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Algebraic geometry 1

problem set 9

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Problem 1

Let $S_\bullet = \bigoplus_{n\geq 0} S_n$ be a graded ring, and for $d \geq 1$ let:

$$S_\bullet^{(d)} = \bigoplus_{n\geq 0} S_{dn}$$

be the subring generated by homogeneous elements of degree divisible by d. Show that the natural inclusion $S_\bullet^{(d)} \subseteq S_\bullet$ induces an isomorphism $\mathrm{Proj}(S_\bullet) \simeq \mathrm{Proj}(S_\bullet^{(d)})$.

Problem 2

Show that $\mathbb{P}(1,1,2)$ admits a closed embedding in $\mathbb{P}^3$.

Problem 3

1. Let $X \to \mathrm{Spec}(k)$ be a finite morphism. Show that (the underlying topological space of) X consists of finitely many points with the discrete topology.
2. Show that finite morphisms have finite fibres.
3. Find an example of a morphism with finite fibres which is not finite.

Problem 4

1. Show that $\mathrm{Proj}(R[t])$ is isomorphic to $\mathrm{Spec}(R)$.
2. Let $A \subseteq B$ be a finite ring extension. Define a graded ring $S_\bullet$ with $S_0 = A$ and $S_n = B$ for all $n \geq 1$. Show that $\mathrm{Proj}(S_\bullet) \simeq \mathrm{Spec}(B)$. Show that $S_\bullet$ is a finitely generated graded A-algebra.
3. Use the previous point to show that finite morphisms are proper.