Optimization HW 5

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Question 1:
No Capacity Random

- Price: 500
- Pricing below $500 gives too small of a unit revenue, and pricing above $500 costs us too many customers. Optimal solution is to price at the mean of the price range.
- Mathematically, this can be proven as follows. Assuming we set the price at $x. Then , the expected $Revenue = \frac{(1000-x)}{1000} x \times 100$
- To obtain the maximum expected revenue we differentiate the equation and equate it to 0 to get the optimal value of x.

$$\frac{\partial Revenue}{\partial x} = \frac{\partial}{\partial x} \frac{1}{10} \times (1000x - x^2) = 0$$
$$x = 500$$

- On average, we expect revenues to be distributed around the value of $25,000; we ran the simulation multiple times and, because of the randomness of the simulation, we observed values below $21k and above $30k.

Question 2:
With Capacity Random

- Price: 850
- In this case, we are supposed to catch at most 15 customers out of the 100 total potential customers.
- Since their willingness to pay is distributed uniformly between 0 and 1000, the optimal price is generated by computing 1000*(1-15%) = 850

Question 3:
No Capacity Time Dependent

- Low Price: 250
- High Price: 500
- Assuming the two groups of customers are independent of each other, the two optimal prices are generated by maximizing the expected value of the revenue.
- For tier 1, if the cutoff point is x, $Revenue = \frac{(500-x)}{500} x \times 50$

$$\frac{\partial Revenue}{\partial x} = \frac{\partial}{\partial x} \frac{1}{10} \times (500x - x^2) = 0$$
$$x = 250$$

The expected revenue from tier 1 is $6,250.

- For tier 2, if the cutoff point is x, $Revenue = \frac{(1000-x)}{500} x \times 50$

$$\frac{\partial Revenue}{\partial x} = \frac{\partial}{\partial x} \frac{1}{10} \times (1000x - x^2) = 0$$

$$x = 500$$

  The expected revenue from tier 2 is $25,000.
- Therefore, the total expected revenue is $31,250.

Question 4:
With Capacity Time Dependent

- Low Price: 501
- High Price: 850
- We have more customers with a high willingness to pay than capacity; hence, it is not profit maximizing to sell to the customers with a low willingness to pay. We set 501 as the low price to make sure that nobody from the first group of customers will purchase the product.
- Since the willingness to pay is uniformly distributed, the high price is generated by 1000 - 500 * (15/50) = 850

Question 5:
Q4 with cap of 40

- Low Price: 501
- High Price: 600
- As in the previous question, we have more customers with a high willingness to pay than capacity; hence, it is not profit maximizing to sell to the customers with a low willingness to pay.
- Since the price is uniformly distributed, the high price is generated by 1000 - 500 * (40/50) = 600

Introduction to the Theory and Practice of Revenue Management: Problem 2

A. Protection Level is 46 and booking limit is 184
   F(Q*) >= (440-218)/440 = 0.5045; Q* = 46
   230 - 46 = 184

B. The protection level will be higher. Now that unsold seats can be sold at the last minute, the overage penalty C is smaller, i.e. overage is less costly. Hence, it is now optimal to set a higher protection level

Introduction to the Theory and Practice of Revenue Management: Problem 4

- A.
    - Critical Ratio = (\$10,000 - \$4,000)/\$10,000 = 0.60
    - Based on the table, Q* should be 13, thus the protection level is 13 and booking limit is 25 - 13 = 12.
    - They should sell 12 in advance at discount price.
    - Alternatively, we could have solved this problem in terms of underage and overage cost; C = 4000, B = 6000; critical ratio = B / (B+C) = 6000/10000 = 0.6

- B.
    - (1-F(Q)) * \$10,000 + F(Q) * \$2,500 <= \$4,000
    - F(Q) >= 0.8
    - Based on the table, Q* should be 15, thus the protection level is 15 and the booking limit is 25 - 15 = 10.
    - They should only sell 10 in advance at discount price.
    - Alternatively, we could have solved this problem in terms of underage and overage cost; C = 1500, B = 6000; critical ratio = B / (B+C) = 6000/7500 = 0.8