Problem set 1

In the solutions, don't just give the answer, explain how you got it.

Coins

Using a normal quarter coin, you threw T, T, T, T. Which is true after this?

1. It is more likely to throw a T.
2. It is more likely to throw an H.
3. It is equally likely to throw an H or a T.

The bus problem

Warning, this is a difficult problem. The solution is simple, but it may be hard to find. It's ok if you don't solve it. Consider it food for thought.

One version You arrive at a bus stop at a random time. Buses arrive every 10 minutes. What is the expected time you will wait for the next bus?

Another version You are in Dustland, a remote country where, every now and then, a bus comes. You walk to a bus stop in the middle of nowhere. You have no idea of the bus schedule, or even if a bus passes by every day. But, there is another person there, looking bored. You ask how long they have been waiting, and they tell you, oh, x hours.

What is the expected time to the next bus? Why?

Hint: The solution is simple. If you think something complicated, you are likely following a wrong path.

The Blood Test

Suppose that there's a very rare disease, that affects only 1 person in 10,000,000 people. That is very rare indeed!

To test for the disease, there is a blood test that is 99% accurate.

1. How many tests are needed to be 90% confident that a person has the disease?

2. If a person does 5 tests, and 4 come out positive and one negative, what is the probability that the person has the disease?

Asssume that the tests are independent, of course.

Determining the bias of a coin

You need to determine the bias of a coin. The bias is x when the coin has probability x of coming up heads.

As a reminder of what is done in class, if you toss the coin and get k heads and m tails, the posterior distribution of the bias x is given by

$$P(x|k,m) \propto x^k(1-x)^m$$

and in particular, the Beta distribution Beta (α, β) with $\alpha = k+1$ and $\beta = m+1$.

The variance of the Beta (α, β) distribution is

$$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

Suppose you want to know the bias within a standard deviation of 0.1. How many tosses do you need to do? Find a number of tosses that guarantees that the standard deviation is at most 0.1, no matter what the actual toss outcomes are.