Week 9

Introduction to Programming and Numerical Analysis

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Overview

- Hints and overview of the tasks
- Work on problem set 1

Function

- **Task:** Implement a Python version of the utility function:

$$u(x_1, x_2) = \left(\alpha x_1^{-\beta} + (1-\alpha)x_2^{-\beta}\right)^{-1/\beta}$$

- **Hint:** Remember that raising a number to a power is done with '**'. Test if you get the expected results

Printing

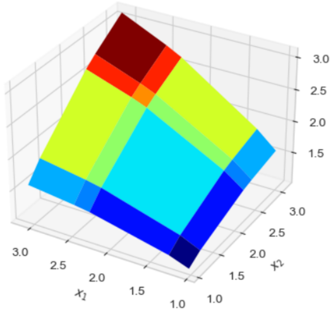
- **Task:** Construct a Python function `print_table(x1_vec, x2_vec)` to print values of $u(x_1, x_2)$ looking like this:

	0	1	2	3	4
0	1.050	1.162	1.442	1.479	1.569
1	1.162	1.300	1.661	1.711	1.832
2	1.442	1.661	2.300	2.396	2.641
3	1.479	1.711	2.396	2.500	2.768
4	1.569	1.832	2.641	2.768	3.100

- **Explanation:** You can use f-strings to format the output table. Specify the number of decimal points using the ‘:m.nf‘ syntax, where ‘n‘ is the desired number of decimal points and m of the width of the whole number.

Plotting

- **Task:** Reproduce the figure below of $u(x_1, x_2)$ using the ‘meshgrid‘ function from NumPy and the ‘plot_surface‘ function from Matplotlib.
- **Explanation:**
  1. x1_grid,x2_grid = np.meshgrid(x1_vec,x2_vec) creates two 2D grid arrays x1_grid, x2_grid
  2. Create a figure and an axis using ‘plt.figure()‘ and ‘add_subplot()‘
  3. Plot your data using appropriate plot functions (e.g. ’plot_surface’)
  4. Add labels, adjust colors etc.

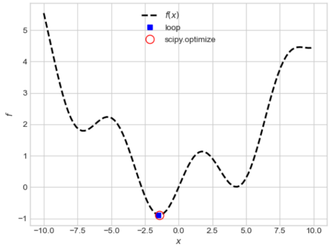


Optimization

- **Task:** Consider the following minimization problem:

$$\min_x f(x) = \min_x \sin(x) + 0.05 \cdot x^2$$

- **Explanation:**
  - Define the objective function.
  - Use a loop to iteratively compare all possible x-values
  - Utilize scipy.optimize to find the optimal solution using optimization algorithms.



Utility Maximization

- **Problem:** M goods with exogenous income I and price vector p:

$$V(p_1, p_2, \ldots, p_M, I) = \max_{x_1, x_2, \ldots, x_M} x_1^{\alpha_1} x_2^{\alpha_2} \ldots x_M^{\alpha_M}$$

- **Constraints:** $\sum_{i=1}^{M} p_i x_i \leq I$, with $p_1, p_2, \ldots, p_M, I > 0$ and $x_1, x_2, \ldots, x_M \geq 0$.

1. **Loop Solution:**
    - Iterate over possible combinations of $x_1, x_2, \ldots, x_M$.
    - Calculate the utility for each combination and keep track of the maximum utility.
    - Ensure that the budget constraint is satisfied for each combination.
2. **Numerical Optimization:**
    - Define the objective function representing utility.
    - Formulate the optimization problem with constraints.
    - Use a numerical optimization method to find the optimal solution.

Extra Problem: Cost Minimization

- **Problem:** 2-good cost minimization problem with required utility u_0 and price vector $p = (p_1, p_2)$:

$$E(p_1, p_2, u_0) = \min_{x_1, x_2} p_1 x_1 + p_2 x_2$$

- **Constraints:** $x_1^{\alpha} x_2^{1-\alpha} \geq u_0$ and $x_1, x_2 \geq 0$.
- **Guidelines:**
  1. Follow the same approach as for the utility maximization
  2. This time the objective function is expenditures()
  3. The constraint is the utility so define a minimum utility
  4. Either define a utility constraint or a penalty if the utility is too low

Classy Solution: Implementing the Problem in a Class

- **Problem:** Implement your solution to the utility maximization problem and/or the cost minimization problem above in a class as seen in Lecture 3.
- **Guidelines:**
  - Define a class to encapsulate the problem-solving process.
  - Implement the same methods as above within the class
  - Always think about the scope of your variables
  - Save the results of the optimization as attributes of the class