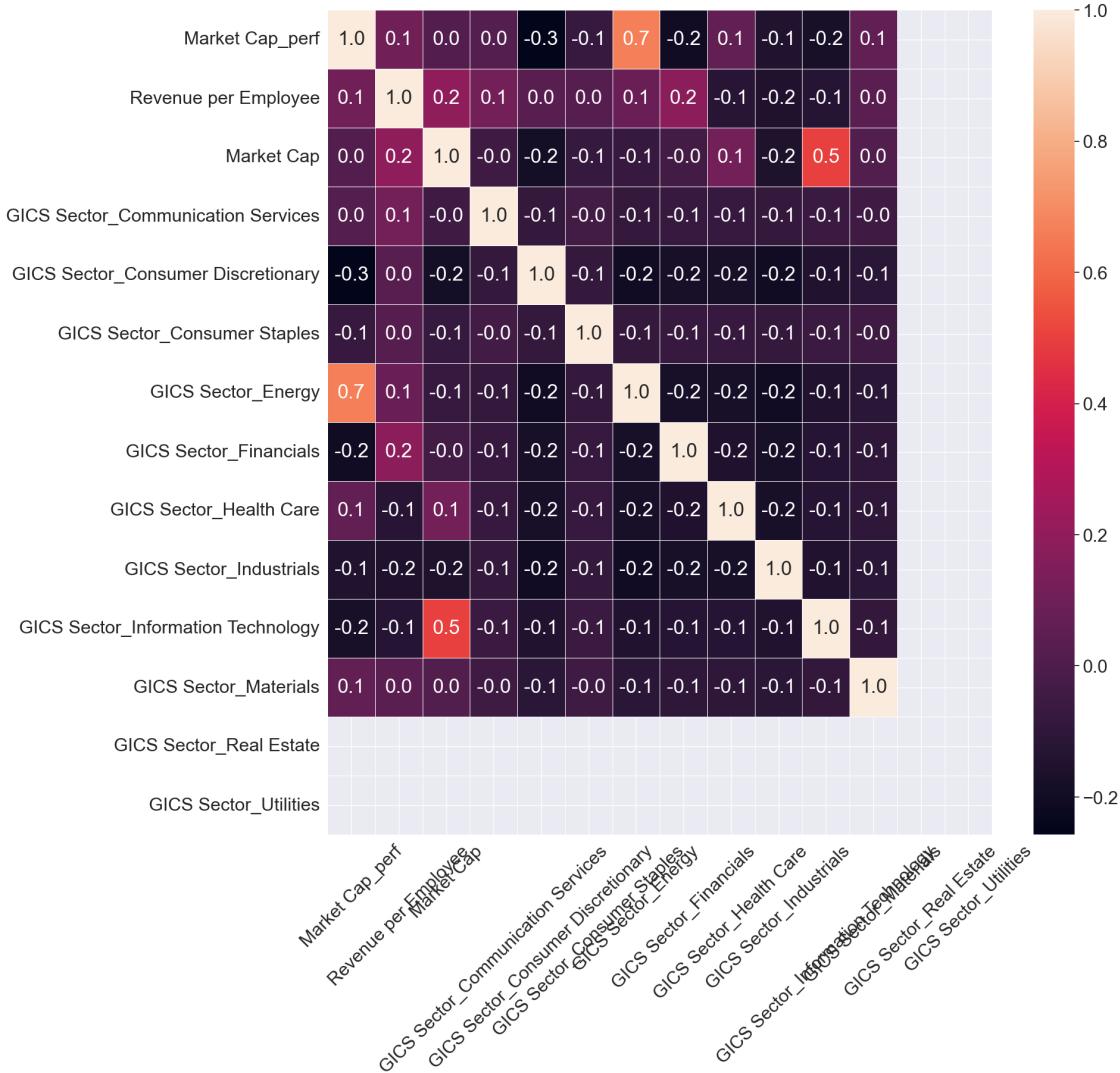
```

GICS Sector

|                        |   |
|------------------------|---|
| Energy                 | 5 |
| Industrials            | 5 |
| Consumer Discretionary | 5 |
| Health Care            | 4 |
| Financials             | 4 |
| Information Technology | 3 |
| Materials              | 2 |
| Communication Services | 1 |
| Consumer Staples       | 1 |

Name: count, dtype: int64

|       | Market Cap_perf | Revenue per Employee | Market Cap   |
|-------|-----------------|----------------------|--------------|
| count | 30.000000       | 3.000000e+01         | 30.000000    |
| mean  | 2.280571        | 5.911873e+05         | 3358.930623  |
| std   | 0.977822        | 4.769266e+05         | 6469.283667  |
| min   | 1.308081        | 1.006522e+05         | 245.865200   |
| 25%   | 1.500476        | 2.704801e+05         | 473.586450   |
| 50%   | 1.992901        | 4.660659e+05         | 1135.774550  |
| 75%   | 2.539989        | 8.024246e+05         | 2422.908625  |
| max   | 5.033335        | 2.353037e+06         | 33785.250000 |



### 1.3 3. Machine learning

#### 1.3.1 Basic Decision Tree

```
[ ]: from sklearn.metrics import confusion_matrix, accuracy_score

# function to quickly retrieve minimal evaluation data for the classifier

def evaluate(clf, X_train, X_test, y_train, y_test):
    print("Train Accuracy : ", accuracy_score(y_train, clf.predict(X_train)))
    print("Train Confusion Matrix:")
    print(confusion_matrix(y_train, clf.predict(X_train)))
    print("Test Accuracy : ", accuracy_score(y_test, clf.predict(X_test)))
```

```

print("Test Confusion Matrix:")
print(confusion_matrix(y_test, clf.predict(X_test)))

[ ]: from sklearn.model_selection import train_test_split
from sklearn.tree import DecisionTreeClassifier
from sklearn.tree import plot_tree

def decision_tree(df, title):
    df["Compared Performance"] = np.where(
        (df["Market Cap_perf"] > df["Market Cap_perf"].mean()),
        "Outperformed",
        "Not Outperformed",
    )
    y = df["Compared Performance"]
    X = df.drop(
        columns=["Compared Performance",
                  "Market Cap_perf", "Name", "GICS Sector"]
    )

    X_train, X_test, y_train, y_test = train_test_split(
        X, y, test_size=0.1, random_state=222
    )

    fig, ax = plt.subplots(figsize=(6, 4))
    sns.countplot(x="Compared Performance", data=df)
    plt.title("Target distribution")
    plt.show()

    print(f"X_train : {X_train.shape}")
    print(f"y_train : {y_train.shape}")
    print(f"X_test : {X_test.shape}")
    print(f"y_test : {y_test.shape}")

    print(title)
    dt = DecisionTreeClassifier(max_depth=2)

    dt.fit(X_train, y_train)

    fig, ax = plt.subplots(figsize=(30, 30))
    plot_tree(
        dt,
        feature_names=X.columns,
        class_names=["Outperformed", "Not Outperformed"],
        filled=True,
        proportion=False,
        fontsize=35,
    )

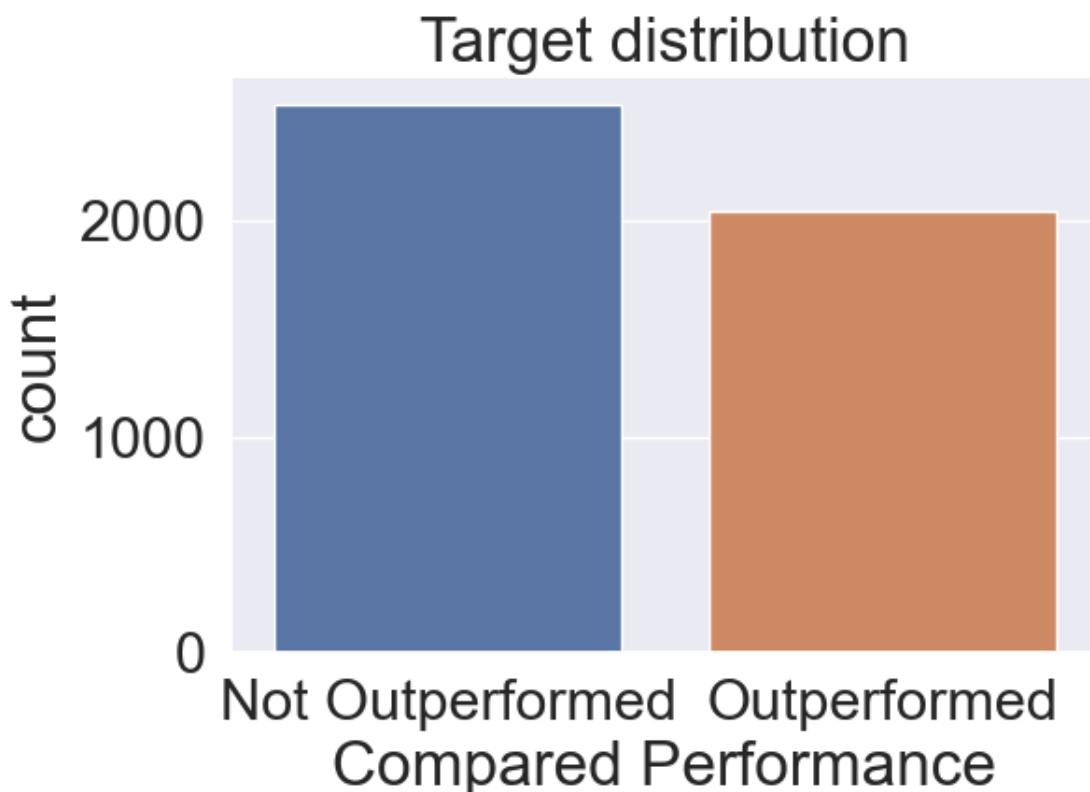
```

```

    )
evaluate(dt, X_train, X_test, y_train, y_test)

[ ]: decision_tree(
    all_df, "Using the data of all periods (including the current, ongoing one)"
)

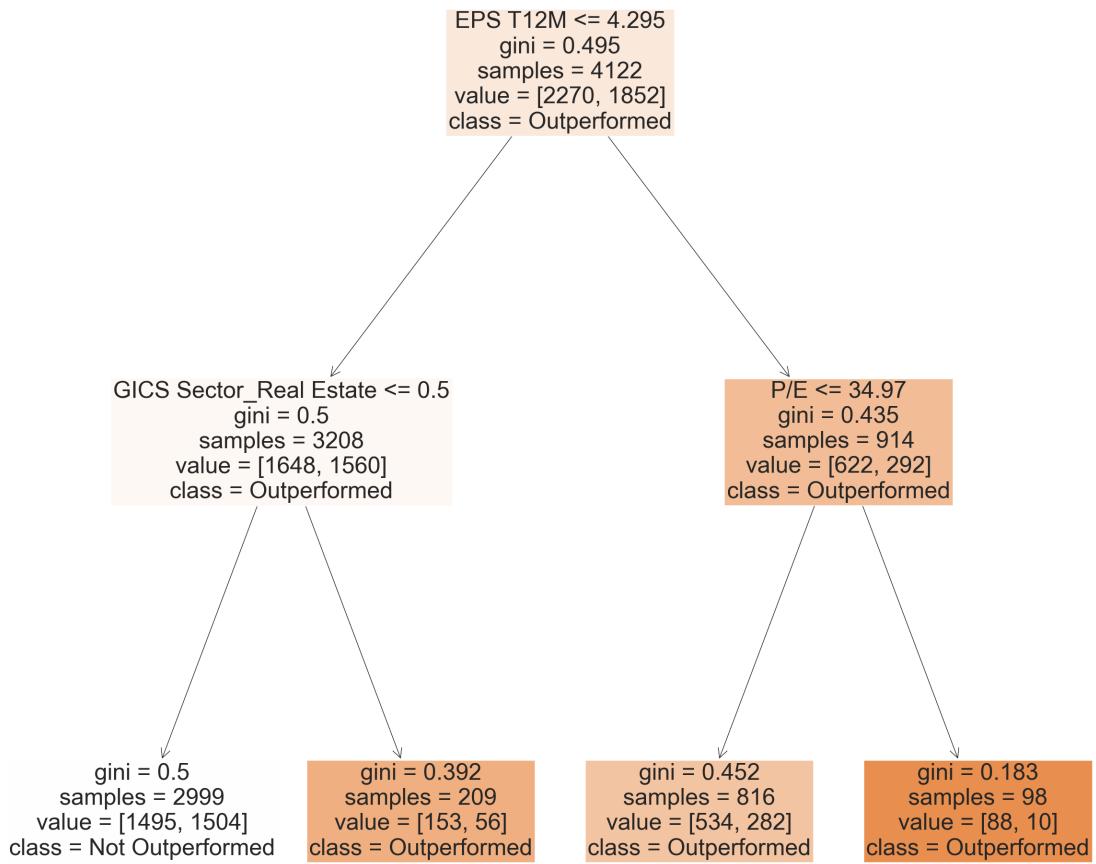
```



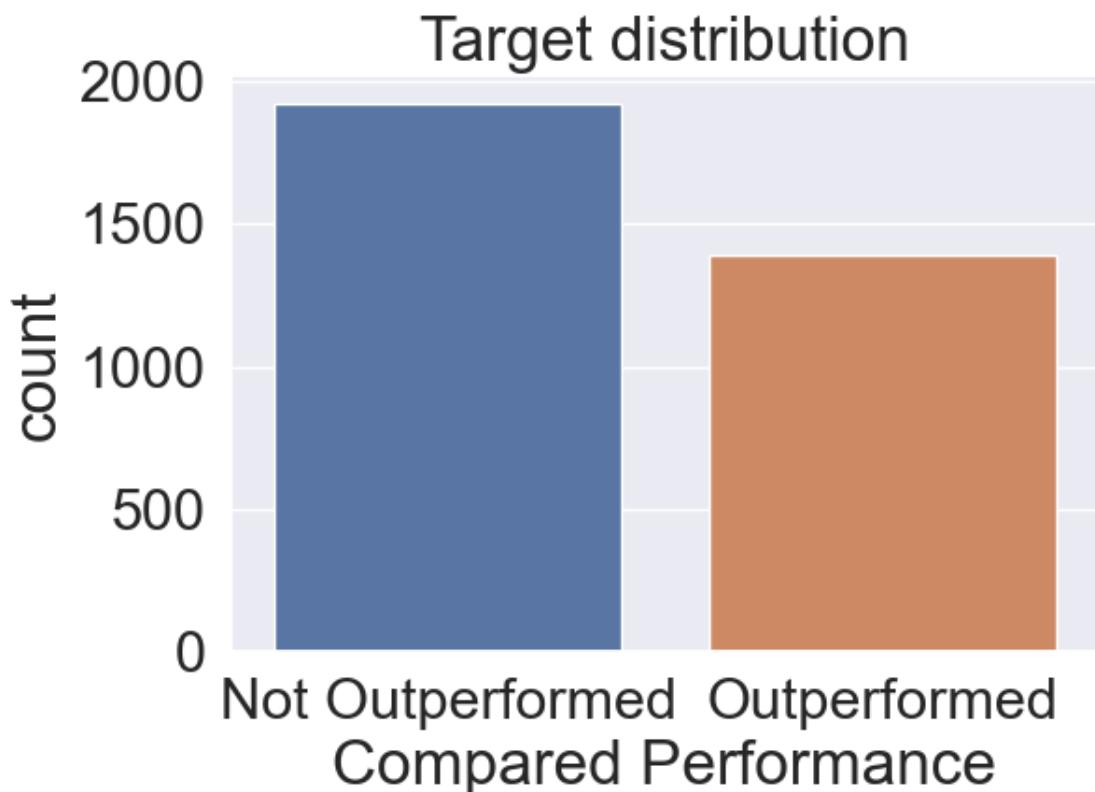
```

X_train : (4122, 20)
y_train : (4122,)
X_test : (458, 20)
y_test : (458,)
Using the data of all periods (including the current, ongoing one)
Train Accuracy : 0.5528869480834546
Train Confusion Matrix:
[[ 775 1495]
 [ 348 1504]]
Test Accuracy : 0.5
Test Confusion Matrix:
[[ 80 185]
 [ 44 149]]

```

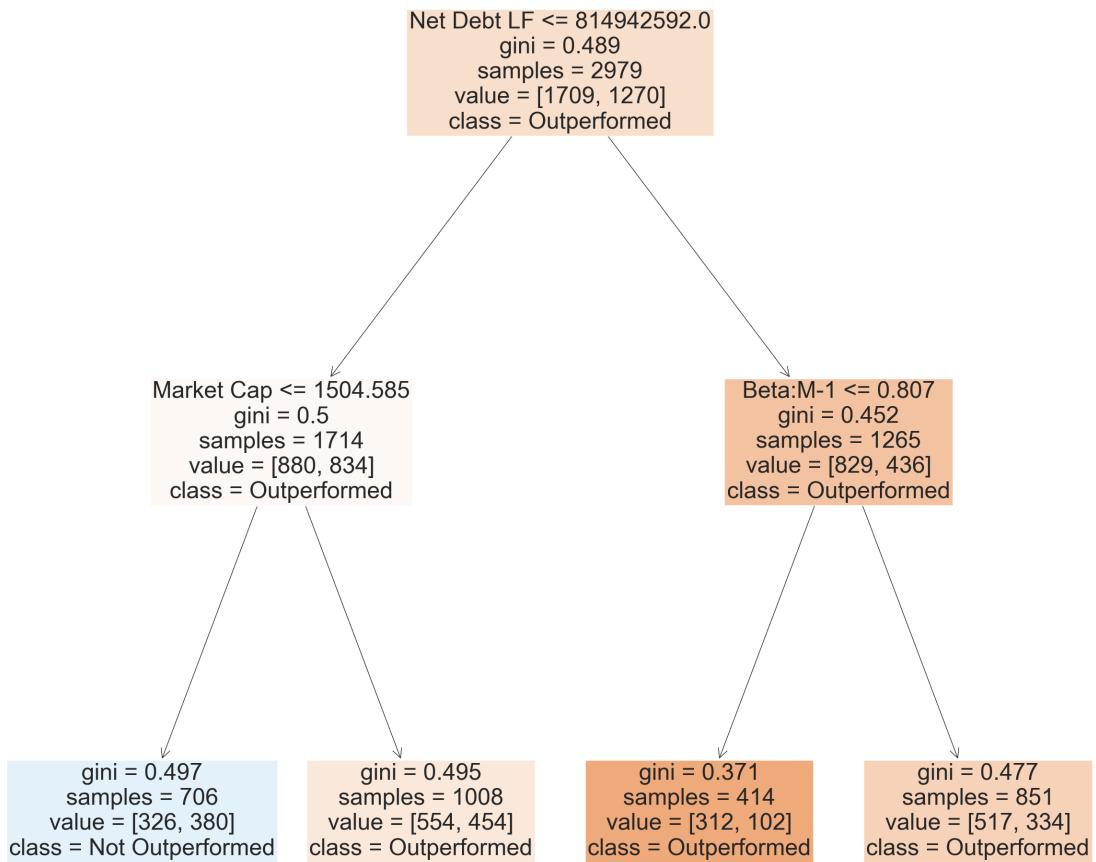


```
[ ]: decision_tree(concluded_df, "Using the data of only concluded periods")
```



```
X_train : (2979, 20)
y_train : (2979,)
X_test : (331, 20)
y_test : (331,)

Using the data of only concluded periods
Train Accuracy : 0.5918093319906008
Train Confusion Matrix:
[[1383 326]
 [ 890 380]]
Test Accuracy : 0.5981873111782477
Test Confusion Matrix:
[[164 48]
 [ 85 34]]
```



```

[ ]: concluded_df["Compared Performance"] = np.where(
    (concluded_df["Market Cap_perf"] > concluded_df["Market Cap_perf"].mean()),
    "Outperformed",
    "Not Outperformed",
)
y_train = concluded_df["Compared Performance"]
X_train = concluded_df.drop(
    columns=["Compared Performance", "Market Cap_perf", "Name", "GICS Sector"]
)

CrP["Compared Performance"] = np.where(
    (CrP["Market Cap_perf"] > CrP["Market Cap_perf"].mean()),
    "Outperformed",
    "Not Outperformed"
)
  
```

```

    "Not Outperformed",
)
y_test = CrP["Compared Performance"]
X_test = CrP.drop(
    columns=["Compared Performance", "Market Cap_perf", "Name", "GICS Sector"]
)

fig, ax = plt.subplots(figsize=(6, 4))
sns.countplot(x="Compared Performance", data=all_df)
plt.title("Target distribution")
plt.show()

print(f"X_train : {X_train.shape}")
print(f"y_train : {y_train.shape}")
print(f"X_test : {X_test.shape}")
print(f"y_test : {y_test.shape}")

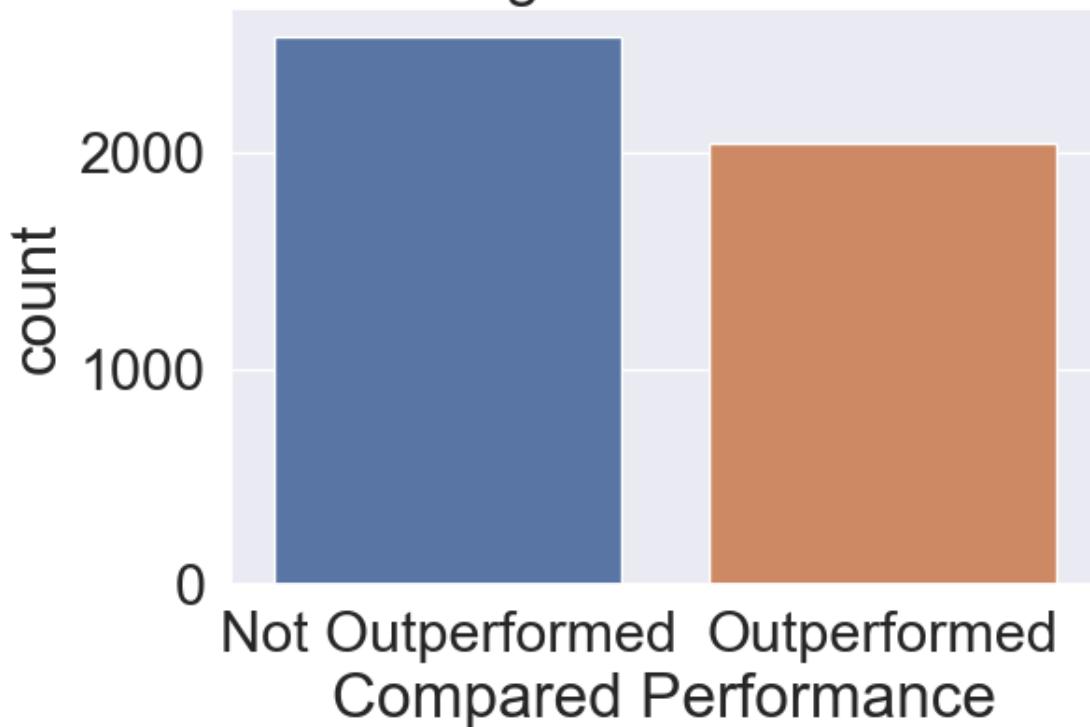
print(
    "Using the data of concluded periods as training data and the current, ↴ongoing period data, as test data"
)
dt = DecisionTreeClassifier(max_depth=2)

dt.fit(X_train, y_train)

fig, ax = plt.subplots(figsize=(30, 30))
plot_tree(
    dt,
    feature_names=X_train.columns,
    class_names=["Outperformed", "Not Outperformed"],
    filled=True,
    proportion=False,
    fontsize=35,
)
evaluate(dt, X_train, X_test, y_train, y_test)

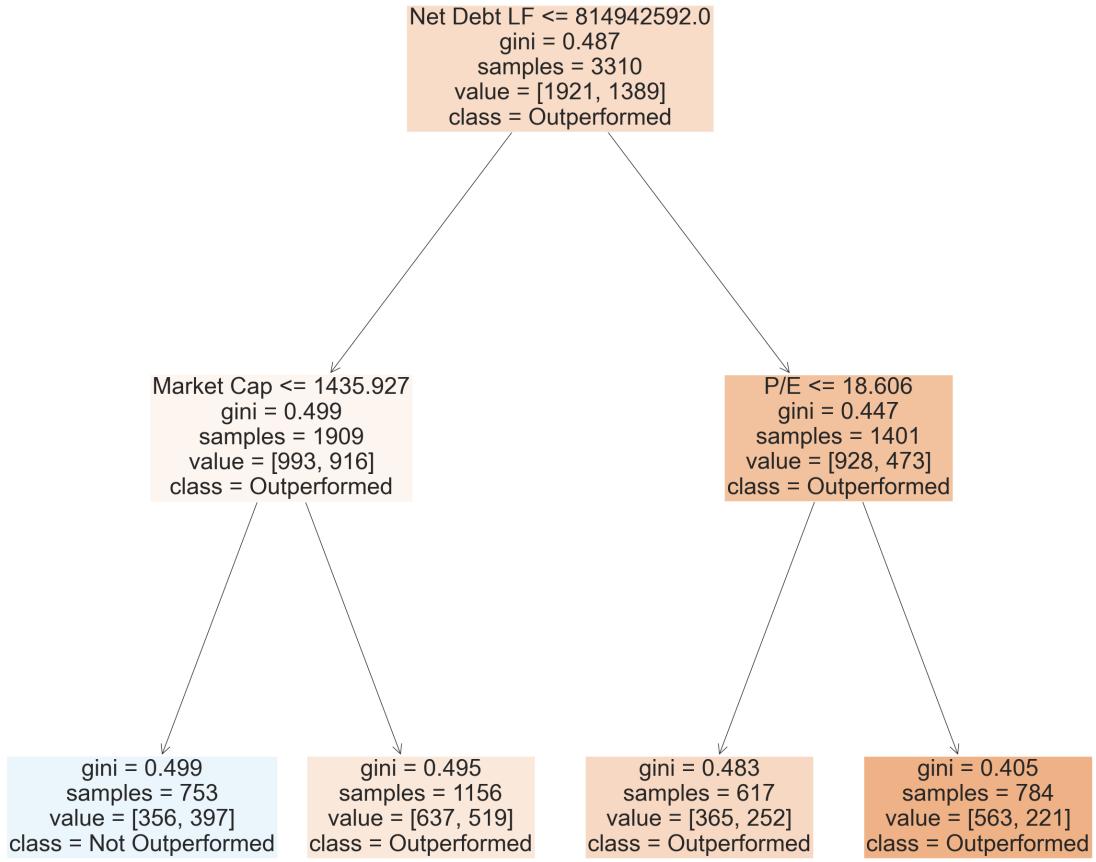
```

## Target distribution



```
X_train : (3310, 20)
y_train : (3310,)
X_test : (1270, 20)
y_test : (1270,)

Using the data of concluded periods as training data and the current, ongoing
period data, as test data
Train Accuracy : 0.592749244712991
Train Confusion Matrix:
[[1565  356]
 [ 992  397]]
Test Accuracy : 0.5062992125984253
Test Confusion Matrix:
[[571  85]
 [542  72]]
```



### 1.3.2 3.2 Random Forest with Hyperparametertuning

```
[ ]: n_estimators = np.arange(50, 250, 50)
max_features = ["auto", "sqrt"]
max_depth = np.arange(2, 20, 1)
min_samples_leaf = [1, 5, 25, 50]
min_samples_split = [2, 5, 25, 50]
max_leaf_nodes = [50, 100, 250, 500]
bootstrap = [True, False]

params_arr = {
    "n_estimators": n_estimators,
    "max_features": max_features,
```

```

    "max_depth": max_depth,
    "min_samples_leaf": min_samples_leaf,
    "min_samples_split": min_samples_split,
    "max_leaf_nodes": max_leaf_nodes,
    "bootstrap": bootstrap,
}

[ ]: from treeinterpreter import treeinterpreter as ti
import os
from sklearn.tree import export_graphviz
import six
import pydot
from sklearn.ensemble import RandomForestClassifier
from sklearn.model_selection import GridSearchCV

rf = RandomForestClassifier()

def visualize_best(gridname, grid, X_test, X_train, y_test, y_train):
    params = {
        "n_estimators": [grid.best_params_["n_estimators"]],
        "max_features": [grid.best_params_["max_features"]],
        "max_depth": [grid.best_params_["max_depth"]],
        "min_samples_leaf": [grid.best_params_["min_samples_leaf"]],
        "min_samples_split": [grid.best_params_["min_samples_split"]],
        "max_leaf_nodes": [grid.best_params_["max_leaf_nodes"]],
        "bootstrap": [grid.best_params_["bootstrap"]],
    }

    clf = RandomForestClassifier(
        n_estimators=params["n_estimators"][0],
        max_features=params["max_features"][0],
        max_depth=params["max_depth"][0],
        min_samples_leaf=params["min_samples_leaf"][0],
        min_samples_split=params["min_samples_split"][0],
        max_leaf_nodes=params["max_leaf_nodes"][0],
        bootstrap=params["bootstrap"][0],
    )

    clf.fit(X_train.values, y_train.values)

    prediction, bias, contributions = ti.predict(clf, X_test.values)

    N = len(X_test.columns)

    outperformed = []
    not_outperformed = []

```

```

for j in range(2):
    list_ = [outperformed, not_outperformed]
    for i in range(N - 1):
        val = contributions[0, i, j]
        list_[j].append(val)

outperformed.append(prediction[0, 0] / N)
not_outperformed.append(prediction[0, 1] / N)

fig, ax = plt.subplots()
ind = np.arange(N)

width = 0.5

p1 = ax.bar(ind, outperformed, width, color="green", bottom=0)
p2 = ax.bar(ind + width, not_outperformed, width, color="red", bottom=0)

ax.set_title("Feature importance for performance result")
ax.set_xticks(ind + width / 2)
ax.set_xticklabels(X_train.columns, rotation=90)
ax.legend((p1[0], p2[0]), ("Outperformed",
                           "Not Outperformed"), loc="upper left")
ax.autoscale_view()

fig.set_figwidth(15)
plt.show()

dotfile = six.StringIO()

i = 0
for tree_in_forest in clf.estimators_:
    export_graphviz(
        tree_in_forest,
        out_file="trees/" + gridname + "tree.dot",
        feature_names=X_train.columns,
        filled=True,
    )
    (graph,) = pydot.graph_from_dot_file("trees/" + gridname + "tree.dot")
    name = gridname + "tree_" + str(i)
    graph.write_png("trees/" + name + ".png")
    os.system("dot -Tpng tree.dot -o tree.png")
    i += 1

```

```
[ ]: def params(X_train, y_train):
    rf_Grid = GridSearchCV(
        estimator=rf, param_grid=params_arr, cv=4, verbose=3, n_jobs=-1
```

```

    )
rf_Grid.fit(X_train.values, y_train.values)

params = {
    "n_estimators": [rf_Grid.best_params_["n_estimators"]],
    "max_features": [rf_Grid.best_params_["max_features"]],
    "max_depth": [rf_Grid.best_params_["max_depth"]],
    "min_samples_leaf": [rf_Grid.best_params_["min_samples_leaf"]],
    "min_samples_split": [rf_Grid.best_params_["min_samples_split"]],
    "max_leaf_nodes": [rf_Grid.best_params_["max_leaf_nodes"]],
    "bootstrap": [rf_Grid.best_params_["bootstrap"]],
}

print(params)

return params

```

```

[ ]: # current is test
current_is_test_params = params(X_train, y_train)

current_test_Grid = GridSearchCV(
    estimator=rf, param_grid=current_is_test_params, cv=4, verbose=3, n_jobs=-1
)
current_test_Grid.fit(X_train.values, y_train.values)

evaluate(
    current_test_Grid, X_train.values, X_test.values, y_train.values, y_test.
    ↴values
)

visualize_best("current_test_", current_test_Grid,
    X_train, X_test, y_train, y_test)

CrP.drop(["Compared Performance"], axis=1, inplace=True)

current_test_Grid

```

Fitting 4 folds for each of 18432 candidates, totalling 73728 fits

c:\Users\lucar\anaconda3\envs\stuffed\lib\site-packages\sklearn\ensemble\\_forest.py:424: FutureWarning:

`max\_features='auto'` has been deprecated in 1.1 and will be removed in 1.3. To keep the past behaviour, explicitly set `max\_features='sqrt'` or remove this parameter as it is also the default value for RandomForestClassifiers and ExtraTreesClassifiers.

{'n\_estimators': [50], 'max\_features': ['auto'], 'max\_depth': [2],

```
'min_samples_leaf': [50], 'min_samples_split': [25], 'max_leaf_nodes': [500],
'bootstrap': [False]}
Fitting 4 folds for each of 1 candidates, totalling 4 fits

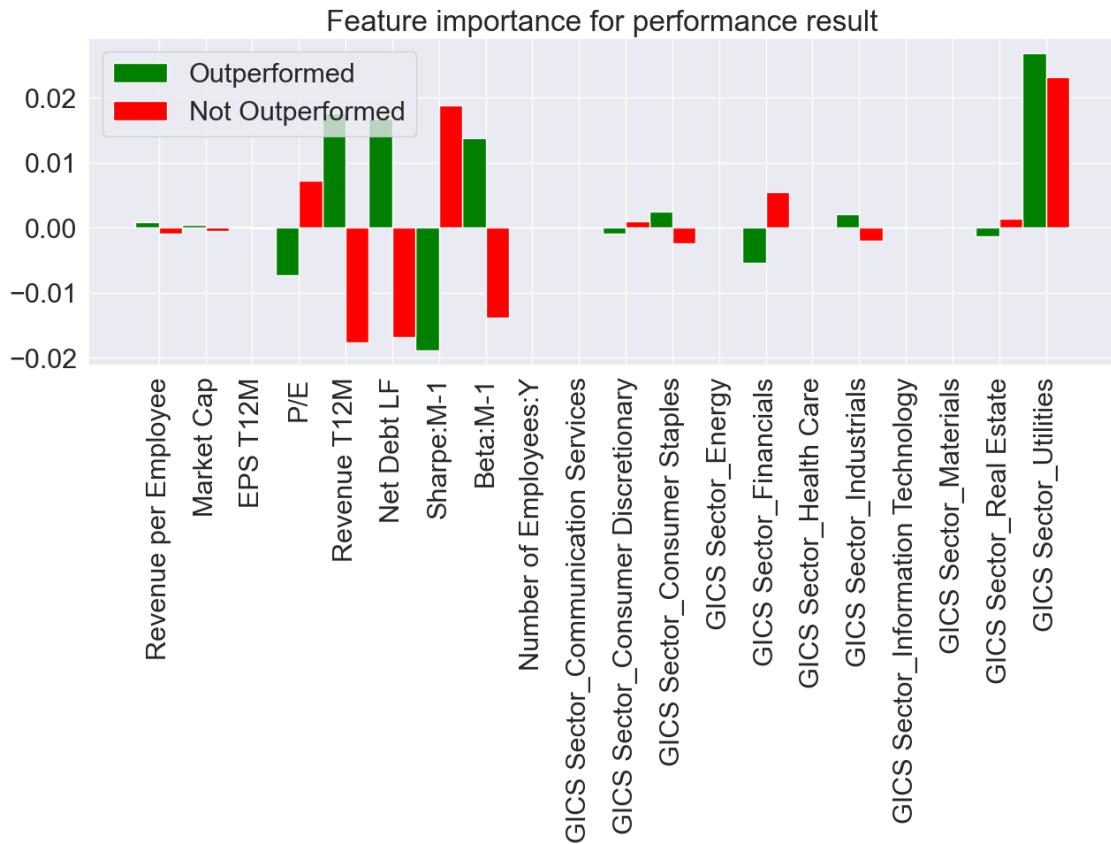
c:\Users\lucar\anaconda3\envs\stuffed\lib\site-
packages\sklearn\ensemble\_forest.py:424: FutureWarning:

`max_features='auto'` has been deprecated in 1.1 and will be removed in 1.3. To
keep the past behaviour, explicitly set `max_features='sqrt'` or remove this
parameter as it is also the default value for RandomForestClassifiers and
ExtraTreesClassifiers.

Train Accuracy : 0.5806646525679758
Train Confusion Matrix:
[[1917    4]
 [1384    5]]
Test Accuracy : 0.5149606299212598
Test Confusion Matrix:
[[651    5]
 [611    3]]

c:\Users\lucar\anaconda3\envs\stuffed\lib\site-
packages\sklearn\ensemble\_forest.py:424: FutureWarning:

`max_features='auto'` has been deprecated in 1.1 and will be removed in 1.3. To
keep the past behaviour, explicitly set `max_features='sqrt'` or remove this
parameter as it is also the default value for RandomForestClassifiers and
ExtraTreesClassifiers.
```



```
[ ]: GridSearchCV(cv=4, estimator=RandomForestClassifier(), n_jobs=-1,
                  param_grid={'bootstrap': [False], 'max_depth': [2],
                  'max_features': ['auto'], 'max_leaf_nodes': [500],
                  'min_samples_leaf': [50], 'min_samples_split': [25],
                  'n_estimators': [50]}, verbose=3)
```

```
[ ]: # all_df
y = all_df[["Compared Performance"]]
X = all_df.drop(
    columns=["Compared Performance", "Market Cap_perf", "Name", "GICS Sector"])
)

X_train, X_test, y_train, y_test = train_test_split(
    X, y, test_size=0.1, random_state=222
)

all_params = params(X_train, y_train)
all_Grid = GridSearchCV(
```

```

estimator=rf, param_grid=all_params, cv=4, verbose=3, n_jobs=-1)
all_Grid.fit(X_train.values, y_train.values)

evaluate(all_Grid, X_train.values, X_test.values,
        y_train.values, y_test.values)

visualize_best("all_", all_Grid, X_train, X_test, y_train, y_test)

all_Grid

```

Fitting 4 folds for each of 18432 candidates, totalling 73728 fits

c:\Users\lucar\anaconda3\envs\stuffed\lib\site-packages\sklearn\ensemble\\_forest.py:424: FutureWarning:

`max\_features='auto'` has been deprecated in 1.1 and will be removed in 1.3. To keep the past behaviour, explicitly set `max\_features='sqrt'` or remove this parameter as it is also the default value for RandomForestClassifiers and ExtraTreesClassifiers.

```
{'n_estimators': [50], 'max_features': ['auto'], 'max_depth': [9],
'min_samples_leaf': [5], 'min_samples_split': [5], 'max_leaf_nodes': [500],
'bootstrap': [False]}
```

Fitting 4 folds for each of 1 candidates, totalling 4 fits

c:\Users\lucar\anaconda3\envs\stuffed\lib\site-packages\sklearn\ensemble\\_forest.py:424: FutureWarning:

`max\_features='auto'` has been deprecated in 1.1 and will be removed in 1.3. To keep the past behaviour, explicitly set `max\_features='sqrt'` or remove this parameter as it is also the default value for RandomForestClassifiers and ExtraTreesClassifiers.

Train Accuracy : 0.8027656477438136

Train Confusion Matrix:

```
[[1898  372]
 [ 441 1411]]
```

Test Accuracy : 0.6179039301310044

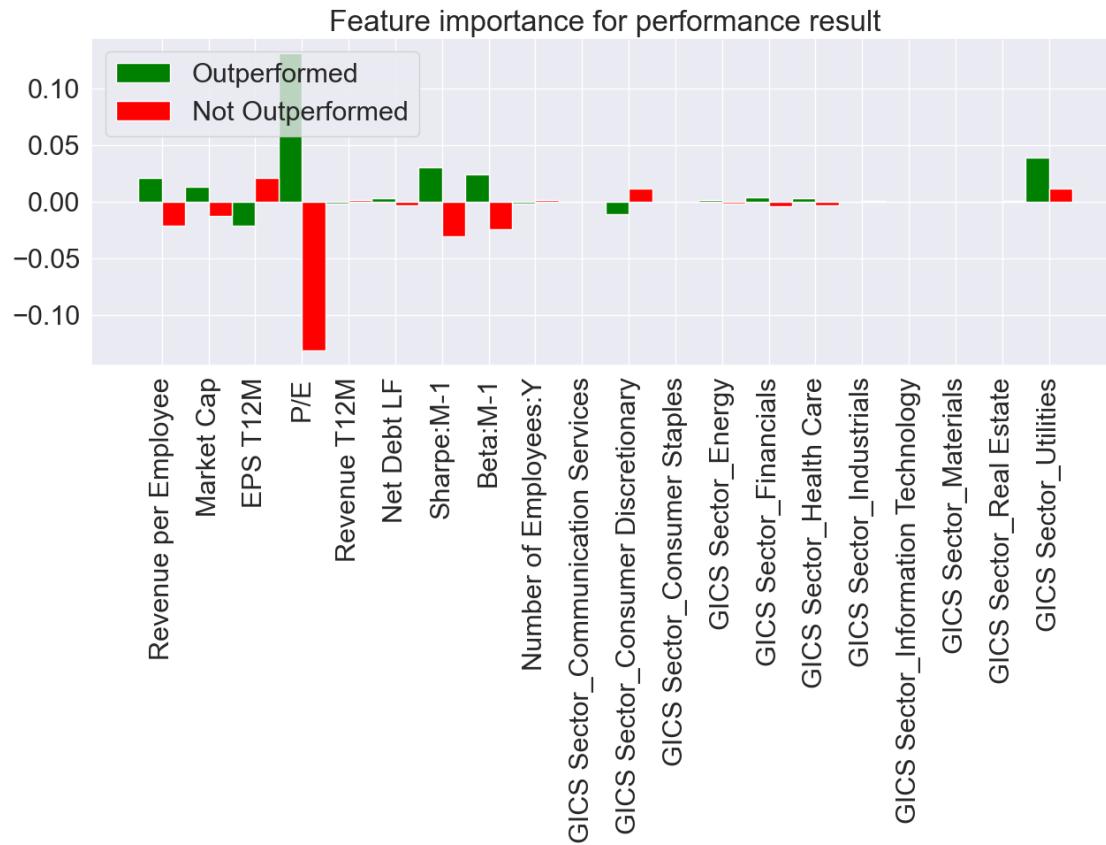
Test Confusion Matrix:

```
[[185  80]
 [ 95  98]]
```

c:\Users\lucar\anaconda3\envs\stuffed\lib\site-packages\sklearn\ensemble\\_forest.py:424: FutureWarning:

`max\_features='auto'` has been deprecated in 1.1 and will be removed in 1.3. To keep the past behaviour, explicitly set `max\_features='sqrt'` or remove this parameter as it is also the default value for RandomForestClassifiers and

ExtraTreesClassifiers.



```
[ ]: GridSearchCV(cv=4, estimator=RandomForestClassifier(), n_jobs=-1,
                  param_grid={'bootstrap': [False], 'max_depth': [9],
                               'max_features': ['auto'], 'max_leaf_nodes': [500],
                               'min_samples_leaf': [5], 'min_samples_split': [5],
                               'n_estimators': [50]}, verbose=3)
```

```
[ ]: # concluded_df
y = concluded_df["Compared Performance"]
X = concluded_df.drop(
    columns=["Compared Performance", "Market Cap_perf", "Name", "GICS Sector"])
)

X_train, X_test, y_train, y_test = train_test_split(
    X, y, test_size=0.1, random_state=222
)

concluded_params = params(X_train, y_train)
```

```

concluded_Grid = GridSearchCV(
    estimator=rf, param_grid=concluded_params, cv=4, verbose=3, n_jobs=-1
)
concluded_Grid.fit(X_train.values, y_train.values)

evaluate(concluded_Grid, X_train.values,
         X_test.values, y_train.values, y_test.values)

visualize_best("concluded_", concluded_Grid, X_train, X_test, y_train, y_test)

concluded_Grid

```

Fitting 4 folds for each of 18432 candidates, totalling 73728 fits

c:\Users\lucar\anaconda3\envs\stuffed\lib\site-packages\sklearn\ensemble\\_forest.py:424: FutureWarning:

`max\_features='auto'` has been deprecated in 1.1 and will be removed in 1.3. To keep the past behaviour, explicitly set `max\_features='sqrt'` or remove this parameter as it is also the default value for RandomForestClassifiers and ExtraTreesClassifiers.

```
{'n_estimators': [50], 'max_features': ['auto'], 'max_depth': [17],
'min_samples_leaf': [5], 'min_samples_split': [5], 'max_leaf_nodes': [500],
'bootstrap': [True]}
```

Fitting 4 folds for each of 1 candidates, totalling 4 fits

c:\Users\lucar\anaconda3\envs\stuffed\lib\site-packages\sklearn\ensemble\\_forest.py:424: FutureWarning:

`max\_features='auto'` has been deprecated in 1.1 and will be removed in 1.3. To keep the past behaviour, explicitly set `max\_features='sqrt'` or remove this parameter as it is also the default value for RandomForestClassifiers and ExtraTreesClassifiers.

Train Accuracy : 0.9315206445115811

Train Confusion Matrix:

```
[[1676  33]
 [ 171 1099]]
```

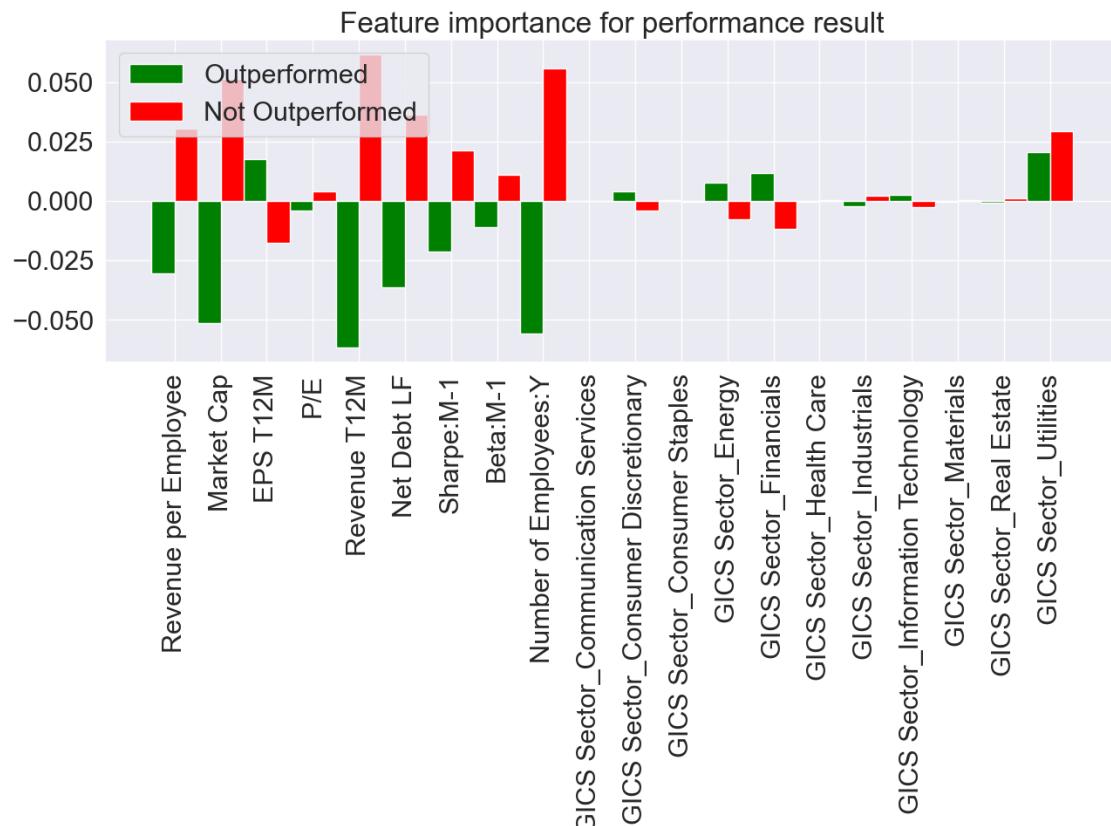
Test Accuracy : 0.6706948640483383

Test Confusion Matrix:

```
[[165  47]
 [ 62  57]]
```

c:\Users\lucar\anaconda3\envs\stuffed\lib\site-packages\sklearn\ensemble\\_forest.py:424: FutureWarning:

```
`max_features='auto'` has been deprecated in 1.1 and will be removed in 1.3. To
keep the past behaviour, explicitly set `max_features='sqrt'` or remove this
parameter as it is also the default value for RandomForestClassifiers and
ExtraTreesClassifiers.
```



```
[ ]: GridSearchCV(cv=4, estimator=RandomForestClassifier(), n_jobs=-1,
                  param_grid={'bootstrap': [True], 'max_depth': [17],
                               'max_features': ['auto'], 'max_leaf_nodes': [500],
                               'min_samples_leaf': [5], 'min_samples_split': [5],
                               'n_estimators': [50]}, verbose=3)
```

## 1.4 3.3 Predictor implementation

COMPARED TO INVESTING INTO THE SP1500 ONLY

Using the findings from above we can determine the companies predicted to outperform

```
[ ]: # we add all companies that have been identified as "Outperformed" and assign
      ↵equal portfolio weights
all_classifier_performance = []
```

```

concluded_classifier_performance = []
current_test_classifier_performance = []

# calculating the portfolio performances
for df in all_dfs:
    current_test_outperformed = []
    concluded_outperformed = []
    all_outperformed = []

    compare_df = df.drop(["Market Cap_perf", "Name", "GICS Sector"], axis=1)
    for i in range(df.shape[0]):
        if current_test_Grid.predict([compare_df.iloc[i].values]) == "Outperformed":
            current_test_outperformed.append(df.iloc[i, 0])
        if all_Grid.predict([compare_df.iloc[i].values]) == "Outperformed":
            all_outperformed.append(df.iloc[i, 0])
        if concluded_Grid.predict([compare_df.iloc[i].values]) == "Outperformed":
            concluded_outperformed.append(df.iloc[i, 0])

    print(current_test_outperformed)
    print(all_outperformed)
    print(concluded_outperformed)

    current_test_classifier_performance.append(1 + np.mean(current_test_outperformed))
    all_classifier_performance.append(1 + np.mean(all_outperformed))
    concluded_classifier_performance.append(1 + np.mean(concluded_outperformed))

```

```

[0.5021601011182174]
[-0.5414174630173221, 0.11610251740509339, 0.1506381395146691,
-0.045177658784108496, 0.13700589878376465, -0.06245373392666296,
-0.5242943917359884, 1.6368671160494397, -0.4445325980698409,
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-0.08623746326322505]
```

```
[ ]: # importing comparative indices/assets, gold is still saved
sp1500 = yf_import("SP1500", periods.iloc[0]["Start"], periods.
    ↪iloc[-1]["Last"])
nasdaq = yf_import("IXIC", periods.iloc[0]["Start"], periods.iloc[-1]["Last"])
WTI = web.DataReader(
    "WTISPLC", "fred", periods.iloc[0]["Start"], periods.iloc[-1]["Last"])
)
```

```
[*****100%*****] 1 of 1 completed
[*****100%*****] 1 of 1 completed
```

```
[ ]: performance = periods
gold_performance = []
sp1500_performance = []
nasdaq_performance = []
WTI_performance = []

# creating a data-frame to compare performances of the portfolios with common
    ↪indices and "crisis" resources
for index, row in periods.iterrows():
    gold_performance.append(
        (
            1
            + ((gold.loc[row["Last"]].values) - (gold.loc[row["Start"]].values))
            / (gold.loc[row["Start"]].values)
        ).item()
    )

    sp1500_performance.append(
        (
            1
            + ((sp1500.loc[row["Last"]].values) - (sp1500.loc[row["Start"]].
    ↪values))
            / (sp1500.loc[row["Start"]].values)
        ).item()
    )

    nasdaq_performance.append(
        (
            1
            + ((nasdaq.loc[row["Last"]].values) - (nasdaq.loc[row["Start"]].
    ↪values))
            / (nasdaq.loc[row["Start"]].values)
        ).item()
    )
```

```

WTI_performance.append(
    (
        1
        + ((WTI.loc[row["Last"]].values) - (WTI.loc[row["Start"]].values))
        / (WTI.loc[row["Start"]].values)
    ).item()
)

[ ]: performance["Gold"] = gold_performance
performance["Nasdaq"] = nasdaq_performance
performance["Spot Crude Oil Price WTI"] = WTI_performance
performance["S&P 1500"] = sp1500_performance
performance["all_Grid"] = all_classifier_performance
performance["concluded_Grid"] = concluded_classifier_performance
performance["current_test_Grid"] = current_test_classifier_performance
dfi.export(
    performance.style.set_properties(
        **{"background-color": "white", "color": "black", "border-color": "#948b8b"}
    ),
    "performance.png",
)
performance

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|   | Name                     | Start      | Last       | Duration       | Gold     | Nasdaq   | \ |
|---|--------------------------|------------|------------|----------------|----------|----------|---|
| 0 | Period 1                 | 2004-04-01 | 2006-08-01 | 27             | 1.626804 | 1.047715 |   |
| 1 | Period 2                 | 2016-09-01 | 2017-08-01 | 11             | 1.000984 | 1.201220 |   |
| 2 | Period 3                 | 2017-09-01 | 2018-07-01 | 10             | 0.957095 | 1.206716 |   |
| 3 | Period 4                 | 2022-01-01 | 2023-04-01 | 15             | 1.125556 | 0.830855 |   |
|   | Spot Crude Oil Price WTI | S&P 1500   | all_Grid   | concluded_Grid | \        |          |   |
| 0 | 1.991006                 | 1.148445   | 1.465244   | 1.722937       |          |          |   |
| 1 | 1.063302                 | 1.135936   | 1.264555   | 1.383578       |          |          |   |
| 2 | 1.424729                 | 1.124224   | 1.282267   | 1.421511       |          |          |   |
| 3 | 0.954698                 | 0.900910   | 1.043568   | 0.817482       |          |          |   |
|   | current_test_Grid        |            |            |                |          |          |   |
| 0 | 1.502160                 |            |            |                |          |          |   |
| 1 | 1.159184                 |            |            |                |          |          |   |
| 2 | 1.455724                 |            |            |                |          |          |   |
| 3 | 0.983550                 |            |            |                |          |          |   |