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Algebraic geometry 1

problem set 2

Keywords: Irreducibility, dimension, regular functions, and morphisms

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Every problem is worth 2 points.

Problem 1

Let X be a Hausdorff topological space (i.e. for every pair of distinct points $x, y \in X$ there exist disjoint open subsets $U, V \subseteq X$ such that $x \in U, y \in V$). A subspace of X is irreducible if and only if it is a singleton.

Problem 2

Let $f\colon X \to Y$ be a continuous and open map of topological spaces. Suppose that Y is irreducible, and so is every fibre of f. Then X is irreducible as well.

Problem 3

Let X be an irreducible topological space of finite dimension n. For any subspace $Y \subseteq X$, $\dim(Y) \leq \dim(X)$.

Problem 4

Let $\varphi\colon R \to S$ be a homomorphism of local rings. Show that $\varphi^{-1}(\mathfrak{m}_S) = \mathfrak{m}_R$ if and only if $\varphi(\mathfrak{m}_R) \subseteq \mathfrak{m}_S$.

Problem 5

Let $GL_n(k)$ denote the set of invertible $n \times n$ matrices over k. Show that it is an affine algebraic variety of dimension n^2.