Politecnico di Milano

Financial Engineering

Assignment 2 RM - Group 1

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Results

Let us start with the baseline case, a portfolio composed by 200 fixed rate 2y bonds issued by different independent IG names ($\rho = 0$). The issuers, starting from rating IG, could migrate to other ratings (HY or default), following a transition matrix Q. The objective is to evaluate the Present Value in a year's time according to the CreditMetrics approach.

The main assumption is knowing the forward discounts curve. In order to obtain it from our data, we can first derive the discounts curve from the zeta rates, then we can apply the following formula to get the forward discount rates at 1 year:

$$B(t_0; t_m, t_n) = \frac{B(t_0, t_n)}{B(t_0, t_m)}$$

Firstly, we need to compute the Present Value in a year's time conditional on having rating j at time $t = 1$ year, for every j. For $j = IG, HY$ the value of the coupon bearing corporate bond at $t = 1$ year is:

$$FV_j(t) = \sum_{n=k}^{N} c_n \overline{B}(t_0; t, T_n) + \pi \sum_{n=k}^{N} \left[P_j(t, T_{n-1}) - P_j(t, T_n)\right] B(t_0; t, T_n)$$

where T_k is the first payment date after 1 year, c_n is the cash-flow at time T_n. To compute the survival probabilities we exploit the fact that the issuers follow a time-homogeneous Markov chain process, hence $P(t_m, t_n) = P(t_m + \alpha, t_n + \alpha)$. Thanks to this, we can take $P_j(1, 2) = P_j(0, 1)$ from the transition matrix (last column, first row for IG, second for HY). Moreover, if we assume piece-wise constant hazard rates, we can compute the survival probabilities for semiannual dates as:

$$P_j(1, 3/2) = P_j(0, 1/2) = \exp\{-\lambda_{j,1} \cdot 1/2\} = \sqrt{P_j(0, 1)}$$

If at 1 year we are in the case of default, the conditional FV corresponds simply to the recovery value. Finally, the Present Value in a year's time is:

$$\mathbb{E}[FV] = \sum_j FV_j \cdot q_{1j}$$

The result is in the second column of Table 1.

The FV's and the expected value are needed in order to compute the losses for a single IG issuer in the different cases (up, down, unchanged, default).

Thanks to a Monte Carlo simulation with 10^6 scenarios, we compute the average number of each case by imposing the Merton model's barrier for default, up and down. With these quantities we deduce the loss in the default case and in the migration case (default, down and unchanged). The VaR is thus computed by taking its 0.999 quantile.

The results are in the third and fourth column of Table 1.

Correlation	Present value	VaR 1y 99.9 %(only defaults)	VaR 1y 99.9%(default and migration)
0	99.7584	1.4940	1.2706

Table 1: Baseline case results

We now set different parameters in order to evaluate the results under other circumstances; particularly we are interested in knowing how the correlation between the companies and the number of issuers impact the obtained values. The results for these cases are in Table 2 and Table 3. With

R_{LC} we mean the correlation given by the Internal Ratings Based formula for large corporates and sovereign:

$$R_{LC} = R_{min} \frac{1 - e^{-k \cdot PD}}{1 - e^{-k}} + R_{max} \frac{e^{-k \cdot PD}}{1 - e^{-k}}$$

where $R_{min} = 0.12$, $R_{max} = 0.24$, $k = 50$ and PD is the one-year default probability.

Correlation	VaR 1y 99.9% (only default)	VaR 1y 99.9% (default and migration)
$\sqrt{0.12}$	3.8843	4.2425
$\sqrt{0.24}$	7.1710	7.8255
IRB	6.2746	6.9750

Table 2: Results for different values of correlation with 200 issuers

Correlation	VaR 1y 99.9% (only default)	VaR 1y 99.9% (default and migration)
$\sqrt{0.12}$	8.9638	8.8448
$\sqrt{0.24}$	8.9638	9.8853
IRB	8.9638	9.6772

Table 3: Results for different values of correlation with 20 issuers

Discussion

1) FALSE: based on our previous results, we can say that considering the migration risk on top of the default risk impacts on the VaR, even if the portfolio is well diversified. Indeed, comparing the second and the third column of Table 2 we can clearly see different values of the VaR. This is due to the fact that, by taking into consideration the rating migration risk, we have an additional potential source of losses on top of the default risk.

2) TRUE: comparing the results in Table 1 with the correspondent ones in Table 2 and Table 3 we notice that the values of VaR with zero correlation are significantly smaller than the ones with a non-zero correlation, meaning that AVR correlation is significant in the computation of VaR. Moreover, looking at the second and third column of Table 2 and Table 3, we can notice that a different value of AVR correlation leads to a significant change in the value of VaR, in particular between the cases with correlation of $\sqrt{0.12}$ and $\sqrt{0.24}$. We also notice that considering the last two rows in both tables we have a less sensitive variation in the value of VaR, given by the fact that the two correlations are very similar ($\sqrt{IRB} = \sqrt{0.21}$).

3) FALSE: as we can see in Table 2, including the migration risk causes an increase of VaR. Nevertheless, we can not draw the same conclusion in case of zero correlation between the issuers, as we can see in Table 1, then we cannot say that including the migration risk increases the VaR under any correlation assumptions.

4) FALSE: as it is shown in Table 2 and Table 3, the concentration risk has a significant impact on the Credit Portfolio Model since for the same value of correlation we can notice different values of VaR between the cases with 20 and 200 issuers. In particular we have a growth in the values of VaR in case of 20 issuers with respect to the case with 200 issuers, due to the action of this concentration risk.