1a)

Matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 10^{-4} & 0 & 10^{-4} \end{pmatrix}$ $b = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$$\left\| \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 10^{-4} & 0 & 10^{-4} \end{pmatrix} \right\|_\infty = \max\{1+0+2,\ 0+1+0,\ 10^{-4}+0+10^{-4}\} = 3$$

Matrix $A \to A^{-1} = \begin{pmatrix} -1 & 0 & 20000 \\ 0 & 1 & 0 \\ 1 & 0 & -10000 \end{pmatrix}$

$$\left\| \begin{pmatrix} -1 & 0 & 20000 \\ 0 & 1 & 0 \\ 1 & 0 & -10000 \end{pmatrix} \right\|_\infty = \max\{1+0+2\cdot 10^4,\ 0+1+0,\ 1+0+1\cdot 10^4\} = 20001$$

$$\text{cond}(A)_\infty = \|A\|_\infty \cdot \|A^{-1}\|_\infty = 60003$$

1b) $\tilde{b} = \begin{pmatrix} 1 \\ 1 \\ \varepsilon \end{pmatrix}$ $\varepsilon > 0$ $\quad \frac{\|x - \tilde{x}\|_\infty}{\|x\|_\infty} \leq 1\%$

$$b\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \tilde{b}\begin{pmatrix} 1 \\ 1 \\ \varepsilon \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0-\varepsilon \end{pmatrix} \to \|b - \tilde{b}\|_\infty = \max\{0, 0, 0+\varepsilon\} = \varepsilon$$

$$\|b\|_\infty = \max\{1, 1, 0\} = 1$$

$$60003 \cdot \frac{\varepsilon}{1} = 0.01 \to 60003\,\varepsilon = 0.01$$

$$\varepsilon = \frac{1}{6000300}$$

1c) $x = A \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 10^{-4} & 0 & 10^{-4} \end{pmatrix} \cdot b \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$$\tilde{x} = A \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 10^{-4} & 0 & 10^{-4} \end{pmatrix} \cdot \tilde{b} \begin{pmatrix} 1 \\ 1 \\ \frac{1}{6000360} \end{pmatrix} = \begin{pmatrix} -0.99666683 \\ 1 \\ 0.99833342 \end{pmatrix}$$

$$x - \tilde{x} = \begin{pmatrix} -0.00333317 \\ 0 \\ 0.00166658 \end{pmatrix}$$

$$\|x - \tilde{x}\|_\infty = 0.00333317 \qquad \frac{\|x - \tilde{x}\|_\infty}{\|x\|_\infty} = 0.3\% \text{ gestört.}$$

1d)