Optimization HW3

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1. *Problem #2 from Chapter 1 of SCND*
   A producer of dog food is trying to decide whether they should change the number and locations of their warehouses to better meet projected demand over the next three years. They do a study and determine that their transportation and warehousing costs will be $51 million if they stick with their current structure. They have determined that if they close two warehouses and open two new warehouses, their costs will drop to $50.5 million. Assume that all other costs stay the same. Should they make the change?

   It is impossible to give a definite answer, as we do not have enough data. However, we believe that the information provided points toward not implementing the change being the best approach.
   First and foremost, we should consider whether this difference in cost is statistically different from zero. The change in cost is less than 1% and it is therefore unlikely to be significant. If the difference was not to be significant, we would not be able to exclude -with an acceptable statistical confidence- that there is any difference at all. In this case, we would definitely suggest to not implement the change.
   Even in the presence of a statistically significant difference in cost, it would not necessarily be optimal to change the structure of our supply chain. We are assuming that all costs will remain the same. Have the fixed costs associated to the restructuring been factored in? If not, they might more than outweigh the $500k savings. Moreover, we should also factor in the effect of this change on revenues. If revenues take a hit because of relocation, then this change should be a given a second thought.
   There are also a few other issues and risks that should be carefully considered before implementing the change in the supply chain structure. In particular:
   - What would be the effect on the service level?
   - Would the company be able to find people with the right skills in the new locations?
   - What are the organizational challenges? Would the company be culturally and logistically ready for this change?
   - Are there risks associated to moving warehouses in terms of public relations? Is the change going to be bad publicity for the company?
   - Would this change jeopardize our future relationship with trucking companies or other shipping services we use?
   - What would be the response of our competitors to our change in the supply chain? Would this start a price war? Would they be able to steal some of our market share?

2. *Problem #3 from Chapter 1 of SCND*
   You need to set up a mathematical optimization model. Assume you are modeling a supply chain for a business with ten warehouses and 1,000 customers. If you set up the model to minimize cost, set the decision variables to decide which warehouse should serve which customers, and set up no constraints, why would you expect the minimal cost to come back as $0?

   You need a constraint that says that customer demand is always met (number of customers being served = 1000). Otherwise, the model will suggest distributing no product at all (at zero cost), as this is the solution that minimizes the cost. No warehouse would serve any customer and costs, revenues and profits would all be equal to 0. However, this would clearly be a non-optimal solution.
   Alternatively, we might frame the problem to maximize profit (instead of minimizing cost), which would force the optimization to take both revenues and costs into consideration.

3. *Problem #4 from Chapter 1 of SCND*
   You are helping a firm determine their future transportation costs between their plant in Dallas and their warehouse in Atlanta. Your best estimate, with the data you have, is that the cost will be between $1.70 and $1.80 per mile. You decide to use $1.75 as your cost because it is the midpoint. If you are asked to spend more time seeing whether the number should be closer to $1.70 or $1.80, what would be your argument against further refining this number?

   Given our estimate and our assumption, at most we will be off by only 5 cents. We know that more precision is not necessarily better and that the data just needs to be precise enough for us to make the decision we are making. Collecting more precise data will require time and this might delay the actual decision making; we could end up losing more by not acting promptly than what we would gain thanks to a more precise estimate of the cost. In other words, the cost of further refining the estimate might outweigh the benefits of having a more precise estimate.
   Moreover, it is important to remember that we are making predictions about the future and predictions inevitably come with some degree of uncertainty. Another important question to ask is whether the extra precision in the prediction would have any impact on the decision we make. If the decision we are making only requires a direction rather than an absolutely spot-on number, then it doesn't make sense to invest resources into further improving our estimate.

4. *Problem #6 from Chapter 2 of SCND*
   What if the western part of Logistica decided it needed a capital city as well? Assume that City 15 was the easternmost city in this region. What is the best location for the capital of the western half of Logistica? Why?

   We believe that the capital of the western part of Logistica should be City 18. This is the city with the lowest weighted average distance, the highest percentage of citizens within 100 miles and the highest percentage of citizens within 300 miles. While it is not the best city in terms of the percentage of citizens within 200 miles -that would be City 17- we believe that City 18 is the best capital as it is the best performers according to 75% of the possible criteria. City 17 would be our second choice, or the top choice if the metric was percentage of citizens within 200 miles.

5. *Best items*
   a. *Set up the problem as an integer program and solve it so that you maximize the total value of the items you pick while respecting the capacity.*
      The best solution with integer programming has a total value 1123 and a total volume of 498, so we are wasting only 2 units of space.

   b. *Describe how your heuristic works relative to the IP you set up in (a). What is the IP doing that your heuristic is not?*
      The best heuristic we could build lead to a total value of 1116 and a total volume of 493. We are wasting 5 more units of space than with the optimal solution and losing $7 in profit.
      The heuristic works as follows. First of all, we took the ratio of value over size. We then sorted observations by their ‘value per unit of volume’. We then added items one at a time, starting from the one with the highest value per unit of volume, until we reached the item that would cause us to break the total volume constraint of 500. This led us to a solution with total volume equal to 442, which meant that 58 units of volume were still available. When choosing how to utilize the remaining 58 units, we had two options:
      - Adding -among the products that would fit- the product with the highest total value; this would add 56 units of value;
      - Adding -among the products that would fit- the product with the highest value per unit of volume; this would end up allowing for yet another product to be added and it would add a total of 37 + 15 = 52 units of value.

      We therefore opted for the first option mentioned above and reached our final solution.
   c. *Go back to your model in (a) and relax the integer constraints and discuss how realistic the solution is relative to your answers in (a) and (b). Note that there are several ways to relax the integer variable—feel free to try a few of these approaches.*
      The best solution with linear programming without integer constraint has a total value of 1136.03 and a total volume of 500. In this case, the decision variable for item 31 has a value of 0.78, and the other decision variables are either 0 or 1. This is not realistic as we cannot use 78% of an item. However, all solutions (including this unrealistic one) change somewhere after reaching 442 units, exactly where the linear program began including fractional items. Another way to

relax the integer constraints would be to allow any number of items picked (including more than 1). This solution picked item 26 250 times to achieve a maximum value of 24250. Again this might be an unrealistic solution but let's us know what the absolute maximum total value can be.

6. *Ambulances*
   In our optimization model, we minimize the total number of ambulances across different locations and make sure each location has at least one ambulance within a distance that could be covered in 3 minutes or less. In addition, we assume that each starting location can have at most 1 ambulance.
   We were able to find multiple solutions that meet the constraints. All the solutions have 3 ambulances in total. The different possible solutions we were able to achieve are:
   a. 3, 9, 10
   b. 2, 3, 9
   c. 1, 3, 5
   d. 2, 5, 9
   e. 5, 9, 10
   Of these solutions, some are better than others depending on which metric you wish to improve upon. For example, 2, 5, 9 is the solution if you maximize the number of stations reached within 3 minutes or less. 5, 9, 10 is the solution if you are minimizing the average time it takes for any of the three ambulances to reach any of the 10 zones. 1, 3, 5 is the solution that minimizes the total minimum time it takes for an ambulance to reach any destination.