

# A novel approach to particle characterization for discrete element method by means of artificial neural networks

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## Abstract

Discrete Element Method (*DEM*) simulations are widely used to model and understand particle behavior. It is important to note that each combination of DEM-micro parameters gets different bulk-macro behavior. As a consequence, a straight-forward trial-and-error calibration procedure is prohibitively computationally expensive to fathom the micro-macro transition relationship. A limited number of combinations have been simulated, through 2000 shear cell numeric test and 300 *static – angle – of – repose* (*SAOR*) numeric test. The DEM parameters of the simulations have been used as inputs of feed forward Multilayer Perceptron Neural Networks (*MLPNN*), while the bulk values and behavior as targets for the Neural Networks (*NN*). A backpropagation reinforcement learning training algorithm has been used (scaled conjugate gradient). A NN has been created for each bulk parameter investigated. 15% of the simulations have been excluded from the training processes. They have been used to define per each *NN* the correct number of neurons in the hidden layer, based on an  $R^2$  maximization. Then each trained *NN* received as input one million different combinations. The bulk solids were characterized using shear cell testers. The DEM coefficients were obtained by fitting *NN* outputs to experimental data (within a 5% error). Further, we validated the *DEM* parameters by means of *SAOR* experiments and *SAOR* simulations-trained NN. The validation agreement was also within reliable limits (5% error). The calculated *DEM* coefficients of iron ore, limestone and silibeads accord well with published data and in-house experiments.

**Keywords:** Meshless methods (DEM), Rheology, experimental validation studies, process industries, process metallurgy, LIGGGHTS, Material characterization, Artificial Neural Networks

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## 1. Introduction

Particles in various forms - ranging from raw materials to food grains and pharmaceutical powders - play a major role in a variety of industries, including process industry and metallurgy. In his book, Holdich [5] stated that "between 1 and 10% of all the energy is used in comminution, i.e. the processes of crushing, grinding, milling, micronising". However, a univocal method to characterize these particles has so far not been established. From the experimental point of view, the main issues are the difficult setups and the general reliability and reproducibility of the tests. From the numerical point of view, no general procedure is available, and the existence of a mathematically unique solution describing macro/micro particle contact has yet to be

proved. Moreover, in a recent study, Krantz et al. [7] implied "that the dynamic properties of a powder cannot be applied to universally predict the static properties of a powder, and, likewise, the static properties cannot be used to predict dynamic properties".

Discrete Element Method (DEM) simulations are widely used to understand particle behavior. Mishra and Rajamani [8] defined the *DEM* as "a special class of numerical schemes for simulating the behavior of discrete, interacting bodies". The force that particle *i* exerts on particle *j* is defined as:

$$m\ddot{x}_{ij} + c\dot{x}_{ij} + kx_{ij} = F_i. \quad (1)$$

Further details on the method can be found in Poschel and Schwager [10]. *LIGGGHTS* (LAMMPS improved for general granular and granular heat transfer simulations) is one of the most powerful open source *DEM* simulation software packages available. The models it can analyze are described in detail in the literature, see

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Kloss et al. [6]. In combination with shear cell tester simulation developed by Aigner et al. [1], *LIGGGHTS* has correctly defined the coefficient of sliding friction for coarse round particles - a critical parameter describing inter-particle friction in medium to dense granular flows simulations.

Since the bulk solid is represented by perfect spheres, the only parameter the software uses to describe its shape is the radius of the particle ( $R$ ). However, since the shape is one of the most relevant aspects defining particle behavior, we consider the coefficient of rolling friction ( $\mu_r$ ) as an additional *DEM* shape parameter. It is proportional to the torque counteracting the rotation of the particle and defined as (Eq. 9):

$$\mu_r = \tan(\iota). \quad (2)$$

The last *DEM* - *micro* parameter investigated is the *coefficient of restitution (COR)*, given its centrality in the whole model. *DEM* simulations have recently been used to reduce the bias of the experiments, and more precise devices such as the Schulze ring shear cell tester (SRSCCT)(see Schulze et al. [11]) have been built. A dedicated workflow that combines experiments and simulations must now be devised following the Design of Experiments method, as illustrated by Grossman and Vecchio [3].

The main goal of this new procedure should be the characterization of non-spherical particles, especially the *DEM* coefficients of friction and restitution, following standardizable steps. With this objective in mind, we profited from the shear cell experimental and numerical setup in combination with *LIGGGHTS* simulation to improve the accuracy and the range of applicability of particle characterization. Nevertheless, *DEM* simulations require tens of thousand of particles to achieve the necessary reliability for a straight-forward trial-and-error calibration procedure. The calibration compels to identify the *DEM* - *micro* combination of parameters that numerically grants the same *bulk* - *macro* behavior experimentally registered, measured as *steady-state-flow/pre-shear coefficient-of-internal-friction* ( $\mu_{ie-ps}$ ), *incipient-flow/shear coefficient-of-internal-friction* ( $\mu_{ie-s}$ ) and *bulk density* ( $\rho_b$ ). Furthermore, we used Hertz' Law for the particle-particle and particle-wall interactions. Its complexity increased the computational effort to fathom the micro-macro transition relationship. Thus, the time necessary to perform all the possible *DEM* - *micro* parameters combinations became boundless. In order to overcome this doomed situation we decided to operate artificial neural networks, as suggested by Antony et al. [2]. A limited number of combinations have been simulated, de-

signed to maximize the representativity. Following the indications of Vaferi et al. [12], *feed forward Multilayer Perceptron Neural Networks (MLPNN)* have been handled. Their trustworthiness, together with a backpropagation reinforcement learning training algorithm(scaled conjugate gradient), has been widely demonstrated in the literature, see Haykin [4]. The *DEM* parameters of the simulations have been used as inputs of the Neural Networks (*NN*), while the bulk values and behavior as targets for them. Furthermore, the best practice suggested by Vaferi et al. [12] demands to establish the most appropriate number of neurons inside the hidden layer of each *NN*. As in literature, this goal has been achieved by first excluding 15% of the simulations from the training processes. We then fed the *NN* with the same *DEM* - *micro* parameters of these simulations. Subsequently, we controlled the square regression error between the *bulk* - *macro* behavior in these simulations and in the output of the *NN*. Finally, we selected for each *bulk* - *macro* behavior property ( $\mu_{ie-ps}$ ,  $\mu_{ie-s}$  and  $\rho_b$ ) the *NN* with the number of neurons that provided the maximum  $R^2$ . Later, each of these three trained *NN* received as insertion 1M different combinations *DEM*-*micro* parameters. We then compared their outputs against the values provided by the shear cell experiments (within a 5% error), gaining the *DEM*-*micro* coefficients range. Oberkampf and Roy [9] prescribes an ulterior validation step. To accomplish his demand, we realized a *static-angle-of-repose (SAOR)* experiment and *SAOR* simulation. Furthermore, we performed the *SAOR* simulation, again with the same limited number of combinations. These allowed to determine two *bulk* - *macro* behavior properties, the *angle-of-repose (AOR)* and the *bulk density* ( $\rho_b$ ). Likewise, we then trained *NN* and optimized the number of their neurons. Later, each of these two trained *NN* received as insertion the *DEM* - *micro* coefficients range previously determined. We then compared their outputs against the values provided by the *SAOR* experiment (again within a 5% error), gaining a narrower *DEM* - *micro* coefficients range. Since this study was supported by the metallurgical industry, the materials examined were: silibeads (2 mm), coke, iron ore, limestone (all 0-3.15 mm). For the same reason, cohesive materials have been excluded from this study.

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## 2. Method

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$$e = mc^2 \quad (3)$$

### 2.1. Discrete element method

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$$F_{ij} = \begin{cases} F_{n,ij} + F_{t,ij} = (k_n \delta_{n,ij} + \gamma_n v_{n,ij}) + (k_t \delta_{t,ij} + \gamma_t v_{t,ij}) \\ 0 \end{cases} \quad (4)$$

### 2.2. Artificial Neural Networks

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$$\begin{aligned} M_r &= M_r^k, \\ M_{r,ti+\Delta ti}^k &= M_{r,ti}^k - k_r \Delta \theta_r, \\ |M_{r,ti+\Delta ti}^k| &\leq M_r^m = \mu_r R_{eq} F_n. \end{aligned} \quad (5)$$

### 2.3. Experimental setup

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$$\begin{aligned} \frac{1}{E_{eq}} &= \frac{1 - \nu_i^2}{E_i} + \frac{1 - \nu_j^2}{E_j}, \\ \frac{1}{G_{eq}} &= \frac{2(2 + \nu_i)(1 - \nu_i)}{E_i} + \frac{2(2 + \nu_j)(1 - \nu_j)}{E_j}, \\ \text{if } r < d, \frac{1}{R_{eq}} &= \frac{1}{R_i} + \frac{1}{R_j}, \\ \frac{1}{m_{eq}} &= \frac{1}{m_i} + \frac{1}{m_j}, \\ \beta &= \frac{\ln(e)}{\sqrt{\ln^2(e) + \pi^2}}, \\ S_n &= 2E_{eq} \sqrt{R_{eq} \delta_n}, \\ S_t &= 8G_{eq} \sqrt{R_{eq} \delta_n}, \\ k_r &= k_t R_{eq}^2. \end{aligned} \quad (6)$$

### 3. Results and discussion

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$$F_{t,ij} \leq \mu_s F_{n,ij}. \quad (7)$$

#### 3.1. DEM Simulations

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$$\begin{aligned} k_n &= \frac{4}{3} E_{eq} \sqrt{R_{eq} \delta_n}, \\ \gamma_n &= 2 \sqrt{\frac{5}{6}} \beta \sqrt{S_n m_{eq}}, \\ k_t &= 8 G_{eq} \sqrt{R_{eq} \delta_n}, \\ \gamma_t &= 2 \sqrt{\frac{5}{6}} \beta \sqrt{S_t m_{eq}}. \end{aligned} \quad (8)$$

#### 3.2. ANN model development

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$$\mu_r = \tan(\iota). \quad (9)$$

#### 3.3. ANN identification

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$$m\ddot{x}_{ij} + c\dot{x}_{ij} + kx_{ij} = F_i. \quad (10)$$

#### 3.4. ANN application

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$$\phi_{e-psh} = \arctan\left(\frac{\tau_{ps h}}{\sigma_{n,ps h}}\right), \quad (11)$$

$$\mu_{ps h} = \tan(\phi_{e-ps h}).$$

$$\phi_{e-sh} = \arctan\left(\frac{\tau_{sh}}{\sigma_{n,sh}}\right), \quad (12)$$

$$\mu_{sh} = \tan(\phi_{e-sh}).$$

$$SC = \sum_{k=1}^4 \sum_{l=1}^4 C_{kl}. \quad (13)$$

#### 4. Conclusions

We have discussed the determination of *DEM* inter-particle friction and restitution parameters that are critical in medium to dense granular flow simulation. In particular, we have characterized the materials more commonly used in steel-making, with special regard to their non-sphericity and contact properties. We used a *SRSCT*, a *JSCT* and an *AOR* tester to acquire experimental data. Then we employed a numerical shear cell tester and a numerical *AOR* tester for the simulations. We extended our numerical results by mean of artificial neural networks. Since our results for iron ore, limestone and glass silibeads are in very good agreement with published data and in-house experiments, we can conclude that the described experimental and simulation setup successfully defined the *DEM* parameters for these materials. Furthermore, the effectiveness of *LIGGGHTS* as simulation software has again been demonstrated.

#### 5. References

- [1] Aigner, A., Schneiderbauer, S., Kloss, C., Pirker, S., 2013 2013. Determining the coefficient of friction by shear tester simulation. pp. 335–342.
- [2] Antony, S. J., Zhou, C. H., Wang, X., 1 2006. An integrated mechanistic-neural network modelling for granular systems. *Applied Mathematical Modelling* 30 (1), 116–128.
- [3] Grossman, R. F., Vecchio, R. J. D., 2008. Chapter 22 : Design of Experiments. *Handbook of Vinyl Formulating*. Wiley - Interscience, p. 515.
- [4] Haykin, S. S., 2009. *Neural Networks and Learning Machines*. No. v. 10. Prentice Hall, 2008034079. URL <http://books.google.com.pk/books?id=K7P361KzI.QC>
- [5] Holdich, R. G., 2002. *Fundamentals of Particle Technology*. Midland Information and Technology and Publishing.
- [6] Kloss, C., Goniva, C., Hager, A., Amberger, S., Pirker, S., 2012. Models, algorithms and validation for opensource dem and cfd-dem. *Progress in Computational Fluid Dynamics, an International Journal* 12 (2), 140–152. URL <http://dx.doi.org/10.1504/PCFD.2012.047457>
- [7] Krantz, M., Zhang, H., Zhu, J., 9/15 2009. Characterization of powder flow: Static and dynamic testing. *Powder Technology* 194 (3), 239–245.
- [8] Mishra, B. K., Rajamani, R. K., 11 1992. The discrete element method for the simulation of ball mills. *Applied Mathematical Modelling* 16 (11), 598–604.
- [9] Oberkampf, W. L., Roy, C. J., 2010. *Verification and Validation in Scientific Computing*. Cambridge University Press. URL <http://books.google.com.pk/books?id=7d26zLEJ1FUC>
- [10] Poschel, T., Schwager, T., 2004. *Computational Granular Dynamics*. Springer.
- [11] Schulze, D., Heinrich, H., Zetzener, H., 2001. The ring shear tester as a valuable tool for silo design and powder characterization. *Powder Handling and Processing* 13 (19-26).
- [12] Vaferi, B., Samimi, F., Pakgohar, E., Mowla, D., 11 2014. Artificial neural network approach for prediction of thermal behavior of nanofluids flowing through circular tubes. *Powder Technology* 267 (0), 1–10.