Politecnico di Milano

Financial Engineering

Assignment 3 - Group 1

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March 15, 2024

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1 Asset Swap

The Asset Swap is a financial instrument that permits to have the same behavior of a Floater Coupon Bond. It is useful when an asset manager wants an exposure to a certain company through a floating bond, but this bond is not available on the market and there are only fixed coupon bonds. The asset manager could enter in an Asset Swap Package with an investment bank. An Asset Swap package is a combination of a defaultable fixed coupon bond and an interest-rate swap contract, that swaps the coupon of the F.C. bond with the Euribor rate plus a spread. This particular spread is called "spread in asset swap": s^{asw}. We are in the hypothesis of no counterparty risk.

The goal is to compute the Asset Swap Spread Over Euribor 3m. In our case the bond has maturity of 3 years starting from February 19, 2008 (t_0), with annual coupon $\bar{c} = 3.9\%$, and its price $\bar{C}(t_0)$ is 101 (101% of the face value). The spread is given by:

$$s^{asw} = \frac{C(t_0) - \bar{C}(t_0)}{BPV^f(t_0)}$$

where $C(t_0)$ is the price of an interbank bond with the same coupon and $BPV^f(0)$ is the Basis Point Value of the floating leg.

$C(0)$ is computed through:

$$C(0) = \bar{c} \sum_{i=1}^{N} \delta(t_{i-1}, t_i) B(t_0, t_i) + B(t_0, t_N)$$

where $\{t_i\}_{i=1,\dots,N}$ are the fixed leg payments.

In order to compute $BPV^f(t_0)$, we have to find the floating leg payments. To do this, we first increment the settlement date by 3 months at time, up to maturity 3 years. Then we check that the found dates are not holidays or weekends. We consider as festive days: New Year's Day, Good Friday, Easter Monday, First of May, Christmas Day, Boxing Day. Finally, we can apply the formula

$$BPV^f(t_0) = \sum_{i=1}^{N} \delta(t_{i-1}, t_i) B(t_0, t_i)$$

All the discount factors are found by interpolation of the bootstrap curve.

We obtain a spread equal to -34.1027 bp. It is negative since $\bar{C}(t_0) > C(t_0)$, so we can deduce that the company YY has a rating better than AA, and in fact their bonds are traded above par.

2 CDS Bootstrap

The family of deterministic intensity based models allows to price single-name credit products. It is based on the relation between the survival probability $P(t_0, T)$ of an issuer and the intensity/hazard rate $\lambda(t)$:

$$P(t_0, T) = \exp\left(-\int_{t_0}^{T} \lambda(t)\, dt\right) \tag{1}$$

In order to calibrate these kind of models we can use the bootstrap of intensities from quoted CDS spreads. We are given the CDS spreads on Intesa Sanpaolo (ISP) for expiries from 1 year to 7 years, with the 6 years' one missing. In order to complete this set we compute the missing spread by using spline interpolation, obtaining that $\bar{S}_{6y} = 39.5391$ bp.

Having the complete set of spreads, we compute $\lambda(t)$, assuming it is a piece-wise constant function (constant within every year). We can do this by considering the accrual term or not. Let us start with the approximated result (neglecting the accrual): we iteratively find $P(t_0, t_k)$, starting from $t_1 = 1y$, by inverting

$$\bar{s}_k \sum_{i=1}^{k} \delta(t_{i-1}, t_i) B(t_0, t_i) P(t_0, t_i) = (1 - \pi) \sum_{i=1}^{k} B(t_0, t_i) \left[P(t_0, t_{i-1}) - P(t_0, t_i)\right]$$

where the year fraction δ is computed with day convention 30/360 (European). From the set of survival probabilities we can derive the set of intensities by inverting Formula 1 and by exploiting the fact that $\lambda(t)$ is piece-wise constant. Let us proceed with the exact result (considering the accrual): it is the same procedure as before, but this time we have an extra term deriving from the maturation of payment that is owed, from the last payment date to the default time:

$$\bar{s}_k \sum_{i=1}^{k} \delta(t_{i-1}, t_i) B(t_0, t_i) P(t_0, t_i) + \bar{s}_k \sum_{i=1}^{k} \frac{\delta(t_{i-1}, t_i)}{2} B(t_0, t_i) \left[P(t_0, t_{i-1}) - P(t_0, t_i)\right] =$$

$$= (1 - \pi) \sum_{i=1}^{k} B(t_0, t_i) \left[P(t_0, t_{i-1}) - P(t_0, t_i)\right]$$

The approximated and exact results are in Table 1.

The Mean Squared Error between the approximated and the exact result is $6.3755 \cdot 10^{-10}$. We deduce that the contribution of the accrual is negligible, as we can expect since its term is two order magnitude less than the other term on the left-hand side of the equation. The plot of the intensities computed in the two ways is in Figure 1.

An useful rule of thumb for the computation of the intensity is considering the Jarrow-Turnbull approximation (a constant λ over the time horizon and a continuously paid CDS spread). Thanks

Time horizon	No accrual (approximation)	Accrual (exact)
$1y$	48.0852	48.2014
$2y$	58.5105	58.6792
$3y$	68.7350	68.9657
$4y$	86.7014	87.0477
$5y$	73.7514	74.0215
$6y$	61.4519	61.6393
$7y$	85.4962	85.8475

Table 1: Intensities ISP (bp)

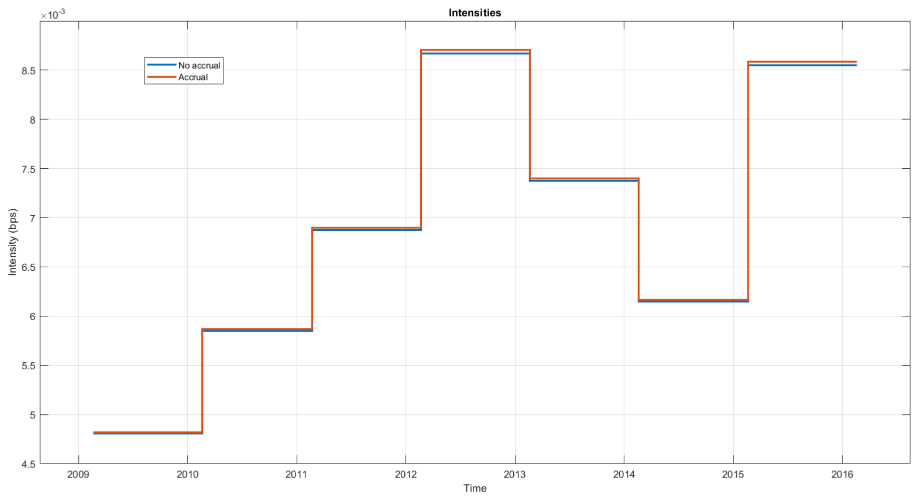

Figure 1: Intensities ISP

to this approximation, we can easily compute the intensities as:

$$\lambda = \frac{\bar{s}}{1-\pi}$$

The reason why it is considered a thumb rule is that the following relation with the bootstrapped λ holds:

$$1y : \lambda_{JT_1} \approx \lambda_{1y}$$

$$2y : \lambda_{JT_2} \approx \frac{\lambda_{1y} + \lambda_{2y}}{2}$$

and so on. λ_{JT_n} is the intensity computed by using the Jarrow-Turnbull approximation over a time horizon of n years and λ_{ny} is the intensity within year n, computed by using the set of CDS spreads (considering the accrual). To simplify the notation, we call as $\bar{\lambda_n}$ the right-hand side of the approximation above for a time horizon of n years. The results are in table 2.

As we can see, the results are very similar to the ones obtain before.

Time horizon	λ_{JT}	$\bar{\lambda}$
$1y$	48.3333	48.2016
$2y$	53.3333	53.4403
$3y$	58.3333	58.6154
$4y$	65.0000	65.7235
$5y$	66.6667	67.3831
$6y$	65.8985	66.4258
$7y$	68.3333	69.2003

Table 2: Intensities ISP (bp) - JT approximation

3 Price First to Default

Let us now introduce another company, UniCredit (UCG), with its CDS spreads (annual bond) taken from the market. Since the 6 years' one is missing, we interpolate as before to obtain it.

Thus, we have a reference portfolio (RP) with two obligors, ISP and UCG, with two different recovery rates (40% for the first and 45% for the second). The objective is to price a First to Default on this RP with maturity 4 years starting from the settlement date. We consider the Li model and a Gaussian copula with correlation $\rho = 0.2$. We use a Monte Carlo approach in order to find the FTD spread.

We first simulate the Gaussian copula with the following steps:

- we simulate a vector of two independent standard normal random variables $\underline{y} = [y_1, y_2]$
- we apply the linear transformation $\underline{x} = \underline{\mu} + A\underline{y}$, where $\underline{\mu}$ is the mean (null vector) and A is such that $AA^T = \Sigma$, where Σ is the covariance matrix (2x2 matrix with 1 on the diagonal and ρ outside). A is a symmetric positive definite lower triangular and we obtain it through the Cholesky factorization.
- we apply the normal CDF to all the components of $\underline{x}$ and we obtain the vector $\underline{u} = [u_1, u_2]$.

Thanks to the copula we describe the default dependencies between the two obligors. The components u_i of the vector $\underline{u}$ represent the survival probability of company i ($u_i = P_i(t_0, \tau)$), hence, by inverting Formula 1, we can obtain the times to default τ_i. Regarding the intensities, we consider the exact results, obtained including the accrual. Since we have to price a First to Default we are interested only in the lowest τ_i, that we call τ, and in the correspondent defaulted obligor recovery rate (π).

We repeat this simulation procedure multiple times and at each iteration we compute the NPV of the fee leg and the NPV of the contingent leg. Regarding the fee leg, the NPV is the sum of the discounted cash flows (given by the spread) up to τ if τ is before the maturity of the contract, otherwise up to the maturity. The spread (our unknow s) can be collected.

The NPV of the contingent leg, instead, is the discounted Loss Given Default $(1-\pi)$ if the first default τ happens before the maturity, zero otherwise.

Now that we have the NPV's of every simulation, we take the mean of all the NPV^{fee}'s, as a function of s, $(s\bar{X})$ and the mean of all the $NPV^{contingent}$'s $(\bar{Y})$. The Monte Carlo spread is such that it makes the average NPV's equal, i.e. $s = \frac{\bar{Y}}{\bar{X}}$.

The $(1-\alpha)$ confidence interval for a ratio of means (Fieller's confidence interval) is:

$$\text{Lower Bound} = \frac{\bar{X}\bar{Y} - t_q^2\hat{\sigma}_{\bar{X}\bar{Y}} - \sqrt{(\bar{X}\bar{Y} - t_q^2\hat{\sigma}_{\bar{X}\bar{Y}})^2 - (\bar{X}^2 - t_q^2\hat{\sigma}_{\bar{X}}^2)(\bar{Y}^2 - t_q^2\hat{\sigma}_{\bar{Y}}^2)}}{\sqrt{\bar{X}^2 - t_q^2\hat{\sigma}_{\bar{X}}^2}} \tag{2}$$

$$\text{Upper Bound} = \frac{\bar{X}\bar{Y} - t_q^2\hat{\sigma}_{\bar{X}\bar{Y}} + \sqrt{(\bar{X}\bar{Y} - t_q^2\hat{\sigma}_{\bar{X}\bar{Y}})^2 - (\bar{X}^2 - t_q^2\hat{\sigma}_{\bar{X}}^2)(\bar{Y}^2 - t_q^2\hat{\sigma}_{\bar{Y}}^2)}}{\sqrt{\bar{X}^2 - t_q^2\hat{\sigma}_{\bar{X}}^2}} \tag{3}$$

where $\hat{\sigma}_{\bar{X}}^2$ is the variance of X, $\hat{\sigma}_{\bar{Y}}^2$ is the variance of Y, $\hat{\sigma}_{\bar{X}\bar{Y}}^2$ is the covariance of X and Y, t_q is the t percent point value with N - 1 degrees of freedom (N is the number of simulation).

The obtained spread with 10^4 simulations is 87.0373 bp with a 95% confidence interval of [80.1467; 93.9448]. We notice that the sum of the two CDS spreads of each obligor, given a 4 years horizon, is bigger than the spread of the First to Default, as expected, since having two the two CDS gives a total protection as opposed to just having the FtD.

If we plot the spread, varying the correlation ρ from -0.99 to 0.99 (we have to exclude $\rho = \pm 1$ beacuse for this values the matrix A is no more positive definite), we obtain the plot in Figure 2.

We notice that the spread is not increasing with respect to the correlation. Moreover, when ρ approaches -1, we have a very high spread, since the default of one obligor implies a very low probability of default of the other. As a consequence, the First to Default gives an almost full protection on the reference portfolio. Instead, when ρ is close to 1, it is likely that if one company defaults, the other will follow. For this reason, we have a very low spread, since, even if we have an FtD, we will be covered only for the first bankruptcy, and so we will incur in a loss. We also point out that for $\rho \approx 1$ the spread is bigger than the maximum between the spreads of the single CDS.

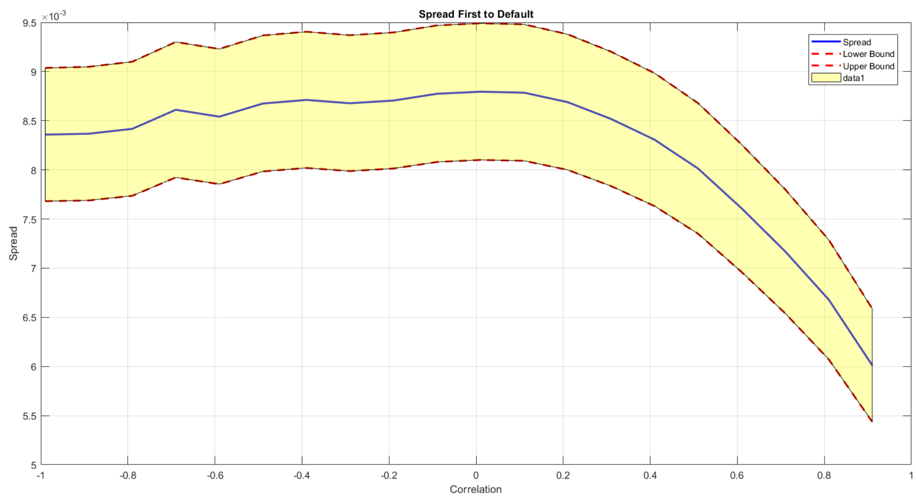

Figure 2: FtD spread varying ρ