Quantum Computing Notes
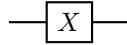
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1 Fundamentals

2 Gates

2.1 NOT gate

2.2 Definizione

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{1}$$

$$
\begin{aligned}
X\,|0\rangle &= |1\rangle \\
X\,|1\rangle &= |0\rangle \\
X(\alpha\,|0\rangle + \beta\,|1\rangle) &= \alpha\,|1\rangle + \beta\,|0\rangle
\end{aligned}
\tag{2}
$$

2.3 Identity gate

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{3}$$

2.4 Hadamard gate
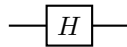
2.4.1 Definizione

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{4}$$

2.4.2 Azione

$$|0\rangle \mapsto \frac{|0\rangle + |1\rangle}{\sqrt{2}}, |1\rangle \mapsto \frac{|0\rangle - |1\rangle}{\sqrt{2}} \tag{5}$$

Si dimostra utilizzando la base $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$:

$$H\left|0\right\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1 \\ 1 & -1\end{bmatrix}\begin{bmatrix}1 \\ 0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 \\ 1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 \\ 0\end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix}0 \\ 1\end{bmatrix} = \frac{\left|0\right\rangle + \left|1\right\rangle}{\sqrt{2}}$$
$$H\left|1\right\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1 \\ 1 & -1\end{bmatrix}\begin{bmatrix}0 \\ 1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 \\ -1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 \\ 0\end{bmatrix} - \frac{1}{\sqrt{2}}\begin{bmatrix}0 \\ 1\end{bmatrix} = \frac{\left|0\right\rangle - \left|1\right\rangle}{\sqrt{2}} \tag{6}$$