Fiscal Compact and Debt Consolidation Dynamics

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Abstract

The paper analyses the macroeconomic effects of a debt consolidation policy in the European Union mimicking the Fiscal Compact rule. The rule requires the signatory states to target a debt-to-GDP ratio below 60%. Within the context of Dynamic Stochastic General Equilibrium models, we use a fully micro-founded New Keynesian model to understand the implications of including a debt consolidation policy. We show that in case of a negative shock hitting nominal variables (cost-push shock and monetary shock) the effects are negligible, while in case of a negative shock hitting real variables (productivity shock, public spending shock, taxation shock) the macroeconomic framework worsens as a function of the rigidity of the debt consolidation rule. As a limiting case, we show that the effects on output, employment and consumption are sizable.

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1 Introduction

Since March 2012 all, except two member states of the European Union (EU) have signed the *Treaty of Stability, Coordination and Governace* (TSCG).[1] The result has been a tightening in the provisions of the *Stability and Growth Pact* (SGP) which requires member states not to exceed a 3% threshold of the primary deficit-to-GDP ratio each year. The tightening is reflected in the adoption of the *Fiscal Compact Rule* (FCR) that requires the signatory states to follow a structural balanced budget rule and to reduce the debt-to-GDP ratio to 60% within twenty years.

Against this politico-economic background, we analyse the impact of a debt consolidation policy mimicking the EU Fiscal Compact rule within a structural New-Keynesian (NK) model. Our study is motivated by the large interest triggered in Europe by the sovereign debt crisis and by the intense debate around the effects of the debt consolidation policy. In particular, a crucial issue is whether implementing such a rule in a context of negative economic shocks is still desirable. In economic theory, the reasons behind the disagreement in the adoption of a debt consolidation policy can be related at least to two parallel research fields. The first is linked to the fiscal multiplier literature. The discussion goes back to the debate on the Keynesian and the non-Keynesian effects of public spending (Giavazzi and Pagano, 1990) and on the contrasting arguments about its effectiveness in presence of high debt level (Reinhart and Rogoff, 2010). The point in question is the sign and the magnitude of the fiscal multiplier and its relationship between the stock of the public debt. Since Keynes, the common wisdom is that the fiscal multiplier is greater than one. From this point of view, tightening public spending will depress production in the short-run and in the case of a negative shock even amplify its effects. While contrasting evidence provided by a report by the European Commission (2003) suggests about half of EU countries starting a fiscal consolidation route between 1970 and 2000 have been experiencing immediate output growth. Even if puzzling, this result is explained as a *non-Keynesian fiscal policy effect*. The idea is the following: if agents obey the *Ricardian Equivalence*, they anticipate that an actual debt reduction implies lower future tax rates. This is equivalent to a positive wealth shock and might cause a present increase in private spending.[2]

Instead, the second research field is linked to the optimal fiscal policy literature (Barro, 1979; Lucas and Stokey, 1983; Aiyagari *et al.*, 2002). This approach tends to predict a countercyclical debt policy; in case of a negative shock ("bad times") the government should run a deficit to finance the provision of public goods, while in "good times" it should accumulate surplus. The main idea is that deficit and surplus may serve as a buffer to avoid fluctuation in distortionary tax rates. However, Aiyagari *et al.* (2002) showed that when the government cannot fully anticipate their spending needs and when it has no limits to debt accumulation, the best policy is to accumulate assets so that taxation could be equal to zero forever. Although this provision is theoretically intriguing, it does not help to understand why most countries are in a perpetual state of debt. Also, imposing a rule on debt return in this setting would not produce any effect, because the optimal policy would simply be to accumulate assets, meaning running surplus in each period. Following this provision, any rules that investigates a debt consolidation policy is automatically full-filled and optimal. Nonetheless, looking at the recent sovereign debt

[1]The United Kingdom and The Czech Republic. Also Croatia joined the EU (July 2013) without signing the treaty.

[2]For a more extensive literature review on these topics refer to European Commission (2003).

crises and analysing countries that started consolidating, there are evidences that debt consolidation could be a painful process and must be carefully considered when implemented. All these gaps represent important challenges for the fiscal policy literature and our paper try to take a step in this direction.[3]

A branch of the fiscal policy literature attempting to solve this issue spawned by the 2008 article by Battaglini and Coate. The authors propose an intriguing argument to asses the welfare impact of consolidation policies. In the wake of Aiyagari *et al.* argument, Battaglini and Coate (2008) endogenise the lower-bound of government bonds holding mixing the optimal taxation approach with the political economy literature. In their environment a benevolent planner is substituted by a democratically elected legislator which set tax, public spending and debt. It can also redistribute resources as *"pork-barrel"* toward it's group (*"district"*). The introduction of this friction helps catching the idea of *"bias toward spending"* legislator. In this setting, debt could be used as in Aiyagari *et al.* to smooth taxation, but the government will face a perpetual debt situation. In fact, if resources were available as in a surplus situation, the coalition in charge would redistribute it back to their district. The idea behind this process is exactly that of setting an endogenous floor for government bonds holding. This model was implemented in further extensions by Azzimonti *et al.* (2016) to quantitatively analyse the effects on welfare by introducing a BBR, by Battaglini and Coate (2015) to analyse the interaction between fiscal policy and unemployement and by Barshegyan and Coate (2013) to analyse the effects of a productivity shock.

In this paper, differently to the outlined literature, we decide to follow a different route: we rely on the literature of New Keynesian model, in which the role of the planner is limited to decide the consumption of public good for each level of expenditure and abide by the government budget constraint. This is fulfilled using lump-sum taxes as form of financing. We do not include a maximising welfare social planner, shifting our focus from consolidation policy welfare effects to impulse response analysis. We are mainly interested in understanding the interaction between macroeconomic variables under: 1) a debt consolidation rule and 2) a one time negative shock hitting the economy. The novelty of the paper is twofold: first, we study the effect of a linear debt consolidation rule mimicking the actual Fiscal Compact Rule in the EU. Secondly, we include the rule into the model extending the standard NK model in a non-standard way. The reason is that in the NK literature, public spending is always treated as an exogenous random variable distributed as an auto-regressive process. We extend this rule by adding to this stochastic part a deterministic component. In this way, the public spending rule is made by two separate elements: the first is random and represents the standard government spending shock; the second is deterministic and embeds the consolidation policy rule. The meaning of the deterministic component is that it constrains the government to implement a debt consolidation policy, to rule-out deficit spending and to impose a debt cut whenever the government deviates from the debt target. We explain in more detail this point in the following section of the paper.

We show that in case of a negative shock hitting nominal variables (cost-push shock and monetary shock) the effects are negligible, while in case of a negative shock hitting real

[3] These and other difficulties in treating a debt consolidation argument in macroeconomic models are deeply considered in the article *The Research Agenda: Marco Bassetto on the Quantitative Evaluation of Fiscal Policy Rules* and we refer the reader to this for further discussions on issues relating to fiscal consolidation in macroeconomic models.

variables (productivity shock, public spending shock, taxation shock) the macroeconomic framework worsens as a function of the rigidity of the debt consolidation rule. As a limiting case, we show that the effects on output, employment and consumption are sizable.

The paper is structured as follows: section 2 illustrates the motivation behind this study and illustrates the type of consolidation rule we cover in the analysis. Section 3 develops the micro-founded model and section 4 reports the final equations used in the investigation. Section 5 describes the solution method. Section 6 comments on the impulse responses and policy analysis. Finally section 7 concludes.

2 Motivation

Figure 1 shows the distribution of the debt-to-GDP ratio for the 28 EU countries in 2012. The black horizontal line represents the maximum level of debt attainable after the ratification of the treaty and also the target debt that countries have to reach in 20 years. By looking at the chart it is easy to see that countries are highly heterogeneous with respect to their debt levels. Having a different stock of debt leads to different obligations in terms of public spending. Signatory countries that have a debt-to-GDP ratio above the horizontal line are required to follow a debt consolidation path according to the FCR, while countries below the horizontal line can keep their budget balanced (or in case running a minor deficit). For this reason the FCR has different implications for different countries. According to the rule *high debt countries*, i.e. those having a debt-to-GDP level above the horizontal line, have to reduce the amount of debt below the horizontal line by spreading it linearly in the following years.

To examine the implication of this particular consolidation path we modeled the FCR as a linear cut depending on a parameter $\alpha \in [0, 1]$, which reflects both the amount and the duration of the reduction.

$$\frac{b_{t+1}}{y_{t+1}} \leq \frac{b_t}{y_t} + \alpha \left(\frac{b^*}{y^*} - \frac{b_0}{y_0} \right) \tag{1}$$

Where b_t/y_t is debt-to-GDP, b^*/y^* is the target debt-to-GDP and b_0/y_0 is the initial stock of debt-to-GDP when the rule enters into force. According to this formulation α is the inverse of the number of years ($\delta > 0$) needed to implement the debt reduction to the target level. As $\delta \longrightarrow \infty$, $\alpha \longrightarrow 0$ and the rule implies a balanced budget rule (BBR). While, as $\delta \longrightarrow 1$, also $\alpha \longrightarrow 1$, meaning the target must be reached in one period. Also, the term into brackets implies that as the target is smaller than the initial level of debt $b^*/y^* < b_0/y_0$, the rule implies a debt reduction. While in the opposite situation $b^*/y^* > b_0/y_0$ allows for deficit spending.[4]

Using equation (1) we compute the expected consolidation path for the 25 EU signatory countries and we show in Table 1 their total debt reduction and the annual surplus required every year to remain on that path. Mimicking the FCR we set $b^*/y^* = 60\%$,

[4]In order to rule-out deficit spending a piece-wise linear rule would be needed:

$$\begin{cases} \frac{b_{t+1}}{y_{t+1}} \leq \frac{b_t}{y_t} + \alpha \left(\frac{b^*}{y^*} - \frac{b_0}{y_0} \right) & \text{if} \quad \frac{b^*}{y^*} < \frac{b_0}{y_0} \\ \\ \frac{b_{t+1}}{y_{t+1}} \leq \frac{b_t}{y_t} & \text{if} \quad \frac{b^*}{y^*} \geq \frac{b_0}{y_0} \end{cases} \tag{2}$$

but in the analysis we will stick to the simpler linear case.

$\alpha = 0.05$ and b_0/y_0 to the 2012 gross central government debt for each country.[5] The lines marked in red refer to countries that have to implement some form of debt consolidation. High debt countries such as Greece, Italy, Portugal and Ireland are required to run a considerable surplus every year in order to fulfill the consolidation rule, while low debt countries such as Finland, Denmark and Sweden can simply balance their budget.[6] Figure 2 shows the debt path for selected high and low debt EU countries, abstracting from GDP growth. Data untill 2012 is actual, while data after 2012 are forecasted assuming the dynamics in equation (1) and shown in the shadow area. In the chart we also show as a bold red line the median of the 28 EU countries and its median consolidation path. The difference between two consecutive periods in the shadow area represents the surplus that a country has to run in order to fulfill the rule. The chart shows that there is a marked difference between the two types of countries. In particular, the high debt countries show a really steep path; this implies that a really high surplus is demanded each year. For this reason we believe that the full implementation of such a rule could have some relevant implication on the macroeconomic outlook of such countries.

From the economic theory standpoint, our idea is that having some rigid constraint on the fiscal side may have non-negligible effects on the dynamics of other macroeconomic variables. On one side, restricting public spending may have negative effect on output, consumption and employment; on the other, it may limit the government ability to counteract exogenous shocks.

In order to highlight some shadows on this topic, in the next section we develop a general equilibrium model which includes a government sector. The government has to abide by a debt consolidation rule similar to the one shown in this section in an environment hit by negative shocks. Anticipating the results contained in the next sections, we confirm our idea that including a debt consolidation path could have non-negligible effects on the economy of these countries.

3 Model

In order to study whether a debt consolidation path, as the one implied by the FCR, may affect the main macroeconomic variables in the economy, we study the effects of including such provision in a fully microfounded NK model. Our interest is mainly to analyse the effect of the FCR on the economy outside the equilibrium, i.e. when the debt level has deviated from its target (which is also assumed to be the steady state level). We are also interested in the dynamics of the other macroeconomic variables in the model and how they respond to shocks. For this reason, after microfounding and solving the model, our analysis will be mainly focussing on the impulse response function of the system.

The model we use for the analysis is a New Keynesian model with infinitely life-time utility maximizer agents with CRRA utility function and monopolistically competitive firms producing differentiated goods using labor and technology. To keep the model as smooth as possible there is no capital. Agents each period choose between consumption and saving and the only asset in the model is a risk-free bond. As anticipated, the government sector has to abide to a linear debt consolidation policy rule.

[5] The data from Eurostat *http://ec.europa.eu/eurostat/data/database*

[6] The original FCR include the possibility to run a 1% deficit/GDP spending policy if the debt-to-GDP ratio is below 60%.

Household

There is a representative infinity-lived household maximising his expected life-time utility at period $t = 0$. We assume a utility function depending on consumption and leisure. Consumers have to minimise expenditure given the consumption level of composite good C_t. We assume that regularity conditions hold and that $\partial U/\partial C_t > 0$, $\partial U/\partial N_t < 0$, $\partial U/\partial C_t^2 < 0$ and $\partial U/\partial N_t^2 < 0$. Moreover, we assume a standard constant relative risk aversion (CRRA) functional form with separable consumption and leisure.

$$\max_{C,N} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right) \tag{3}$$

We also assume that there is a continuum (in the $[0,1]$ interval) of different goods produced under monopolistic competition in the goods market.

$$C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \tag{4}$$

Utility is maximised subject to the household's budget constraint and to a *No-Ponzi Game* condition in the government bonds market.

$$\int_0^1 P_t(i) C_t(i) di + B_t \leq (1 + R_t) B_{t-1} + W_t N_t - T_t \tag{5}$$

$$\lim_{T \to \infty} \mathbb{E}_t \{B_T\} \geq 0, \quad \forall t \tag{6}$$

The representative consumer allocates wealth between consumption and saving: $P_t(i)$ denote the prices of different goods i, $(1 + R_t)$ is the interest on bonds purchased in the previous period, W_t is wage and T_t is a lump-sum tax/transfer which may also captures the dividends coming from firms owned by households.

In order to derive the optimal allocation between goods, the representative agent maximises total consumption subject to any possible level of expenditure. The Lagrangian of the described maximisation problem is the following:

$$\min_{C_t(i)} \quad \mathcal{L}_1 \equiv \int_0^1 P_t(i) C_t(i) di - \psi_t \left(\left[\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} - C_t \right) \tag{7}$$

From the first order conditions (FOC) we can recover the *demand schedule* and the *aggregate price index*

$$\frac{\partial \mathcal{L}_1}{\partial C_t(i)} \equiv C_t(i) = C_t \left(\frac{P_t(i)}{\psi_t} \right)^{-\epsilon} \tag{8}$$

Where ψ_t is the Lagrangian multiplier and from the FOC it is possible to show that given $\psi_t > 0$ the constraint binds. Plugging into the definition of a composite good and solving for ψ_t we obtain the *aggregate price index* and the *demand for good* i.

$$\psi_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}} \equiv P_t \tag{9}$$

$$C_t(i) = C_t \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} \tag{10}$$

Finally, we get an aggregate formulation for a composite consumption good which can be plugged into the original budget constraint yielding Equation (12):

$$\int_0^1 P_t(i)C_t(i)di = P_tC_t \tag{11}$$

$$P_tC_t + B_t \leq (1+R_t)B_{t-1} + W_tN_t - T_t \tag{12}$$

Maximising the utility function w.r.t. (12) we can construct the current-value Lagrangian:

$$\max_{C_t,N_t} \quad \mathcal{L}_2 \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} - \Lambda_t \left(P_tC_t + B_t - (1+R_t)B_{t-1} - W_tN_t + T_t\right) \right) \tag{13}$$

By solving the system of first order conditions we can recover the *Labour Supply* (14) and the *Euler Equation* (15).[7]

$$\frac{W_t}{P_t} = N_t^{\phi}C_t^{\sigma} \tag{14}$$

$$\mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\Pi_{t+1}} \right] = \frac{1}{(1+R_t)} \tag{15}$$

Firms

We assume that firms operate under monopolistic competition and produce differentiated goods by using labour as their only input. Technology A_t is equal among firms and the production function takes the following form:

$$Y_t(i) = A_tN_t(i)^{1-\alpha} \tag{16}$$

Price levels adjust *à la Calvo* with a fraction $1-\theta$ of re-optimizing firms and a fraction θ of non re-optimizing firms with $\theta \in [0,1]$.

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon}di \right)^{\frac{1}{1-\epsilon}} \tag{17}$$

$$P_t = \left(\int_{S(t)} \theta P_{t-1}(i)^{1-\epsilon}di + (1-\theta)P_t^{*1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \tag{18}$$

$$P_t = \left(\theta P_{t-1}^{1-\epsilon} + (1-\theta)P_t^{*1-\epsilon} \right)^{\frac{1}{1-\epsilon}} \tag{19}$$

Where $S(t)$ is the set of non re-optimizing firms. By dividing both sides by P_{t-1} we can rewrite Equation (19) in terms of inflation

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon} \tag{20}$$

Re-optimizing firms solve the following profit maximisation subject to the *Demand Constraint*:

[7] Also in this case the Lagrangian multiplier is positive $\Lambda_t > 0$ and by the complementary slackness we can show that the constraint binds.

$$\max_{P_t^*} \quad \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left\{ Q_{t,t+k} \left(P_t^* Y_{t+k|t} - \Psi_{t+k} \left(Y_{t+k|t} \right) \right) \right\} \tag{21}$$

$$s.t. \quad Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} \tag{22}$$

Where $Q_{t,t+k}$ is the discount factor. Given we are operating in a general equilibrium setting, we assume that the households own the firms implying that the two have the same discount factor. Instead, Ψ_{t+k} is a cost function depending on the production level. For this function we assume that the regularity conditions hold. By directly plugging the *Demand Constraint* into the objective equation and maximising for P_t^* we can rewrite the Lagrangian as:

$$\max_{P_t^*} \quad \mathcal{L}_3 \equiv \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \left\{ Q_{t,t+k} \left(P_t^* \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} - \Psi_{t+k} \left(\left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} \right) \right) \right\} \tag{23}$$

and after some algebraic manipulations we retrieve an equation for P_t^*.[8]

$$P_t^* = \mathcal{M} \frac{\sum_{k=0}^{\infty} (\theta\beta)^k \mathbb{E}_t \left[C_{t+k}^{-\sigma} Y_{t+k} P_{t+k} MC_{t+k} \right]}{\sum_{k=0}^{\infty} (\theta\beta)^k \mathbb{E}_t \left[C_{t+k}^{-\sigma} Y_{t+k} \right]} \tag{24}$$

where $\mathcal{M} = \frac{\epsilon}{\epsilon - 1}$. Notice that as $\theta = 0$ we are in the case of flexible price, thus the optimal price setting is given by $P_t^* = \mathcal{M} \Psi'_{t|t}$. As consumers, also firms face a dualistic problem. They need to choose the optimal price in order to maximize profits and choose the optimal amount of labor to minimize the costs.

$$\min_{N_t(i)} \quad \frac{W_t}{P_t} N_t(i) \tag{25}$$

$$s.t. \quad Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{26}$$

Building-up the Lagrangian function we define the Lagrangian multiplier as the marginal cost of increasing the production by one unit (*shadow price*). Solving the FOC we get an equation for the marginal cost MC_t.

$$\min_{N_t(i)} \quad \mathcal{L}_4 \equiv \frac{W_t}{P_t} N_t(i) - MC_t (Y_t(i) - A_t N_t(i)^{1-\alpha}) \tag{27}$$

$$\frac{\partial \mathcal{L}_4}{\partial N_t(i)} \equiv MC_t = \frac{W_t}{P_t} \frac{1}{(1-\alpha) A_t N_t(i)^{-\alpha}} \tag{28}$$

Government

The government purchases a continuum of different public goods in the $[0, 1]$ range produced by firms under monopolistic competition. As agents, it maximises the consumption of public goods for any level of expenditure.[9]

$$\max_{G_t(i)} \quad G_t = \left(\int_0^1 G_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \tag{29}$$

[8] We divide by P_{t-1}, plug $Q_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}}$ and $MC_{t+k} = \frac{\Psi'_{t+k}}{P_{t+k}}$ we get Equation (24)

[9] We assume that public and private goods have the same elasticity.

$$s.t. \quad \int_0^1 P_t(i)G_t(i)di \tag{30}$$

The Lagrangian takes the form:

$$\max_{G_t(i)} \quad \mathcal{L}_5 \equiv \left(\int_0^1 G_t(i)^{1-\frac{1}{\epsilon}}di\right)^{\frac{\epsilon}{\epsilon-1}} - \nu_t \left(\int_0^1 P_t(i)G_t(i)di\right) \tag{31}$$

and solving the FOC with respect to $G_t(i)$ we can recover the *Demand Schedule for Public Goods*:

$$G_t(i) = G_t \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} \tag{32}$$

Government consumption is financed by *lump-sum taxes* and abide to the *Government Budget Constraint* (GBC).

$$B_t = (1 + R_t)B_{t-1} + G_t - T_t \tag{33}$$

Finally, in our model public spending is not set as a purely exogenous process but is composed by two different elements; the first component is deterministic and determines the debt consolidation process. As anticipated, our consolidation rule is a linear function of the deviation from the debt-to-GDP target. We model the deterministic part of the process according to this rule in order to mimic a simplified version of the EU Fiscal Compact. We call it *Fiscal Compact Rule* (FCR). The second component is a stochastic process that can be considered as an unexpected government spending shock Ω_t. As in the optimal fiscal policy literature, this can be thought as a war or a natural catastrophic event.

$$G_t = T_t + \alpha_g \left(\frac{B^*}{Y^*} - \frac{B_t}{Y_t^F}\right) + \Omega_t \tag{34}$$

Accordingly to the first part of our paper, equation (34) represents the debt consolidation rule that the government has to abide to. The main differences between equation (1) and equation (34) is that we substitute the difference between the debt-to-GDP in two consecutive period with primary deficit defined as $D_t \equiv G_t - T_t$. Secondly, we include in the rule a stochastic process to replicate the effect of an unexpected public spending shock. As in equation (1) $\frac{B^*}{Y^*}$ is the government debt-to-GDP target exogenously assigned by the fiscal compact rule. We assume that the target level is equal to the steady state level B/Y. This assumption will be useful in the log-linearized version of the model, where variables are described as deviation from their steady state level. Assuming that the target level equals the steady state level then the log-deviation from the steady state is also the deviation from the target. Differently to equation (1) we assume that the initial level of debt is computed with respect the natural level of output instead of computing it with respect to actual output. In the paper we compute the natural level of output as the output prevailing in flexible prices.[10] Including potential output into the debt consolidation process allows us to define the deficit as the *structural deficit* in the fiscal compact.

From a different perspective, including a debt consolidation policy in this form might be considered as imposing a constraint on the timing of the system shock absorption. A simple example may help to clarify; first, remember that in a general equilibrium setting,

[10]We follow Smets and Wouters (2007) on this point. We will devote a paragraph on that in the next sections.

in absence of shocks the variables are always at their steady state level. Suppose that the Central Bank decides to unexpectedly increase the interest rate (*monetary policy shock*). Starting from the policy rate, all the variables in the system move away from their steady state level. Now including into the system equation (34) imposes a constraint on the debt dynamics and on the return path to the steady state by determining the amount of public good that the government can purchase. This process depends on the parameter α_g which determines the velocity with which the debt has to come back to their steady state level and indeed the velocity with which the whole system has to come back to its steady state. By changing the size of α_g it is possible to change the velocity with which the shock is absorbed. It is useful to think α_g as the inverse of the number of years in which the debt reduction must be implemented. As $\alpha_g \longrightarrow 0$ the rule implies a balanced budget rule (BBR) $G_t = T_t$. While, as $\alpha_g \longrightarrow 1$, the target must be reached in one period by running a surplus equal to the amount of the debt in excess with respect to the target level $G_t - T_t = -ExcessDebt$. Modifying α_g it is possible to explore the entire spectrum of linear debt consolidation policy which takes this form.[11]

Market clearing and equilibrium

In the environment we set up, an equilibrium is a vector of prices and allocations such that:

- Households maximise utility subject to their budget constraint.
- Firms maximise profits subject to their production function.
- The government abide by the budget constraint and by the fiscal compact rule.
- All the markets clear.

Goods Market

The market clearing condition in the good market is

$$Y_t(i) = C_t(i) + G_t(i) \tag{35}$$

From which we get the *Aggregate Output Equation*

$$Y_t = C_t + G_t \tag{36}$$

Labour Market

The *Aggregate Labour* equation is

$$N_t = \int_0^1 N_t(i)di \tag{37}$$

Solving the *Production Function* (16) for $N_t(i)$, plugging the *Aggregate Labour* and using the definition of $Y_t(i)$, we get:

$$N_t = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di \tag{38}$$

[11]Even if in the particular application, this spectrum must be reduced to the parameter space that ensure the final system of expectation difference equations to converge (Blanchard and Khan, 1980). We are briefly introducing this problem in the next section.

Flexible Price Equilibrium

From the firm price optimization setting we get the flexible price mark-up $P^*_t = \mathcal{M}\Psi'_{t|t}$. Given that under flexible prices $P^*_t = P_t$ we have $MC_t = \frac{1}{\mathcal{M}}$. Now we would like to find an expression for output under flexible price, in order to build the *Dynamic IS* equation, the *New Keynesian Phillips Curve* and to substitute in the FCR.

By plugging the flexible price mark-up into the labor demand equation $MC_t = \frac{W_t}{P_t}\frac{1}{A_t(1-\alpha)N_t^{-\alpha}}$ and using the goods market clearing condition $C_t = Y_t + G_t$, the labor supply $\frac{W_t}{P_t} = C_t^\sigma N_t^\phi$ and the labor market clearing condition $N_t = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}}$ we get:

$$(Y_t - G_t)^\sigma Y_t^{\frac{\phi+\alpha}{1-\alpha}} = \mathcal{M}(1-\alpha)A_t^{\frac{1+\phi}{1-\alpha}} \tag{39}$$

In order to disentangle Y_t we log-linearise the equation around the steady state and we get an equation for the flexible-price output as a function of exogenous and pre-determined variables.[12]

$$\hat{y}^F_t = \Xi\left(\left(\frac{\phi+1}{1-\alpha}\right)\hat{a}_t + \frac{\sigma G}{C}\hat{\tau}_t + \frac{\sigma\alpha_g BR}{C(Y+\alpha_g)}\theta_{i,t} - \frac{\sigma\alpha_g B(1+R)}{C(Y+\alpha_g)}\hat{b}_{t-1} + \frac{\sigma}{C}\omega_t\right) \tag{40}$$

where $\Xi = \left(\frac{(Y+\alpha_g)C(1-\alpha)}{\sigma Y(1-\alpha)+C(\phi+\alpha)(Y+\alpha_g)-(1-\alpha)\sigma\alpha_g B}\right)$, $\hat{a}_t$, $\hat{\tau}_t$, $\hat{b}_{t-1}$ and ω_t are the log-linear-deviation version of A_t, T_t, B_{t-1} and Ω_t. While $\theta_{i,t}$ is a monetary policy shock embedded in the feedback rule we included in the log-linear version of the model. This is described in the next section.

4 The log-linearized model

The log-linearized version of the model is made by the following equations:

$$\tilde{y}_t = \mathbb{E}_t\tilde{y}_{t+1} - \alpha_y(\hat{i}_t - \mathbb{E}_t\pi_{t+1}) - b_y(\mathbb{E}_t\hat{g}_{t+1} - \hat{g}_t) + \epsilon_{y,t} \tag{41}$$

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa\tilde{y}_t + \zeta_{\pi,t} \tag{42}$$

$$\hat{i}_t = \phi_\pi\pi_t + \phi_y\tilde{y}_t + \theta_{i,t} \tag{43}$$

$$\hat{b}_t = (1+a_{b1})\hat{b}_{t-1} + a_{b1}\hat{i}_t + a_{b2}(\hat{g}_t - \hat{\tau}_t) \tag{44}$$

$$\hat{g}_t = \hat{\tau}_t - a_{d1}(\hat{b}_t - \hat{y}^F_t) + a_{d2}\omega_t \tag{45}$$

$$\hat{y}^F_t = \Xi\left(a_{e1}\hat{a}_t + a_{e2}\hat{\tau}_t + a_{e3}\theta_{i,t} - a_{e4}\hat{b}_{t-1} + a_{e5}\omega_t\right) \tag{46}$$

$$\epsilon_{y,t} = \mathbb{E}_t\hat{y}^F_{t+1} - \hat{y}^F_t \tag{47}$$

[12] The computation is slightly cumbersome and we include all the details to get Equation (40) into the technical appendix.

Where $a_y = C/Y\sigma$; $b_y = G/Y$; $\kappa = \lambda\frac{\sigma Y(1-\alpha)+C(\phi+\alpha)}{C(1-\alpha)}$; $\lambda = \frac{(1-\theta)(1-\beta\theta)\Theta}{\theta}$; $\Theta = \frac{1-\alpha}{1-\alpha-\alpha\epsilon}$; $a_{b1} = R$; $a_{b2} = G/B$; $a_{d1} = \alpha_g B/GY$; $a_{d2} = 1/G$; $\Xi = \left(\frac{(Y+\alpha_g)C(1-\alpha)}{\sigma Y(1-\alpha)+C(\phi+\alpha)(Y+\alpha_g)-(1-\alpha)\sigma\alpha_g B}\right)$; $a_{e1} = \frac{\phi+1}{1-\alpha}$; $a_{e2} = \frac{\sigma G}{C}$; $a_{e3} = \frac{\sigma\alpha_g BR}{C(Y+\alpha_g)}$; $a_{e4} = \frac{\sigma\alpha_g B(1+R)}{C(Y+\alpha_g)}$; $a_{e5} = \frac{\sigma}{C}$.

All the algebraic details to rewrite the final system of equation in this form are described into the technical appendix. Lower-case variables with "hat" represent the corresponding upper-case variables in log-deviation from the steady state (except for the nominal interest rate R_t which is denoted as $\hat{i}_t$ to distinguish it from the real interest rate $\hat{r}_t = \hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1}$ and $\tilde{y}_t = \hat{y}_t - \hat{y}_t^F$ that is the *output gap*). Also, $\hat{a}_t$, $\hat{\zeta}_{\pi,t}$, $\hat{\theta}_{i,t}$, $\hat{\tau}_t$ and ω_t are forcing variables distributed as an *autoregressive process* of order one taking the form $x_t = \phi_x x_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim \mathbb{WN}(0, \sigma_\varepsilon^2)$. Finally, we include a standard feedback rule equation for the central bank.

5 Solution Method

In this section we briefly describe the method used to solve the model. The system of linear stochastic difference equations under rational expectations can always be written in companion form as:

$$\mathbf{A}\mathbb{E}_t\mathbf{Y}_{t+1} = \mathbf{B}\mathbf{Y}_t + \mathbf{C}\mathbf{X}_t \tag{48}$$

where $\mathbf{A}$ is the matrix of coefficients related to the forward looking endogenous variables, $\mathbf{B}$ is the matrix of coefficients related to the predetermined variables and backward looking variables; finally, $\mathbf{C}$ is the matrix of coefficients of the exogenous variables. $\mathbf{Y}_{t+1}$ is a vector of forward looking endogenous variables, while $\mathbf{Y}_t$ is a vector of backward looking and predetermined variables. $\mathbf{X}_t$ is a vector of exogenous variables distributed as an AR(1) and dependent on a set of i.i.d. exogenous shock.

In our case, the main problem come from the fact that the Blanchard and Khan (BK) conditions requires that in order to solve the model, the leading matrix $\mathbf{A}$ must be invertible. If this is not the case, the system does not admit a stable solution and we can not solve for $\mathbb{E}_t\mathbf{Y}_{t+1}$. For this reason, in order to deal with this problem we apply a generalized version of BK due to King and Watson (1998).

For example, in the baseline case we have six endogenous variables, two predetermined variables and five shocks. They are then collected in the following vectors:

- $\mathbb{E}_t Y_{t+1} \equiv \mathbb{E}_t\left[y_{t+1}, \pi_{t+1}, i_{t+1}, b_{t+1}, g_{t+1}, y_{t+1}^F, i_t, b_t\right]$
- $Y_t \equiv [y_t, \pi_t, i_t, b_t, g_t, tax_t, i_{t-1}, b_{t-1}]$
- $X_t \equiv [\epsilon_{y,t}, \zeta_{\pi,t}, \theta_{i,t}, \tau_t, \omega_t]$

The method proposed by King and Watson (KW) is a generalization of the method of Blanchard and Khan (1980). Following KW the leading matrix $\mathbf{A}$ is decomposed (in case of singularity) and inverted. $\mathbf{A}$ and $\mathbf{B}$ are rewritten in the form:

$$\mathbf{A} = \mathbf{Q}'\mathbf{\Omega_e}\mathbf{Z}', \quad \mathbf{B} = \mathbf{Q}'\mathbf{\Lambda_e}\mathbf{Z}' \tag{49}$$

where $\mathbf{Q}$ and $\mathbf{Z}$ are "unitary" matrices ($\mathbf{QQ}' = \mathbf{Q}'\mathbf{Q} = \mathbb{I}$) and $\mathbf{\Omega_e}$ and $\mathbf{\Lambda_e}$ are upper-triangular matrices containing the generalized eigenvalues. Than the system can be "de-coupled" into a stable and an unstable part in a way that it resembles the BK method.

$$\mathbb{E}_t \mathbf{Z}_{t+1} = \mathbf{\Lambda_e}^{-1} \mathbf{\Omega_e} \mathbf{Z}_t + \mathbf{R}\mathbf{X}_t \tag{50}$$

where $\mathbf{Z}_t = \mathbf{Z}'\mathbf{Y}_t$, $\mathbf{R} = \mathbf{\Lambda_e^{-1}} \mathbf{QC}$. Following BK, a solution exists if the number of backward looking variables is equal to the number of stable roots, while the number of forward looking variables must be equal to the number of unstable roots.[13] If this condition is met, the variables will return to their long-run equilibrium path after the model has been shocked.[14]

6 Results

In this section we present the impulse responses of the endogenous variables to the following shocks: technology, mark-up, policy rate, public spending and taxation. Variables are hit one at a time by a 1% shock. We report the logarithmic deviation from the steady state of the following variables: output, potential output, output gap, consumption, public debt, public spending, inflation, nominal interest rate, real interest rate, employment (hours worked), real wage and mark-up. For the calibrated values of the parameters the stability conditions of the structural model are satisfied, thus after being shocked, the system will return to its steady state level. As outlined in the above, including a debt consolidation policy like the FCR in equation (34) imposes a constraint on the dynamics of the debt return to the steady state by determining the amount of public goods that the government can purchase. This dynamics depends on the parameter α_g which determines the velocity with which the debt returns to its steady state level and indeed the velocity with which the whole system returns to its steady state.

In the following charts we show the *impulse response functions* (IRFs) for three levels of α_g (0.01, 0.05, 0.2). The size of α_g affects the velocity of absorbtion of the shock by the system. In particular, by observing the debt dynamics, we notice that varying the level of α_g affects the curvature of the public debt and public spending impulse response functions. It is still useful to think of α_g as the inverse of the number of years in which the debt reduction must be implemented and to consider the curvature of the debt impulse response function as an inverse function of this.

Figure 3 describes the effects of a 1% negative technology shock. We report the results for three levels of α_g (0.01, 0.05, 0.2). In particular, the black solid lines show the baseline rule $\alpha_g = 0.05$, the blue dashed lines represent the strict rule $\alpha_g = 0.2$ and red dash-dotted lines represent the loose rule ($\alpha_g = 0.01$). First, notice that the model is robust to different calibration of α_g. The IRFs reflect always a similar behaviour. The main differences are in the magnitude and the persistence of the responses of the endogenous variables, but the sign and the path are unchanged across calibrations. Consistent with

[13]On the contrary, when the number of stable roots is greater than the number of predetermined variables there will be multiple solutions. While, when the number of unstable roots is greater than the number of forward looking variables, we have no solutions.

[14]The method of King and Watson (1998) is implemented in MATLAB: we adapt three different M-files; the first, called file *system*, contains the log-linearised model and the parameters, as calibrated in Tables 2 and 3 at the end of the paper. The second, called *driver*, contains the calibration of the shocks as set in Table 4. Finally, the third file calls the *system* and *driver* files and provides a routine to generate the impulse responses of the endogenous variables to the shocks. All the codes are available upon request.

the theory, a negative technology shock depresses output and potential output. Given that potential output is more affected by the shock, the output gap is positive and inflation raises. Employment increases to maintain a constant level of production, but at the same time real wages and consumption decrease. The central bank raises the policy rate in order to stabilize the inflation and the output gap. Given the worsening in potential output, the debt-to-GDP ratio is now higher than its target level; thus, the debt must be reduced following the FCR and by running a surplus. Analyzing the path for the three rules, we notice that as α_g increases, also the curvature of the IRFs increase. This is evident by looking at public debt and spending. When $\alpha_g = 0.2$ the hump of the public debt IRF is much more pronounced, while as $\alpha_g = (0.05, 0.01)$ the shape is smoother. A similar reasoning holds for the monotonically increasing public spending IRF. For this variable an increase in α_g increases the steepness of the function; this, in turn, implies that also the velocity with which the variable returns to its steady state level increases.

By looking at the three impulse response functions it is also worth noticing that as the rule is stricter, we observe a worsening in the macroeconomic framework. When α_g increases, output, consumption, employment and real wages are more affected by the shock. The reason is still linked to the role of the FCR. In fact, when a negative productivity shock hits the system the debt-to-GDP ratio worsens and deviates from its target level. Then, depending on the debt consolidation rule, the government will need to run a larger/smaller surplus. As the rule is stricter the surplus must be larger in order to reach the target. Running a surplus of different sizes creates a gap between the impulse response functions with different levels of α_g. When the surplus is larger the other variables are negatively affected and the shock is boosted out, while when the surplus can be spread in many years this channel can be neglected. As a limiting case consider $\alpha_g \to \infty$, meaning that the target must be reached immediately. Figure 4 shows that consumption, output, employment and real wage decrease significantly confirming that when the rule is stricter the macroeconomic framework keeps worsening. It also shows that the difference between this framework and one with a very loose rule is sizable (-0.9% consumption, -1.5% employment, -2.4% output and -2.5% real wage).

Figure 5 and 6 show the IRFs for a negative government spending and taxation shock. The IRFs present dynamics similar to the technology shock. Also in this charts, we notice that when the rule is getting stricter, output, consumption, employment and real wage are responding in a stronger way. This is particularly evident in the government spending shock, but can be also noticed in the real wage for the taxation shock.

Finally, Figure 7 and 8 show the IRFs for a negative cost-push and monetary policy shock. As anticipated, the same dynamics does not hold for shocks hitting nominal variables. The reason is linked to the way in which we define our debt consolidation rule using the definition of structural deficit. In fact, we computed the relevant debt-to-GDP ratio considering potential output instead of current output. Potential output is a measure not affected by changes in nominal variables. This choice is consistent with the EU fiscal compact provision, which use the same definition.

Summarizing our findings, we show that including a debt consolidation rule in a New Keynesian framework can be considered as imposing a constraint on the velocity of the system to return to its steady state. Given that the number of periods to return to the steady state are strictly linked to the parameter α_g, we computed a sensitivity analysis varying this parameter. The IRFs show that when the shock hits nominal variables, changing α_g has a completely negligible effect on the dynamics of the system. Instead, when the shock hits real variables, varying α_g has considerable effects. In particular, as the parameter increases, the rule becomes stricter and the shock is amplified through the surplus imposed by the rule. On the other side, as the parameter decreases the rule becomes looser and the amplification effect is negligible as negligible is the surplus that the government has to run. Finally, we consider a limiting case where the $\alpha_g \longrightarrow \infty$ and we show that the amplification effect is not negligible.

7 Conclusions

We have analysed the macroeconomic effects of a debt consolidation policy in the European Union mimicking the Fiscal Compact rule. The rule requires the signatory states to target a debt-to-GDP ratio below 60%. Within the context of Dynamic Stochastic General Equilibrium models we use a fully micro-founded New Keynesian model to understand the implications of including a debt consolidation policy. We show that in case of a negative shock hitting nominal variables (cost-push shock and monetary shock) the effects are negligible, while in case of a negative shock hitting real variables (productivity shock, public spending shock, taxation shock) the macroeconomic framework worsens as a function of the rigidity of the debt consolidation rule. As a limiting case, we show that the effects on output, employment and consumption are sizable.

As a final point. We thought that understanding the implication of a debt consolidation rule in case of a negative shock was the most interesting case from an economic policy point of view. However, we suspect that positive and negative shocks under a debt consolidation policy rule may not have a symmetric effect. The reason is simply because a positive shock can help dealing with a consolidation path creating more resources, while a negative shock absorbs additional resources and may further constraint the economy response in response to shocks. We think that understanding this point would be really interesting and will provide a deeper understanding of the mechanisms relating debt constraints with exogenous shocks. Of course, in order to examine this point a non-linear model is needed and could be interesting to consider it for further extensions.

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Figure

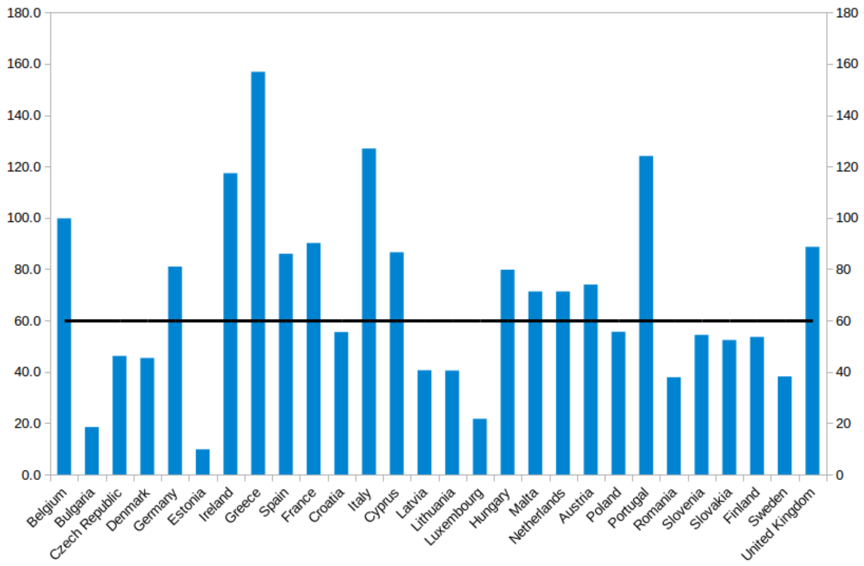

Figure 1: Debt-to-GDP distribution for 28 EU countries in 2012. The black solid line shows both the maximum level of debt attainable and the debt-to-GDP target level following the FCR.

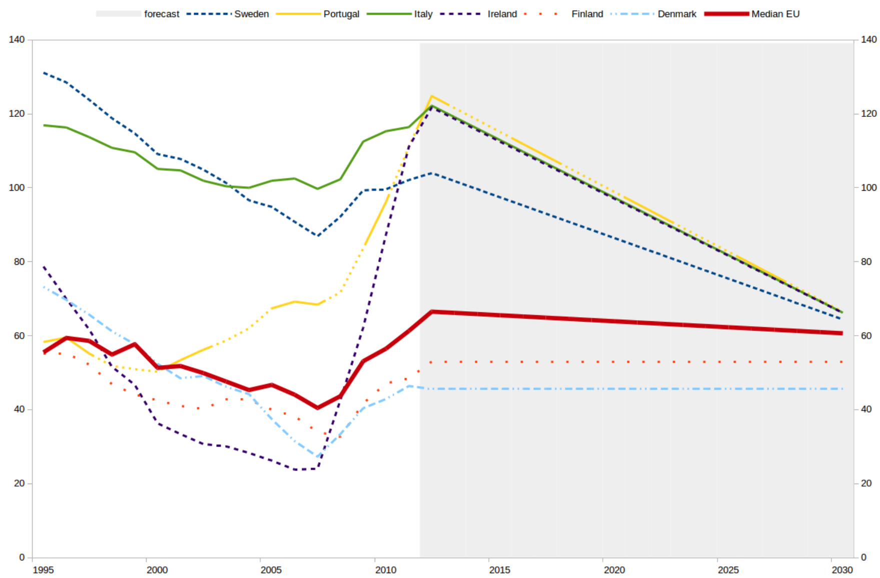

Figure 2: Debt-to-GDP ratio for selected EU countries. Data before 2012 are realized, while after 2012 are computed using Equation (1) and showed in the shadow area. We set $\alpha = 0.05$ and $b^*/y^* = 60\%$.

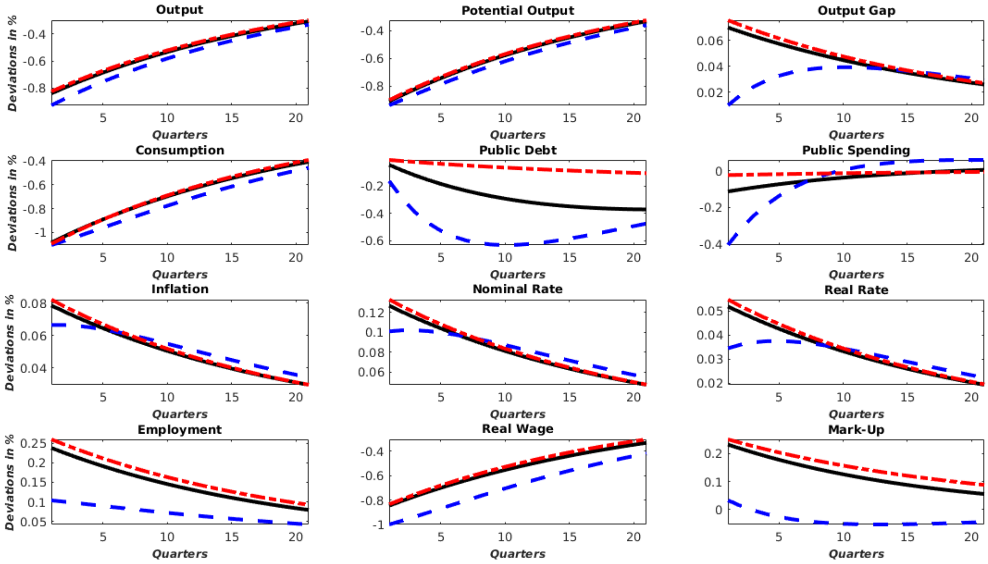

Figure 3: Impulse response function to a negative 1% technology shock. Black solid lines show the baseline rule $\alpha_g = 0.05$. Blue dashed lines represent the strict rule $\alpha_g = 0.2$ and red dash-dotted lines represent the loose rule ($\alpha_g = 0.01$).

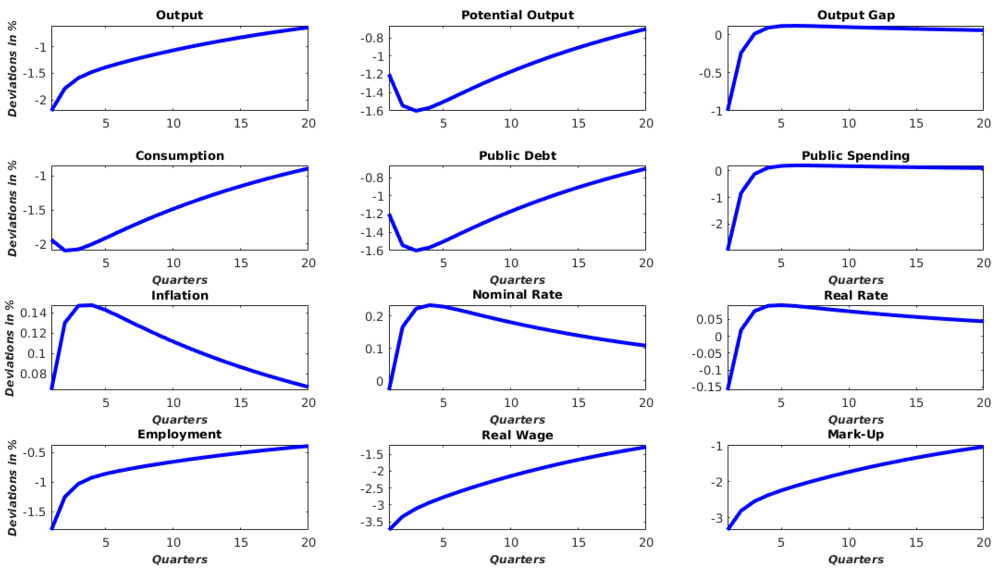

Figure 4: Impulse response function to a negative 1% technology shock (limiting case, $\alpha_g = 10000$).

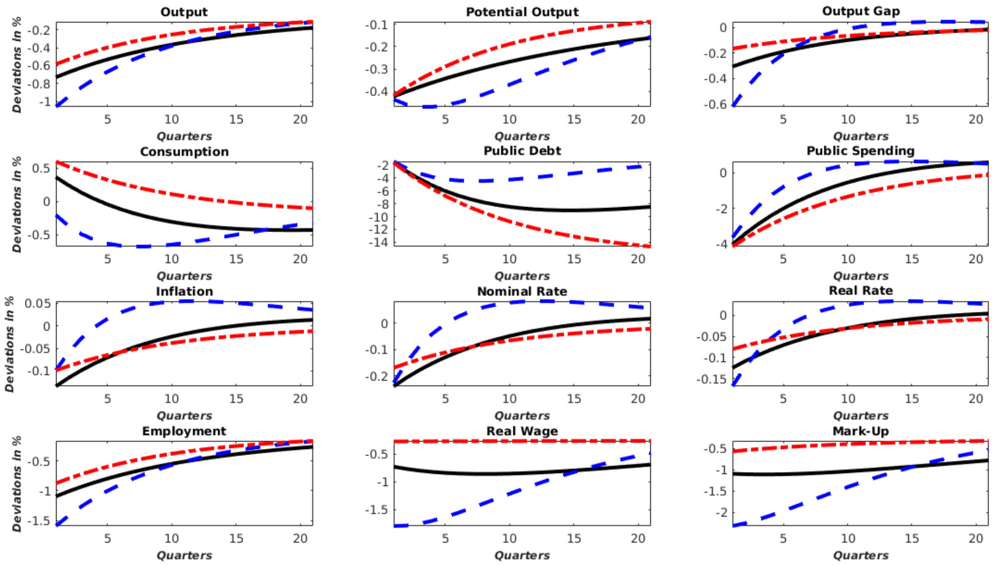

Figure 5: Impulse response function to a negative 1% public spending shock. Black solid lines show the baseline rule $\alpha_g = 0.05$. Blue dashed lines represent the strict rule $\alpha_g = 0.2$ and red dash-dotted lines represent the loose rule ($\alpha_g = 0.01$).

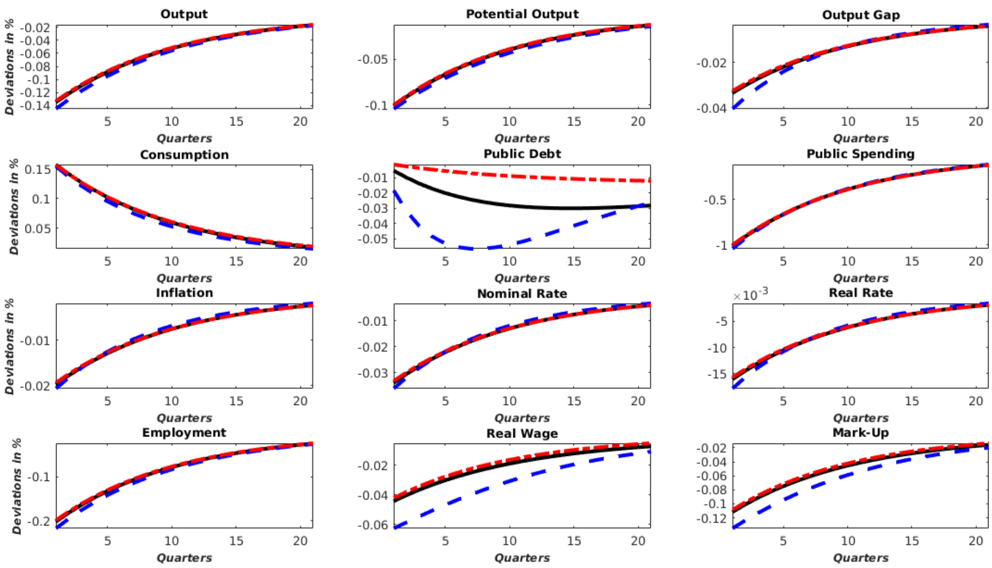

Figure 6: Impulse response function to a negative 1% taxation shock. Black solid lines show the baseline rule $\alpha_g = 0.05$. Blue dashed lines represent the strict rule $\alpha_g = 0.2$ and red dash-dotted lines represent the loose rule ($\alpha_g = 0.01$).

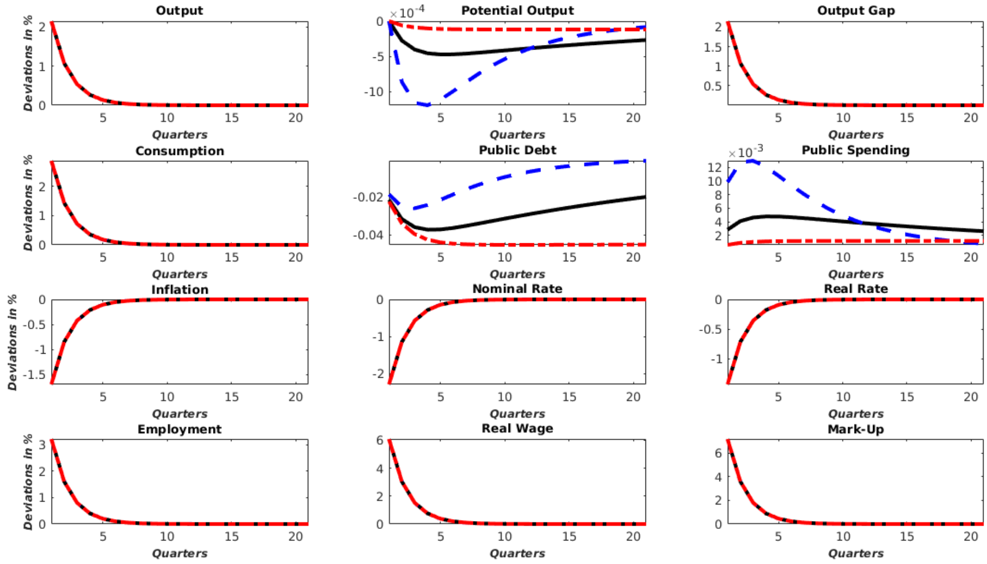

Figure 7: Impulse response function to a negative 1% cost-push shock. Black solid lines show the baseline rule $\alpha_g = 0.05$. Blue dashed lines represent the strict rule $\alpha_g = 0.2$ and red dash-dotted lines represent the loose rule ($\alpha_g = 0.01$).

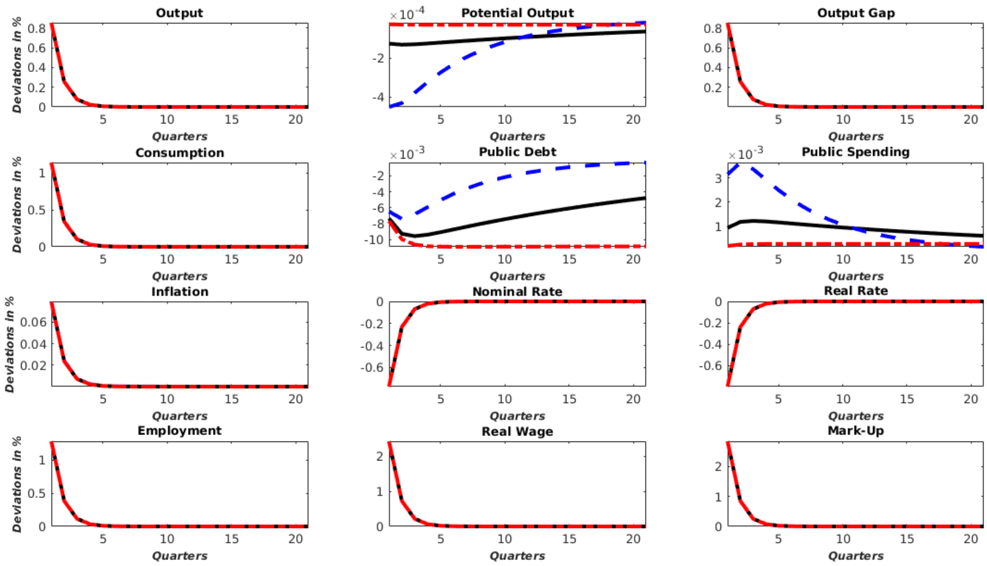

Figure 8: Impulse response function to a negative 1% monetary policy shock. Black solid lines show the baseline rule $\alpha_g = 0.05$. Blue dashed lines represent the strict rule $\alpha_g = 0.2$ and red dash-dotted lines represent the loose rule ($\alpha_g = 0.01$).

Table

Table 1: FCR Debt Consolidation Path.

Country	Debt/GDP	Tot. Reduction	Annual Reduction
Belgium	104.0	44.0	2.2
Bulgaria	18.0	0.0	0.0
Denmark	45.6	0.0	0.0
Germany	79.0	19.0	1.0
Estonia	9.7	0.0	0.0
Ireland	121.7	61.7	3.1
Greece	156.9	96.9	4.8
Spain	84.4	24.4	1.2
France	89.2	29.2	1.5
Italy	122.2	62.2	3.1
Cyprus	79.5	19.5	1.0
Latvia	40.9	0.0	0.0
Lithuania	39.9	0.0	0.0
Luxembourg	21.4	0.0	0.0
Hungary	78.5	18.5	0.9
Malta	67.9	7.9	0.4
Netherlands	66.5	6.5	0.3
Austria	81.7	21.7	1.1
Poland	54.4	0.0	0.0
Portugal	124.8	64.8	3.2
Romania	37.3	0.0	0.0
Slovenia	53.4	0.0	0.0
Slovakia	52.1	0.0	0.0
Finland	53.0	0.0	0.0
Sweden	36.4	0.0	0.0

Table 2: Structural parameters

Parameter	Description	Value
σ	Coefficient of relative risk aversion	1
β	Discount factor	0.99
ϕ	Inverse of Frish elasticity	1
θ	Non-reoptimizing firms	2/3
ϵ	Demand elasticity	6
α	Output elasticity w.r.t. labour	1/3
ϕ_π	Reaction coefficient on inflation	1.50
ϕ_y	Reaction coefficient on output	0.5/4
α_g	Fiscal Compact debt-return rate	0.05

Table 3: Steady state.

Parameter	Description	Value
C	Consumption	0.72
G	Government spending	0.24
Y	Output	0.96
B	Public debt	0.6
I	Interest rate	0.01

Table 4: Shock persistence and volatility.

Persistence	Description	Value
ρ_ϵ	Productivity	0.95
ρ_ζ	Cost-push	0.5
ρ_θ	Policy rate	0.3
ρ_ω	Public spending	0.9
ρ_τ	Tax	0.9
Volatility	Description	Value
σ_ϵ	Productivity	0.0072
σ_ζ	Cost-push	0.01
σ_θ	Policy rate	0.0082
σ_ω	Public spending	0.01
σ_τ	Tax	0.01