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# Algebraic geometry 1

## problem set 13

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### Problem 1

Let $G(2,4) \subset \mathbb{P}^5$ be the Grassmannian of lines in $\mathbb{P}^3$. We denote the homogenoeus Plücker coordinates of $G(2,4)$ in $\mathbb{P}^5$ by $x_{i,j}$ for $1 \leq i < j \leq 4$. Show:

$$G(2,4) = V(x_{1,2}x_{3,4} - x_{1,3}x_{2,4} + x_{1,4}x_{2,3}).$$

### Problem 2

Let $L \subset \mathbb{P}^3$ be a fixed line. Let $H_L \subset G(2,4)$ be the locus of lines which meet $L$. Consider the incidence correspondence

$$I = \{(p,l) \in \mathbb{P}^3 \times G(2,4) : p \in l\} \subset \mathbb{P}^3 \times G(2,4).$$

Show that $I$ is a closed subvariety of $\mathbb{P}^3 \times G(2,4)$. Conclude that $H_L$ is a closed subvariety of $G(2,4)$.

### Problem 3

Prove that the Zariski tangent space at the point $L \in G(k,n)$ is canonically isomorphic to $L^* \otimes K^n/L$ (or equivalently to $\mathrm{Hom}(L, K^n/L)$).