Machine Learning Exercise Sheet 05

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Problem 1

a) The posterior distribution $p(y|x)$ is a Bernoulli Distribution because we are reasoning about the variable y which is a variable that has only 2 possible values, 0 and 1.

b) We know that:

$$P(y=1|x) = \frac{P(x|y=1)P(y=1)}{P(x|y=1)P(y=1)+P(x|y=0)P(y=0)} =$$

$$= \frac{expo(x|\lambda_1)\frac{1}{2}}{\frac{1}{2}expo(x|\lambda_0)+\frac{1}{2}expo(x|\lambda_0)} = 1 + \frac{\lambda_1 e^{-\lambda_1 x}}{\lambda_0 e^{-\lambda_0 x}}$$

$$= \frac{1}{1+exp(-a)}$$

Where $a = ln\frac{\lambda_1 e^{-\lambda_1 x}}{\lambda_0 e^{-\lambda_0 x}}$

The values of x that are classified as class 1 are the ones for which holds $a > 0$:

$$ln\frac{\lambda_1 e^{-\lambda_1 x}}{\lambda_0 e^{-\lambda_0 x}} > 0$$

$$ln(\lambda_1) + ln(e^{-\lambda_1 x}) - ln(\lambda_0) - ln(e^{-\lambda_0 x}) > 0$$

$$ln(\lambda_1) - \lambda_1 x - ln(\lambda_0) + \lambda_0 x > 0$$

$$x(\lambda_0 - \lambda_1) > ln(\lambda_0) - ln(\lambda_1)$$

$$x > ln(\frac{\lambda_0}{\lambda_1})\frac{1}{\lambda_0 - \lambda_1}$$

Problem 2

a) If the dataset is linearly separable we have for each data point x_i:

$$w^T x_i > 0$$

or

$$w^T x_i < 0$$

The MLE solution occurs when $\sigma = \frac{1}{2}$ and the hyperplane $w^T x = 0$ separates the two classes. The magnitude of w goes to infinity in this case.

b) The problem is that if we use a dataset that is linerly separable MLE leads to overfitting. A possible solution for this problem is the usage of the weights regularization. In this case the loss function is

$$E(w) = -ln\ p(y|w, x) + \lambda ||w||_q^2$$

Another possible solution: we can compute the prior probability and then the MAP, in this way we can solve the overfitting problem.

Problem 3

We define the softmax as:

$$\sigma(x)_i = \frac{exp(x_i)}{\sum_{k=1}^{k} exp(x_k)}$$

To show that is equal to the sigmod in the 2-class case I consider the $P(y_i = x_1)$ where the y_i is the prediction and x_1 is the predicted class.

$$P(y_1 = x_1) = \frac{exp(x_1)}{exp(x_1) + exp(x_2)} = 1 + \frac{exp(x_1)}{exp(x_2)} = 1 + exp(x_1 - x_2) = \frac{1}{1 + exp(x2 - x1)}$$

If $a = ln(x_1 - x_2)$ we obtain:

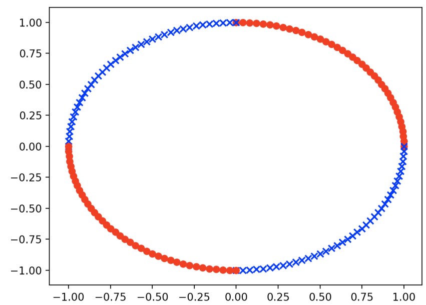
$$\frac{1}{1 + exp(-a)}$$

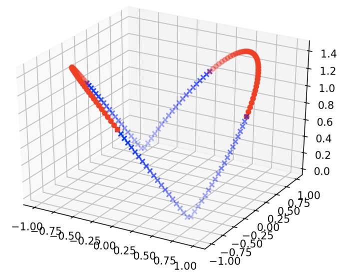
Problem 4

The basis function that makes the data linearly separable is:

$$\phi(x_1, x_2) = \begin{bmatrix} x_1 \\ x_2 \\ |x_1 + x_2| \end{bmatrix} \tag{1}$$

I've plotted the original data and new generated data to check if they are linearly separable:


1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.00 −0.75 −0.50 −0.25 0.00 0.25 0.50 0.75 1.00


1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0