Appunti di quantum computing

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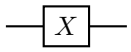
1 Fondamenti

2 Gates

2.1 NOT gate



2.1.1 Definizione matriciale

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{1}$$

2.1.2 Funzionamento

$$\begin{aligned} &X\left|0\right\rangle = \left|1\right\rangle \\ &X\left|1\right\rangle = \left|0\right\rangle \\ &X(\alpha\left|0\right\rangle + \beta\left|1\right\rangle) = \alpha\left|1\right\rangle + \beta\left|0\right\rangle \end{aligned} \tag{2}$$

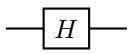
Dimostriamo l'ultima relazione utilizzando la base $\left|0\right\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\left|1\right\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$:

$$\begin{aligned} X(\alpha\left|0\right\rangle + \beta\left|1\right\rangle) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \left(\alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \\ \begin{bmatrix} \beta \\ \alpha \end{bmatrix} = \alpha \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \alpha\left|1\right\rangle + \beta\left|0\right\rangle \end{aligned} \tag{3}$$

2.2 Identity gate

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{4}$$

2.3 Hadamard gate



2.3.1 Definizione matriciale

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{5}$$

2.3.2 Funzionamento

$$|0\rangle \mapsto \frac{|0\rangle + |1\rangle}{\sqrt{2}}, |1\rangle \mapsto \frac{|0\rangle - |1\rangle}{\sqrt{2}} \tag{6}$$

Si dimostra utilizzando la base $|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}$, $|1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$:

$$\begin{aligned} H|0\rangle &= \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\end{bmatrix} + \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\end{bmatrix} = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \\ H|1\rangle &= \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\end{bmatrix} - \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\end{bmatrix} = \frac{|0\rangle - |1\rangle}{\sqrt{2}} \end{aligned} \tag{7}$$