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# Algebraic geometry 1

## problem set 10

**Keywords:** projectivities, linear subspaces, conics, and Hilbert polynomials

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**Problem 1** *(3 pt)*

Linear subspaces and projectivities.

1. Let $r$ and $s$ be disjoint lines in $\mathbb{P}^3(k)$. For every point $p \in \mathbb{P}^3(k) \setminus (r \cup s)$, there exists a unique line $l_p$ passing through $p$ and intersecting both $r$ and $s$.
2. Let $\phi$ be a projectivity of $\mathbb{P}^n(k)$. Show that the locus of fixed points of $\phi$ is a union of linear supspaces $L_1, \ldots, L_r$, such that $L_i$ does not meet the linear span of $(L_j)_{j \neq i}$.
3. Suppose that the fixed locus of $\phi \in PGL_4(k)$ consists of two lines $r$ and $s$. For every point $p \in \mathbb{P}^3(k) \setminus (r \cup s)$, the line joining $p$ to $\phi(p)$ meets both $r$ and $s$.

**Problem 2** *(2 pt)*

Let $r$ be a line in $\mathbb{P}^2(\mathbb{C})$. Show that $r$ contains either one or infinitely many real points. (Hint: use conjugation.)

**Problem 3** *(3 pt)*

Let $k$ be algebraically closed. Let $p_1, \ldots, p_5$ be distinct points of $\mathbb{P}^2(k)$.

1. Show that there exists at least a conic $C$ passing through $p_1, \ldots, p_5$.
2. $C$ is non-degenerate (reduced and irreducible) if and only if any three of the points are not collinear. Moreover, in this case $C$ is unique.
3. If $C$ is degenerate, then it is unique if and only if any four of the points are not collinear.

**Problem 4** *(3 pt)*

Compute the degrees of the Segre and Veronese embeddings.