



CD LAB - PARTICULATE FLOW MODELLING

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Materials2Simulation2Application

Author:

Luca BENVENUTI
luca.benvenuti@jku.at

Supervisors:

DI Dr. Christoph GONIVA
DI Dr. Christoph KLOSS
DI Dr. Stefan PIRKER

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1 DEM Characterization Workflow Coarse Particles

1.1 Jenike's Shear Cell tester

We have performed from 6 to 16 experiments per bulk material.

1.1.1 SCT simulation

The script has been modified to run on 32 cores over *Mach*. As of now each shear cell experiment is simulated approx. 250 times with different combinations of parameters. As can be seen in Fig. 1, a wall effect appears in the simulations, and the coefficient of internal friction decreases if the cell dimension grows. Since a bigger dimension means more particles and more computational demand, only a part of the simulations have been realized with a larger cell's diameter. The Neural Network, see section 3.2 will account also for this *wall* effect.

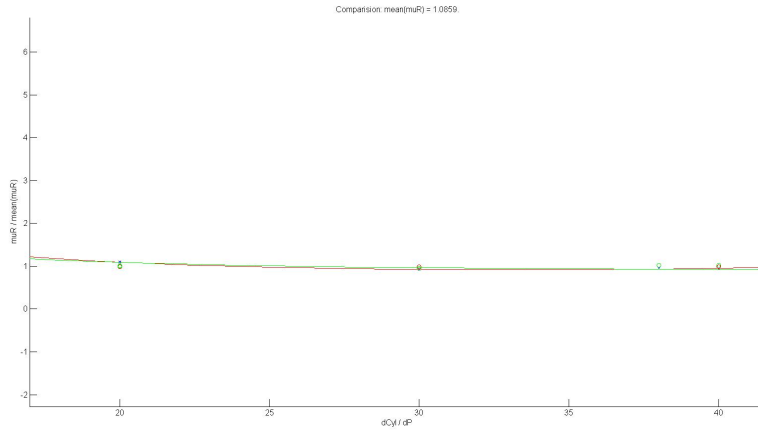


Figure 1: increased geometry effect

1.2 Angle of Repose

For each bulk have been performed at least 2 experiments, of which the mean, median and variance values have been considered.

1.2.1 AOR simulation

The script has been modified to run on 32 cores over *Mach*. As of now each material is simulated approx. 108 times with different combinations of parameters.

1.3 Granulometric curves

I have completed sieving of all materials (except from 0 to 1.25 mm iron ore) and I have for them:

- the granulometric curve;
- the mean radius (R);

1.4 COR characterization

A new system based on frequency has been suggested. I will start working on it ASAP.

1.5 Hollow spheres

This project will remain in stand-by until further notice.

1.6 Rolling drum

Both the experiment and the simulation manage to rotate the spheres at a given velocity. To identify the slope in the middle of the drum have been suggested:

- to put a laser measurement system in the middle of the beam, that rotates together with the drum, but at least once per turn it registers correctly the slope's angle;
- an "unstable" system connected to a load cell: when in a steady-state the load on the cell will be 0, when in movement the load over the cell would allow to determine the mass of the "inclined" material;

2 CFDEM Characterization Workflow Coarse Particles

2.1 Pressure drop tester

I have completed the realization of a second pressure drop tester, with a diameter 50 % larger than the old one. The pressure drop registered are still not reliable, the main culprits should be leakages or a wrong void ratio.

2.1.1 Pressure drop tester simulation

The GUI of the simulation is up and running (thx Daniel Q). I have compared an old Nicolaus' experiment with both analytical and simulation in Fig. 2.

2.2 Venturi device

Initially conceived as an extension of the mass hopper flow tester, I am still stuck in the design phase and I am evaluating if dropping it completely.

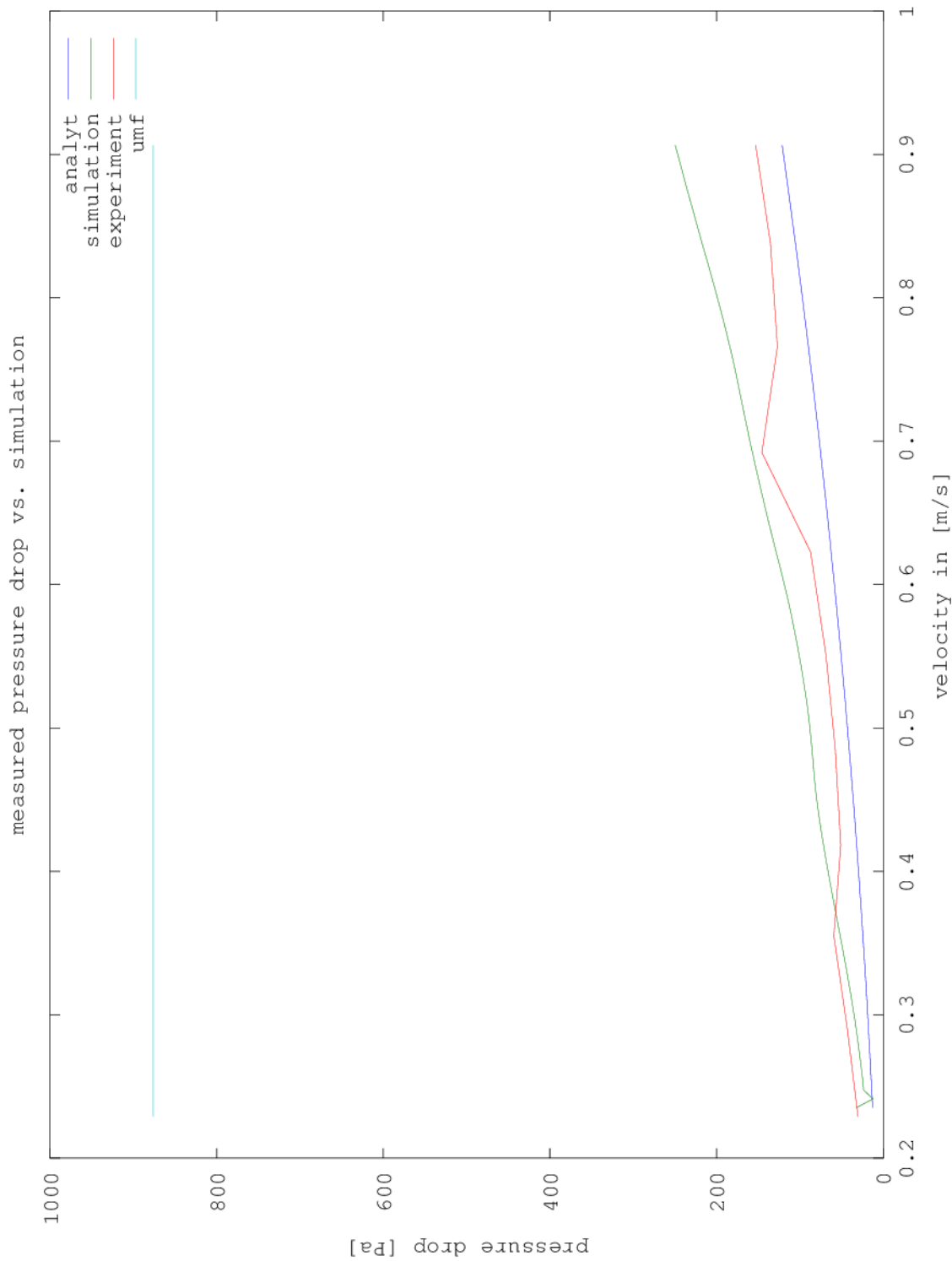


Figure 2: mono glass d2 h1300 v2 R02.xlsx 5

3 Behavior Investigation

This is the third draft of what should be the main topic of my PhD Thesis. The ToC is developed accordingly.

3.1 Investigation topics

In order to define the scientific core of my PhD Thesis the following themes will be investigated:

1. the influence of variations (distributions) of input parameters and poly-dispersity,
2. the possibility to extrapolate (e.g. given 3 different fraction distributions, with known behaviors, extrapolate the behavior of a fourth fraction distribution).

3.2 Micro - DEM parameters

The main micro parameters in a simulation are:

- (A) the particle diameter distribution (*radii* (R),%) (0.00025 - 0.05 [m]),
- (B) the particle density (ρ_p) (2000-4000 [kg/m³]),
- (C) the Young modulus (*E*) (5-10 [MPa]),
- (D) the Poisson ratio (ν) (0-0.5 [-]),
- (E) the coefficient of restitution (*e*) (0.1-1.0 [-]),
- (F) the sliding friction (*sf*) (0.1-1.0 [-]),
- (G) the rolling friction (*rf*) (0.1-1.0 [-]),
- (H) the domain edge dimension (D_{cyl}), proportional to R (76, 100, 124 times bigger).

The parameter that drives the simulation time is D_{cyl} . The number of particles is cubically proportional to its size.

3.3 Macro - bulk parameters

The main macro parameters we experimentally determine for a bulk material are:

- (a) the particle diameter distribution (*radii*) (0.00001 - 0.05 [m]),(%)
- (b) the bulk density (ρ_b) (1000-3000 [kg/m³]),
- (c) the Young modulus (*E*) (5-10 [MPa]),
- (d) the Poisson ratio (ν) (0-0.5 [-]),
- (e) the coefficient of restitution (*e*) (0.1-1.0 [-]),
- (f) the cohesion in different loading conditions (*c'*) (0-100 [kPa]),
- (g) the internal friction angle in different loading conditions (ϕ) (25-50 [°]).

Angle of Repose simulation, individual contacts micro-DEM parameters: Sliding friction: from 0.05 to 1, 0.05 interval = 20 possibilities; Rolling friction: from 0.05 to 1, 0.05 interval = 20 possibilities; Coefficient of restitution: from 0.5 to 1, 0.1 interval = 6 possibilities; Particle density: e.g. from 2500 to 3500 kg/m^3 , 100 interval = 11 possibilities; (Particle to geometry ratio: 50, 100, 150 = 3 possibilities;) $20 \times 20 \times 6 \times 11 \times 3 = 26400$ possible combinations of parameter Each combination would require a simulation, each simulation requires 9 hours over 32 cores = 9900 days! All for only one material with a defined size distribution (e.g. from 3 to 10 mm)! We only consider $p2g = 50$, because otherwise with e.g. 100 the number of particles is 8 times higher, and so the time 8 times longer

Shear cell simulation, individual contacts micro-DEM parameters: Sliding friction: from 0.05 to 1, 0.05 interval = 20 possibilities; Rolling friction: from 0.05 to 1, 0.05 interval = 20 possibilities; Coefficient of restitution: from 0.5 to 1, 0.1 interval = 6 possibilities; Particle density: e.g. from 2500 to 3500 kg/m^3 , 100 interval = 11 possibilities; Normal stress: 1, 2, 5, 10 kPa = 4 possibilities; Shear (Particle to geometry ratio: 20, 36, 38, 40, 100, 150 = 6 possibilities;) $20 \times 20 \times 6 \times 11 \times 4 \times 4 = 422400$ possible combinations of parameter Each combination would require a simulation, each simulation requires 64 minutes over 32 cores = 18774 days! All for only one material with a defined size distribution (e.g. from 3 to 10 mm)!

3.4 Artificial neural network

The representation of a multi-sphere experiment through a mathematical model presents numerous complexities. Do we really need all these simulations? How to identify the effect (weight) each parameter? A different approach involves an artificial neural network, that can be realized to understand the relationship between the input and the output parameters. An artificial neural network is composed of many artificial neurons that are linked together according to a specific network architecture. The objective of the neural network is to transform the inputs into meaningful outputs. The inputs are:

1. A , from real data a but simplified,
2. B, H ,
3. C, D , from literature,
4. E , from real data e ,
5. F, G , the main calibration parameters,

The outputs are f and g .

The idea would be to use a *feed forward Multi Layer Perceptron Neural Network*. A backpropagation reinforcement learning training algorithm has been used (scaled conjugate gradient). A Neural Network has been created for each bulk parameter investigated ($\mu_{e,ps}$, $\mu_{e,s}$, ρ_b). 15% of the simulations have been excluded from the training processes. They have been used to define per each NN the correct number of neurons in the hidden layer, based on an R^2 maximization. Then each trained NN received as input one million different combinations. Now that the networks have been trained, we can feed the networks with all the combination we need and receive reliable macro-BULK parameters as response. Especially, we can increase the parameter that

account for the geometry dimension, without paying completely the price for it. The DEM coefficients were obtained by fitting NN outputs to experimental data (within a 5% error). Further, we validated the DEM parameters by means of static angle-of-repose experiments and AOR simulations-trained NN. The validation agreement was also within reliable limits (5% error).

Between the 1000 simulations remained, as hypothesis 100 of them are experimentally validated. I could use 80 of them to further refine the weights of the neural network functions, and the remaining 20 to validate this second step. At this point I should be able to *extrapolate the behavior of bulk with a volume too large to be simulated* with this complete neural network and to compare the results with the real scale experiments (Leoben).

The reliability of this work is deeply based on the already performed simulations: further numerical investigations as suggested in draft 1 could improve it.

By reserving a portion of the simulations, we can use them to establish the most effective number of neurons inside the hidden layer, for each bulk parameter, through maximization of R2.

References

- [1] Jun Ai et al. “Assessment of rolling resistance models in discrete element simulations”. In: *Powder Technology* 206.3 (2011), pp. 269–282.
- [2] Andreas Aigner et al. “Comparison of simulation and experimental results of a simplified jenike shear tester”. In: Colorado School of Mines, 2013.
- [3] Andreas Aigner et al. “Determining the coefficient of friction by shear tester simulation”. In: 2013, pp. 335–342.
- [4] A. Alenzi et al. “DEM validation using an annular shear cell”. In: *Powder Technology* 248.0 (Nov. 2013), pp. 131–142.
- [5] S. J. Antony, C. H. Zhou, and X. Wang. “An integrated mechanistic-neural network modelling for granular systems”. In: *Applied Mathematical Modelling* 30.1 (Jan. 2006), pp. 116–128.
- [6] D. Aole, M. K. Jain, and M. Bruhis. “New characterization methods for powder die fill process for producing powder metallurgical components”. In: *Powder Technology* 232.0 (Dec. 2012), pp. 7–17.
- [7] International ASTM. *ASTM D6128 - 06 - Standard Test Method for Shear Testing of Bulk Solids Using the Jenike Shear Cell*. 2013.
- [8] International ASTM. *ASTM D6773 - 02 - Standard Shear Test Method for Bulk Solids Using the Schulze Ring Shear Tester*. 2013.
- [9] Reza Barati, Seyed Ali Akbar Salehi Neyshabouri, and Goodarz Ahmadi. “Development of empirical models with high accuracy for estimation of drag coefficient of flow around a smooth sphere: An evolutionary approach”. In: *Powder Technology* 257.0 (May 2014), pp. 11–19.
- [10] Dana Barrasso, Ashutosh Tamrakar, and Rohit Ramachandran. “A reduced order PBM-ANN model of a multi-scale PBM-DEM description of a wet granulation process”. In: *Chemical Engineering Science* 0 ().
- [11] Gabriel K. P. Barrios et al. “Contact parameter estimation for DEM simulation of iron ore pellet handling”. In: *Powder Technology* 248.0 (Nov. 2013), pp. 84–93.
- [12] Niranjana Behera et al. “Modeling and analysis of dilute phase pneumatic conveying of fine particles”. In: *Powder Technology* 249.0 (Nov. 2013), pp. 196–204.
- [13] R. J. Berry and M. S. A. Bradley. “Investigation of the effect of test procedure factors on the failure loci and derived failure functions obtained from annular shear cells”. In: *Powder Technology* 174.1-2 (2007), pp. 60–63.
- [14] Tathagata Bhattacharya and J. J. McCarthy. “Chute flow as a means of segregation characterization”. In: *Powder Technology* 256.0 (Apr. 2014), pp. 126–139.
- [15] Giuseppe Boccignone. *Computazione per interazione naturale: Regressione lineare Bayesiana*.
- [16] L. J. Briggs. “Methods for measuring the coefficient of restitution and the spin of a ball”. In: *Journal of Research of the National Bureau of Standards* 34 (1945).
- [17] Chris Brown and Randal Nelson. *Fitting Experimental Data*. 2011.

- [18] Petra Bubakova, Martin Pivokonsky, and Petr Filip. “Effect of shear rate on aggregate size and structure in the process of aggregation and at steady state”. In: *Powder Technology* 235.0 (Feb. 2013), pp. 540–549.
- [19] Jeffrey W. Bullard and Edward J. Garboczi. “Defining shape measures for 3D star-shaped particles: Sphericity, roundness, and dimensions”. In: *Powder Technology* 249.0 (Nov. 2013), pp. 241–252.
- [20] Aykut Canakci, Sukru Ozsahin, and Temel Varol. “Modeling the influence of a process control agent on the properties of metal matrix composite powders using artificial neural networks”. In: *Powder Technology* 228.0 (Sept. 2012), pp. 26–35.
- [21] John W. Carson and Harald Wilms. “Development of an international standard for shear testing”. In: *Powder Technology* 167.1 (2006), pp. 1–9.
- [22] A. Lumin Chen, B. Zhen Chen, and C. Ansheng Feng. “Image analysis algorithm and verification for on-line molecular sieve size and shape inspection”. In: *Advanced Powder Technology* 0 ().
- [23] Sebastian Chialvo, Jin Sun, and Sankaran Sundaresan. “Bridging the rheology of granular flows in three regimes”. In: *Phys.Rev.E* 85.2 (2012), p. 021305. URL: <http://link.aps.org/doi/10.1103/PhysRevE.85.021305>.
- [24] Arthur Christopoulos and Michael J. Lew. “Beyond eyeballing: fitting models to experimental data”. In: *Critical reviews in biochemistry and molecular biology* 35.5 (2000), pp. 359–391.
- [25] Paul W. Cleary, Rob Morrisson, and Steve Morrell. “Comparison of DEM and experiment for a scale model SAG mill”. In: *International Journal of Mineral Processing* 68.1-4 (Jan. 2003), pp. 129–165.
- [26] Paul W. Cleary and Mark L. Sawley. “DEM modelling of industrial granular flows: 3D case studies and the effect of particle shape on hopper discharge”. In: *Applied Mathematical Modelling* 26.2 (Feb. 2002), pp. 89–111.
- [27] C. J. Coetzee and R. G. Nel. “Calibration of discrete element properties and the modelling of packed rock beds”. In: *Powder Technology* 264.0 (Sept. 2014), pp. 332–342.
- [28] Kevin D. Costa, Steven Kleinstein, and Uri Hershberg. *Model Fitting and Error Estimation*. 2010.
- [29] Shyamal C. Das et al. “Importance of particle size and shape on the tensile strength distribution and de-agglomeration of cohesive powders”. In: *Powder Technology* 249.0 (Nov. 2013), pp. 297–303.
- [30] Niklas Engblom et al. “Segregation of powder mixtures at filling and complete discharge of silos”. In: *Powder Technology* 215-216.0 (Jan. 2012), pp. 104–116.
- [31] Zhi-Gang Feng and Samuel Gem Musong. “Direct numerical simulation of heat and mass transfer of spheres in a fluidized bed”. In: *Powder Technology* 262.0 (Aug. 2014), pp. 62–70.
- [32] P. Frankowski et al. “Material characterisation for Discrete Element Modelling calibration”. In: *III International Conference on Particle-based Methods - Fundamentals and Applications PARTICLES 2013* (2013).
- [33] V. Ganesan, K. A. Rosentrater, and K. Muthukumarappan. “Flowability and handling characteristics of bulk solids and powders, a review with implications for DDGS”. In: *Biosystems Engineering* 101.4 (Dec. 2008), pp. 425–435.

- [34] D. Geldart, E. C. Abdullah, and A. Verlinden. “Characterisation of dry powders”. In: *Powder Technology* 190.1-2 (2009), pp. 70–74.
- [35] LaTosha M. Gibson et al. “Image analysis measurements of particle coefficient of restitution for coal gasification applications”. In: *Powder Technology* 247.0 (Oct. 2013), pp. 30–43.
- [36] Christoph Goniva et al. “Influence of rolling friction on single spout fluidized bed simulation”. In: *Particuology* 10.5 (Oct. 2012), pp. 582–591.
- [37] Yan Grasselli and Hans J. Herrmann. “On the angles of dry granular heaps”. In: *Physica A: Statistical Mechanics and its Applications* 246.3-4 (1997), pp. 301–312.
- [38] Yan Grasselli et al. “Effect of impact energy on the shape of granular heaps”. English. In: 2.2 (2000). journal: Granular Matter, pp. 97–100. URL: <http://dx.doi.org/10.1007/s100350050039>.
- [39] Pierre A. Gremaud, John V. Matthews, and Meghan O’Malley. “On the computation of steady Hopper flows: II: von Mises materials in various geometries”. In: *Journal of Computational Physics* 200.2 (2004), pp. 639–653.
- [40] Pierre A. Gremaud, John V. Matthews, and David G. Schaeffer. “On the computation of steady hopper flows III: Model comparisons”. In: *Journal of Computational Physics* 219.1 (2006), pp. 443–454.
- [41] A. P. Grima and P. W. Wypych. “Investigation into calibration of discrete element model parameters for scale-up and validation of particle-structure interactions under impact conditions”. In: *Powder Technology* 212.1 (2011), pp. 198–209.
- [42] Richard F. Grossman and R. J. Del Vecchio. “Chapter 22 : Design of Experiments”. In: *Handbook of Vinyl Formulating*. Wiley - Interscience, 2008, p. 515.
- [43] Jie Guo, Alan W. Roberts, and Jan-Dirk Prigge. “Experimental investigation of wall pressure and arching behavior under surcharge pressure in mass-flow hoppers”. In: *Powder Technology* 258.0 (May 2014), pp. 272–284.
- [44] V. K. Gupta and Shivani Sharma. “Analysis of ball mill grinding operation using mill power specific kinetic parameters”. In: *Advanced Powder Technology* 0 ().
- [45] K. Gurney. *An Introduction to Neural Networks*. Taylor and Francis, 2003. ISBN: 9780203451519. URL: <http://books.google.com.pk/books?id=sn6oBHq8qQQC>.
- [46] G. Gustafsson et al. “Determination of bulk properties and fracture data for iron ore pellets using instrumented confined compression experiments”. In: *Powder Technology* 241.0 (June 2013), pp. 19–27.
- [47] Kevin J. Hanley et al. “Application of Taguchi methods to DEM calibration of bonded agglomerates”. In: *Powder Technology* 210.3 (2011), pp. 230–240.
- [48] Johannes Hartl and Jin Y. Ooi. “Numerical investigation of particle shape and particle friction on limiting bulk friction in direct shear tests and comparison with experiments”. In: *Powder Technology* 212.1 (2011), pp. 231–239.
- [49] S. S. Haykin. *Neural Networks and Learning Machines*. v. 10. 2008034079. Prentice Hall, 2009. ISBN: 9780131471399. URL: http://books.google.com.pk/books?id=K7P361KzI_QC.

- [50] Jon Hilden et al. “Note on the interpretation of powder shear test data”. In: *Powder Technology* 182.3 (2008), pp. 486–492.
- [51] M. Hirota et al. “Proposal of an approximation equation for the yield locus to evaluate powder properties”. In: *Advanced Powder Technology* 18.3 (2007), pp. 287–302.
- [52] D. Hohner, S. Wirtz, and V. Scherer. “A numerical study on the influence of particle shape on hopper discharge within the polyhedral and multi-sphere discrete element method”. In: *Powder Technology* 226.0 (Aug. 2012), pp. 16–28.
- [53] Richard G. Holdich. *Fundamentals of Particle Technology*. Midland Information, Technology, and Publishing, 2002.
- [54] Etienne Horn. *The Calibration of Material Properties for Use in Discrete Element Models*. 2012.
- [55] S. Humby, U. Tuzun, and A. B. Yu. “Prediction of hopper discharge rates of binary granular mixtures”. In: *Chemical Engineering Science* 53.3 (Feb. 1998), pp. 483–494.
- [56] Kazuyoshi Iwashita and Masanobu Oda. “Rolling Resistance at Contacts in Simulation of Shear band Development by DEM”. In: *Journal of Engineering Mechanics* 124.3 (1998), pp. 285–292.
- [57] R. J. M. Janssen, M. J. Verwijs, and B. Scarlett. “Measuring flow functions with the Flexible Wall Biaxial Tester”. In: *Powder Technology* 158.1-3 (2005), pp. 34–44.
- [58] M. J. Jiang, H. S. Yu, and D. Harris. “A novel discrete model for granular material incorporating rolling resistance”. In: *Computers and Geotechnics* 32.5 (July 2005), pp. 340–357.
- [59] Kerry Johanson. “Effect of particle shape on unconfined yield strength”. In: *Powder Technology* 194.3 (2009), pp. 246–251.
- [60] Mohammad Khalilitehrani, Per J. Abrahamsson, and Anders Rasmuson. “The rheology of dense granular flows in a disc impeller high shear granulator”. In: *Powder Technology* 249.0 (Nov. 2013), pp. 309–315.
- [61] A. H. Kharaz, D. A. Gorham, and A. D. Salman. “Accurate measurement of particle impact parameters”. In: *Measurement Science and Technology* 10.1 (1999), p. 31. URL: <http://stacks.iop.org/0957-0233/10/i=1/a=009>.
- [62] A. H. Kharaz, D. A. Gorham, and A. D. Salman. “An experimental study of the elastic rebound of spheres”. In: *Powder Technology* 120.3 (2001), pp. 281–291.
- [63] JuHyeon Kim and SangHwan Lee. “Modeling drag force acting on the individual particles in low Reynolds number flow”. In: *Powder Technology* 261.0 (July 2014), pp. 22–32.
- [64] Christoph Kloss and Christoph Goniva. *LIGGGHTS Manual*. URL: <http://www.cfdem.com/>.
- [65] Christoph Kloss et al. “Models, algorithms and validation for opensource DEM and CFDDEM”. In: *Progress in Computational Fluid Dynamics, an International Journal* 12.2 (2012), pp. 140–152. URL: <http://dx.doi.org/10.1504/PCFD.2012.047457>.
- [66] R. Kobyłka and M. Molenda. “DEM simulations of loads on obstruction attached to the wall of a model grain silo and of flow disturbance around the obstruction”. In: *Powder Technology* 256.0 (Apr. 2014), pp. 210–216.

- [67] Matthew Krantz, Hui Zhang, and Jesse Zhu. “Characterization of powder flow: Static and dynamic testing”. In: *Powder Technology* 194.3 (2009), pp. 239–245.
- [68] H. Kruggel-Emden, S. Wirtz, and V. Scherer. “An analytical solution of different configurations of the linear viscoelastic normal and frictional-elastic tangential contact model”. In: *Chemical Engineering Science* 62.23 (Dec. 2007), pp. 6914–6926.
- [69] M. Kheiripour Langroudi et al. “An investigation of frictional and collisional powder flows using a unified constitutive equation”. In: *Powder Technology* 197.1-2 (2010), pp. 91–101.
- [70] M. Lashkarbolooki, B. Vaferi, and M. R. Rahimpour. “Comparison the capability of artificial neural network (ANN) and EOS for prediction of solid solubilities in supercritical carbon dioxide”. In: *Fluid Phase Equilibria* 308.1-2 (2011), pp. 35–43.
- [71] H. Li, G. R. McDowell, and I. S. Lowndes. “A laboratory investigation and discrete element modeling of rock flow in a chute”. In: *Powder Technology* 229.0 (Oct. 2012), pp. 199–205.
- [72] Stefan Luding. “Introduction to Discrete Element Methods, Basics of Contact Force Models and how to perform the MicroMacro Transition to Continuum Theory”. In: *EJECE* 12 (2008), pp. 785–826.
- [73] G. Lumay et al. “Measuring the flowing properties of powders and grains”. In: *Powder Technology* 224.0 (July 2012), pp. 19–27.
- [74] Tankeo M., Richard P., and Canot E. “Analytical solution of the mu-i-rheology for fully developed granular flows in simple configurations”. In: *ArXiv e-prints* (2013). 1309.2267; Provided by the SAONASA Astrophysics Data System. URL: <http://adsabs.harvard.edu/abs/2013arXiv1309.2267T>.
- [75] Mohan Medhe, B. Pitchumani, and J. Tomas. “Flow characterization of fine powders using material characteristic parameters”. In: *Advanced Powder Technology* 16.2 (2005), pp. 123–135.
- [76] J. Mellmann, T. Hoffmann, and C. Füll. “Flow properties of crushed grains as a function of the particle shape”. In: *Powder Technology* 249.0 (Nov. 2013), pp. 269–273.
- [77] Hiroshi Mio et al. “Validation of Particle Size Segregation of Sintered Ore during Flowing through Laboratory-scale Chute by Discrete Element Method”. In: *ISIJ International* 48.12 (2008). journal: ISIJ International, pp. 1696–1703.
- [78] B. K. Mishra and Raj K. Rajamani. “The discrete element method for the simulation of ball mills”. In: *Applied Mathematical Modelling* 16.11 (Nov. 1992), pp. 598–604.
- [79] Paul A. Moysey, Nadella V. Rama Rao, and Malcolm H. I. Baird. “Dynamic coefficient of friction and granular drag force in dense particle flows: Experiments and DEM simulations”. In: *Powder Technology* 248.0 (Nov. 2013), pp. 54–67.
- [80] Patric Muller et al. “Complex Velocity Dependence of the Coefficient of Restitution of a Bouncing Ball”. In: *Phys.Rev.Lett.* 110.25 (2013), p. 254301. URL: <http://link.aps.org/doi/10.1103/PhysRevLett.110.254301>.
- [81] Emilia Nowak et al. “Measurements of contact angle between fine, non-porous particles with varying hydrophobicity and water and non-polar liquids of different viscosities”. In: *Powder Technology* 250.0 (Dec. 2013), pp. 21–32.

- [82] W. L. Oberkampf and C. J. Roy. *Verification and Validation in Scientific Computing*. Cambridge University Press, 2010. ISBN: 9781139491761. URL: <http://books.google.com.pk/books?id=7d26zLEJ1FUC>.
- [83] J. R. Pillai, M. S. A. Bradley, and R. J. Berry. "Comparison between the angles of wall friction measured on an on-line wall friction tester and the Jenike wall friction tester". In: *Powder Technology* 174.1-2 (2007), pp. 64–70.
- [84] Sophie L. Pirard et al. "Motion of carbon nanotubes in a rotating drum: The dynamic angle of repose and a bed behavior diagram". In: *Chemical Engineering Journal* 146.1 (2009), pp. 143–147.
- [85] S. J. Plimpton. "Fast Parallel Algorithms for Short-Range Molecular Dynamics". In: *J Comp Phys* 117 (1995), pp. 1–19.
- [86] Yury V. Polezhaev and I. V. Chircov. "DRAG COEFFICIENT". In: *Thermopedia* (2011). URL: <http://www.thermopedia.com/content/707/>.
- [87] Thorsten Poschel and Thomas Schwager. *Computational Granular Dynamics*. Springer, 2004.
- [88] C J Reagle et al. "Measuring the coefficient of restitution of high speed microparticle impacts using a PTV and CFD hybrid technique". In: *Measurement Science and Technology* 24.10 (2013), p. 105303. URL: <http://stacks.iop.org/0957-0233/24/i=10/a=105303>.
- [89] Alberto Di Renzo and Francesco Paolo Di Maio. "Comparison of contactforce models for the simulation of collisions in DEMbased granular flow codes". In: *Chemical Engineering Science* 59.3 (Feb. 2004), pp. 525–541.
- [90] Rudiger Schmitt and Hermann Feise. "Influence of Tester Geometry, Speed and Procedure on the Results from a Ring Shear Tester". In: *Particle and Particle Systems Characterization* 21.5 (2004), pp. 403–410.
- [91] Dietmar Schulze. *Flow Properties of Powders and Bulk Solids*. 2011.
- [92] Dietmar Schulze. *Powders and bulk solids: behavior, characterization, storage and flow*. Springer, 2008.
- [93] Dietmar Schulze, H. Heinrich, and H. Zetzener. "The Ring Shear Tester as a Valuable Tool for Silo Design and Powder Characterization". In: *Powder Handling and Processing* 13.19-26 (2001).
- [94] Jorg Schwedes. "Measurement of flow properties of bulk solids". In: *Powder Technology* 88.3 (Sept. 1996), pp. 285–290.
- [95] Jorg Schwedes. "Review on testers for measuring flow properties of bulk solids". English. In: 5.1 (2003). journal: Granular Matter, pp. 1–43. URL: <http://dx.doi.org/10.1007/s10035-002-0124-4>.
- [96] A. Schweiger and I. Zimmermann. "A new approach for the measurement of the tensile strength of powders". In: *Powder Technology* 101.1 (1999), pp. 7–15.
- [97] Abd-Krim Seghouane, Yassir Moudden, and Gilles Fleury. "Regularizing the effect of input noise injection in feedforward neural networks training". English. In: *Neural Computing and Applications* 13.3 (2004). J2: Neural Comput and Applic, pp. 248–254. URL: <http://dx.doi.org/10.1007/s00521-004-0411-6>.
- [98] Erdem Simsek et al. "Influence of particle diameter and material properties on mixing of monodisperse spheres on a grate: Experiments and discrete element simulation". In: *Powder Technology* 221.0 (May 2012), pp. 144–154.

- [99] U. Sindel and I. Zimmermann. “Measurement of interaction forces between individual powder particles using an atomic force microscope”. In: *Powder Technology* 117.3 (2001), pp. 247–254.
- [100] Dale M. Snider. “Three fundamental granular flow experiments and CPFD predictions”. In: *Powder Technology* 176.1 (2007), pp. 36–46.
- [101] R. Sondergaard, K. Chaney, and C. E. Brennen. “Measurements of Solid Spheres Bouncing Off Flat Plates”. In: *Journal of Applied Mechanics* 112.3 (1990), p. 694. URL: <http://resolver.caltech.edu/CaltechAUTHORS:SONjam90>.
- [102] Jin Sun and Sankaran Sundaresan. “Radial hopper flow prediction using a constitutive model with microstructure evolution”. In: *Powder Technology* 242.0 (July 2013), pp. 81–85.
- [103] K. Takenaka et al. “Shape Effects of the Yield Locus on the Rankine Coefficient”. In: *Advanced Powder Technology* 19.1 (2008), pp. 25–37.
- [104] Gabriel I. Tardos. “A fluid mechanistic approach to slow, frictional flow of powders”. In: *Powder Technology* 92.1 (June 1997), pp. 61–74.
- [105] Jose’ M. Torralba et al. “Development of high performance powder metallurgy steels by high-energy milling”. In: *Advanced Powder Technology* 24.5 (Sept. 2013), pp. 813–817.
- [106] B. Vaferi et al. “Artificial neural network approach for prediction of thermal behavior of nanofluids flowing through circular tubes”. In: *Powder Technology* 267.0 (Nov. 2014), pp. 1–10.
- [107] Temel Varol, Aykut Canakci, and Sukru Ozsahin. “Artificial neural network modeling to effect of reinforcement properties on the physical and mechanical properties of Al2024-B4C composites produced by powder metallurgy”. In: *Composites Part B: Engineering* 54.0 (Nov. 2013), pp. 224–233.
- [108] Loc Vu-Quoc and Xiang Zhang. “An accurate and efficient tangential force-displacement model for elastic frictional contact in particle-flow simulations”. In: *Mechanics of Materials* 31.4 (Apr. 1999), pp. 235–269.
- [109] C. M. Wensrich and A. Katterfeld. “Rolling friction as a technique for modelling particle shape in DEM”. In: *Powder Technology* 217.0 (Feb. 2012), pp. 409–417.
- [110] C. M. Wensrich and A. Katterfeld. “Rolling friction as a technique for modelling particle shape in DEM”. In: *Powder Technology* 217.0 (Feb. 2012), pp. 409–417.
- [111] Wikipedia. *Discrete element method*. URL: http://en.wikipedia.org/wiki/Discrete_element_method.
- [112] M. Wojcik, J. Tejchman, and G. G. Enstad. “Confined granular flow in silos with inserts - Full-scale experiments”. In: *Powder Technology* 222.0 (May 2012), pp. 15–36.
- [113] Hideto Yoshida et al. “Particle size measurement of reference particle candidates and uncertainty region of count and mass based cumulative distribution”. In: *Advanced Powder Technology* 0 ().
- [114] Y. C. Zhou et al. “An experimental and numerical study of the angle of repose of coarse spheres”. In: *Powder Technology* 125.1 (2002), pp. 45–54.