Assignment: Tabular Reinforcement Learning
Course: Reinforcement Learning, Master CS, Leiden University
Written by: Thomas Moerland

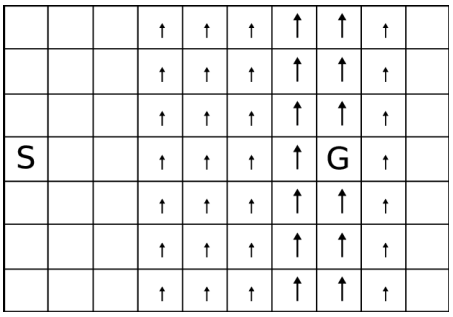
Research Question

In this assignment, you will study a range of basic principles in tabular, value-based reinforcement learning. They serve as a primer for the rest of the course. In particular, we will study the following topics:

- **Dynamic Programming** (DP) (Part 1):
  We first focus on dynamic programming, which is a bridging method between planning and reinforcement learning. DP assumes full access to a model of the environment, i.e., we can get $p(s'|s,a)$ and $r(s,a,s')$ for any state s and action a. DP is guaranteed to find the optimal solution, but it 1) requires a model (which is not always available) and 2) suffers from the curse of dimensionality (which all tabular methods actually do, and to which we get back later in the course).
- **Model-free RL**: We next switch to the reinforcement learning setting, where we do not have access to a model, but can only permanently execute actions from a state, and have to continue from the resulting next state.
  - **Exploration** (Part 2) The first issue this brings up is exploration versus exploitation: we need to sometimes try novel things, but at some point also exploit what we know works well. We will compare two simple ways to ensure exploration: ϵ-greedy and a softmax/Boltzmann policy.
  - **Back-up**: The second main aspect of any RL algorithm is the back-up. We acquired new information, and want to construct a new estimate of the value of a certain state-action pair s, a. There are two important considerations when constructing this back-up:
    - **Off-policy versus on-policy** (Part 3): This difference is best illustrated for one-step back-ups, for which we will compare Q-learning (off-policy) to SARSA (on-policy).
    - **Depth** (Part 4): We can also compute deeper back-ups, where we sum more rewards in a trace. We will compare 1-step back-ups, n-step back-ups, and Monte Carlo back-ups.

Environment

You will study these methods on the *Stochastic Windy Gridworld*, an adapted version of Example 6.5 (page 130) in *Reinforcement Learning: An Introduction* (second edition) by Sutton and Barto (see figure).



The environment consists of a 10x7 grid, where at each cell we can move up, down, left or right. We start at location (0,3) (we start indexing at 0, as is done in Python as well), indicated in the figure by 'S'. Our goal is to move to location (7,3), indicated by 'G'. However, a special feature of the environment is that there is a vertical wind. In columns 3, 4, 5 and 8, we are pushed one additional step up, while in columns 6 and 7, we move up two additional steps. The wind does not always blow, but is randomly present on 80% of the occasions (which makes the environment stochastic!). The reward of the agent at each step is -1, while reaching the goal gives a reward of +40, and terminates the episode.

Preparation

Python You need to install Python 3, the packages `Numpy`, `Matplotlib`, `SciPy` and an IDE of your choice.

Files You are provided with the following Python files:

- `Environment.py`: This file generates the environment. Run the file to see a demonstration of the environment with randomly selected actions. Inspect the class methods and make sure you understand them. With `render()` you can interactively visualize the environment during execution. If you provide `Q_sa` (a Q-value table), the environment will also display the Q-value estimates for each action in each state, while toggling `plot_optimal_policy` will also show arrows for the optimal policy. Play around with these settings, and make sure you understand them.
- `Dynamic_Programming.py`: This file contains placeholder classes and functions for your Dynamic Programming experiments (Part 1). Your goal is to complete these classes and functions.
- `Q-learning.py`: This file contains placeholder classes and functions for your Q-learning implementation.
- `SARSA.py`: This file contains placeholder classes and functions for your SARSA implementation.

- `MonteCarlo.py`: This file contains placeholder classes and functions for your Monte Carlo RL implementation.
- `Nstep.py`: This file contains placeholder classes and functions for your n-step Q-learning implementation.
- `Experiments.py`: In this file you will write all your code for the reinforcement learning experiments (Part 2, 3 and 4).
- `Helper.py`: This file contains some helper classes for plotting and smoothing. You can choose to use them, but are of course free to write your own code for plotting and smoothing as well. Inspect the code and run the file to verify that your understand what the functions do.

Matplotlib rendering Depending on your local software setup and the way you run your code (e.g., from the command line, or within an IDE), you may need to change the Matplotlib backend to allow for interactive rendering. For example, when your code does not give interactive rendering in PyCharm, you may add the following two lines to the top of `Environment.py`:

```
import matplotlib
matplotlib.use('Qt5Agg') # or TkAgg
```

Depending on you own software setup, play around with the backend settings until you find the plot being interactively updated (or run it from the command line, outside of your IDE).

Grading The focus of this assignment is on understanding the basis RL methodology. Therefore, we mostly grade you on showing conceptual understanding, and provide you with relatively much starting code. You are graded on three criteria:

1. A **proper description of your algorithms and methodology in your report**. Include equations, explain what they mean, show that you understood the algorithm.
2. A **proper implementation of the algorithms**. Do not copy your code from others! (We have to punish plagiarism). Systematically making the same mistakes as someone else is also suspicious. You can discuss together how an algorithm works, but you really need to write your own code.
3. A **good interpretation and discussion of your results**. Show that you understood what was going on, and that you thought about different algorithmic decisions.

Handing in The deadline for this assignment is **Sunday March 5, 2023, 23:59**. You need to hand in:

- A **report** (pdf) of **maximum 8 pages**. Use the following LaTeX template. See `https://rl.liacs.nl/assignments` for further details. Be sure your report:
  - Describes your methods (include equations).
  - Shows results (figures).
  - Interprets your results.
- All **code** to replicate your results. Your submission should contain:

- The original `Environment.py` and `Helper.py`
- Your modified `Dynamic_Programming.py`, `Q_learning.py`, `SARSA.py`, `MonteCarlo.py`, `Nstep.py`, and (potentially) `Experiment.py`.
- Executing `Dynamic_Programming.py` and `Experiment.py` from the command line should produce all your plots, and save them to the current folder (with an clear name).

Be sure to verify that your code runs from the command line, and does not give errors!

Warning: common errors (with statistical experiments).

- Average your results over repetitions (since each runs is stochastic)! If necessary, apply additional smoothing to your curves to make them better interpretable.
- In each repetition, really start from scratch, i.e., randomly initialize a new environment, and initialize your policy from scratch. Do not fix any seeds within the loop over your repetitions! Each repetition should really be an independent repetition.

For these experiments, code to average over repetitions and to smooth learning curves, is given in the assignment. This serves as an example, so you can do it yourself in next assignments.

1 Dynamic Programming

You first study Dynamic Programming, in particular the Q-value iteration algorithm (Alg. 1). In this algorithm you sweep through all state-action pairs, each time updating the estimate of a state-action value based on the following equation:

$$Q(s,a) \leftarrow \sum_{s'} \Big[p(s'|s,a) \cdot \big(r(s,a,s') + \gamma \cdot \max_{a'} Q(s',a') \big) \Big] \tag{1}$$

In DP, you have access to the full model of the environment dynamics. Therefore, you may use the `StochasticWindyGridworld.model()` function in your experiments.

You proceed with the following steps:

a) **Implement**:

- Correctly complete the class `QValueIterationAgent()` in the file `DynamicProgramming.py`.
  - In `init()`, initialize a table with means $Q(s,a)$ to 0.
  - In `select_action()`, implement the greedy policy: $\pi(s) = \arg\max_a Q(s,a)$.
  - In `update()`, implement the Q-iteration update, shown in Eq. 1. Make sure you print the maximum absolute error after each full sweep (i.e., each time after you visited each state-action pair once).
- Correctly complete the function `Q_value_iteration()` in the file `DynamicProgramming.py`. This function should execute Q-value iteration, as shown in Algorithm 1. It first initializes an agent, and then sweeps through the state space, each time calling the model and then updating the agent, until convergence.

b) **Experiment**: Verify that your code works by running the file. You should see a visualization of all the Q-value estimates during execution of the algorithm. **Closely inspect the values, how they change, and how they converge**. (You may need to increase the value of `step_pause` to make plotting slower). Do you understand the final values in each cell, and can you interpret them?

c) **Write**: Write a section of you report, in which you:

- Explain your method (with equations/algorithm boxes).
- Show a picture with the progression of Q-value iteration during execution, e.g., the estimates at each state-action at the beginning, midway, and at convergence. Explain the final values you observe, working backwards from the goal.
- Compute $V^\star(s=3)$ at the start state ($s=3$, `location=(0,3)`), i.e., the converged optimal value at the start. Explain what it means.
- Compute the average reward per timestep under the optimal policy. Explain your answer. (Hint: You must use i) the optimal value at the start state, ii) the magnitude of the final reward, iii) the magnitude of the reward on every other step. First try to compute the average number of steps the agent needs to take to reach the goal based on these quantities. Use this number to derive the average reward per step, again using the optimal value at the start state.) **If you do not manage to compute this number, you can assume it is 1.3 in the remainder of your experiments**.
- The goal state in the environment ($s = 52$) is terminal. Explain how your implementation of dynamic programming deals with this issue, i.e., why does it still converge? (Hint: check the code in `Environment.py`). Could you think of another way to solve this issue?

- The goal state is currently located at (x,y) location [7,3], defined in initialization function of the environment. Briefly change the location tot [6,2], run your DP algorithm again, and observe how the agent now behaves under the optimal policy. Is there a noticeable difference. Explain your answer.

Algorithm 1: Tabular Q-value iteration (Dynamic Programming)

Input: Threshold $\eta \in R^+$.
Result: The optimal value function $Q^\star(s,a)$ and/or associated optimal policy $\pi^\star(s)$.
Initialization: A state-action value table $\hat{Q}(s,a) = 0 \quad \forall s \in \mathcal{S}, a \in \mathcal{A}$
repeat
  $\Delta \leftarrow 0$
  for *each* $s \in \mathcal{S}$ **do**
    for *each* $a \in \mathcal{A}$ **do**
      $x \leftarrow Q(s,a)$ `/* Store current estimate */`
      $\hat{Q}(s,a) \leftarrow \sum_{s'} \left[p(s'|s,a) \cdot \left(r(s,a,s') + \gamma \cdot \max_{a'} Q(s',a') \right) \right]$ `/* Eq. 1 */`
      $\Delta \leftarrow \max(\Delta, |x - \hat{Q}(s,a)|)$ `/* Update max error */`
    end
  end
until $\Delta < \eta$;
$Q^\star(s,a) = \hat{Q}(s,a)$ `/* Converged at optimal value function */`
$\pi^\star(s) = \arg\max_a Q^\star(s,a) \quad \forall s \in \mathcal{S}$ `/* Optimal policy is greedy */`
Return $Q^\star(s,a)$ and/or $\pi^\star(s)$.

for each row take the best value of a'

2 Exploration

You will now switch to the (model-free) reinforcement learning setting. In this case, you no longer have access to the model (like the real world, where executing an action permanently brings you to the next state). You therefore no longer have access to the `StochasticWindyGridworld.model()` function, and you can no longer sweep through all states. **Instead, you have to move forward from the current state, where you use the `StochasticWindyGridworld.step()` function**.

Since you cannot sweep through all states anymore, you will proceed in episodes from the start state. Compared to sweeping through the state space, you are now no longer guaranteed to visit all states under a greedy policy. You therefore need to introduce *exploration* into your action selection, to sometimes try something novel.

1. This first crucial step of any RL algorithm is the **action selection**. You decide to compare two types of policies:

   - The **ϵ-greedy policy**:

   $$\pi(a|s) = \begin{cases} 1.0 - \epsilon \cdot \frac{|\mathcal{A}|-1}{|\mathcal{A}|}, & \text{if } a = \arg\max_{b \in \mathcal{A}} \hat{Q}(s,b) \\ \epsilon/(|\mathcal{A}|), & \text{otherwise} \end{cases} \tag{2}$$

   In words, we select with small probability ϵ a random action, which ensures exploration, and otherwise take the greedy action. The parameter ϵ allows you to scale the amount of exploration ($\epsilon = 0$ gives a greedy policy, $\epsilon = 1$ gives a uniform/random policy).

   - The **Boltzmann policy**:

   $$\pi(a|s) = \frac{e^{\hat{Q}(s,a)/\tau}}{\sum_{b \in \mathcal{A}} e^{\hat{Q}(s,b)/\tau}} \tag{3}$$

   where $\tau \in (0, \infty)$ denotes a temperature parameter. This approach gives a higher probability to actions with a higher current value estimate, but still ensures exploration of other actions than the greedy one. The temperature τ allows you to scale the amount of exploration: for $\tau \to \infty$ the policy becomes uniform/random (why?), and for $\tau \to 0$ the policy becomes greedy.

2. The second crucial step of an RL algorithm is the **update**. After executing an action, the environment gives you new data, in the form of the observed reward and next state. Therefore, after timestep t you have observed data $\langle s_t, a_t, r_t, s_{t+1} \rangle$. In the next part of the assignment we will compare different ways to use this data to compute a new estimate for the state-action value at s_t, a_t, but in this assignment we will use the 1-step Q-learning update. We first compute the new **back-up estimate/target** G_t as

   $$G_t = r_t + \gamma \cdot \max_{a'} \hat{Q}(s_{t+1}, a') \tag{4}$$

   and then apply the **tabular learning update**

   $$\hat{Q}(s_t, a_t) \leftarrow \hat{Q}(s_t, a_t) + \alpha \cdot [G_t - \hat{Q}(s_t, a_t)] \tag{5}$$

   where $\alpha \in (0, 1]$ denotes the learning rate. For $alpha \to 0$ learning is slow but stable, while $\alpha \to 1$ makes learning fast but less stable. The optimal learning rate typically

lies somewhere in between. The learning rate is an important parameter in any learning experiment, and typically needs to be tuned extensively.

You proceed with your experiments as follows:

a) **Implement**:

- Correctly complete the class `QLearningAgent()` in the file `Q_learning.py`.
  - In `init()`, initialize a table with means $Q(s, a)$ to 0.
  - In `select_action()`, implement the above ϵ-greedy policy and softmax policy. Note: we already provided `argmax()` and `softmax()` functions for you in `Helper.py`, which are already imported into the file.
  - In `update()`, implement the Q-learning update shown above.
- Correctly complete the function `q_learning()` in the file `Q_learning.py`. This function should execute Q-learning, as shown in Algorithm 2. The function should return a list with all the rewards observed at each timestep.
- Verify that your code works by running `Q_learning.py`. Observe how the agent explores and learns. Plots the value estimates during execution, and observe how they change.

b) **Experiment**: You decide to perform a more systematic experiment, comparing ϵ-greedy and Boltzmann policies with different settings for the exploration parameters (respectively ϵ and temperature parameter τ).

- Write your experiment code in `Experiment.py`, using the `q_learning()` function you wrote above.
- Try ϵ-greedy with `epsilon = [0.02,0.1,0.3]` and softmax with `temps = [0.01,0.1,1.0]`.
- **For each setting, average your results over** 50 **repetitions, and smooth your learning curves**. Plot the learning curves for each setting in the same graph. Add a clear legend!

c) **Write**:

- Explain your method (with equations/algorithm boxes).
- Show a picture which compares both exploration methods for different values of ϵ and τ.
- Interpret your results. Which method do you prefer? Does RL achieve the optimal performance you found from Dynamic Programming? Explain why it does or doesn't.

d) **Bonus**: When you want to show off, you can try to anneal ϵ and/or τ during training, which more gradually shifts from exploration (ϵ close to 1, or high τ), to exploitation (ϵ or τ close to 0). You can find an example annealing function `linear_anneal{}` in `Helper.py`, which you may use. Implement an annealing schedule, and compare its performance to your baseline implementation with fixed ϵ or τ.

Algorithm 2: Tabular Q-learning.

Input: Exploration parameter, learning rate $\alpha \in (0, 1]$, discount parameter $\gamma \in [0, 1]$, total *budget*.

$\hat{Q}(s, a) \leftarrow 0, \quad \forall s \in \mathcal{S}, a \in \mathcal{A}.$ `/* Initialize Q-value table */`

$s \sim p_0(s)$ `/* Sample initial state */`

while *budget* **do**

  $a \sim \pi(a|s)$ `/* Sample action, e.g., ϵ-greedy, softmax */`

  $r, s' \sim p(r, s'|s, a)$ `/* Simulate environment */`

  $\hat{Q}(s, a) \leftarrow \hat{Q}(s, a) + \alpha \cdot [r + \gamma \cdot \max_{a'} \hat{Q}(s', a') - \hat{Q}(s, a)]$ `/* Q update */`

  if s' *is terminal* **then**

    $s \sim p_0(s)$ `/* Reset environment */`

  else

    $s \leftarrow s'$

  end

end

Return: $\hat{Q}(s, a)$

3 Back-up: On-policy versus off-policy target

The second important part of any RL algorithm is the way we back-up information. A major distinction is between off-policy back-ups (like Q-learning) and on-policy back-ups (like SARSA). We will first focus on the one-step case. The back-up equation for Q-learning was already implemented in the previous assignment:

$$G_t = r_t + \gamma \cdot \max_{a'} \hat{Q}(s_{t+1}, a') \tag{6}$$

$$\hat{Q}(s_t, a_t) \leftarrow \hat{Q}(s_t, a_t) + \alpha \cdot [G_t - \hat{Q}(s_t, a_t)] \tag{7}$$

The back-up equation for SARSA, given observations $\langle s_t, a_t, r_t, s_{t+1}, a_{t+1} \rangle$ is

$$G_t = r_t + \gamma \cdot \hat{Q}(s_{t+1}, a_{t+1}) \tag{8}$$

$$\hat{Q}(s_t, a_t) \leftarrow \hat{Q}(s_t, a_t) + \alpha \cdot [G_t - \hat{Q}(s_t, a_t)] \tag{9}$$

The major difference between these two is the value they bootstrap at the next state. Q-learning plugs in the value of the *best possible action at the next state*, and thereby attempts to learn the value of the optimal policy. SARSA backs up the value of the action we actually take (which may be exploratory and not the one with the currently optimal estimate). SARSA therefore learns the value function of the policy we actually execute (which includes exploration). Both approaches have their benefits and problems (see the textbook). Look closely at both above equations to understand this difference.

- **Implement**:
  - Correctly complete the class `SarsaAgent()` in the file `SARSA.py`.
    - In `init()`, initialize a table with means $Q(s, a)$ to 0.
    - In `select_action()`, copy in your previous ϵ-greedy policy and softmax policy.
    - In `update()`, implement the SARSA update shown above.
  - Correctly complete the function `sarsa()` in the file `SARSA.py`. This function should execute SARSA, as shown in Algorithm 3. The function should return a list with all the rewards observed at each timestep.
  - Run `SARSA.py` to verify that your implementation works. Plots the value estimates during execution, and observe how they change.
- **Experiment**: You decide to perform a more systematic experiment, where you compare Q-learning and SARSA for different learning rates.
  - Write your experiment code in `Experiment.py`, using the `q_learning()` and `sarsa()` functions you wrote above.
  - Try both methods for `learning_rates = [0.02,0.1,0.4]`.
  - **For each setting, average your results over** 50 **repetitions, and smooth your learning curves**. Plot the learning curves for all settings (Q-learning and SARSA for each of the above learning rates) in the same graph. Add a clear legend!
- **Write**:

– Explain your method (with equations/algorithm boxes).
– Show a picture which compares both types of back-ups for different learning rates.
– Interpret your results. Which method do you prefer? Could you think of a situation in which you would prefer the other method?

Algorithm 3: Tabular SARSA.

Input: Exploration parameter, learning rate $\alpha \in (0, 1]$, discount parameter $\gamma \in [0, 1]$, total *budget*.

$\hat{Q}(s, a) \leftarrow 0, \forall s \in \mathcal{S}, a \in \mathcal{A}.$ `/* Initialize Q-value table */`

$s \sim p_0(s)$ `/* Sample initial state */`

$a \sim \pi(a|s)$ `/* Sample action, e.g., ϵ-greedy or softmax */`

while *budget* **do**

  $r, s' \sim p(r, s'|s, a)$ `/* Simulate environment */`

  $a' \sim \pi(a'|s')$ `/* Sample action, e.g., ϵ-greedy */`

  $\hat{Q}(s, a) \leftarrow \hat{Q}(s, a) + \alpha \cdot [r + \gamma \cdot \hat{Q}(s', a') - \hat{Q}(s, a)]$ `/* SARSA */`

  if s' *is terminal* **then**

    $s \sim p_0(s)$ `/* Reset environment */`

    $a \sim \pi(a|s)$

  else

    $s \leftarrow s'$

    $a \leftarrow a'$

  end

end

Return: $\hat{Q}(s, a)$

4 Back-up: Depth of target

The other important aspect of the back-up is its depth. So far, we have only looked at 1-step method, which directly bootstrap a value estimate after one transition. Instead, we can also sum multiple rewards in a trace before we bootstrap, which leads to *n-step methods* (n-step Q-learning or n-step SARSA, depending on the way you bootstrap). You will use n-step Q-learning, which computes the following target:

$$G_t = \sum_{i=0}^{n-1} (\gamma)^i \cdot r_{t+i} + (\gamma)^n \max_a Q(s_{t+n}, a) \tag{10}$$

and again standard tabular update

$$\hat{Q}(s_t, a_t) \leftarrow \hat{Q}(s_t, a_t) + \alpha \cdot [G_t - \hat{Q}(s_t, a_t)] \tag{11}$$

Not that, although it is called n-step Q-learning (due to the maximization over the last action), it is not a full off-policy method, since the first n reward are of course sampled from the current policy (and the target therefore mostly follows our behavioral policy).

On the other extreme, we can also omit bootstrapping altogether, and simply sum all rewards up to the end of the episode (or up to some maximum timestep after which we terminate the episode). The gives a *Monte Carlo update*:

$$G_t = \sum_{i=0}^{\infty} (\gamma)^i \cdot r_{t+i} \tag{12}$$

$$\hat{Q}(s_t, a_t) \leftarrow \hat{Q}(s_t, a_t) + \alpha \cdot [G_t - \hat{Q}(s_t, a_t)] \tag{13}$$

You will run experiments to compare different depths of the back-up target, from one-step up to Monte Carlo targets.

- **Implement**:
  - Correctly complete the class `NstepQLearningAgent()` in the file `Nstep.py`.
    - In `init()`, initialize a table with means $Q(s, a)$ to 0.
    - In `select_action()`, copy in your previous ϵ-greedy policy and softmax policy.
    - In `update()`, implement the n-step Q-learning update for each state-action pair in `states` and `actions`.
  - Run `Nstep.py` to verify that your method works. Observe the agent learning.
  - Correctly complete the function `n_step_Q()` in the file `Nstep.py`. This function should execute n-step Q-learning, as shown in Algorithm 4. The function should return a list with all the rewards observed at each timestep.
  - Correctly complete the class `MonteCarloAgent()` in the file `MonteCarlo.py`.
    - In `init()`, initialize a table with means $Q(s, a)$ to 0.
    - In `select_action()`, copy in your previous ϵ-greedy policy and softmax policy.
    - In `update()`, implement the Monte Carlo update for each state-action pair in `states` and `actions`.

  - Correctly complete the function `monte_carlo()` in the file `MonteCarlo.py`. This function should execute Monte Carlo RL as shown in Algorithm 5. The function should return a list with all the rewards observed at each timestep.

    Run `MonteCarlo.py` to verify that your method works. Observe the agent learning, while you plot the value estimates and optimal policy. Do you see a difference with the previous RL methods?

- **Experiment**: You decide to perform a more systematic experiment, comparing different back-up depths.

  - Write your experiment code in `Experiment.py`, using the `n_step_Q()` and `monte_carlo()` functions you wrote above.
  - For `n_step_Q()`, try different back-up depths: `ns = [1,3,10,30]`. Also run the Monte Carlo method.
  - Plot the learning curves for each setting in the same graph. **For each setting, average your results over** 50 **repetitions, and smooth your learning curves**. Add a clear legend!

- **Write**:

  - Explain your method (with equations/algorithm boxes).
  - Show a picture with the relevant comparisons.
  - Interpret your results. Which method learns faster initially? Which method has better final performance? Give possible explanations for these observations.

Algorithm 4: Tabular n-step Q-learning

Input: Exploration parameter $\epsilon \in (0, 1]$, learning rate $\alpha \in (0, 1]$, discount parameter $\gamma \in [0, 1]$, maximum episode length T, target depth n.

$\hat{Q}(s, a) \leftarrow 0, \forall s \in \mathcal{S}, a \in \mathcal{A}$. /* Initialize Q-value table */

while *budget* **do**

  $s_0 \sim p_0(s)$ /* Sample initial state */

  /// **Collect episode**

  for $t = 0...(T - 1)$ **do**

    $a_t \sim \pi(a|s_t)$ /* Sample action, e.g., ϵ-greedy */

    $r_t, s_{t+1} \sim p(r, s'|s_t, a_t)$ /* Simulate environment */

    if s_{t+1} *is terminal* **then**

      break /* Episode terminates */

    end

  end

  $T_{ep} \leftarrow t + 1$ /* T_{ep} stores episode length */

  /// **Compute n-step targets and update**

  for $t = 0...(T_{ep} - 1)$ **do**

    $m = \min(n, T_{ep} - t)$ /* m is number of rewards left to sum */

    if s_{t+m} *is terminal* **then**

      $G_t \leftarrow \sum_{i=0}^{m-1} (\gamma)^i \cdot r_{t+i}$ /* n-step target without bootstrap */

    else

      $G_t \leftarrow \sum_{i=0}^{m-1} (\gamma)^i \cdot r_{t+i} + (\gamma)^m \cdot \max_a \hat{Q}(s_{t+m}, a)$ /* n-step target */

    end

    $\hat{Q}(s_t, a_t) \leftarrow \hat{Q}(s_t, a_t) + \alpha \cdot [G_t - \hat{Q}(s_t, a_t)]$ /* Update Q-table */

  end

end

Return: $\hat{Q}(s, a)$

Algorithm 5: Tabular Monte Carlo reinforcement learning.

Input: Exploration parameter $\epsilon \in (0, 1]$, learning rate $\alpha \in (0, 1]$, discount parameter $\gamma \in [0, 1]$, maximum episode length T.

$\hat{Q}(s, a) \leftarrow 0, \forall s \in \mathcal{S}, a \in \mathcal{A}$. `/* Initialize Q-value table */`

while *budget* **do**

  $s_0 \sim p_0(s)$ `/* Sample initial state */`

  for $t = 0...(T-1)$ **do**

    $a_t \sim \pi(a|s_t)$ `/* Sample action, e.g., ϵ-greedy */`

    $r_t, s_{t+1} \sim p(r, s'|s_t, a_t)$ `/* Simulate environment */`

    if s_{t+1} *is terminal* **then**

      break `/* Episode terminates */`

    end

  end

  $G_{t+1} \leftarrow 0$ `/* Start reward summation from 0 */`

  for $i = t...0$ **do**

    $G_i \leftarrow r_i + \gamma \cdot G_{i+1}$ `/* Compute Monte Carlo target at each step */`

    $\hat{Q}(s_i, a_i) \leftarrow \hat{Q}(s_i, a_i) + \alpha \cdot [G_i - \hat{Q}(s_i, a_i)]$ `/* Update Q-table */`

  end

end

Return: $\hat{Q}(s, a)$

5 Reflection

You did a lot of experiments with Dynamic Programming and various aspects of tabular model-free RL algorithms. Write a reflection/discussion of your experiments, which discusses the following topics:

- **DP versus RL**: What is a strength of Dynamic Programming compared to RL? And what is a potential weakness?
- **Exploration**: Which exploration method do you prefer: ϵ-greedy of softmax exploration? Could you think of a better way to explore?
- **Back-up, on-policy versus off-policy**: What are the benefits and problems of on-policy (SARSA) or off-policy (Q-learning) updates? Explain their difference. Is n-step Q-learning an on-policy or off-policy method? Explain your answer.
- **Back-up, target depth**: What are the benefits and problems of one-step, n-step and MC methods? Explain the bias-variance trade-off. Which method do you prefer for this task? Which method propagates information faster? Which method may converge on the optimal policy?
- **Curse of dimensionality**: You extensively studied tabular RL algorithms. What is their benefit? And when will they run into trouble? Explain the curse of dimensionality. How may machine learning methods, like deep learning, help to overcome this issue?

6 Bonus (optional)

The assignments on the previous page are enough to pass the assignment with a high grade (when you do well on all of them). Should you however want to impress us (and possibly get a 10), then you are very welcome to show additional experiments. Some ideas:

- **Evaluation**: There is something off with the current type of evaluation. We plot the performance of the agent *with exploration*, i.e., we plot the obtained rewards during training episodes. It might be more fair to run separate *evaluation episodes*, where the agent act completely greedy (or with very little noise). Write a function to log the greedy performance of an agent at a certain timepoint, and for example run this function every `interval=500` steps. You can then plot learning curves based on the logged average return at these interval points, which may show a different performance profile.
- **Annealing**: Maybe you can do additional experiments with learning rates and exploration parameters. You could try the annealing schedule, as already mentioned in the Exploration assignment.
- **Modify the task**: You could also try to play around with the task definition. In the definition of the environment there are `self.goal_locations` and `self.goal_rewards`, which specify the locations and associated rewards of the terminal goal states in this task. Try to play around with these goal locations, maybe adding a second and third goal to the task. How does this alter behaviour? Does your new environment for example require more or less exploration?
- **Exploration**: Maybe you have a better method for exploration, like "count-based exploration", which you would like to implement.
- **Open**: Maybe you have another idea that you would like to try, or you find something online. Feel free to add your own thoughts and experiments.