Homework 1 - Part 2 - Group 4

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Question 7

For this problem, assume there are two coal mines that feed four power plants.

Decision Variables
Xij, total number of units going from mine i to plant j

Minimize:
Total Cost =
65 x X11 + 40 x X12 + 30 x X13 + 15 x X14 +
10 x X21 + 35 x X22 + 40 x X23 + 60 x X24

Subject To:
0 <= X11 + X12 + X13 + X14 <= 230
0 <= X21 + X22 + X23 + X24 <= 150
X11 + X21 >= 80
X12 + X22 >= 100
X13 + X23 >= 70
X14 + X24 >= 130

Solution:
X11 = 0
X12 = 30
X13 = 70
X14 = 130
X21 = 80
X22 = 70
X23 = 0
X24 = 0
Total Cost = 8500

We created a decision variable for the number of units of coal transported from each mine to each plant, for a total of 8 decision variables. The objective function is to minimize the total cost of transportation, while delivering enough coal to each plant and taking into consideration the total coal availability at each mine.

Question 8

a. You need to determine what product should be slotted in which location. Only one product can go in each location and each product can only be slotted once.

Decision Variables

Xij, boolean variables (0/1) representing if product i is in slot j
We have 100 decision variables (10 products x 10 locations)

Maximize Total Profit
$\Sigma X_{ij} P_{ij}$, where Pij is the profit per product

Subject To:
Column Sum Xj = 1, for any j = 0, 1, ..., 10
Row Sum Xi = 1, for any i = 0, 1, ..., 10

Solved in Excel
X1,4 = 1
X2,10 = 1
X3,5 = 1
X4,3 = 1
X5,6 = 1
X6,1 = 1
X7,2 = 1
X8,7 = 1
X9,8 = 1
X10,9 = 1
Total Profit = 284

b. Once you have your model built, determine what the absolute worse slotting would be. What is the business value for calculating this?

We have the same 100 decision variables and the same constraints we had when we were maximizing profits; however, we are now minimizing profits.

Minimize Total Profit
$\Sigma X_{ij} P_{ij}$, where Pij is the profit per product

Solved in Excel, see attachments.
X1,10 = 1
X2,8 = 1
X3,4 = 1
X4,1 = 1
X5,3 = 1
X6,5 = 1
X7,6 = 1
X8,2 = 1
X9,9 = 1
X10,7 = 1
Total Profit = 176

The worst possible organization of products leads to a profit of 176. There are different benefits to knowing the least amount of profit a business can make. This is the minimum profit the company will ever make. No matter how poorly they decide to place the items, they know they will have at least this much profit; obviously, the goal should be to always make more profits than the minimum. Minimum profit tells management what is the worst possible outcome if they were to randomly place items. This has important business consequences: if, for example, the minimum and maximum profit were very close, it could make sense to not invest the extra man-hours needed to figure out the optimal disposition and then place the items in the exact optimal location.

c. Also, determine the profit if each item was slotted in its best possible location - not considering the overall feasibility of the solution. What does this number tell you from a business point of view?

Profit = 336
This can be calculated by just adding the maximum possible profit for each product. This number tells us the maximum worth of the inventory of products we have, if it was possible to position multiple items in the same location. Notice that the optimal solution found in point a slots 3 out of the 10 products at their best possible slots.
This value, while interesting, is intrinsically not feasible given the current availability of shelf space. However, we don't want this value to be too much higher than the maximum achievable profit. If this was the case, we would have an indication that "different products are competing for the same slots" and that there are some slots the outperform others drastically. This might signal that the supermarket would have to consider changing the product mix (what products are offered) and/or the structure of the shelves in an effort to minimize the difference between the absolute best and the best achievable profits.

Question 9

a. Set up the model in Excel (you should at least try this in Excel and if you like try it in Python too) and solve with OpenSolver.

Decision Variables
X1 - Number of American Planes
X2 - Number of British Planes

Maximize:
30000 x 21 x X1 + 20000 x 21 x X2

Subject to:
2 x X1 + 1 x X2 <= 64
X1 >= 0
X2 >= 0
0 <= 9000 x 21 x X1 + 5000 x 21 x X2 <= 7000000
X1 + X2 <= 44

Solved in Excel:

X1 = 20
X2 = 24
Capacity = 22,680,000

b. What if the budget was $5,000,000, what is the solution? What is wrong with the solution? What are at least two ways to resolve this problem?

Solved in Excel:

X1 = 4.5
X2 = 39.5
Capacity = 19,430,000

The solutions are not in the form of integer in this case.

In order to resolve this issue, we can round up and down A and B and compare all possible combinations. The combination with the highest capacity within the constraints is the optimal solution.
When using OpenSolver in Excel, the solution we obtained after adding the integer constraint was not optimal, as it had a lower total capacity than the solution found trying all possible combinations of ceiling and floors.

X1 = 4 and X2 = 38 is the best solution. This solution does not exhaust the constraint on the number of crews, but the budget constraint binds and does not allow to use more American planes. A reduction in budget from $7 million to $5 million cause a substantial change in the solution, as a much higher number of British planes is now utilized and only very few American planes are flown.

Question 10

You are working for a distributor of vegetables. You can see the data on the vegetables in the spreadsheet. You have the price that you purchase, the price you sell, and the minimum quantity you have to sell (by contract), the max you can sell (the most the market will bear), and the cubic feet per carton.
Your warehouse only has room for 18,000 cubic feet of product. And, your supplier only allows you to purchase up to $30,000 of product per week.

a. Set this up and solve as a linear program.

Decision Variables:
X1, X2, X3, ..., X15, the quantity of each vegetable to purchase.

Maximize
Total Profit
$(2.27 - 2.15) \times X_1 +$
$(2.48 - 2.20) \times X_2 +$
$(2.70 - 2.40) \times X_3 +$
$(5.20 - 4.80) \times X_4 +$
$(2.92 - 2.60) \times X_5 +$
$(2.48 - 2.30) \times X_6 +$
$(2.20 - 2.35) \times X_7 +$
$(3.13 - 2.85) \times X_8 +$
$(2.48 - 2.25) \times X_9 +$
$(2.27 - 2.10) \times X_{10} +$
$(3.13 - 2.80) \times X_{11} +$
$(3.18 - 3.00) \times X_{12} +$
$(2.92 - 2.60) \times X_{13} +$
$(2.70 - 2.50) \times X_{14} +$
$(3.13 - 2.90) \times X_{15}$

Subject to:
$1.25 \times (X_1 + X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8 + X_9 + X_{10} + X_{11} + X_{12} + X_{13} + X_{14} + X_{15}) <= 18000$

$2.15 \times X_1 + 2.20 \times X_2 + 2.40 \times X_3 + 4.80 \times X_4 + 2.60 \times X_5 +$
$2.30 \times X_6 + 2.35 \times X_7 + 2.85 \times X_8 + 2.25 \times X_9 + 2.10 \times X_{10} +$
$2.80 \times X_{11} + 3.00 \times X_{12} + 2.60 \times X_{13} + 2.50 \times X_{14} + 2.90 \times X_{15} <= 30000$

$300 <= X_1 <= 1500$
$400 <= X_2 <= 2000$
$250 <= X_3 <= 900$
$0 <= X_4 <= 150$
$300 <= X_5 <= 1200$
$200 <= X_6 <= 800$
$150 <= X_7 <= 600$
$100 <= X_8 <= 300$
$750 <= X_9 <= 3500$
$400 <= X_{10} <= 2000$
$500 <= X_{11} <= 3300$
$100 <= X_{12} <= 500$
$500 <= X_{13} <= 3200$
$100 <= X_{14} <= 500$
$400 <= X_{15} <= 2500$

Solved in Excel:
$X_1 = 300$
$X_2 = 2000$
$X_3 = 900$

X4 = 0
X5 = 1200
X6 = 200
X7 = 150
X8 = 100
X9 = 750
X10 = 400
X11 = 2150
X12 = 100
X13 = 3200
X14 = 100
X15 = 400
Profit = $3,395.50

b. What insights do you get from the solution.

We purchased the maximum amount of creamed corn, black-eyed peas, carrots, and green beans as they have the highest value of profit per unit cost. Lima beans was purchased for 2150 units (more than the minimum requirement) because its profit per unit cost is higher than other products. If we add one extra dollar to the total budget, that dollar is used to buy lima beans, which shows that it's a profitable product and we would buy more of it if we had not hit the budget constraint. The selling cost for Okra is less than the buying price. This might be a data entry error. If not, it shows that the supplier is charging an excessively high price for okra, a price that is higher than the one the final customers are willing to pay for okra.