Scientific Programming: Part B

Trees

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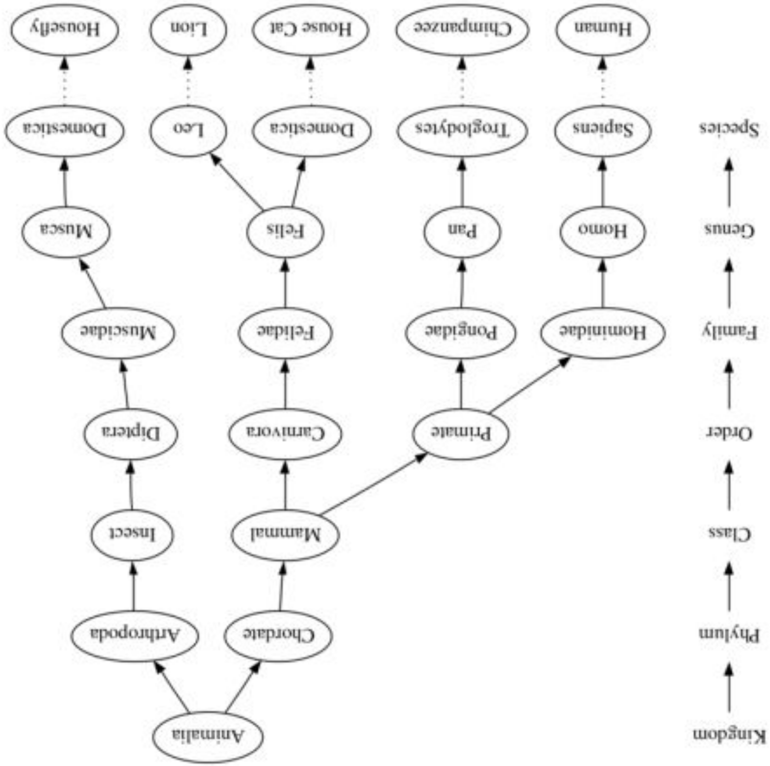
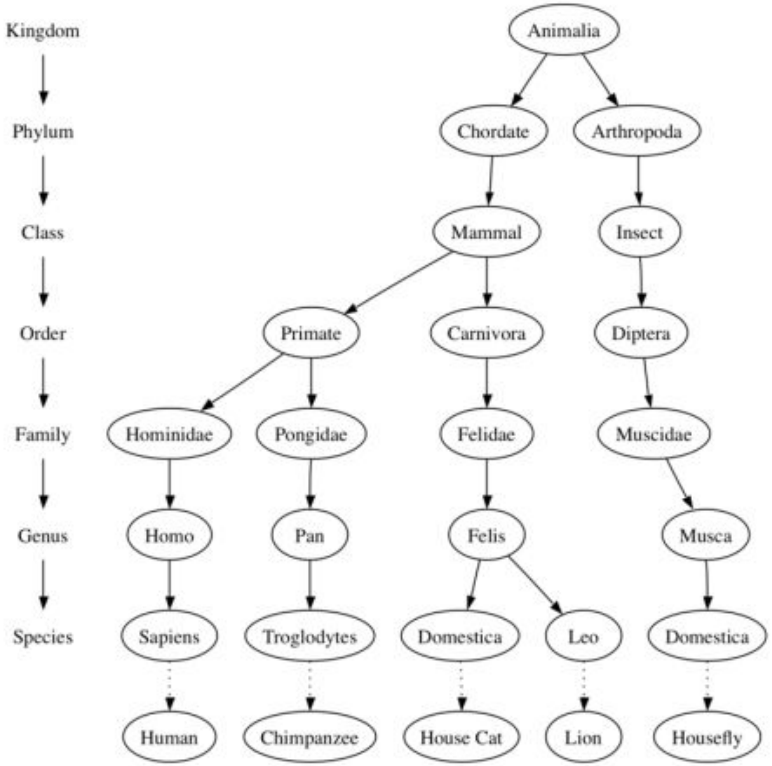
[credits: thanks to Prof. Alberto Montresor]

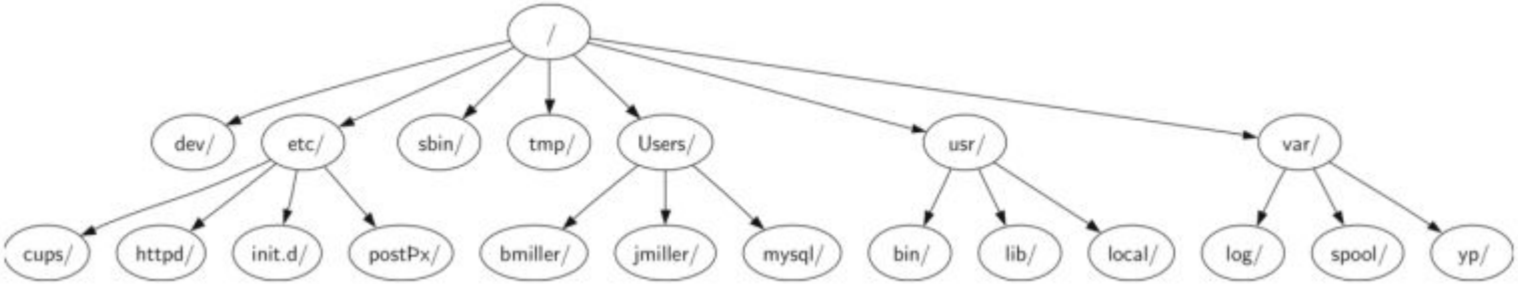
Tree: examples



Tree: examples

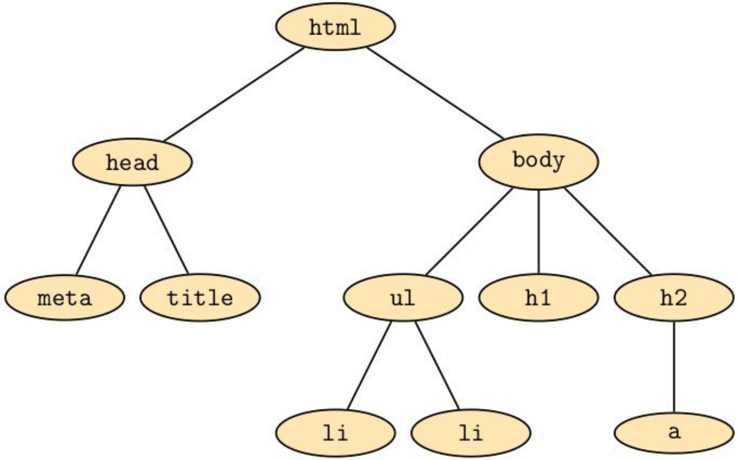


Tree: examples

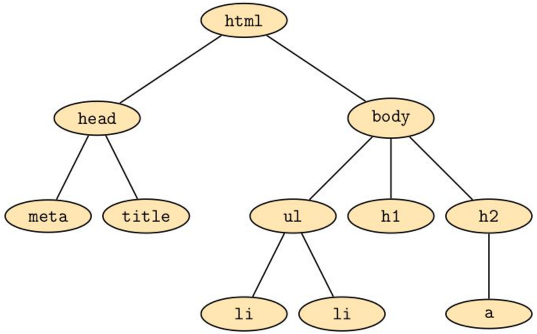


Tree: examples

```html
<html>
<head>
    <meta http-equiv="Content-Type" content="text/html" />
    <title>simple</title>
</head>
<body>
<h1>A simple web page</h1>
<ul>
    <li>List item one</li>
    <li>List item two</li>
</ul>
<h2><a href="http://www.cs.luther.edu">Luther CS </a><h2>
</body>
</html>
```



Definitions



Trees are data structures composed of two elements: ***nodes*** and ***edges***.

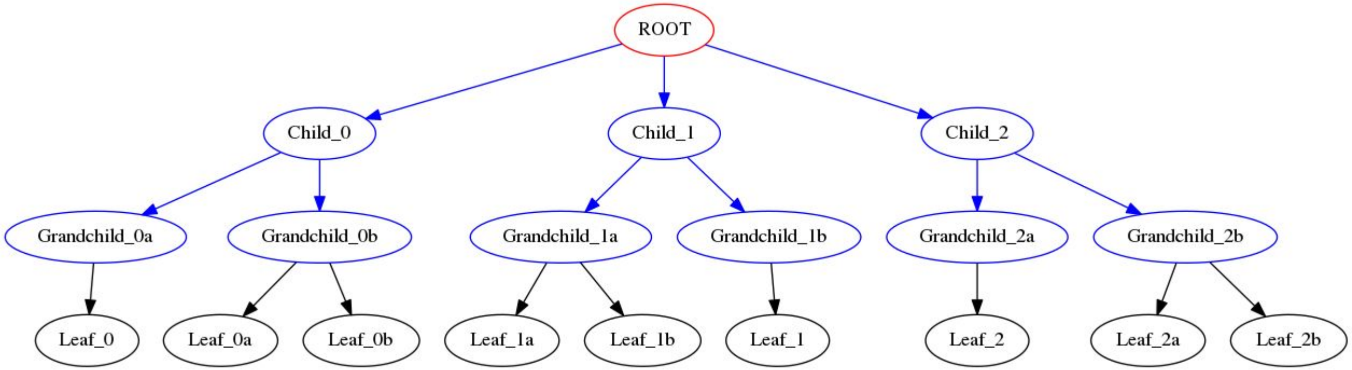
Nodes represent ***things*** and edges represent ***relationships*** (typically non-symmetric) among **two** nodes.

Tree

A tree consists of a set of nodes and a set of edges that connect pairs of nodes, with the following properties:

- One node of the tree is designated as the root node
- Every node n, except the root node, is connected by an edge from exactly one other node p
- A unique path traverses from the root to each node
- The tree is connected

Definitions



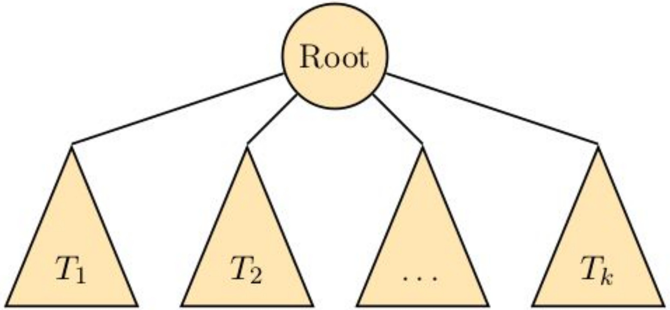
Facts

- One node called the **root** is the top level of the tree and is connected to one or more other nodes;
- If the root is connected to another node by means of one edge, then it is said to be the **parent** of the node (and that node is the **child** of the root);
- Any node can be **parent** of one or more other nodes, the only important thing is that **all nodes have only one parent**;
- The **root is the only exception as it does not have any parent**. Some nodes do not have children and they are called **leaves**;

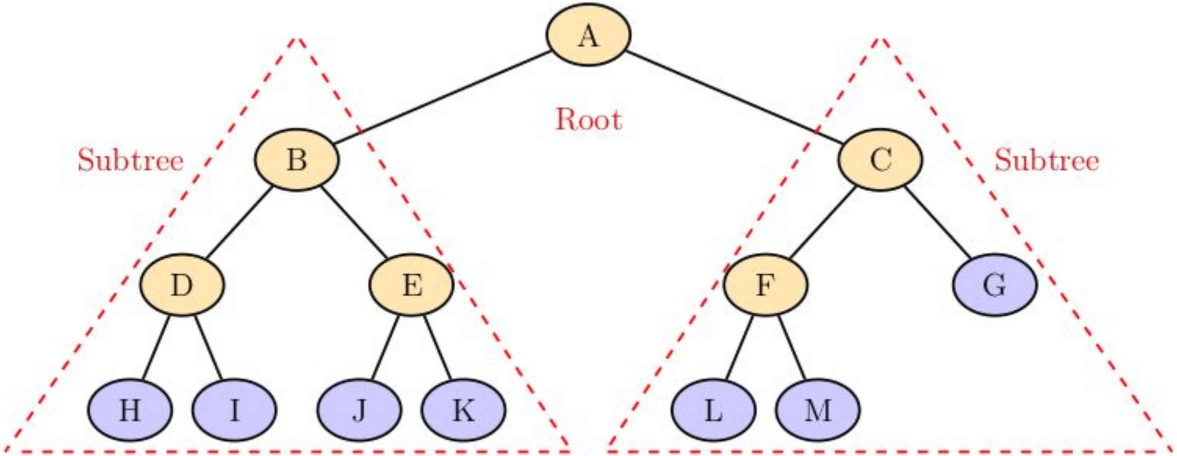
Recursive definition

Tree

A tree is either empty or consists of a root and zero or more subtrees, each of which is also a tree. The root of each subtree is connected to the root of the parent tree by an edge.



Terminology



- A is the tree root
- B, C are roots of their subtrees
- D, E are siblings
- D, E are children of B
- B is the parent of D, E
- Purple nodes are leaves
- The other nodes are internal nodes

Terminology - 2

Depth of a node

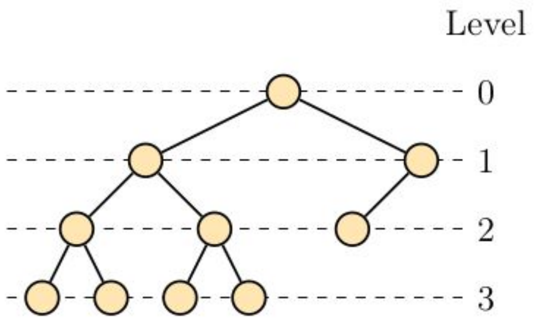
The length of the simple path from the root to the no-de (measured in number of edges)

Level

The set of nodes having the same depth

Height of the tree

The maximum depth of all its leaves



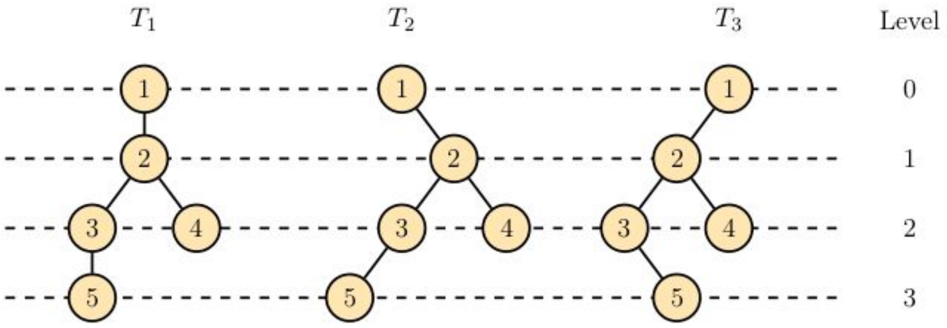
Height of this tree = 3

Binary tree

Binary tree

A binary tree is a tree data structure in which each node has at most two children, which are referred to as the left child and the right child.

Note: *Two trees T and U having the same nodes, the same children for each node and the same root, are said to be different if a node u is a left child of a node v in T and a right child of the same node in U.*



Binary tree: ADT

TREE

% Build a new node, initially containing v, with no children or parent

Tree(OBJECT v)

% Read the value stored in this node

OBJECT getValue()

% Write the value stored in this node

setValue(OBJECT v)

% Return the parent, or **none** if this node is the root

TREE getParent()

% Return the left (right) child of this node; return **none** if absent

TREE getLeft()

TREE getRight()

% Insert the subtree rooted in t as left (right) child of this node

insertLeft(TREE t)

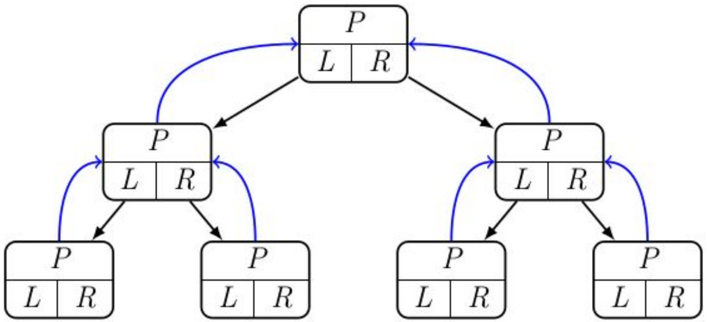
insertRight(TREE t)

% Delete the subtree rooted on the left (right) child of this node

deleteLeft()

deleteRight()

Binary tree: Node

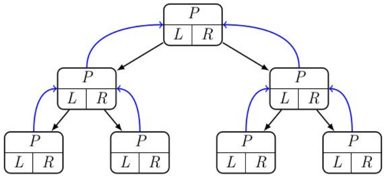


- *parent*: reference to the parent node
- *left*: reference to the left child
- *right*: reference to the left child

When implementing a tree we can define a **node object** and then a **tree object** that stores nodes.

We will use the more compact way which is to **use the recursive definition of a tree**.

Binary tree: the code



TREE

% Build a new node, initially containing v, with no children or parent
Tree(OBJECT v)

% Read the value stored in this node
OBJECT getValue()

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TREE getLeft()
TREE getRight()

% Insert the subtree rooted in t as left (right) child of this node
insertLeft(TREE t)
insertRight(TREE t)

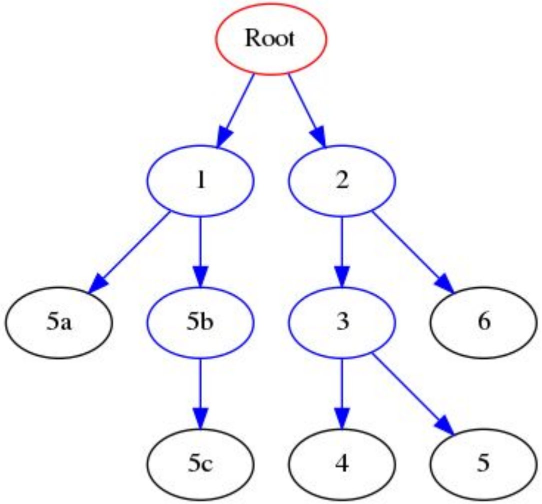
% Delete the subtree rooted on the left (right) child of this node
deleteLeft()
deleteRight()

```python
class BinaryTree:
    #the initializer, set the data
    #all pointers empty
    def __init__(self, value):
        self.__data = value
        self.__right = None
        self.__left = None
        self.__parent = None

    #returns the value
    def getValue(self):
        return self.__data

    #sets the value
    def setValue(self, newval):
        self.__data = newval

    #gets the parent
    def getParent(self):
        return self.__parent

    #sets the parent
    #NOTE: needed because we are using
    #private attributes
    def setParent(self, tree):
        self.__parent = tree

    #gets the right child
    def getRight(self):
        return self.__right

    #gets the left child
    def getLeft(self):
        return self.__left

    #set the right child
    def insertRight(self, tree):
        if self.__right == None:
            self.__right = tree
            tree.setParent(self)

    #sets the left child
    def insertLeft(self, tree):
        if self.__left == None:
            self.__left = tree
            tree.setParent(self)
    #deletes the right subtree
    def deleteRight(self):
        self.__right = None
    #deletes the left subtree
    def deleteLeft(self):
        self.__left = None
```

A sample tree...

```python
if __name__ == "__main__":
    BT = BinaryTree("Root")
    bt1 = BinaryTree(1)
    bt2 = BinaryTree(2)
    bt3 = BinaryTree(3)
    bt4 = BinaryTree(4)
    bt5 = BinaryTree(5)
    bt6 = BinaryTree(6)
    bt5a = BinaryTree("5a")
    bt5b = BinaryTree("5b")
    bt5c = BinaryTree("5c")

    BT.insertLeft(bt1)
    BT.insertRight(bt2)
    bt2.insertLeft(bt3)
    bt3.insertLeft(bt4)
    bt3.insertRight(bt5)
    bt2.insertRight(bt6)
    bt1.insertRight(bt5b)
    bt1.insertLeft(bt5a)
    bt5b.insertRight(bt5c)
```



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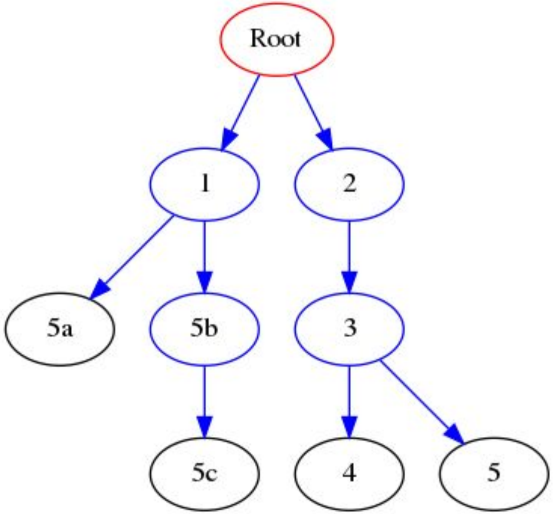
    BT.insertLeft(bt1)
    BT.insertRight(bt2)
    bt2.insertLeft(bt3)
    bt3.insertLeft(bt4)
    bt3.insertRight(bt5)
    bt2.insertRight(bt6)
    bt1.insertRight(bt5b)
    bt1.insertLeft(bt5a)
    bt5b.insertRight(bt5c)
    print("\nDelete right branch of 2")
    bt2.deleteRight()
```



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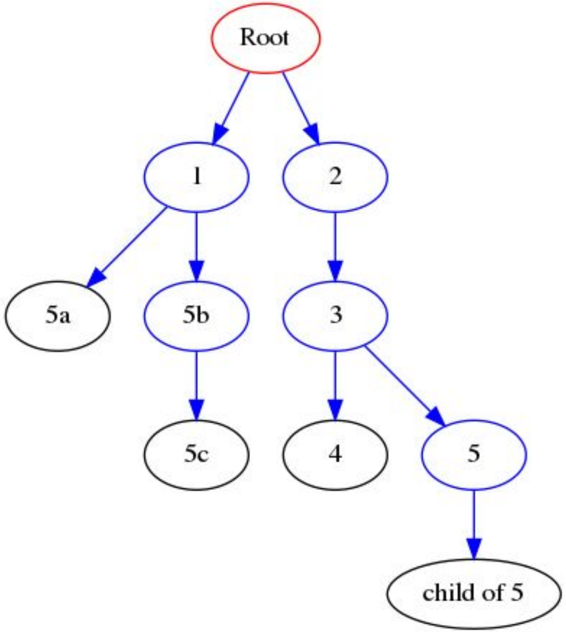
    BT.insertLeft(bt1)
    BT.insertRight(bt2)
    bt2.insertLeft(bt3)
    bt3.insertLeft(bt4)
    bt3.insertRight(bt5)
    bt2.insertRight(bt6)
    bt1.insertRight(bt5b)
    bt1.insertLeft(bt5a)
    bt5b.insertRight(bt5c)
    print("\nDelete right branch of 2")
    bt2.deleteRight()
```



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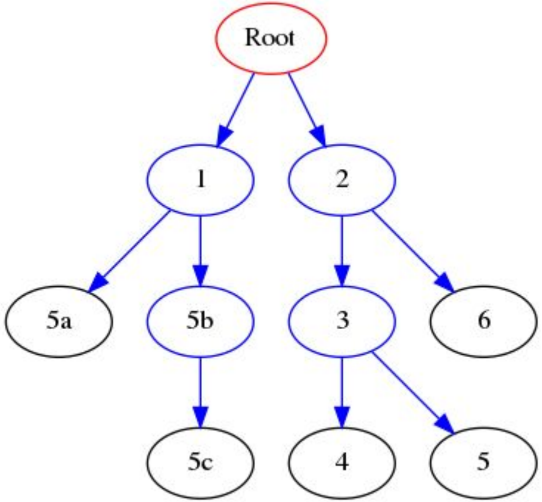
    BT.insertLeft(bt1)
    BT.insertRight(bt2)
    bt2.insertLeft(bt3)
    bt3.insertLeft(bt4)
    bt3.insertRight(bt5)
    bt2.insertRight(bt6)
    bt1.insertRight(bt5b)
    bt1.insertLeft(bt5a)
    bt5b.insertRight(bt5c)
```



Exercise. write a print function that gets the root node and prints the three:

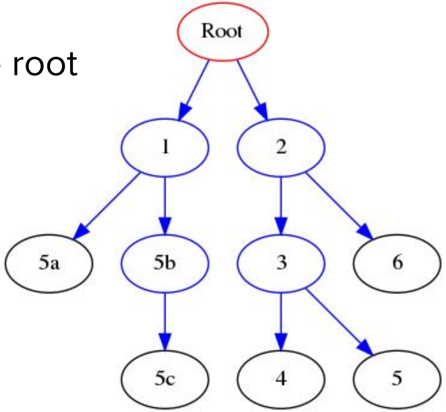
```
Root (r)-> 2
Root (l)-> 1
         1 (r)-> 5b
         1 (l)-> 5a
                 5b (r)-> 5c
         2 (r)-> 6
         2 (l)-> 3
                 3 (r)-> 5
                 3 (l)-> 4
```

A sample tree...

Exercise. write a print function that gets the root node and prints the three:

Tabs depend on depth

```
Root (r)-> 2
Root (l)-> 1
        1 (r)-> 5b
        1 (l)-> 5a
                5b (r)-> 5c
        2 (r)-> 6
        2 (l)-> 3
                3 (r)-> 5
                3 (l)-> 4
```




```python
def printTree(root):
    cur = root
    #each element is a node and a depth
    #depth is used to format prints (with tabs)
    nodes = [(cur,0)]
    tabs = ""
    lev = 0
    while len(nodes) >0:
        cur, lev = nodes.pop(-1)
        if cur.getRight() != None:
            print ("{}{} (r)-> {}".format("\t"*lev,
                                          cur.getValue(),
                                          cur.getRight().getValue()))
            nodes.append((cur.getRight(), lev+1))
        if cur.getLeft() != None:
            print ("{}{} (l)-> {}".format("\t"*lev,
                                          cur.getValue(),
                                          cur.getLeft().getValue()))
            nodes.append((cur.getLeft(), lev+1))
```

A sample tree...

```python
def printTree(root):
    cur = root
    #each element is a node and a depth
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    nodes = [(cur,0)]
    tabs = ""
    lev = 0
    while len(nodes) >0:
        cur, lev = nodes.pop(-1)
        if cur.getRight() != None:
            print ("{}{} (r)-> {}".format("\t"*lev,
                                          cur.getValue(),
                                          cur.getRight().getValue()))
            nodes.append((cur.getRight(), lev+1))
        if cur.getLeft() != None:
            print ("{}{} (l)-> {}".format("\t"*lev,
                                          cur.getValue(),
                                          cur.getLeft().getValue()))
            nodes.append((cur.getLeft(), lev+1))
```
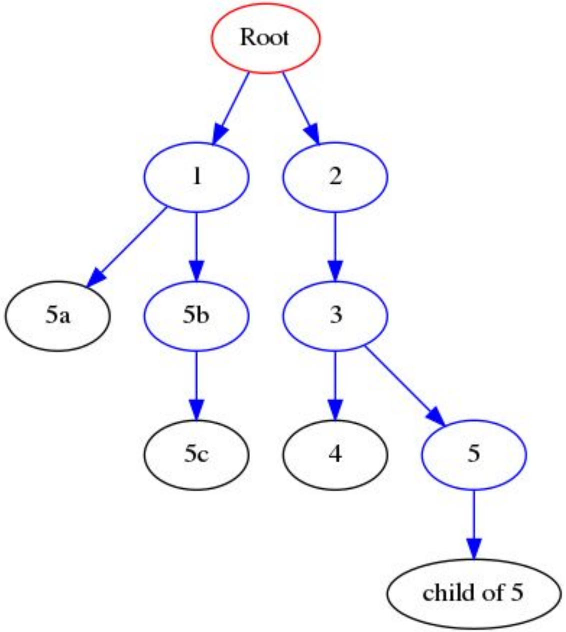



OUTPUT

```
Root (r)-> 2
Root (l)-> 1
	1 (r)-> 5b
	1 (l)-> 5a
		5b (r)-> 5c
	2 (l)-> 3
		3 (r)-> 5
		3 (l)-> 4
			5 (l)-> child of 5
```

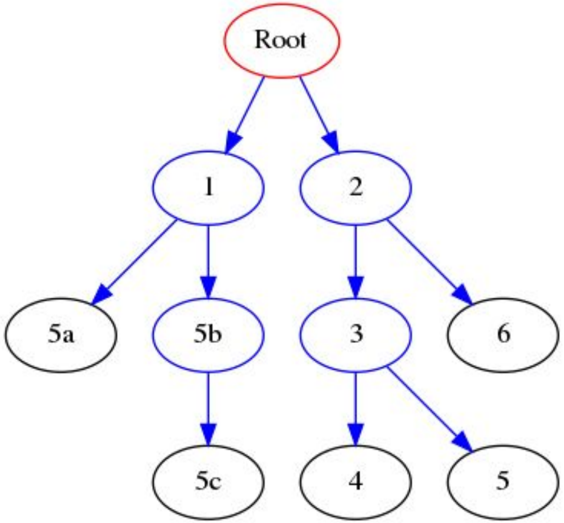
Tree traversals

Tree traversal / search

A strategy to pass through (visit) all the nodes of a tree.

Depth-First Search (DFS)

- Each subtree of the tree is visited, one after another
- Three variants (pre/in/post order)
- Requi res a stack



Recursively

1. visit Root
2. visit left
3. visit right

Preorder:
Root

Stack: (5c right of 5b!)

Root

To store all unfinished calls to DFS(node)

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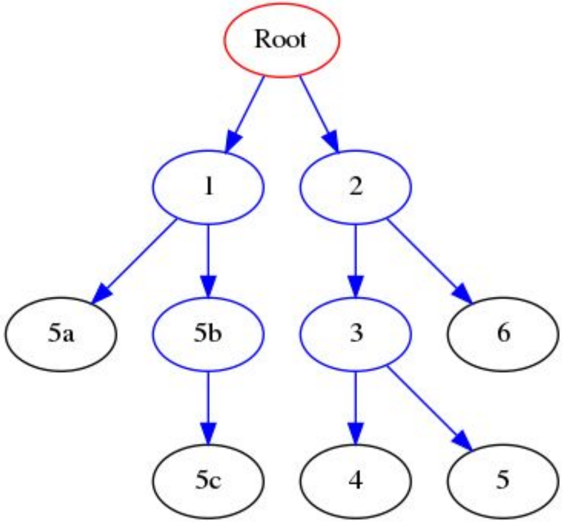

Recursively

1. visit Root
2. visit left
3. visit right

Preorder:
Root
1

Stack: (5c right of 5b!)
1
Root

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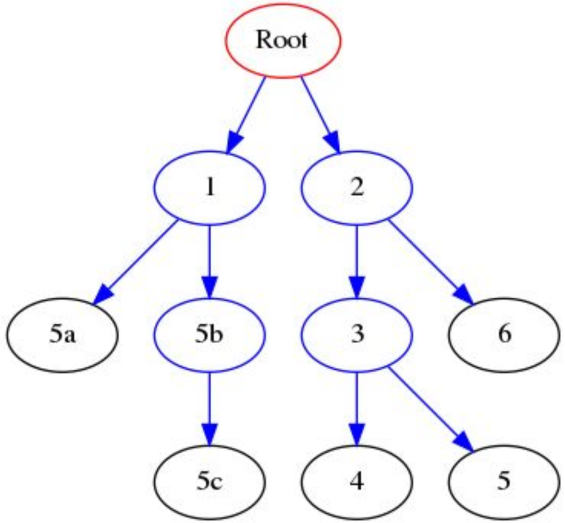

Recursively

1. visit Root
2. visit left
3. visit right

Preorder:
Root
1
5a

Stack: (5c right of 5b!)
5a
1
Root

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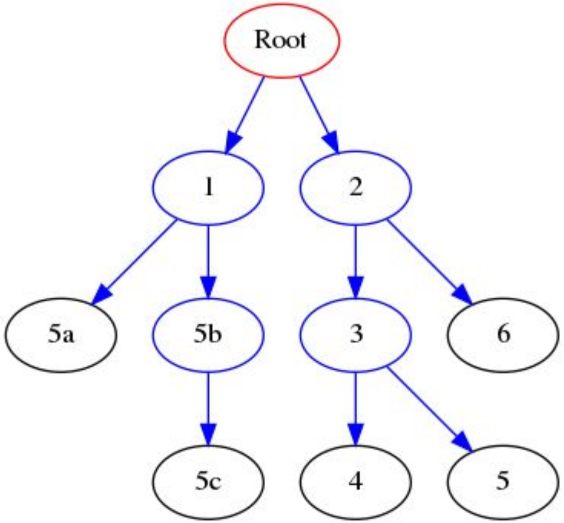

Recursively

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Preorder:
Root
1
5a

Stack: (5c right of 5b!)
1
Root

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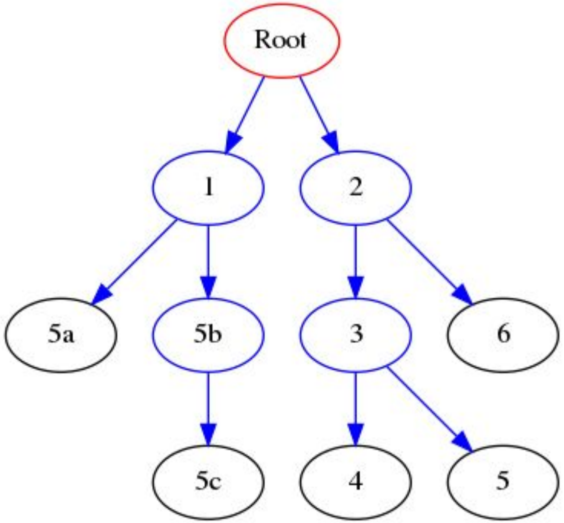

Recursively

1. visit Root
2. visit left
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Preorder:
Root
1
5a
5b

Stack: (5c right of 5b!)
5b
1
Root

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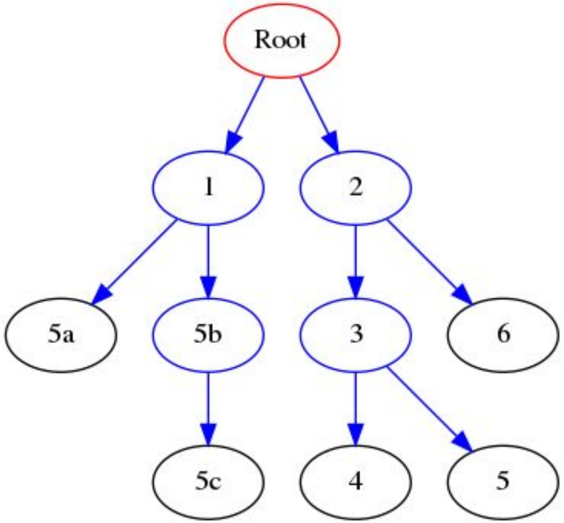

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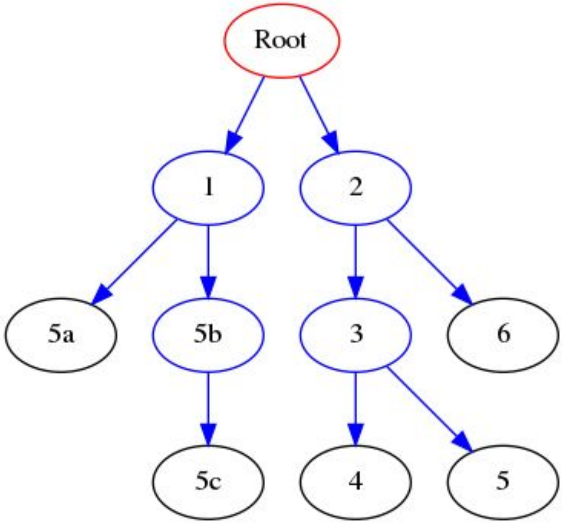

Recursively

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Preorder:
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Stack: (5c right of 5b!)
5b
1
Root

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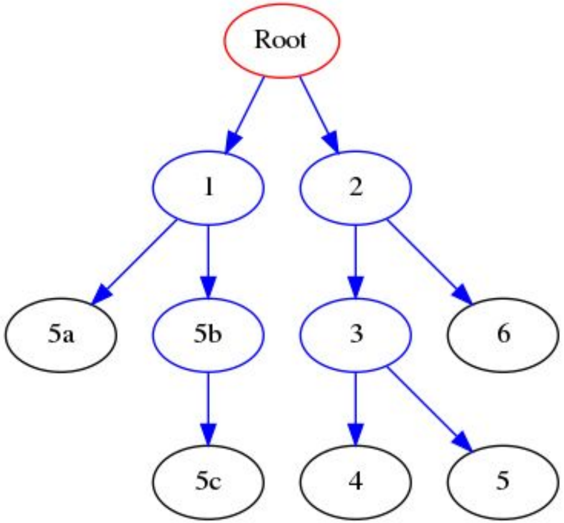

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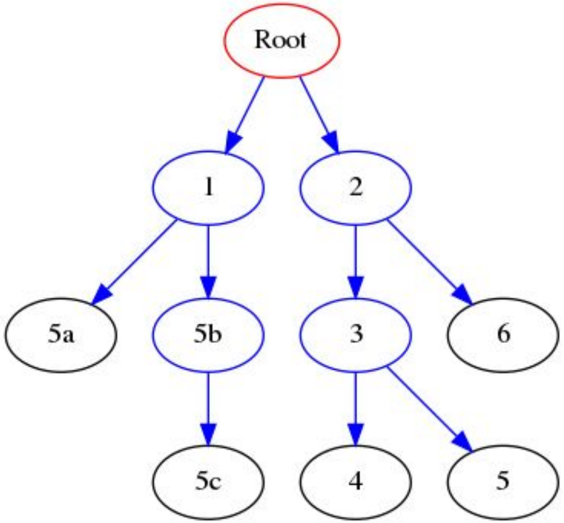

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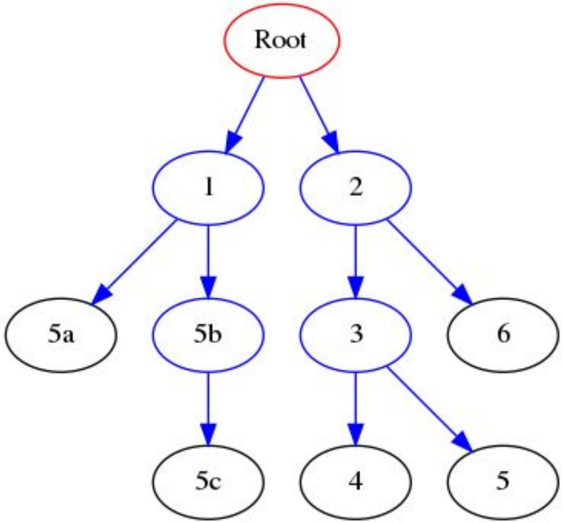


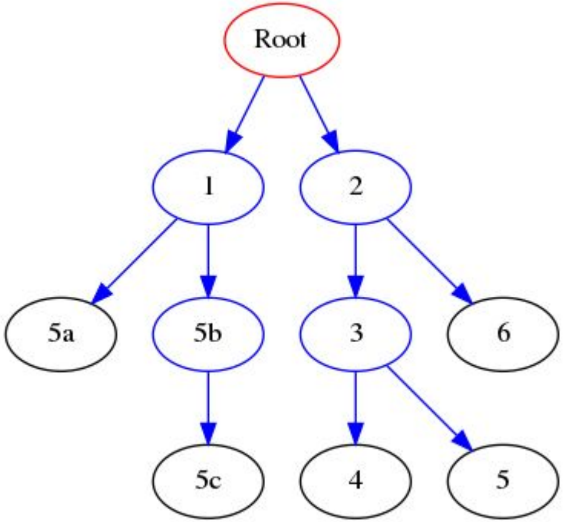
Recursively

1. visit Root
2. visit left
3. visit right

Preorder:
Root
1
5a
5b
5c
2

Stack: (5c right of 5b!)
2
Root

Tree traversals



Tree traversal / search

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5a
5b
5c
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3

Stack: (5c right of 5b!)
3
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Root

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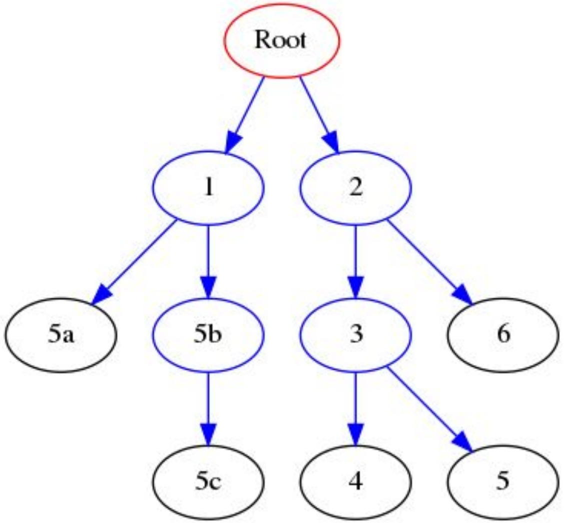

Recursively

1. visit Root
2. visit left
3. visit right

Preorder:
Root
1
5a
5b
5c
2
3
4

Stack: (5c right of 5b!)
4
3
2
Root

Tree traversals

Tree traversal / search

A strategy to pass through (visit) all the nodes of a tree.

Depth-First Search (DFS)

- Each subtree of the tree is visited, one after another
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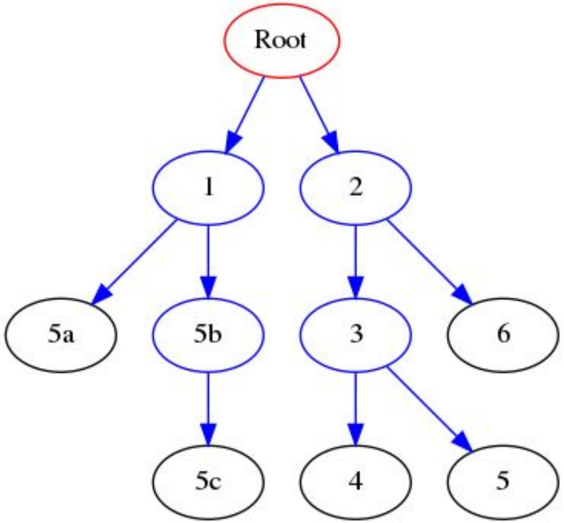

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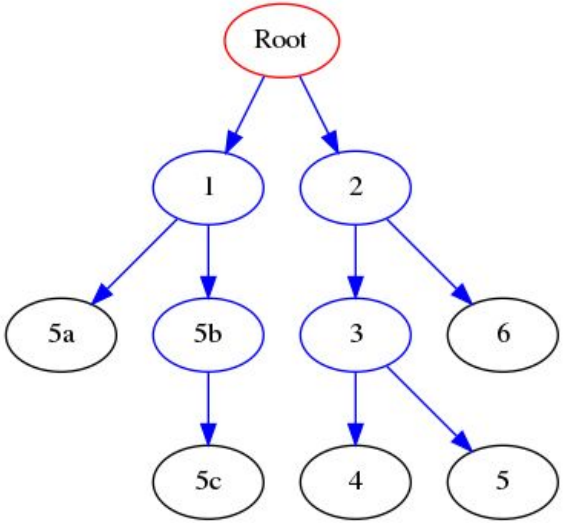

Preorder:
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5a
5b
5c
2
3
4
5

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3
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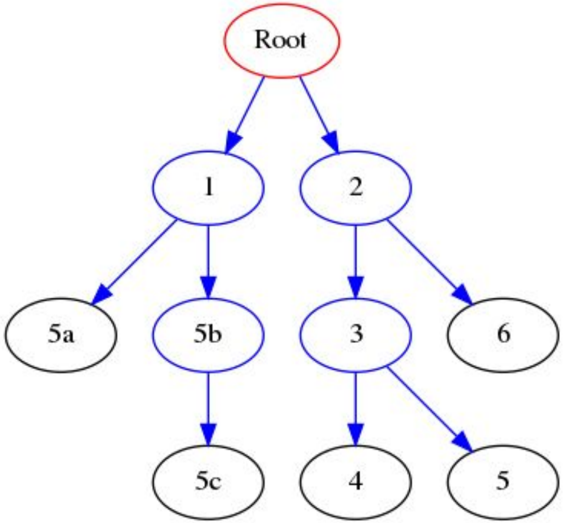

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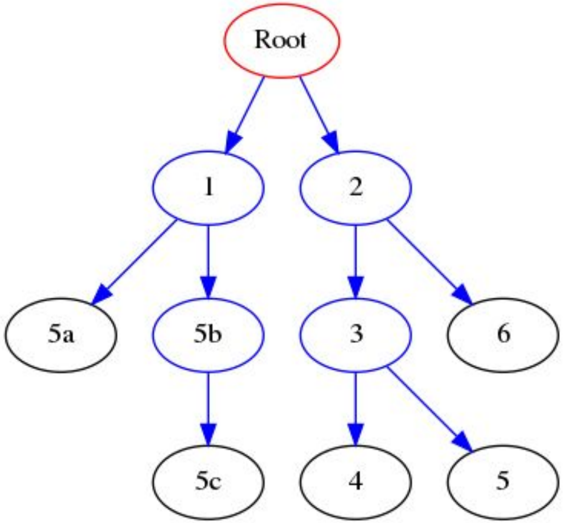

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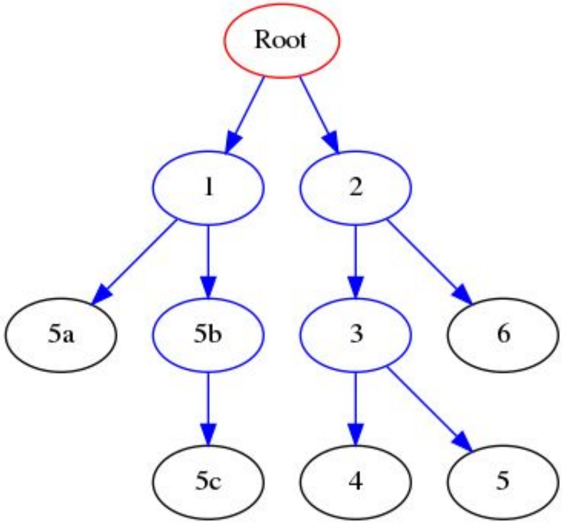

Recursively

1. visit Root
2. visit left
3. visit right

Preorder:
Root
1
5a
5b
5c
2
3
4
5
6

Stack: (5c right of 5b!)
6
2
Root

Tree traversals

Tree traversal / search

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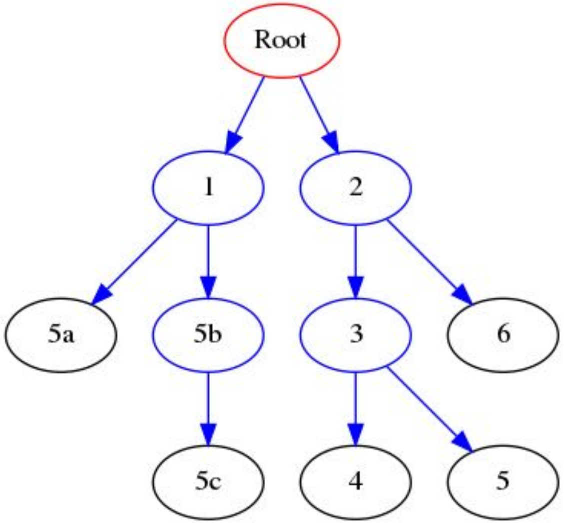

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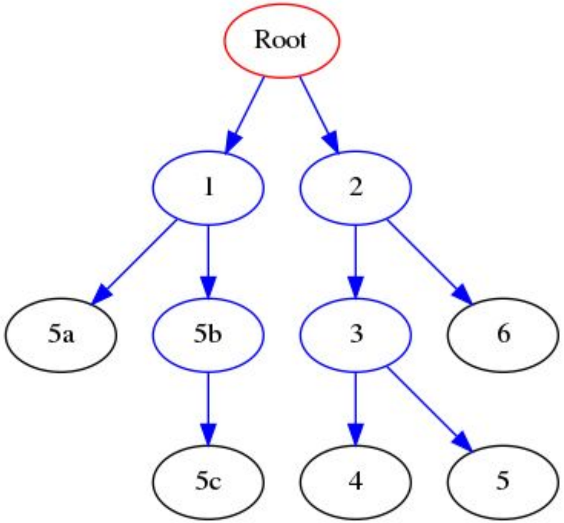

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5a
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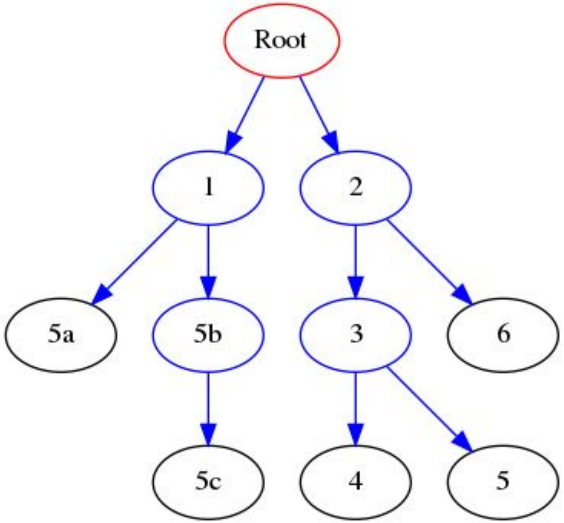

Recursively

1. visit Root
2. visit left
3. visit right

Preorder:
Root
1
5a
5b
5c
2
3
4
5
6

Stack: (5c right of 5b!)

empty! **Done**

Tree traversals

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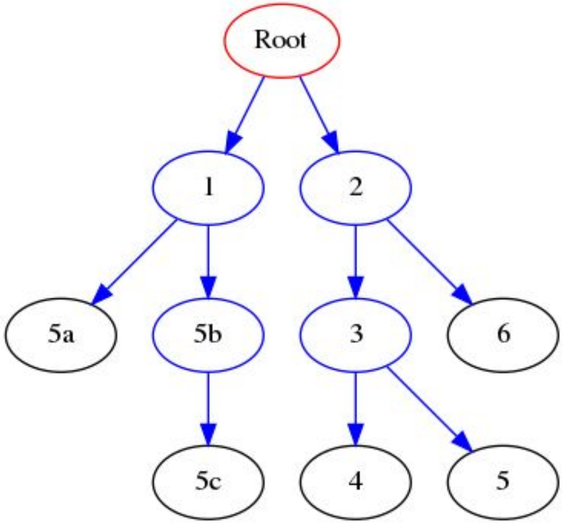

Recursively

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Inorder:

Stack: (5c right of 5b!)
Root

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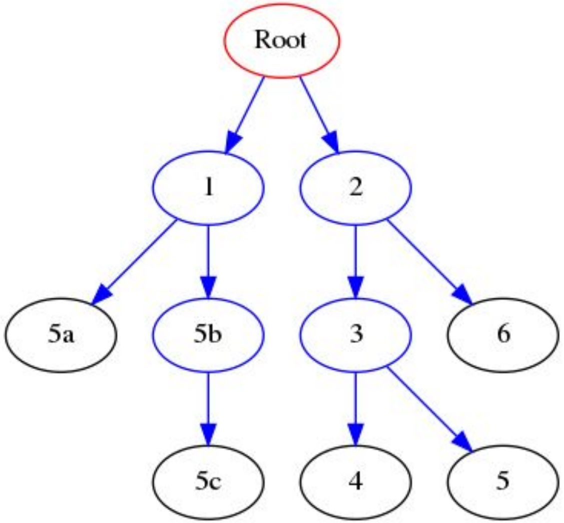

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Inorder:

Stack: (5c right of 5b!)

1

Root

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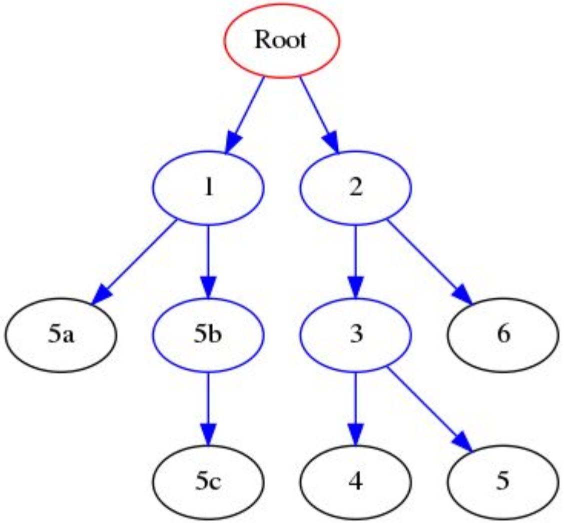

Recursively

1. visit left
2. visit Root
3. visit right

Inorder:
5a

Stack: (5c right of 5b!)
5a
1
Root

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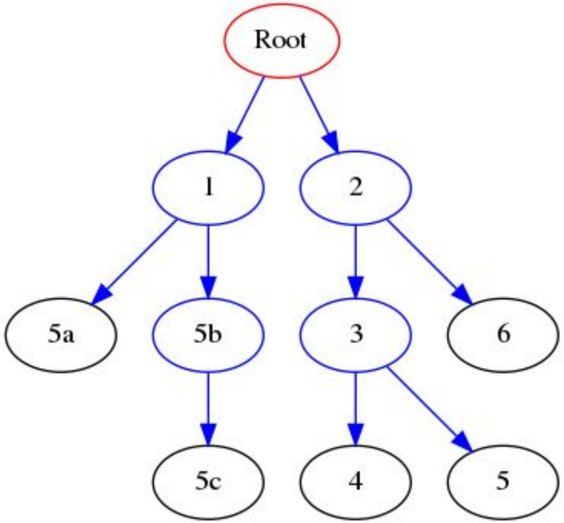

Recursively

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Inorder:
5a

Stack: (5c right of 5b!)
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Root

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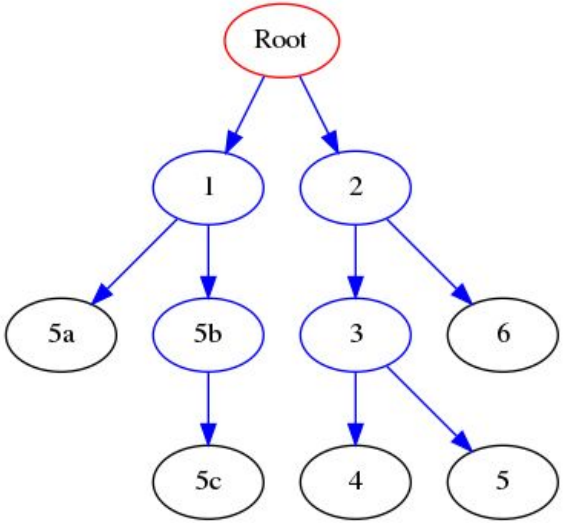

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5a
1

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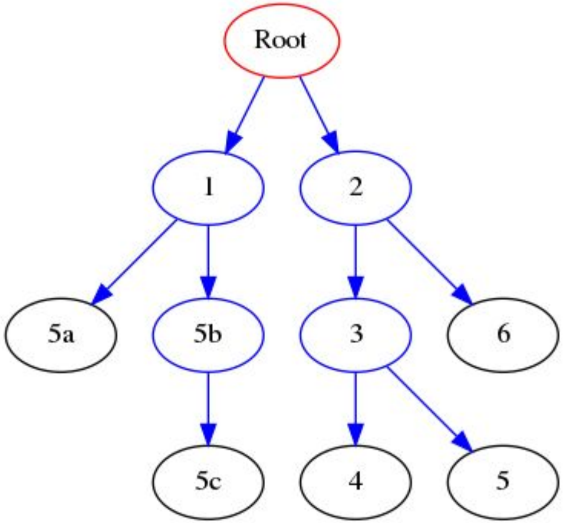

Recursively

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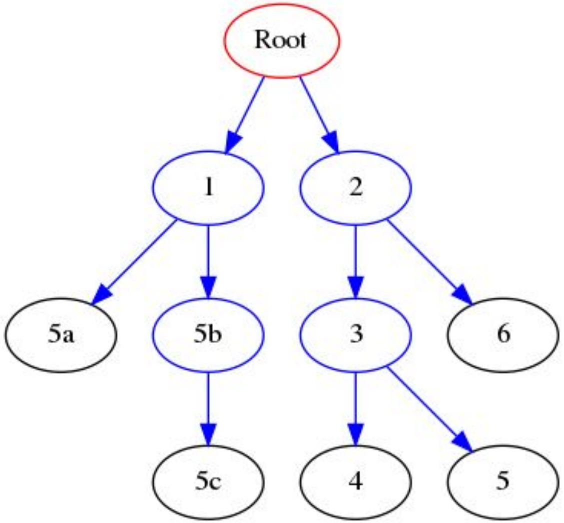
2. visit Root
3. visit right

Inorder:
5a
1

Stack: (5c right of 5b!)
5b
1
Root

Tree traversals



Tree traversal / search

A strategy to pass through (visit) all the nodes of a tree.

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5a
1
5b

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5b
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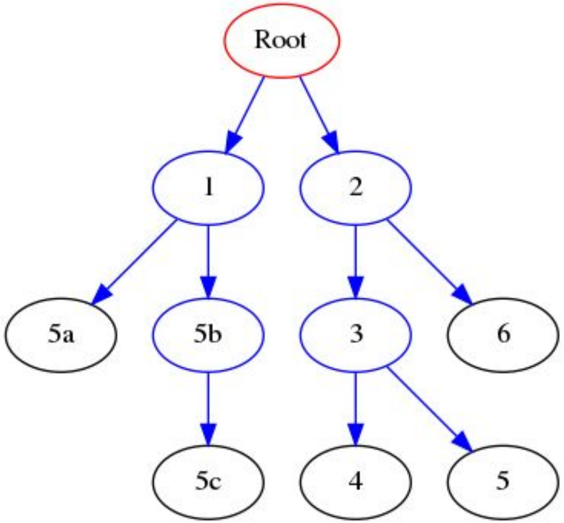

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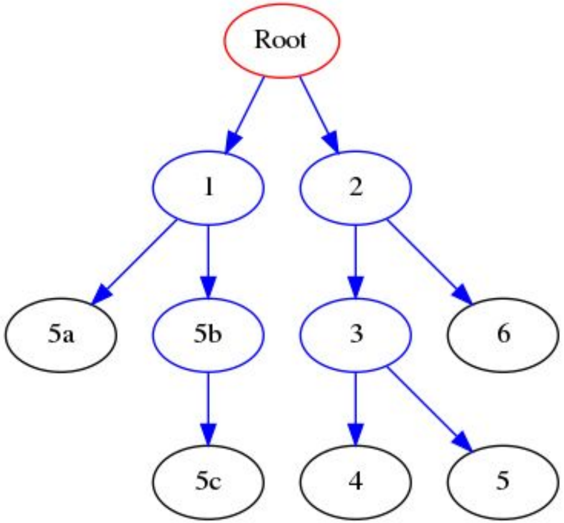

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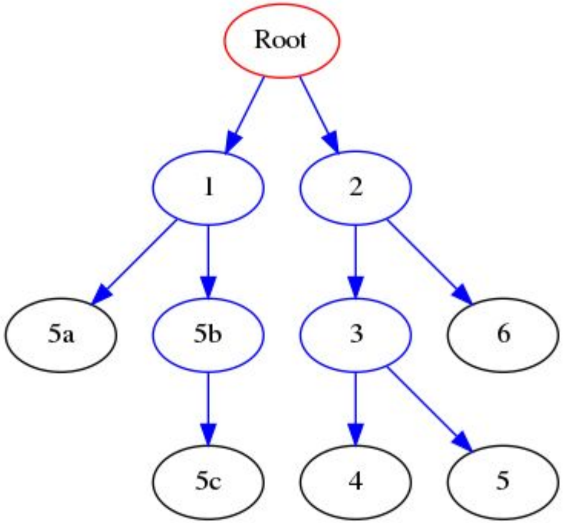

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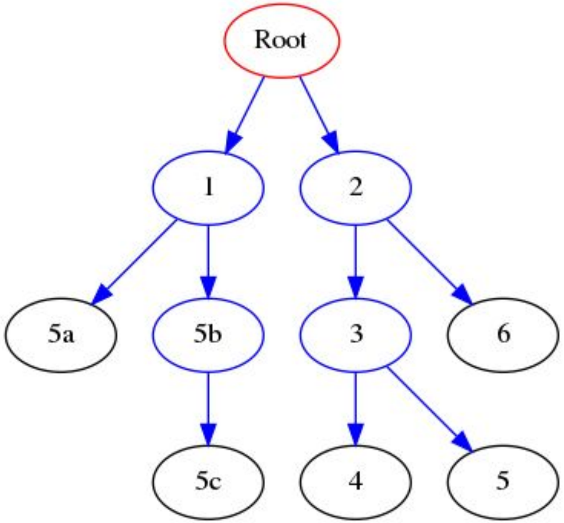

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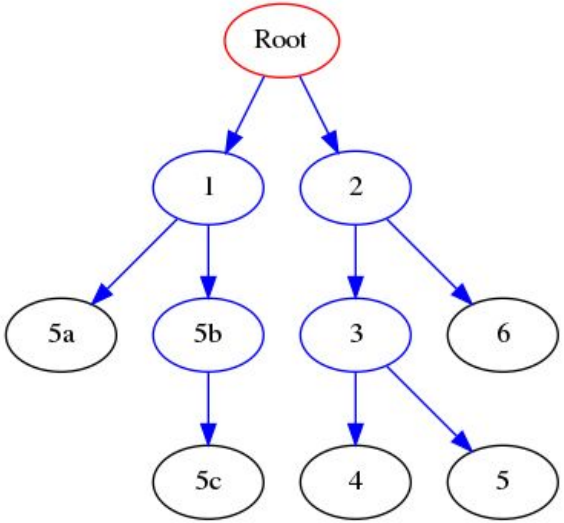

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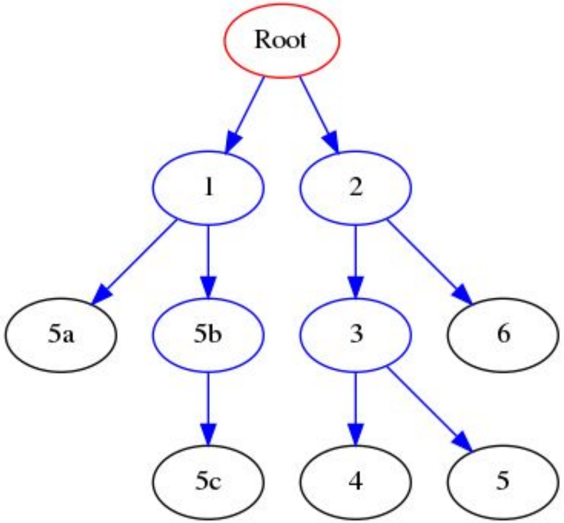
2
Root



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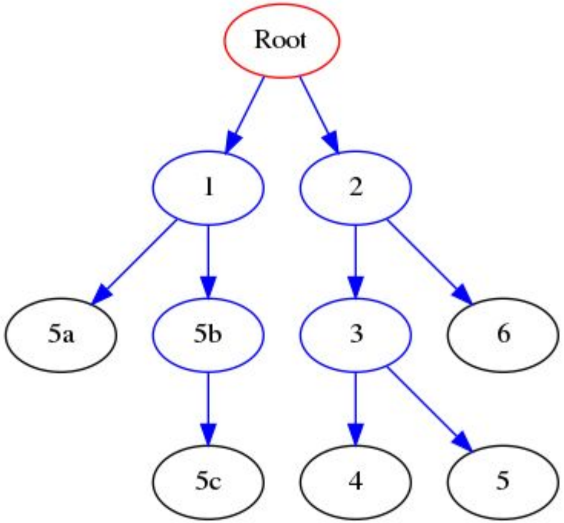

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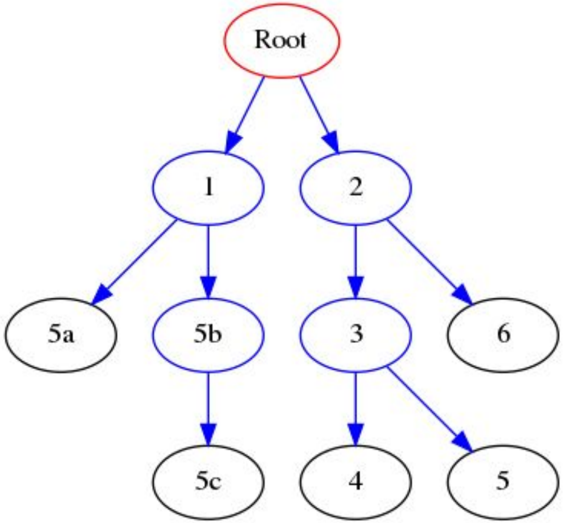

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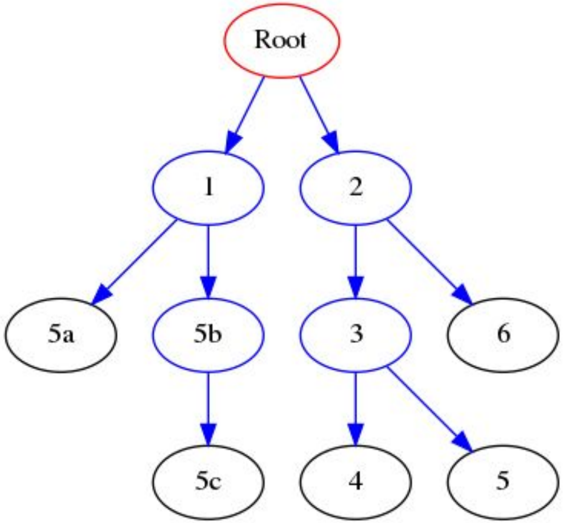

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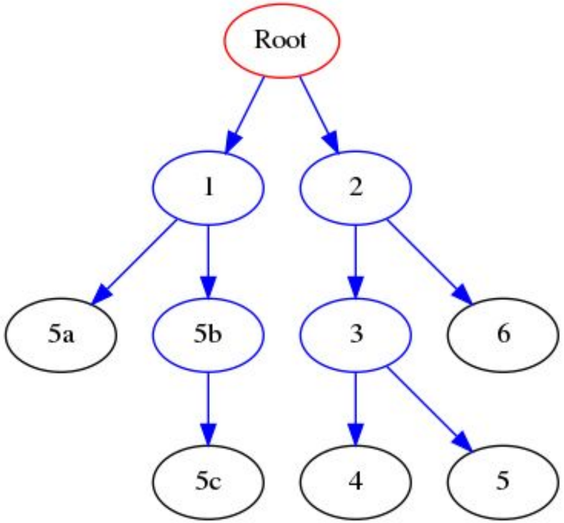

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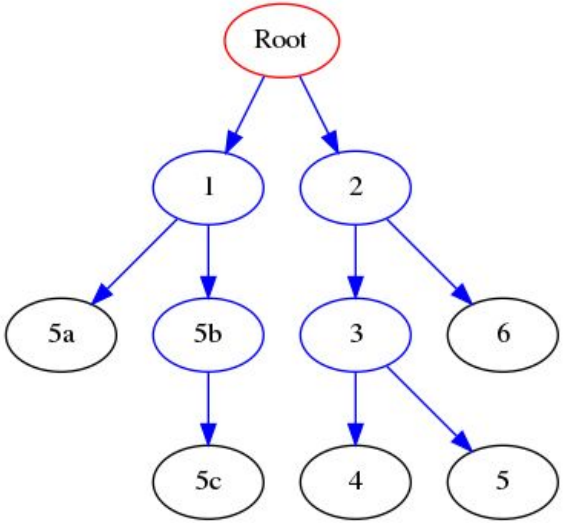

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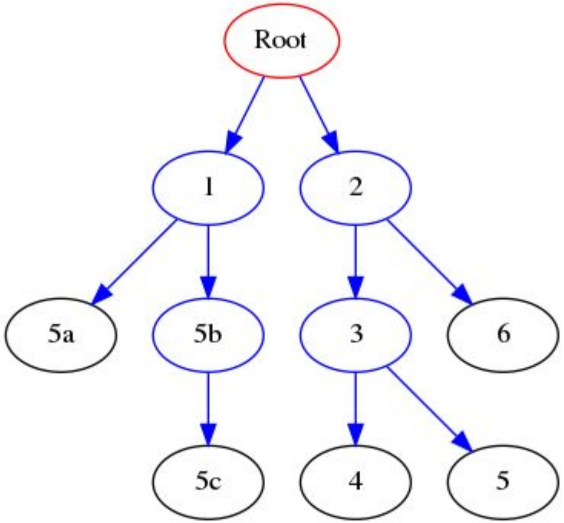

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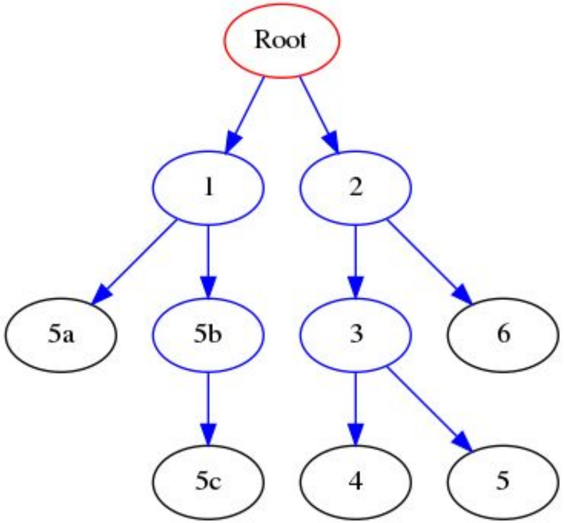

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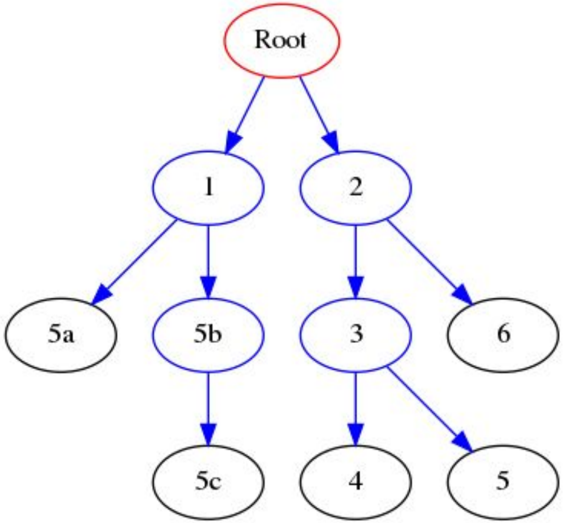

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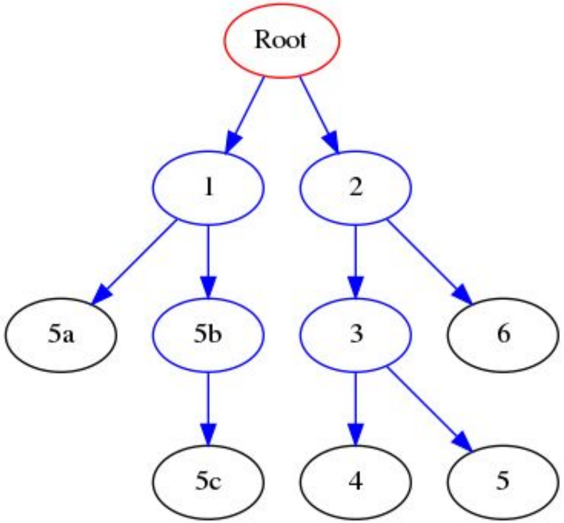

Recursively

1. visit left
2. visit Root
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Inorder:
5a
1
5b
5c
Root
4
3
5
2
6

Stack: (5c right of 5b!)
6
2
Root

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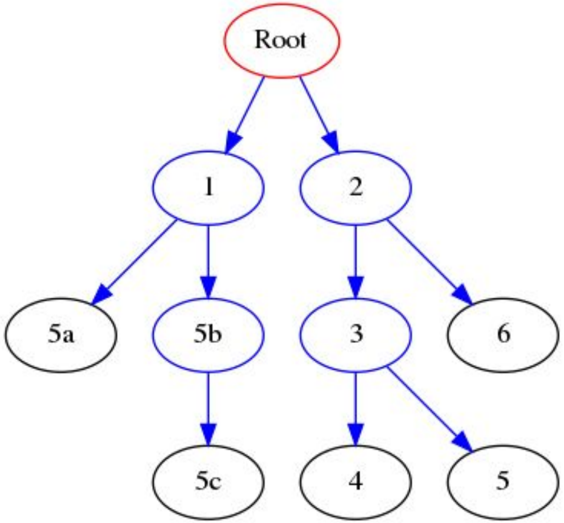

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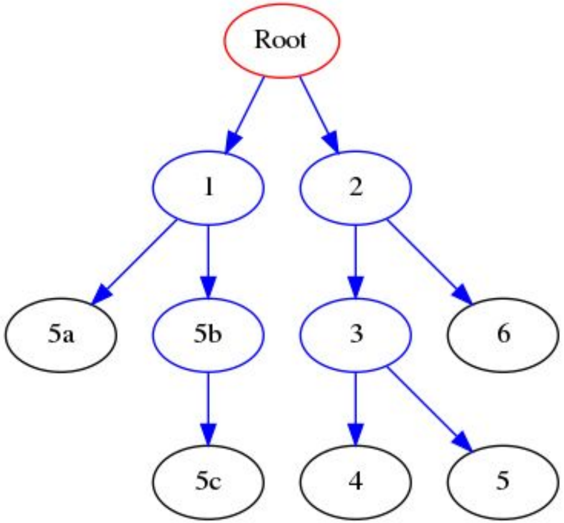

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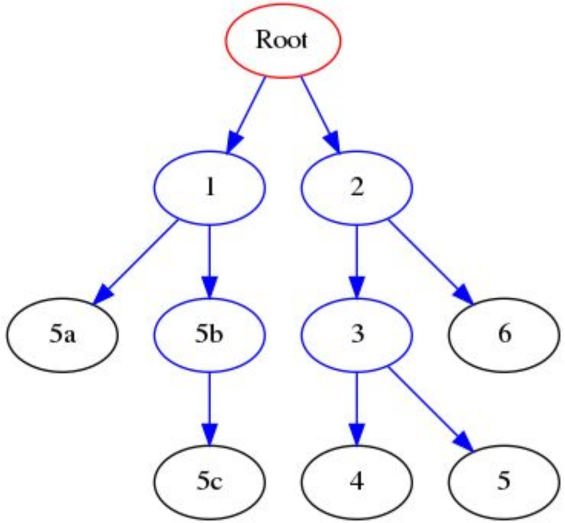
Root

Tree traversals



Tree traversal / search

A strategy to pass through (visit) all the nodes of a tree.

Depth-First Search (DFS)

- Each subtree of the tree is visited, one after another
- Three variants (pre/in/post order)
- Requi res a stack

Recursively

1. visit left
2. visit Root
3. visit right

Inorder:
5a
1
5b
5c
Root
4
3
5
2
6

Stack: (5c right of 5b!)

empty. Done!

Tree traversals

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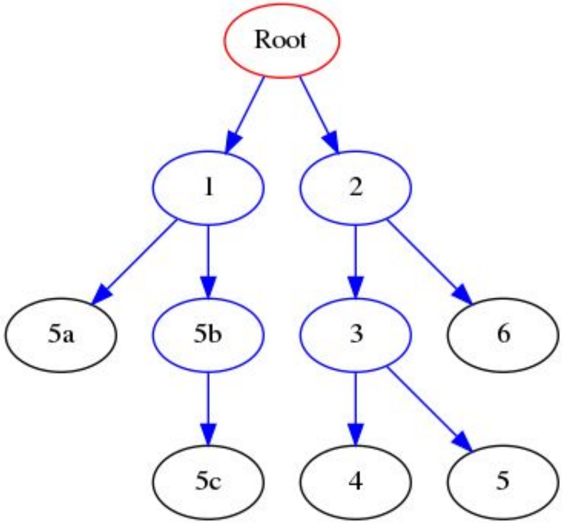


Recursively

1. visit left
2. visit right
3. visit Root

Postorder:

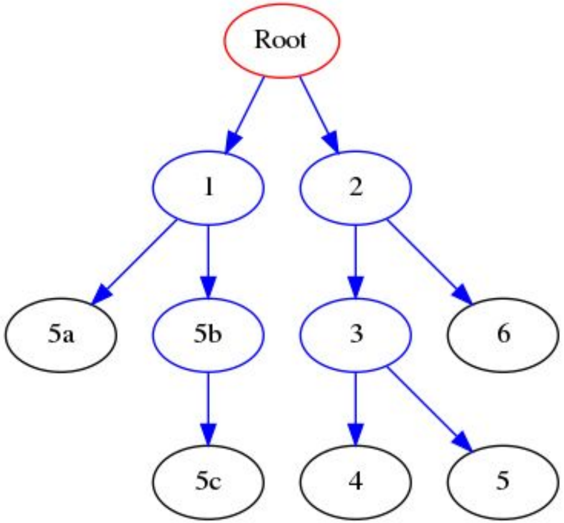
5a
5c (right of 5b)
5b
1
4
5
3
6
2
Root

Stack: Exercise!

DFS: the code

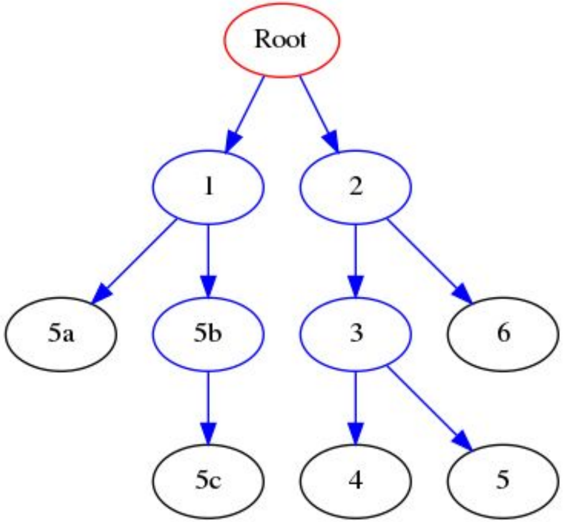
visit means “print”

```python
def DFS(node, kind = "preorder"):
    if node != None:
        if kind == "preorder":
            print("{}".format(node.getValue()))
        DFS(node.getLeft(), kind = kind)
        if kind == "inorder":
            print("{}".format(node.getValue()))
        DFS(node.getRight(), kind = kind)
        if kind == "postorder":
            print("{}".format(node.getValue()))
```



Preorder:	**Inorder:**	**Postorder:**
Root	5a	5a
1	1	5c
5a	5b	5b
5b	5c	1
5c	Root	4
2	4	5
3	3	3
4	5	6
5	2	2
6	6	Root

Tree traversals



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- Each level of the tree is visited, one after the other
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- Requires a queue

Tree traversals

Tree traversal / search

A strategy to pass through (visit) all the nodes of a tree.

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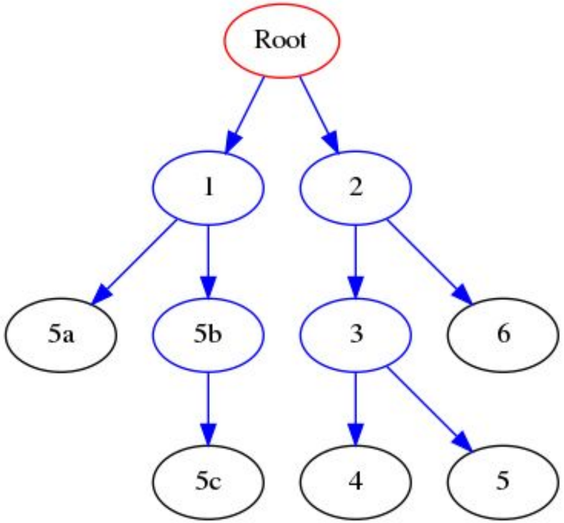
- Each level of the tree is visited, one after the other
- Starts from the root
- Requires a queue

0. Add root to the queue Q

Recursively

1. get node from Q
2. visit the node
3. add all children to Q



Visit order

Queue
Root

Tree traversals

Tree traversal / search

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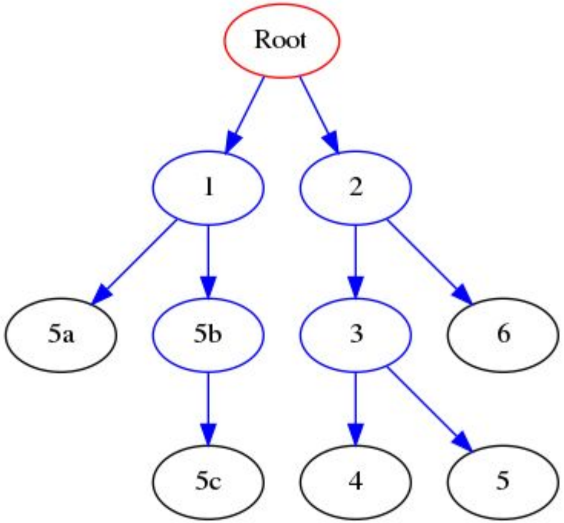

Visit order	**Queue**
Root	1 , 2

Tree traversals

Tree traversal / search

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Recursively

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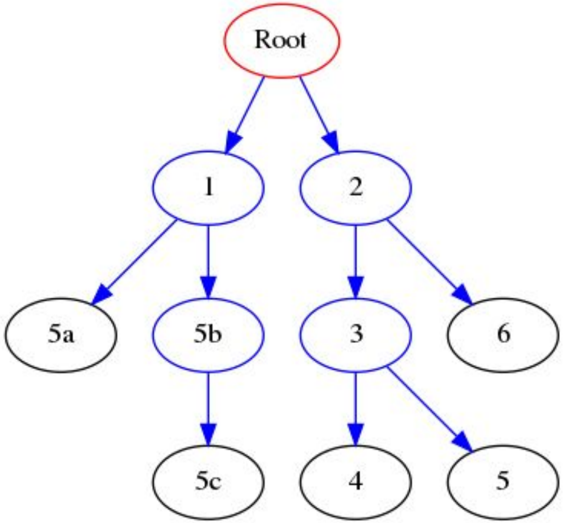


Visit order	Queue
Root	2, 5a, 5b
1	

Tree traversals

Tree traversal / search

A strategy to pass through (visit) all the nodes of a tree.

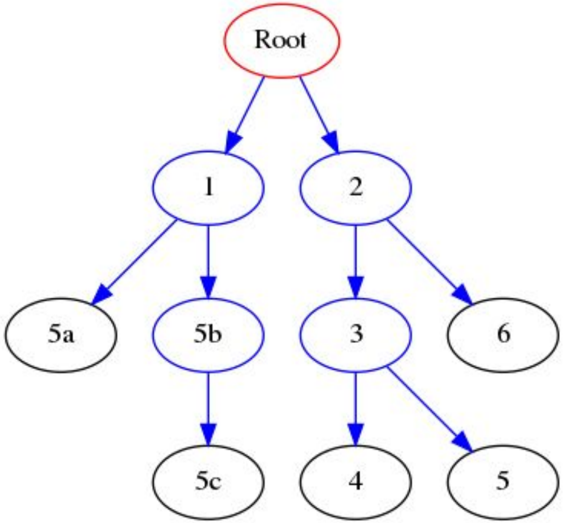
Breadth-First Search (BFS)

- Each level of the tree is visited, one after the other
- Starts from the root
- Requires a queue

0. Add root to the queue Q

Recursively

1. get node from Q
2. visit the node
3. add all children to Q



Visit order	**Queue**
Root	5a, 5b, 3, 6
1	
2	

Tree traversals

Tree traversal / search

A strategy to pass through (visit) all the nodes of a tree.

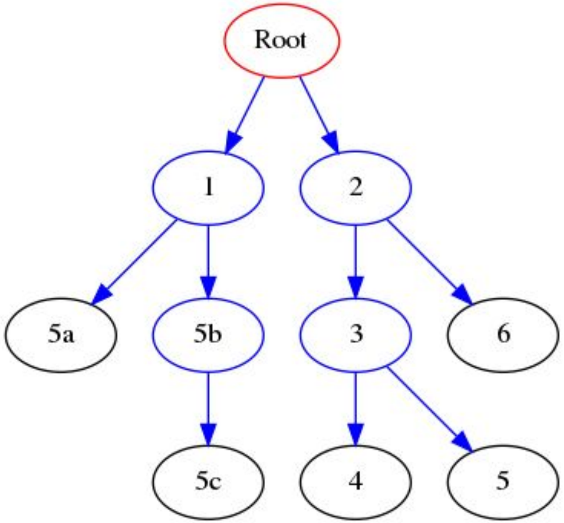
Breadth-First Search (BFS)

- Each level of the tree is visited, one after the other
- Starts from the root
- Requires a queue

0. Add root to the queue Q

Recursively

1. get node from Q
2. visit the node
3. add all children to Q

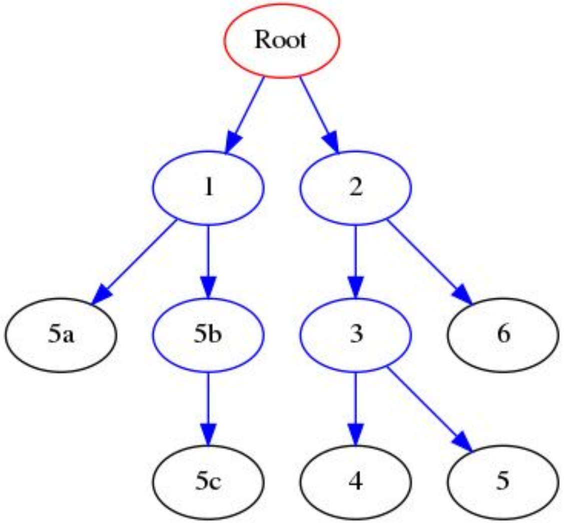


Visit order	**Queue**
Root	5b, 3, 6
1	
2	
5a	

Tree traversals

Tree traversal / search

A strategy to pass through (visit) all the nodes of a tree.



Breadth-First Search (BFS)

- Each level of the tree is visited, one after the other
- Starts from the root
- Requires a queue

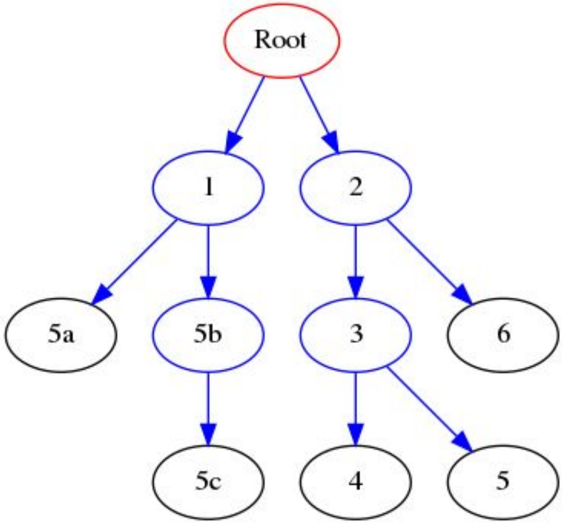
0. Add root to the queue Q

Recursively

1. get node from Q
2. visit the node
3. add all children to Q

Visit order	**Queue**
Root	3, 6, 5c
1	
2	
5a	
5b	

Tree traversals



Tree traversal / search

A strategy to pass through (visit) all the nodes of a tree.

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- Requires a queue

0. Add root to the queue Q

Recursively

1. get node from Q
2. visit the node
3. add all children to Q

Visit order	**Queue**
Root	6, 5c, 4, 5
1	
2	
5a	
5b	
3	

Tree traversals

Tree traversal / search

A strategy to pass through (visit) all the nodes of a tree.

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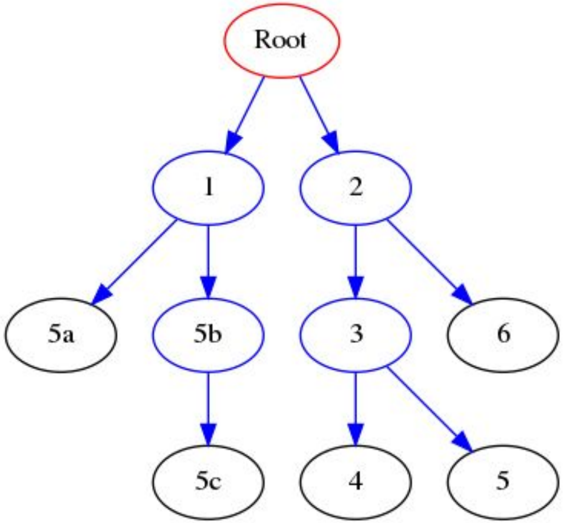
- Each level of the tree is visited, one after the other
- Starts from the root
- Requires a queue



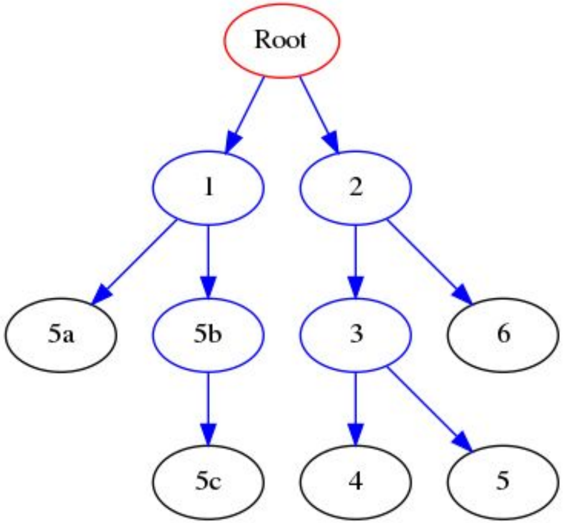
0. Add root to the queue Q

Recursively

1. get node from Q
2. visit the node
3. add all children to Q

Visit order	**Queue**
Root	5c, 4, 5
1	
2	
5a	
5b	
3	
6	

Tree traversals



Tree traversal / search

A strategy to pass through (visit) all the nodes of a tree.

Breadth-First Search (BFS)

- Each level of the tree is visited, one after the other
- Starts from the root
- Requires a queue

0. Add root to the queue Q

Recursively

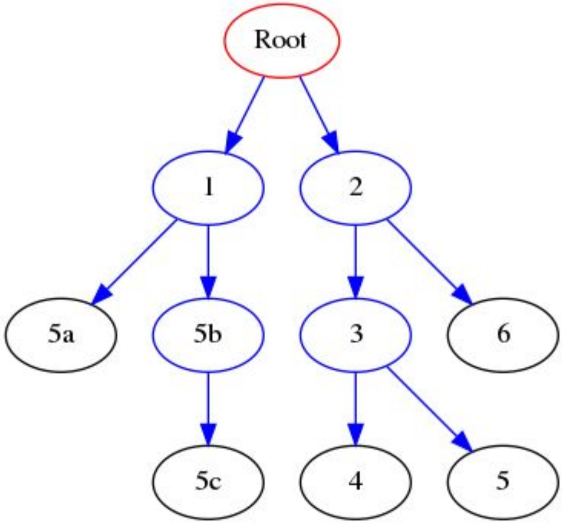
1. get node from Q
2. visit the node
3. add all children to Q

Visit order	**Queue**
Root	4, 5
1	
2	
5a	
5b	
3	
6	
5c	

Tree traversals

Tree traversal / search

A strategy to pass through (visit) all the nodes of a tree.



Breadth-First Search (BFS)

- Each level of the tree is visited, one after the other
- Starts from the root
- Requires a queue

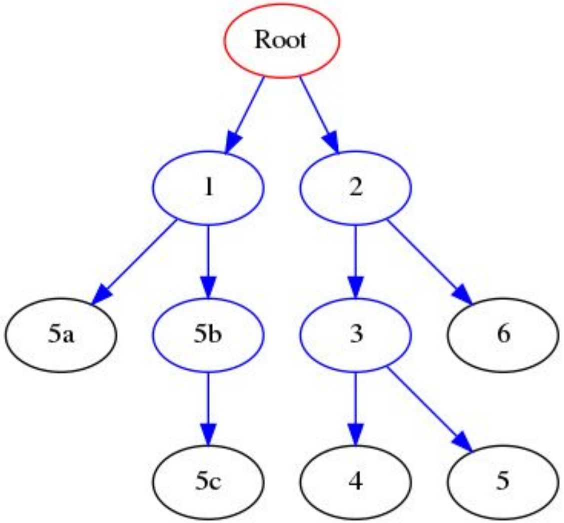
0. Add root to the queue Q

Recursively

1. get node from Q
2. visit the node
3. add all children to Q

Visit order	**Queue**
Root	5
1	
2	
5a	
5b	
3	
6	
5c	
4	

Tree traversals



Tree traversal / search

A strategy to pass through (visit) all the nodes of a tree.

Breadth-First Search (BFS)

- Each level of the tree is visited, one after the other
- Starts from the root
- Requires a queue

0. Add root to the queue Q

Recursively

1. get node from Q
2. visit the node
3. add all children to Q

Visit order	Queue
Root	
1	Empty. Done
2	
5a	
5b	
3	
6	
5c	
4	
5	

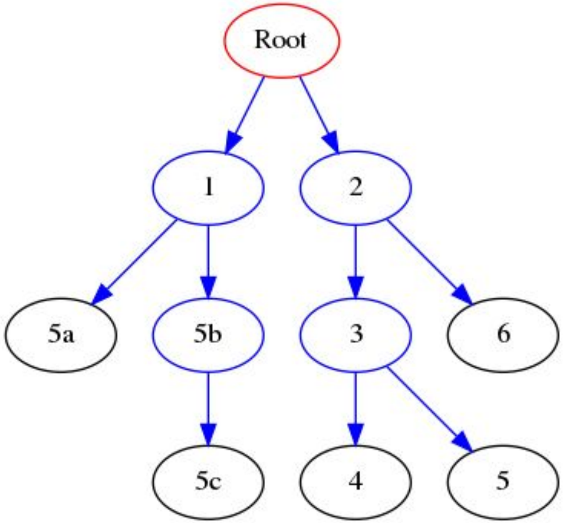
Tree traversals

Tree traversal / search

A strategy to pass through (visit) all the nodes of a tree.

Breadth-First Search (BFS)

- Each level of the tree is visited, one after the other
- Starts from the root
- Requires a queue



0. Add root to the queue Q

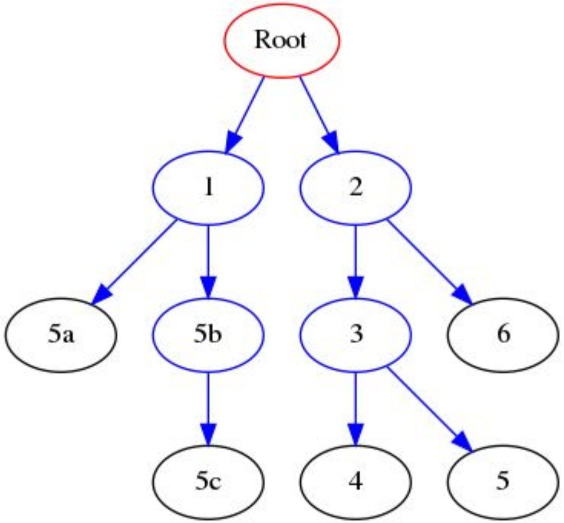
Recursively

1. get node from Q
2. visit the node
3. add all children to Q

Visit order	**Level**
Root	0
1	1
2	1
5a	2
5b	2
3	2
6	2
5c	3
4	3
5	3

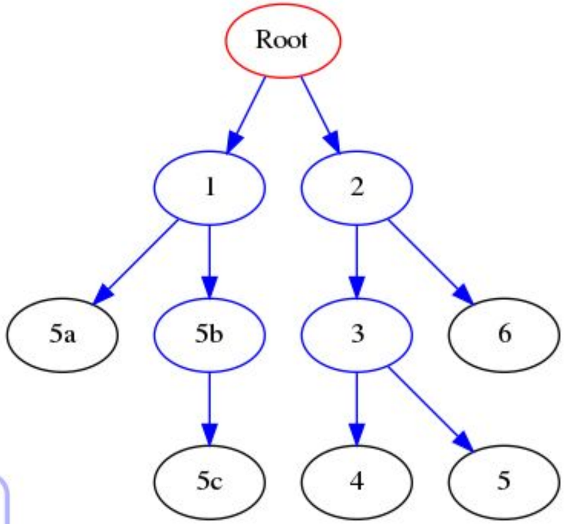
Tree traversals: BFS

```
from collections import deque

def BFS(node):
        Q = deque()
        if node != None:
            Q.append(node)

        while len(Q) > 0:
            curNode = Q.popleft()
            if curNode != None:
                print("{}".format(curNode.getValue()))
                Q.append(curNode.getLeft())
                Q.append(curNode.getRight())
```



BFS visit:
Root
1
2
5a
5b
3
6
5c
4
5

Tree traversals: complexity

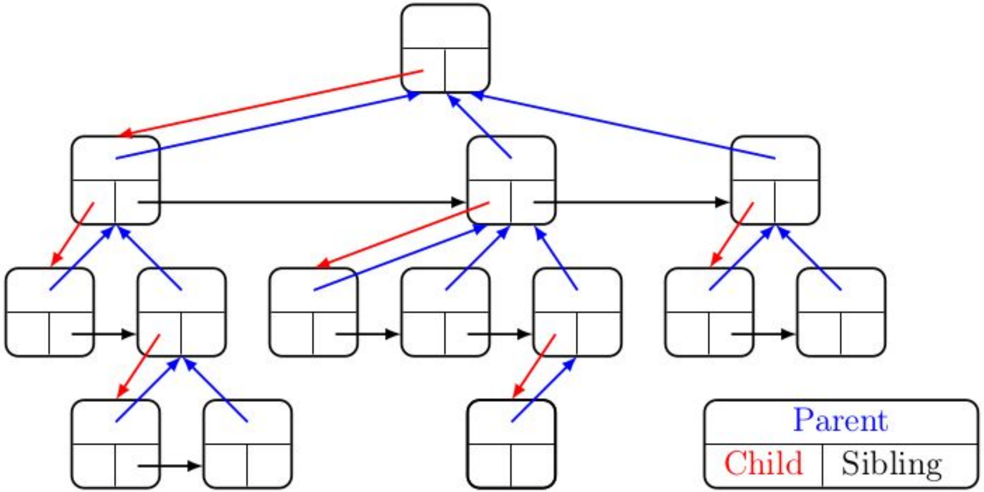


The cost of a visit of a tree containing n nodes is $\Theta(n)$, because each node is visited exactly once.

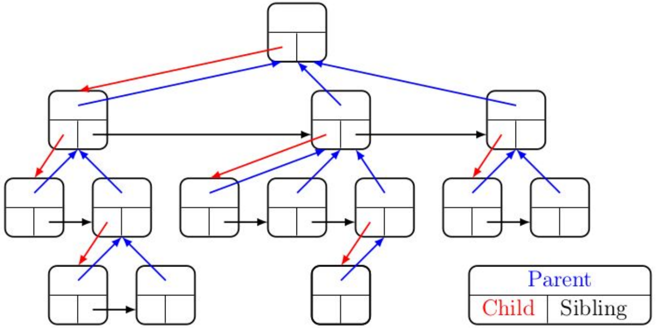
Generic trees

Generic Trees are like binary trees, but **each node can have more than 2 children**. One possible implementation is that each node (that is a subtree in itself) has a **value**, a link to its **parent** and a **list of children**.

Another implementation is that each node has a **value**, a link to its **parent**, a link to its **next sibling** and a link to its **first child**.



Generic trees



TREE

% Build a new node, initially containing v, with no children or parent

Tree(OBJECT v)

% Read the value stored in nodes

OBJECT getValue()

% Write the value stored in nodes

setValue(OBJECT v)

% Returns the parent, or `None` if this node is root

TREE getParent()

% Returns the first child, or `None` if this node is leaf

TREE leftmostChild()

% Returns the next sibling, or `None` if there is none

TREE rightSibling()

% Insert the subtree t as first child of this node

insertChild(TREE t)

% Insert the subtree t as next sibling of this node

insertSibling(TREE t)

% Destroy the subtree rooted in the first child

deleteChild()

% Destroy the subtree rooted in the next sibling

deleteSibling()

Exercise!

Exercise

The visit order of a binary tree containing 9 nodes are the following:

- A, E, B, F, G, C, D, I, H (pre-order)
- B, G, C, F, E, H, I, D, A (post-order)
- B, E, G, F, C, A, D, H, I (in-order)

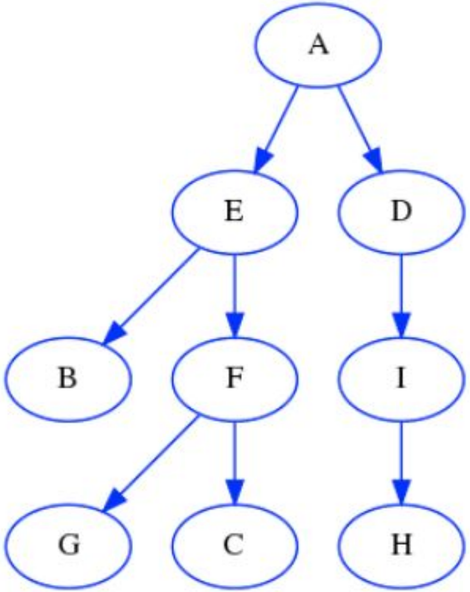
What is the corresponding binary tree? Explain.

Exercise

The visit order of a binary tree containing 9 nodes are the following:

- A, E, B, F, G, C, D, I, H (pre-order)
- B, G, C, F, E, H, I, D, A (post-order)
- B, E, G, F, C, A, D, H, I (in-order)

What is the corresponding binary tree? Explain.

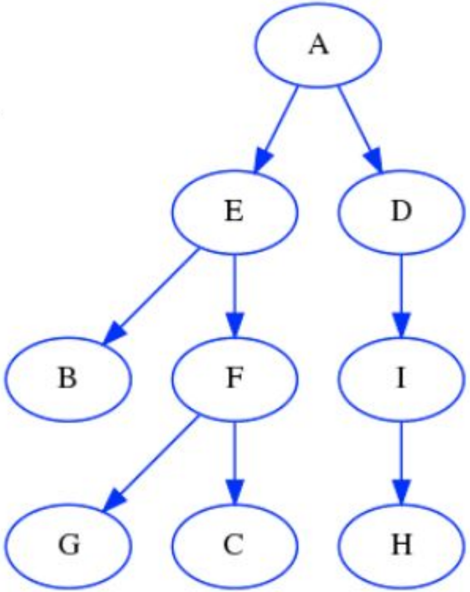


Preorder visit	Postorder visit	Inorder visit
A	B	B
E	G	E
B	C	G
F	F	F
G	E	C
C	H	A
D	I	D
I	D	H
H	A	I

where I is on the right of D and H is on the left of I

Exercises

- The width of a binary tree is the largest number of nodes that belong to the same level. Write a function that given a tree t, returns the width of t.
- The minimal height of a binary tree t is the minimal distance between node v and any of the leaf in its subtree. Write a function that given a tree t, returns the minimal height of t.
- Write a function that given a binary tree t and an integer k, returns the number of nodes at level k



Width: 3
Minimal height: 2
k = 2 → output: 3