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Algebraic geometry 1

problem set 5

Keywords: Fibres and sheafification.

Problem 1

1. Let $f\colon X \to Y$ be a map of sets, and let $y \in Y$ be a point. Show that the fibre product $y \times_Y X$ can be identified with the fibre $f^{-1}(y)$.
2. Let $f_i\colon X_i \to S$, $i = 1, 2$, be morphisms of schemes, and let $x \in X_1$ be a point with $k(x) = k(f_1(x))$. Show that the fibre of $X_1 \times_S X_2$ over x is isomorphic to the fibre of f_2 over $f_1(x)$.

Problem 2

Let F be a presheaf (of sets) on X, and $x \in X$ a point. Show that the stalk of the sheafification F^{sh} at x is isomorphic to F_x.

Problem 3

Let $a\colon F \to G$ be a morphism of sheaves (of sets) on X. Show that a is an epimorphism if and only if $a_x\colon F_x \to G_x$ is surjective for every $x \in X$.

Problem 4

Let $s \in F(U)$ be a section of the sheaf of Abelian groups F over the open subset $U \subseteq X$. Show that the support of s, i.e. the set $\{x \in U | 0 \neq s_x \in F_x\}$, is closed in U.

Problem 5

Construct a scheme whose underlying topological space consists of three points $X = \{p, q, \eta\}$, such that p and q are closed, and η is open.