Politecnico di Milano

Financial Engineering

Assignment 1 RM - Group 1

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Exercise 1

Given market data (risk free discount factors, cashflows and dirty prices of coupon-bearing defaultable bonds), the objective is to deduce the hazard rate curve for each issuer and the Z-spread.

Firstly, we assume that the hazard curve $h(t,T)$ is step-wise constant: $h(t,T) = h \ \forall \ T$ s.t. $T_m \leq T \leq T_n$, where T_m and T_n are two consecutive bond maturities. For every company we compute the hazard rate by inverting the Risk-Neutral formula for pricing coupon-bearing defaultable bonds (Formula 1) and by substituting Formula 2 for defaultable ZC bonds. In this way we obtain an expression for h_i (in our case h_1 and h_2).

$$C(t) = \sum_{n=1}^{N} \overline{c_n}\overline{B}(t,T_n) + \overline{B}(t,T_N) + \pi \sum_{n=1}^{N} [P(t,T_{n-1}) - P(t,T_n)]\overline{B}(t,T_n) \tag{1}$$

$$\overline{B}(t,T) = B(t,T) \cdot P(t,T) = B(t,T) \cdot e^{-\int_t^T h(t,s)\,ds} \tag{2}$$

We report the results in Table 1, having verified on Matlab that they make the model prices match market prices.

h_1 **IG**	h_2 **IG**	h_1 **HY**	h_2 **HY**
46.9	128.0	396.8	253.1

Table 1: Hazard rates (BP)

Regarding the Z-spread, we assume that this score is a unique constant for each issuer. We calculate the Z-spread by inverting the "Fixed-Income trader formula" (Formula 5) and the formula for defaultable ZC bonds using Z-spread (Formula 4):

$$C(t) = \sum_{n=1}^{N} \overline{c_n}\hat{B}(t,T_n) + \hat{B}(t,T_N) \tag{3}$$

$$\hat{B}(t,T) = \exp\left(-\int_t^T [f(t,s) + z(t,s)]\,ds\right) = \exp\left(\ln B(t,T) - \ln B(t,t) - z(T-t)\right) \tag{4}$$

In 4 we exploit the formula of the forward $f(t,s)$ and we apply the Fundamental Theorem of Calculus, knowing that z is constant. The results are shown in Table 2.

Z_1 **IG**	Z_2 **IG**	Z_1 **HY**	Z_2 **HY**
28.6	52.8	239.6	197.0

Table 2: Z-spread (BP)

Exercise 2

We deduce the market-implied rating transition matrix by exploiting the property that the stochastic rating process is a Homogeneous Markov chain. Hence, thanks to the Chapman-Kolmogorov Equation, we can write:

$$Q^{(2)} = Q * Q \tag{5}$$

where Q is the transition matrix associated to the Markov chain: $\begin{pmatrix} 1-x-PD_1 & x & PD_1 \\ 1-y-PD_2 & y & PD_2 \\ 0 & 0 & 1 \end{pmatrix}$.

$PD1$ and $PD2$ are the 1y default probabilities for each issuer.

$Q^{(2)}$ is the transition matrix for a time horizon of 2 years, whose elements (1,3) and (2,3) are equal to the 2y default probability $1-e^{-[h_1+h_2]}$, respectively for IG and HY bonds.

Using Matlab we solve for x and y obtaining the rating matrix: $\begin{pmatrix} 0.762 & 0.234 & 0.004 \\ 0.391 & 0.570 & 0.039 \\ 0 & 0 & 1 \end{pmatrix}$.